Direct lattice-QCD calculation of pion valence quark distribution

Jianhui Zhang
University of Regensburg

Lattice 2018, 24 July, 2018,
Michigan State University, East Lansing

Direct lattice-QCD calculation of pion valence quark distribution

Jianhui Zhang
University of Regensburg

Collaborators: J.-W. Chen, L. Jin, H.-W. Lin, Y.-S. Liu, A. Schäfer, Y. Yang and Y. Zhao

Direct lattice-QCD calculation of pion valence quark distribution

Jianhui Zhang
University of Regensburg

Collaborators: J.-W. Chen, L. Jin, H.-W. Lin, Y.-S. Liu, A. Schäfer, Y. Yang and Y. Zhao

Lattice parton physics project (LP^3)

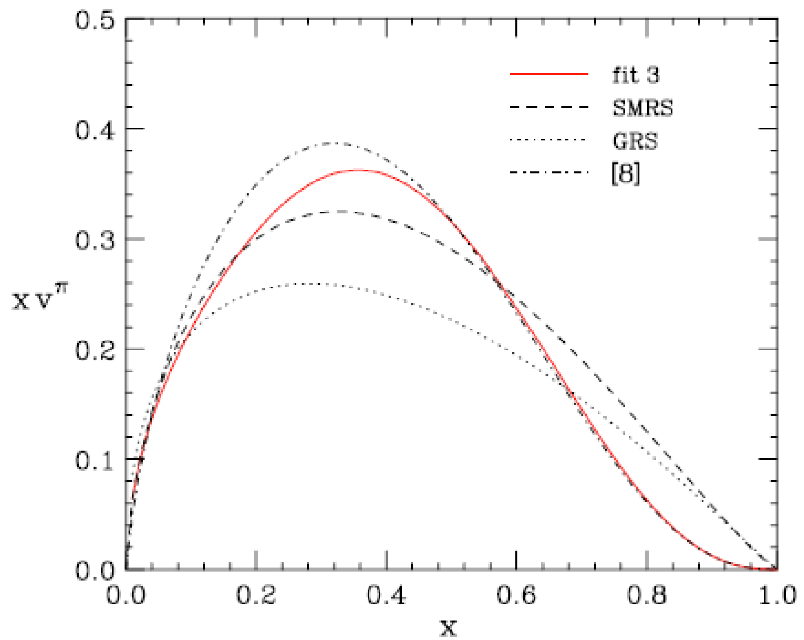
Contents

- Introduction
- Pion parton structure from first principles
- Results on pion valence quark distribution
- Summary and outlook

Introduction

- Pion plays a fundamental role in QCD
  - Lightest quark-antiquark bound state
  - Goldstone boson associated with dynamical chiral symmetry breaking
  - Explains the flavor asymmetry in the nucleon quark sea

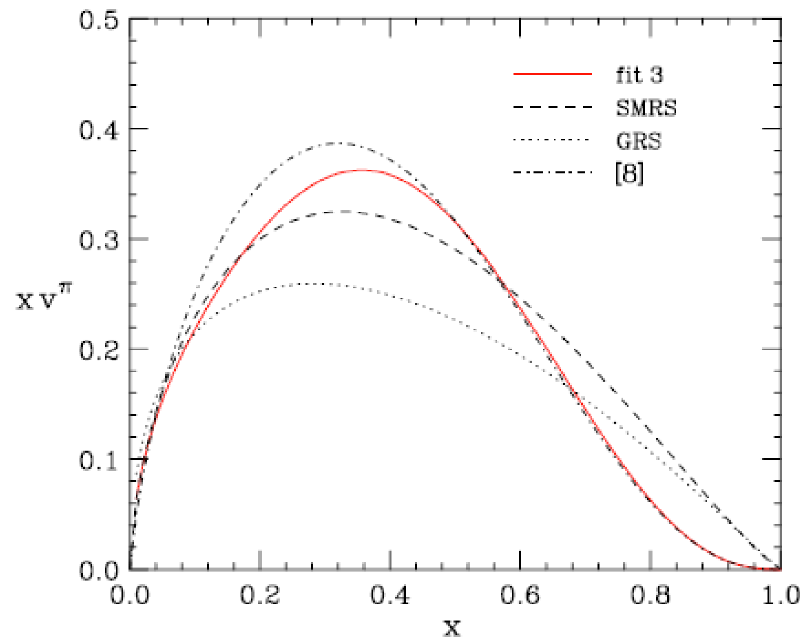
- Its parton structure mainly from Drell-Yan data on πN scattering
  - Soft gluon resummation renders q_v^π softer at large x, $\sim (1-x)^2$ [Aicher, Schäfer and Vogelsang, PRL 10']
  - Consistent with perturbative QCD [
  - Farrar and Jackson, PRL 79', Berger and
  - Brodsky, PRL 79'] and Dyson-Schwinger
  - Equation [Hecht, Roberts and Schmidt,
  - PRC 01']



Introduction

- Pion plays a fundamental role in QCD
  - Lightest quark-antiquark bound state
  - Goldstone boson associated with dynamical chiral symmetry breaking
  - Explains the flavor asymmetry in the nucleon quark sea

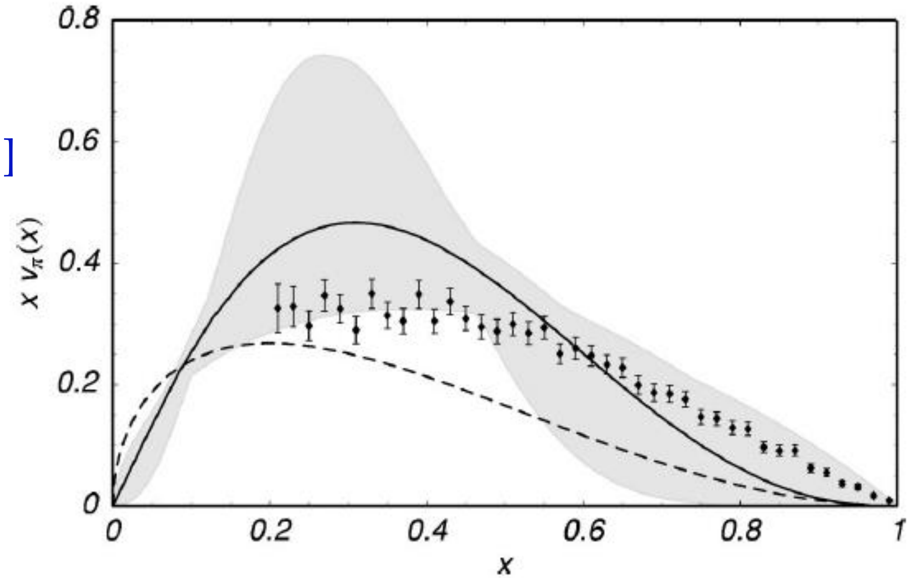
- Its parton structure mainly from Drell-Yan data on πN scattering
  - Soft gluon resummation renders q_v^π softer at large x, $\sim (1-x)^2$ [Aicher, Schäfer and Vogelsang, PRL 10']
  - Consistent with perturbative QCD [
  - Farrar and Jackson, PRL 79', Berger and
  - Brodsky, PRL 79'] and Dyson-Schwinger
  - Equation [Hecht, Roberts and Schmidt,
  - PRC 01']
  - Quark models favor a linear dependence $(1-x)$ at large x



Introduction

- Pion plays a fundamental role in QCD
  - Lightest quark-antiquark bound state
  - Goldstone boson associated with dynamical chiral symmetry breaking
  - Explains the flavor asymmetry in the nucleon quark sea

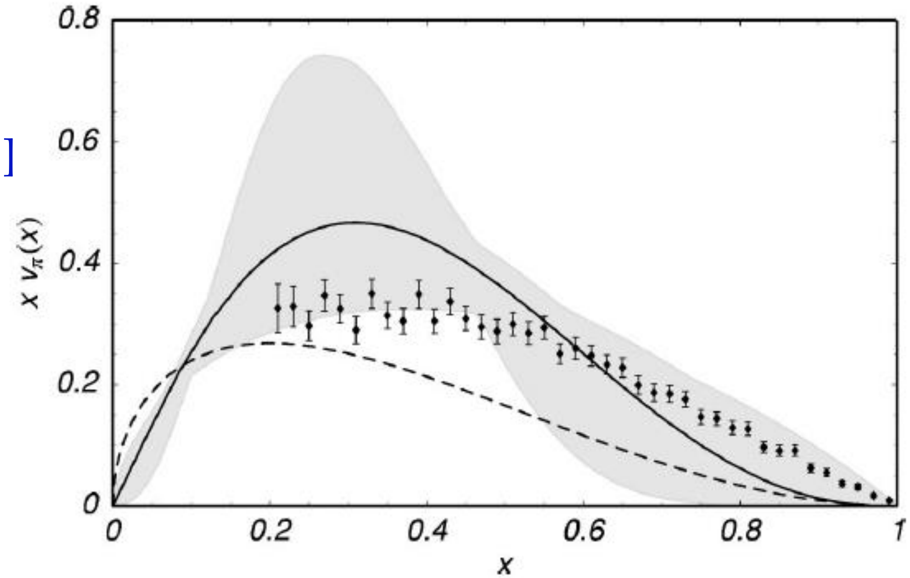
- Its parton structure mainly from Drell-Yan data on πN scattering
  - Lattice QCD is only able to access the first few moments of pion PDF [Detmold, Melnitchouk, Thomas, PRD 03']



Introduction

- Pion plays a fundamental role in QCD
  - Lightest quark-antiquark bound state
  - Goldstone boson associated with dynamical chiral symmetry breaking
  - Explains the flavor asymmetry in the nucleon quark sea

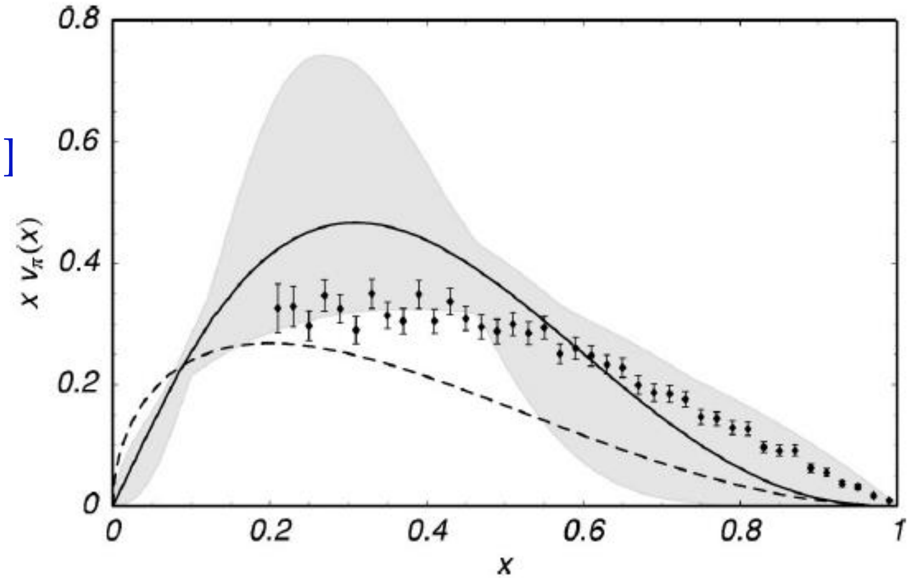
- Its parton structure mainly from Drell-Yan data on πN scattering
  - Lattice QCD is only able to access the first few moments of pion PDF [Detmold, Melnitchouk, Thomas, PRD 03']
  - It can shed more light on pion parton structure if its computational potential can be extended beyond that



Introduction

- Pion plays a fundamental role in QCD
  - Lightest quark-antiquark bound state
  - Goldstone boson associated with dynamical chiral symmetry breaking
  - Explains the flavor asymmetry in the nucleon quark sea

- Its parton structure mainly from Drell-Yan data on πN scattering
  - Lattice QCD is only able to access the first few moments of pion PDF [Detmold, Melnitchouk, Thomas, PRD 03']



  - It can shed more light on pion parton structure if its computational potential can be extended beyond that
  - This becomes possible due to large momentum effective theory [Ji, PRL 13', Sci. China Phys. Mech. Astron. 14']

Large momentum effective theory (LaMET)

- Parton picture arises in high-energy collisions where hadrons/probe move nearly at the speed of light, or with infinite momentum

Large momentum effective theory (LaMET)

- Parton picture arises in high-energy collisions where hadrons/probe move nearly at the speed of light, or with infinite momentum
- Parton physics usually formulated in terms of light-cone quantization [Dirac]
  - light-cone coordinates $\xi^{\pm} = (t \pm z)/\sqrt{2}$
  - Example: [Collins and Soper, NPB 82']

$$q(x,\mu^2) = \int \frac{d\xi^-}{4\pi} e^{-ix\xi^- P^+} \langle P|\overline{\psi}(\xi^-)\gamma^+ \exp\left(-ig\int_0^{\xi^-} d\eta^- A^+(\eta^-)\right)\psi(0)|P\rangle$$

Large momentum effective theory (LaMET)

- Parton picture arises in high-energy collisions where hadrons/probe move nearly at the speed of light, or with infinite momentum
- Parton physics usually formulated in terms of light-cone quantization [Dirac]
  - light-cone coordinates $\xi^{\pm} = (t \pm z)/\sqrt{2}$
  - Example: [Collins and Soper, NPB 82']

$$q(x, \mu^2) = \int \frac{d\xi^-}{4\pi} e^{-ix\xi^- P^+} \langle P|\overline{\psi}(\xi^-)\gamma^+ \exp\left(-ig\int_0^{\xi^-} d\eta^- A^+(\eta^-)\right)\psi(0)|P\rangle$$

- However, it was originally introduced by Feynman as the infinite momentum limit of frame-dependent quantities

$$q(x) = \lim_{P_z \to \infty} \tilde{q}(x, P_z)$$

Large momentum effective theory (LaMET)

- Parton picture arises in high-energy collisions where hadrons/probe move nearly at the speed of light, or with infinite momentum
- Parton physics usually formulated in terms of light-cone quantization [Dirac]
  - light-cone coordinates $\xi^{\pm} = (t \pm z)/\sqrt{2}$
  - Example: [Collins and Soper, NPB 82']

$$q(x,\mu^2) = \int \frac{d\xi^-}{4\pi} e^{-ix\xi^- P^+} \langle P|\overline{\psi}(\xi^-)\gamma^+ \exp\left(-ig\int_0^{\xi^-} d\eta^- A^+(\eta^-)\right)\psi(0)|P\rangle$$

- However, it was originally introduced by Feynman as the infinite momentum limit of frame-dependent quantities

$$q(x) = \lim_{P_z \to \infty} \tilde{q}(x, P_z)$$

- Large momentum effective theory [Ji, PRL 13', Sci. China Phys. Mech. Astron. 14']
  - Appropriately chosen $\tilde{q}(x, P_z)$ can be calculated on the Euclidean lattice
  - A finite but large P_z already offers a good approximation, where (leading) frame-dependence can be removed through a factorization procedure

Pion PDF from LaMET

- Pion PDF

$$q_f^\pi(x) = \int \frac{d\lambda}{4\pi} e^{-ix\lambda n\cdot P} \langle \pi(P) \left| \bar{\psi}_f(\lambda n) \not{n} \Gamma(\lambda n, 0) \psi_f(0) \right| \pi(P) \rangle$$

  - $P^\mu = (P_0, 0,0, P_z), n^\mu = (1,0,0,-1)/\sqrt{2}$

- Pion quasi-PDF [Ji, PRL 13']

$$\tilde{q}_f^\pi(x) = \int \frac{d\lambda}{4\pi} e^{-ix\lambda \tilde{n}\cdot P} \langle \pi(P) \left| \bar{\psi}_f(\lambda \tilde{n}) \not{\tilde{n}} \Gamma(\lambda \tilde{n}, 0) \psi_f(0) \right| \pi(P) \rangle$$

  - $\tilde{n}^\mu = (0,0,0,-1)$, $\not{\tilde{n}} = \gamma^z$ can also be replaced by γ^t

- Nonperturbative renormalization of quasi-PDF [Ji, JHZ and Zhao, PRL 18', Ishikawa, Ma, Qiu and Yoshida, PRD 17', Green, Jansen and Steffens, 17']

$$\tilde{h}_R(\lambda\tilde{n}) = Z_1 Z_2 e^{\delta m \lambda} \tilde{h}(\lambda \tilde{n})$$

  - δm can be calculated from Wilson loop corresponding to static quark-antiquark potential

Pion PDF from LaMET

- Pion PDF

$$q_f^\pi(x) = \int \frac{d\lambda}{4\pi} e^{-ix\lambda n\cdot P} \langle \pi(P) | \bar{\psi}_f(\lambda n) \not{n} \Gamma(\lambda n, 0) \psi_f(0) | \pi(P) \rangle$$

  - $P^\mu = (P_0, 0,0, P_z), n^\mu = (1,0,0,-1)/\sqrt{2}$

- Pion quasi-PDF [Ji, PRL 13′]

$$\tilde{q}_f^\pi(x) = \int \frac{d\lambda}{4\pi} e^{-ix\lambda \tilde{n}\cdot P} \langle \pi(P) | \bar{\psi}_f(\lambda \tilde{n}) \not{\tilde{n}} \Gamma(\lambda \tilde{n}, 0) \psi_f(0) | \pi(P) \rangle$$

  - $\tilde{n}^\mu = (0,0,0,-1)$, $\not{\tilde{n}} = \gamma^z$ can also be replaced by γ^t

- Nonperturbative renormalization of quasi-PDF [Ji, JHZ and Zhao, PRL 18′, Ishikawa, Ma, Qiu and Yoshida, PRD 17′, Green, Jansen and Steffens, 17′]

$$\tilde{h}_R(\lambda\tilde{n}) = Z^{-1}(\lambda\tilde{n}, p_z^R, 1/a, \mu_R) \tilde{h}(\lambda\tilde{n})$$

  - RI/MOM [Stewart and Zhao, PRD 17′, Alexandrou et al, NPB 17′, LP3, PRD 17′]

$$Z(\lambda\tilde{n}, p_z^R, 1/a, \mu_R) = \left. \frac{\mathrm{Tr}[\not{p} \sum_s \langle p,s| \bar{\psi}_f(\lambda\tilde{n}) \not{\tilde{n}} \Gamma(\lambda\tilde{n},0) \psi_f(0) |p,s\rangle]}{\mathrm{Tr}[\not{p} \sum_s \langle p,s| \bar{\psi}_f(\lambda\tilde{n}) \not{\tilde{n}} \Gamma(\lambda\tilde{n},0) \psi_f(0) |p,s\rangle_{tree}]} \right|_{\substack{p^2 = -\mu_R^2 \\ p_z = p_z^R}}$$

Pion PDF from LaMET

- Pion PDF

$$q_f^\pi(x) = \int \frac{d\lambda}{4\pi} e^{-ix\lambda n\cdot P} \left\langle \pi(P) \left| \bar{\psi}_f(\lambda n) \not{n} \Gamma(\lambda n, 0) \psi_f(0) \right| \pi(P) \right\rangle$$

  - $P^\mu = (P_0, 0,0, P_z), n^\mu = (1,0,0,-1)/\sqrt{2}$

- Pion quasi-PDF [Ji, PRL 13′]

$$\tilde{q}_f^\pi(x) = \int \frac{d\lambda}{4\pi} e^{-ix\lambda \tilde{n}\cdot P} \left\langle \pi(P) \left| \bar{\psi}_f(\lambda \tilde{n}) \not{\tilde{n}} \Gamma(\lambda \tilde{n}, 0) \psi_f(0) \right| \pi(P) \right\rangle$$

  - $\tilde{n}^\mu = (0,0,0,-1)$, $\not{\tilde{n}} = \gamma^z$ can also be replaced by γ^t

- Factorization [Ji, PRL 13′, Xiong, Ji, JHZ and Zhao, PRD 14′, Stewart and Zhao, PRD 18′, Ma and Qiu, 14′ & PRL 18′]

$$\tilde{q}_{v,R}^\pi(x, \tilde{n}\cdot P, \tilde{\mu}) = \int_0^1 \frac{dy}{y} C\left(\frac{x}{y}, \frac{\tilde{\mu}}{\mu}, \frac{\mu}{y\tilde{n}\cdot P}\right) q_{v,R}^\pi(y, \mu) + \mathcal{O}\left(\frac{m_\pi^2}{(\tilde{n}\cdot P)^2}, \frac{\Lambda_{\rm QCD}^2}{(\tilde{n}\cdot P)^2}\right)$$

  - $q_{u,v}^\pi(x) = q_u^\pi(x) - q_{\bar{u}}^\pi(x) = q_u^\pi(x) - q_d^\pi(x)$ due to isospin symmetry

Other proposals

- **They all share the same property of computing correlations at spacelike separations**

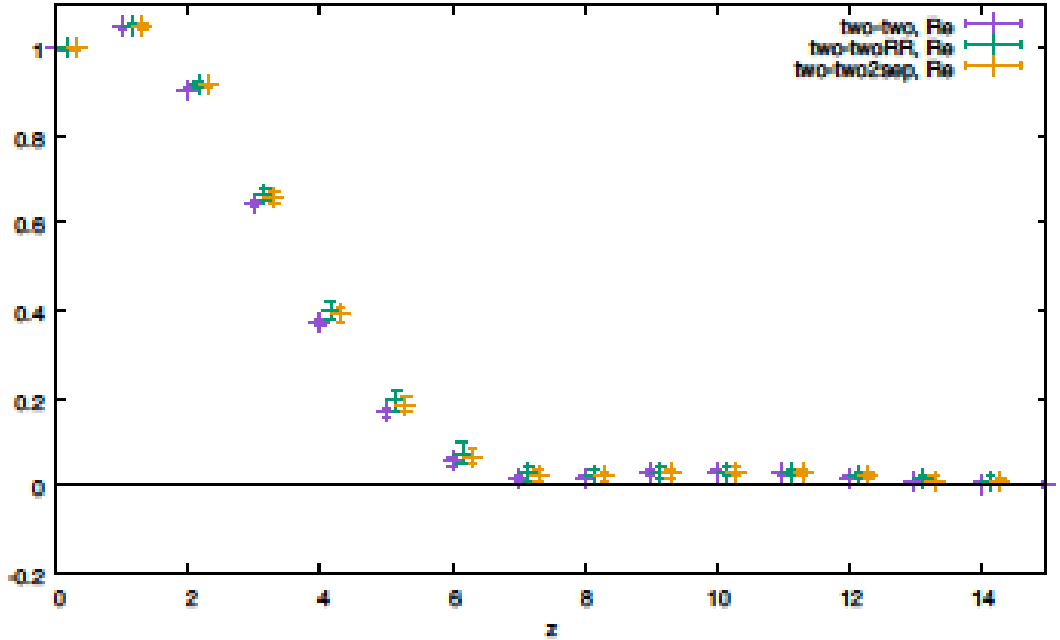
- Current-current correlation functions
- [Liu and Dong, PRL 94']
- [Detmold and Lin, PRD 06']
- [Braun and Müller, EPJC 08']
- [Davoudi and Savage, PRD 12']
- [Chambers et al., PRL 17']

- Lattice cross sections
- [Ma and Qiu, 14' & PRL 17']

- Ioffe-time /pseudo-distribution
- [Radyushkin, PRD 17']

Results on pion valence quark PDF

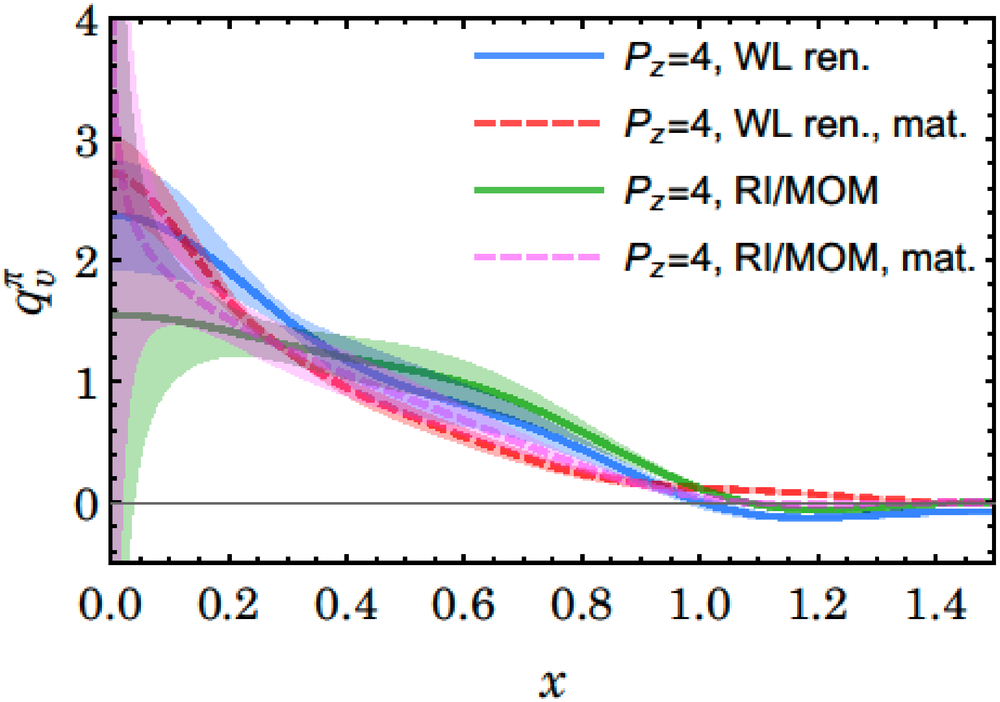
- Renormalized matrix element



LP3, 1804.01483, $m_\pi \approx 310\ MeV, a = 0.12\ fm, L \approx 3fm$

Results on pion valence quark PDF

- One-loop matching effect

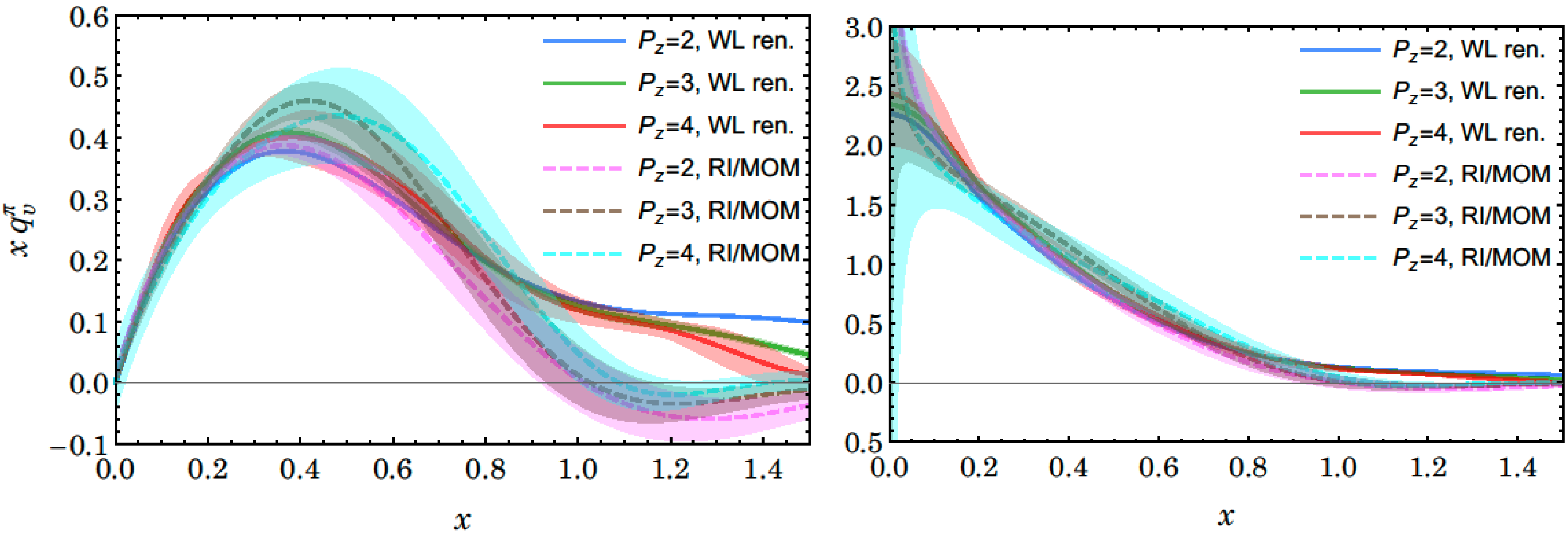


LP3, 1804.01483, $m_\pi \approx 310\ MeV, a = 0.12\ fm, L \approx 3 fm$

- Matching has a sizeable effect, and cannot be ignored, as was done in [Xu, Chang, Roberts and Zong, PRD 18'], where they observed that for $P_z \geq 2$ GeV, by further increasing pion momentum the quasi-PDF shrinks to the physical region very slowly

Results on pion valence quark PDF

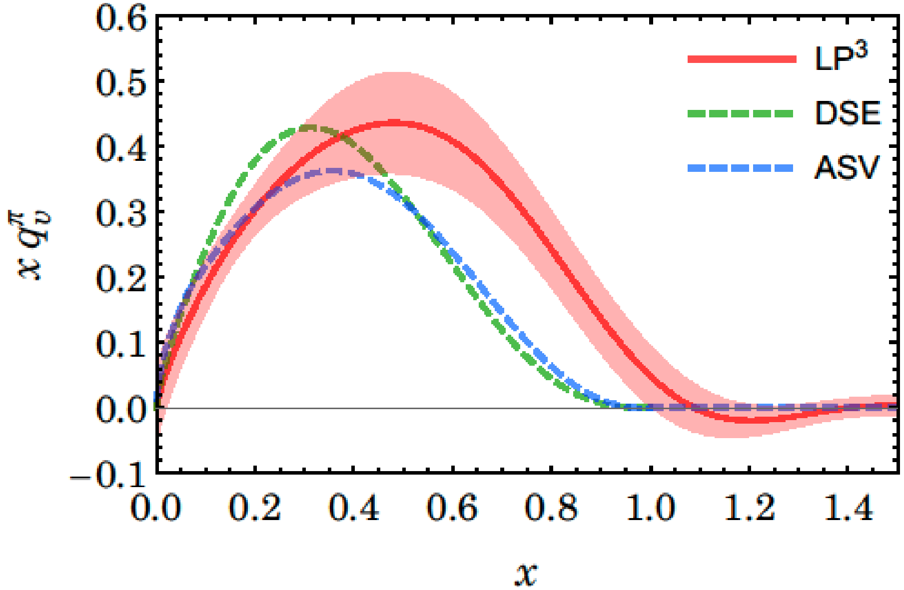
- Pion momentum dependence

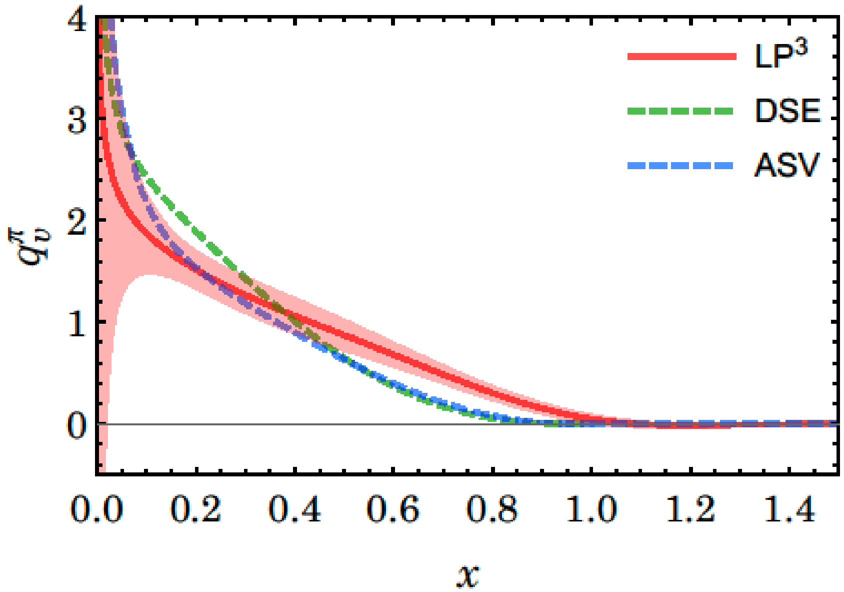


LP3, 1804.01483,
$m_\pi \approx 310\ MeV, a =$
$0.12\ fm, L \approx 3 fm$

Results on pion valence quark PDF

- Final result





LP3, 1804.01483, $m_\pi \approx 310\ MeV, a = 0.12\ fm, L \approx 3fm$

Summary and outlook

- **Large momentum effective theory** opens a new door for *ab initio* studies of hadron structure
- It has been applied to computing dynamical properties of hadrons, like nucleon PDFs, meson PDFs & DAs, and yields encouraging results
- Systematic studies of uncertainties or artifacts are required:
  - Physical pion mass
  - Continuum extrapolation
  - Finite volume effects
  - Discretization effects
  - Higher-order matching

Backup Slides