Beyond the Standard Model Kaon Mixing with physical light quarks

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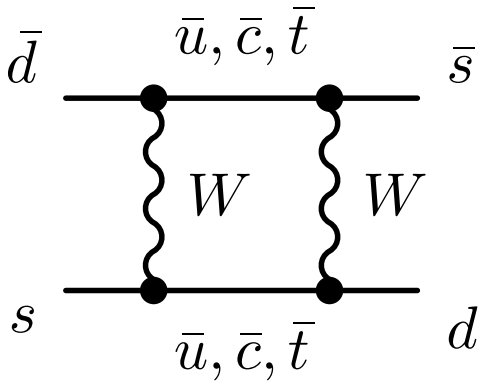
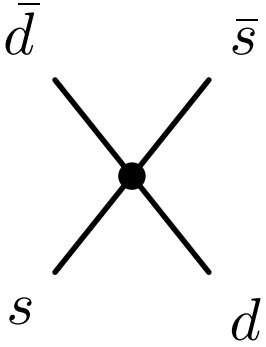
Table of contents

Kaon Mixing in the Standard Model

- Oscillation of K^0 to $\bar{K}^0$
- One-loop FCNC
- Mediated by $W^{\pm}$
- Related to indirect CP violation





- OPE separates out a long-distance 4quark operator
- Matrix elements of $O^{\Delta S=2}$ calculated with LQCD

$$O^{\Delta S=2} = [\bar{s}_a \gamma_\mu (1-\gamma_5) d_a][\bar{s}_b \gamma_\mu (1-\gamma_5) d_b]$$

Beyond the Standard Model

Generalized Weak Hamiltonian:

$$\mathcal{H}^{\Delta S=2} = \sum_{i=1}^{5} C_i(\mu) O_i + \sum_{i=1}^{3} \tilde{C}_i(\mu) \tilde{O}_i$$

Basis of 5 model independent parity-even four-quark operators.

$$O_1 = [\bar{s}_a \gamma_\mu (1-\gamma_5) d_a][\bar{s}_b \gamma_\mu (1-\gamma_5) d_b]$$
$$O_2 = [\bar{s}_a (1-\gamma_5) d_a][\bar{s}_b (1-\gamma_5) d_b]$$
$$O_3 = [\bar{s}_a (1-\gamma_5) d_b][\bar{s}_b (1-\gamma_5) d_a]$$
$$O_4 = [\bar{s}_a (1-\gamma_5) d_a][\bar{s}_b (1+\gamma_5) d_b]$$
$$O_5 = [\bar{s}_a (1-\gamma_5) d_b][\bar{s}_b (1+\gamma_5) d_a]$$

Past calculations by SWME[1], ETM [2] and RBC-UKQCD [3].

[1] Jang et al 15, Bae et al 14
[2] Carrasco et al 15,Bertone et al 10
[3] Garron,Hudspith,Lytle 16, Blum et al 14

Simulations

- 2+1f DWF QCD with Iwasaki Gauge Action
- 3 lattice spacings
- 2 ensembles with physical pions
- New: $a^{-1} = 2.774 GeV$ $m_\pi \approx 230 MeV$

name	L/a	T/a	kernel	source	a^{-1}[GeV]	m_π[MeV]	$n_{configs}$	am_l^{uni}	am_s^{sea}	am_s^{val}	am_s^{phys}
C0	48	96	M	Z2GW	1.7295(38)	139	90	0.00078	0.0362	0.0358	0.03580(16)
C1	24	64	S	Z2W	1.7848(50)	340	100	0.005	0.04	0.03224	0.03224(18)
M0	64	128	M	Z2GW	2.3586(70)	139	82	0.000678	0.02661	0.0254	0.02539(17)
M1	32	64	S	Z2GW	2.3833(86)	303	83	0.004	0.03	0.02477	0.02477(18)
M2	32	64	S	Z2GW	2.3833(86)	360	76	0.006	0.03	0.02477	0.02477(18)
F1	48	96	M	Z2GW	2.774(10)	234	98	0.002144	0.02144	0.02132	0.02132(17)

M and F stand for coarse, medium and fine, respectively, M and S for Moebius and Shamir kernels. Propagators had either Z2 wall (Z2W) or Z2 Gaussian Wall (Z2GW) sources, with latter including source smearing.

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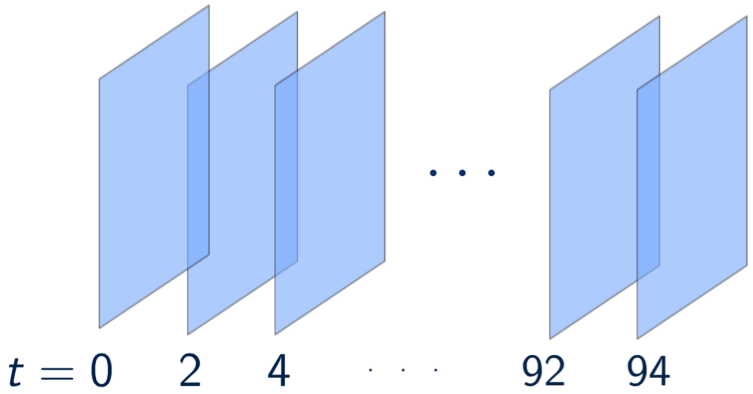
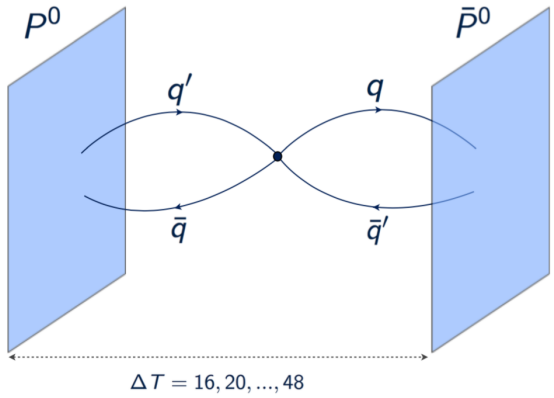
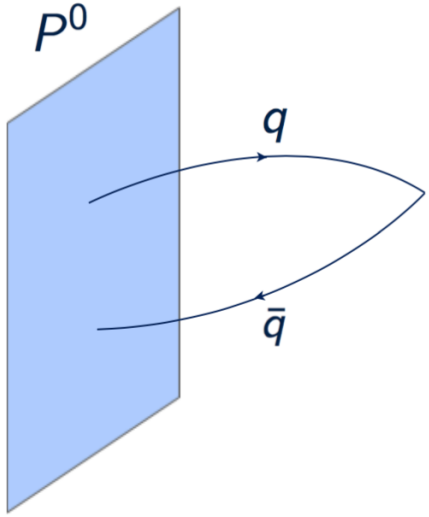
Source-Sink Time Separations

- Z_2 (Gaussian) wall sources at every other time slice.
- Several different ΔT
- Bin all data with same ΔT







Quantities Measured

The SM bag parameters:

$$B_1(\mu) = \frac{\langle \bar{P}|\mathcal{O}_1(\mu)|P\rangle}{\frac{8}{3} m_K^2 f_K^2} \qquad B_{i>1}(\mu) = \frac{(m_s(\mu)+m_d(\mu))^2}{N_i m_K^4 f_K^2} \langle \bar{P}|\mathcal{O}_i(\mu)|P\rangle$$

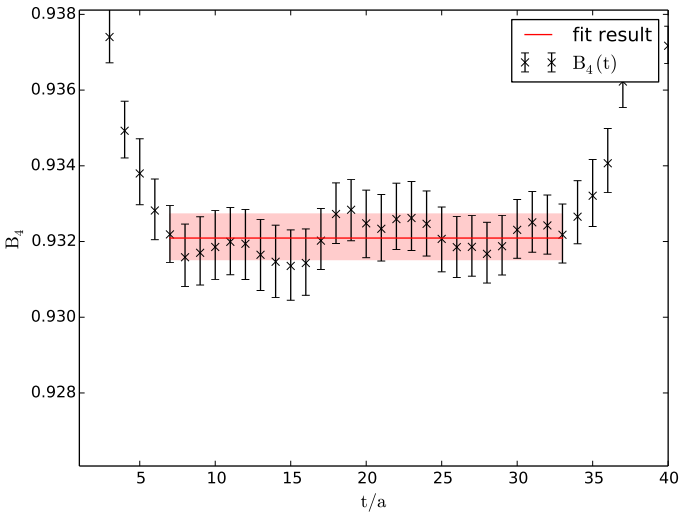
Define a ratio parameter:

$$R_i(\frac{m_P^2}{f_P^2}, a^2, \mu) = \left[\frac{f_K^2}{m_K^2}\right]_{exp} \left[\frac{m_P^2}{f_P^2} \frac{\langle \bar{P}|\mathcal{O}_i(\mu)|P\rangle}{\langle \bar{P}|\mathcal{O}_1(\mu)|P\rangle}\right]_{lat}$$

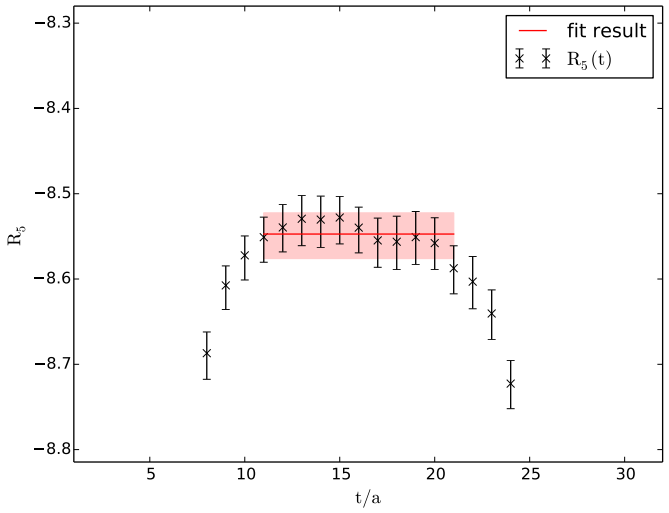
such that when $a^2 \to 0$ and $m_P^2/f_P^2 \to m_K^2/f_K^2$ it reduces to:

$$R_i(\mu) = \frac{\langle \bar{K}|\mathcal{O}_i(\mu)|K\rangle}{\langle \bar{K}|\mathcal{O}_1(\mu)|K\rangle}$$

Correlator Fitting



(a) $\mathcal{B}_4(t)$ for C0



(b) $\mathcal{R}_5(t)$ for M0.

$$\frac{C^{3pt}(t,t_{sink})}{C^{AP}_{2pt}(t)C^{AP}_{2pt}(t_{sink}-t)} \rightarrow \frac{\langle \bar{P}|O_i|P\rangle}{\langle P|A\rangle\langle A|P\rangle} \quad \frac{C^{3pt}_i(t,t_{sink})}{C^{3pt}_1(t,t_{sink})} \rightarrow \frac{\langle \bar{P}|O_i|P\rangle}{\langle \bar{P}|O_1|P\rangle}$$

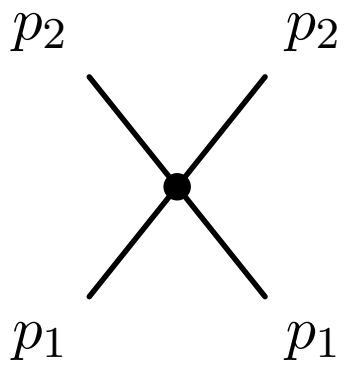
Examples of the fits of correlation functions to measure lattice quantites.

Non-Perturbative Renormalisation

- We use the Rome-Southampton method with non-exceptional kinematics (RI-SMOM).

$$O_i(\mu)^{\overline{MS}} = C_{ij}^{\overline{MS}\leftarrow MOM}(\mu)\left(\lim_{a^2\to 0}\frac{Z_{jk}^{RI}(\mu)}{Z_q^2}O_k(a)\right)$$

$$Z^{RI}(\mu)\hat{P}[\Lambda(p^2)]|_{p^2=\mu^2} = \Lambda(p^2)^{tree}$$



$$p_1 \neq p_2$$

$$p_1^2 = p_2^2 = (p_1 - p_2)^2$$

Non-Perturbative Renormalisation

- Block diagonal structure due to chiral symmetry

$$Z_{O^{\Delta S=2}} = \begin{pmatrix} Z_{11} & 0 & 0 & 0 & 0 \\ 0 & Z_{22} & Z_{23} & 0 & 0 \\ 0 & Z_{32} & Z_{33} & 0 & 0 \\ 0 & 0 & 0 & Z_{44} & Z_{45} \\ 0 & 0 & 0 & Z_{54} & Z_{55} \end{pmatrix}$$

- Define two intermediate schemes (γ, γ) and $(\not{q}, \not{q})$ distiguished by their projectors. The difference between them allows us to quantify a systematic error

Extrapolation to Physical Point and Continuum

Simultaneously extrapolate to the continuum and chiral limit in a global fit with form:

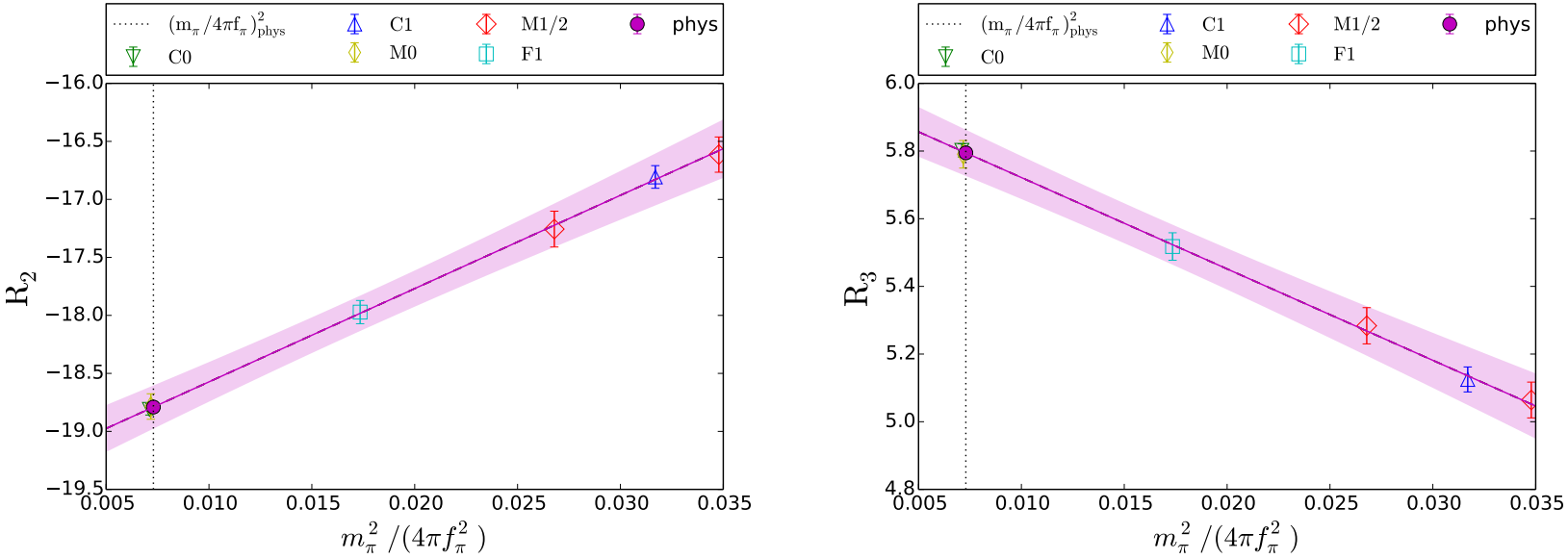
$$Y\left(a^2, \frac{m_{ll}^2}{f_{ll}^2}, \delta_{m_s^{sea}}\right) = Y\left(0, \frac{m_\pi^2}{f_\pi^2}, 0\right)\left[1 + \alpha_i a^2 + \beta_i \frac{m_{ll}^2}{f_{ll}^2} + \gamma_i \delta_{m_s^{sea}}\right]$$

where we include a term linear strange sea-quark mass:

$$\delta_{m_s^{sea}} = \frac{(m_s^{sea} - m_s^{phys})}{m_s^{phys}}$$

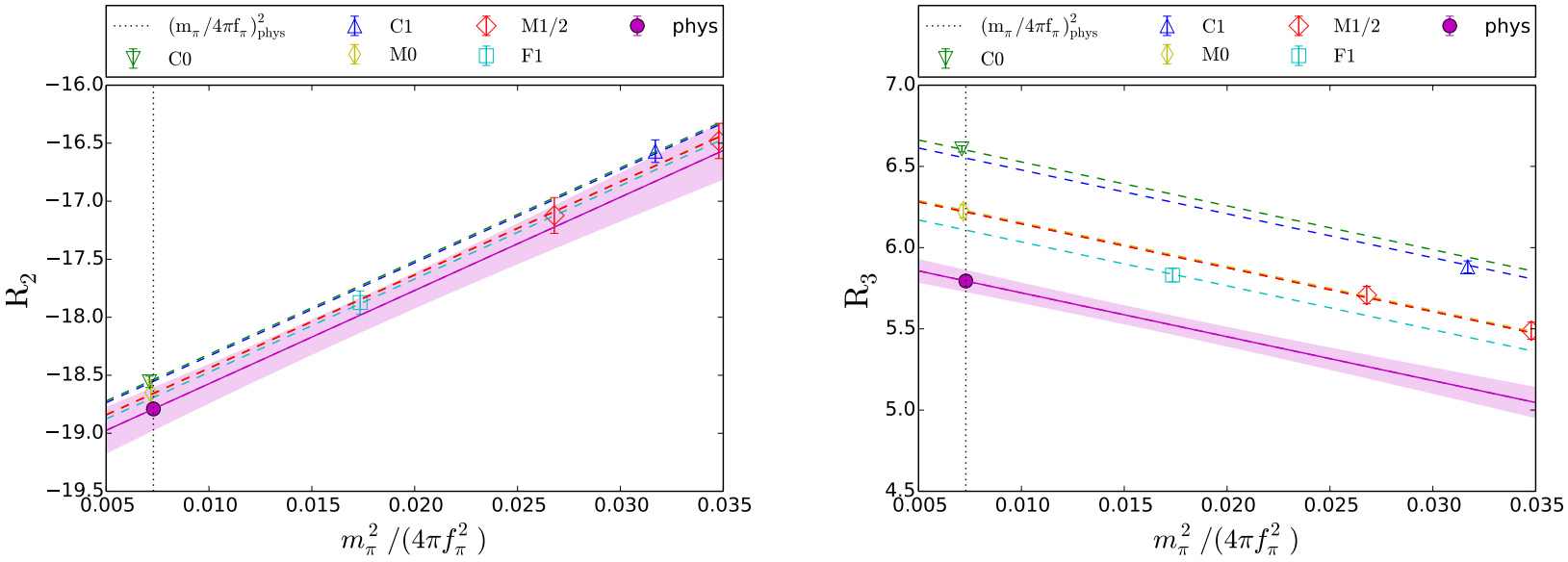
Ratio Parameter Fits

Figure: Preliminary results for R_2 and R_3 in $\overline{MS}$ at 3 GeV RI-SMOM$^{\gamma,\gamma}$ intermediate scheme. Data points have been adjusted to the physical strange-mass and continuum using the fit form and parameters gained from the fit.



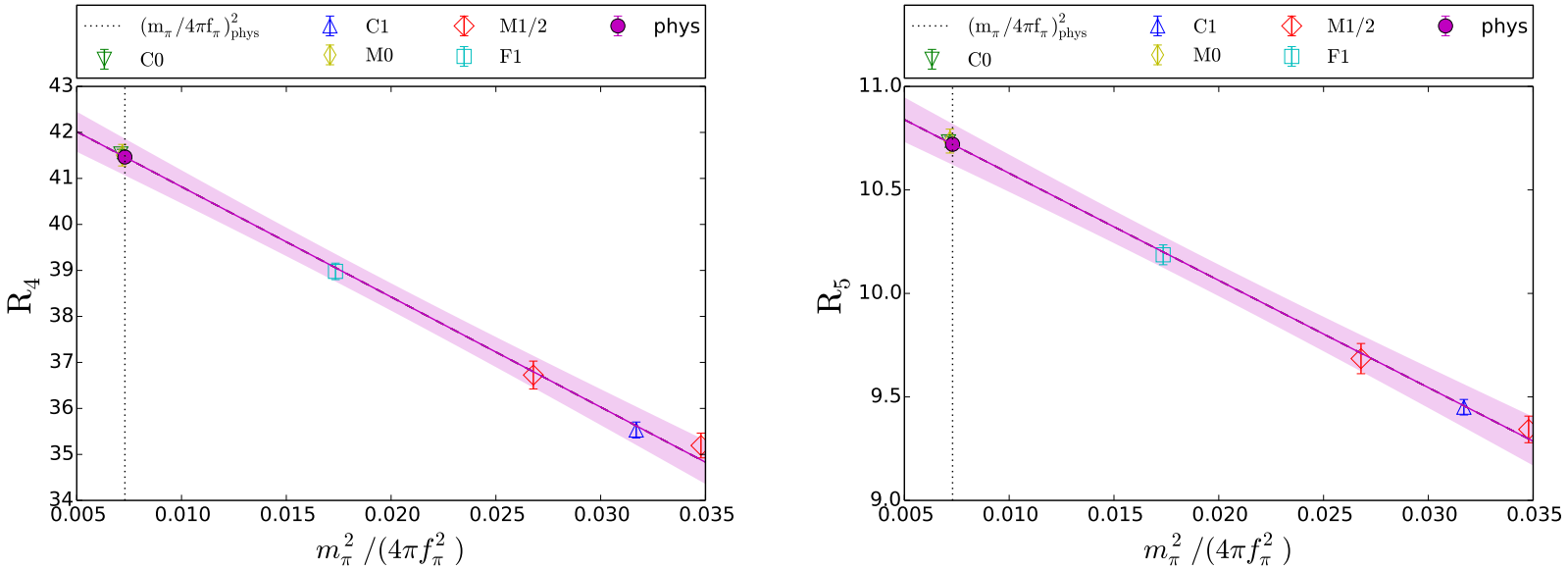
Ratio Parameter Fits (not corrected to continuum)

Figure: Preliminary results for R_2 and R_3 in $\overline{MS}$ at 3 GeV RI-SMOM$^{(\gamma,\gamma)}$ intermediate scheme.



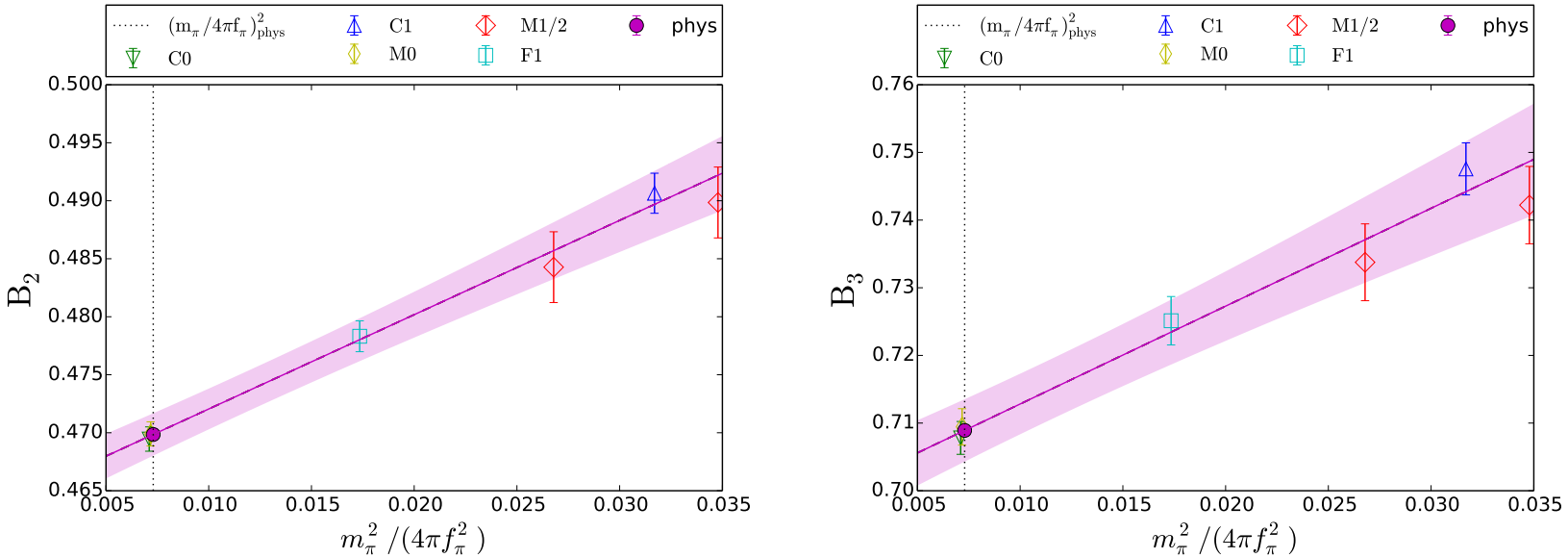
Ratio Parameter Fits

Figure: Preliminary results for R_4 and R_5 in $\overline{MS}$ at 3 GeV RI-SMOM$^{\gamma,\gamma}$ intermediate scheme. Data points have been adjusted to the physical strange-mass and continuum.



Bag Parameter Fits

Figure: Preliminary results for B_2 and B_3 in $\overline{MS}$ at 3 GeV RI-SMOM$^{\gamma,\gamma}$ intermediate scheme. Data points have been adjusted to the physical strange-mass and continuum.



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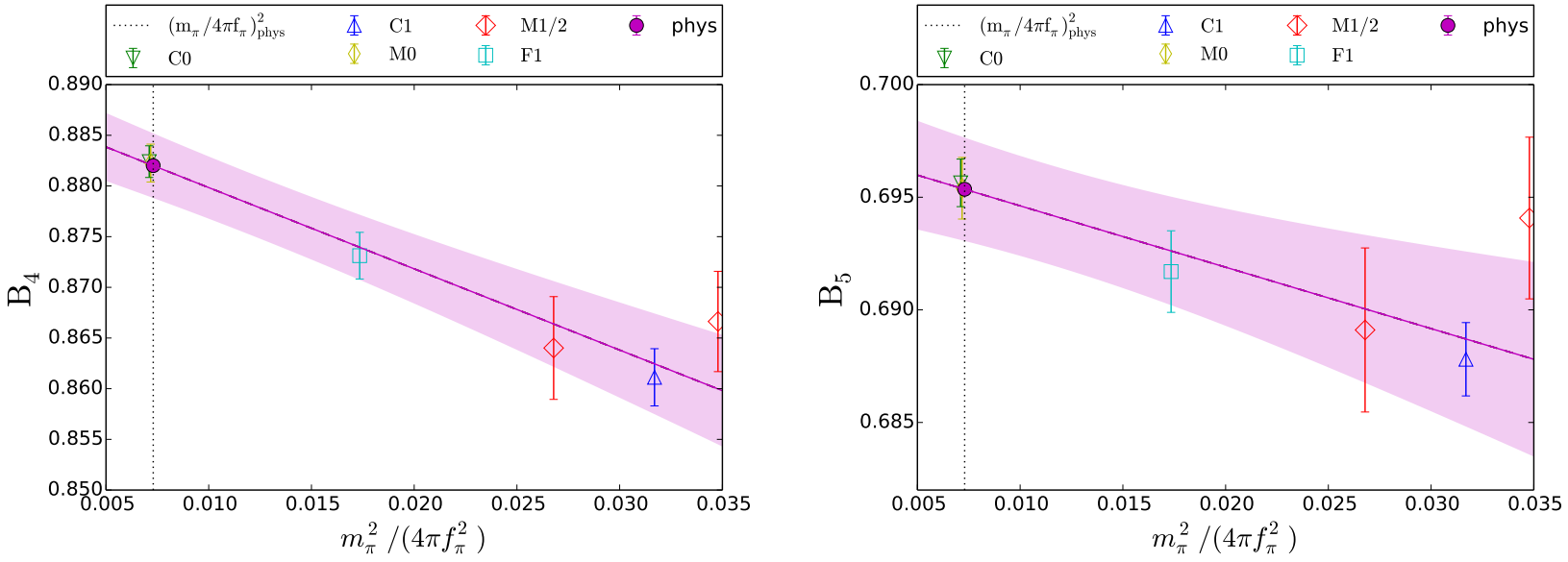


Table of Results

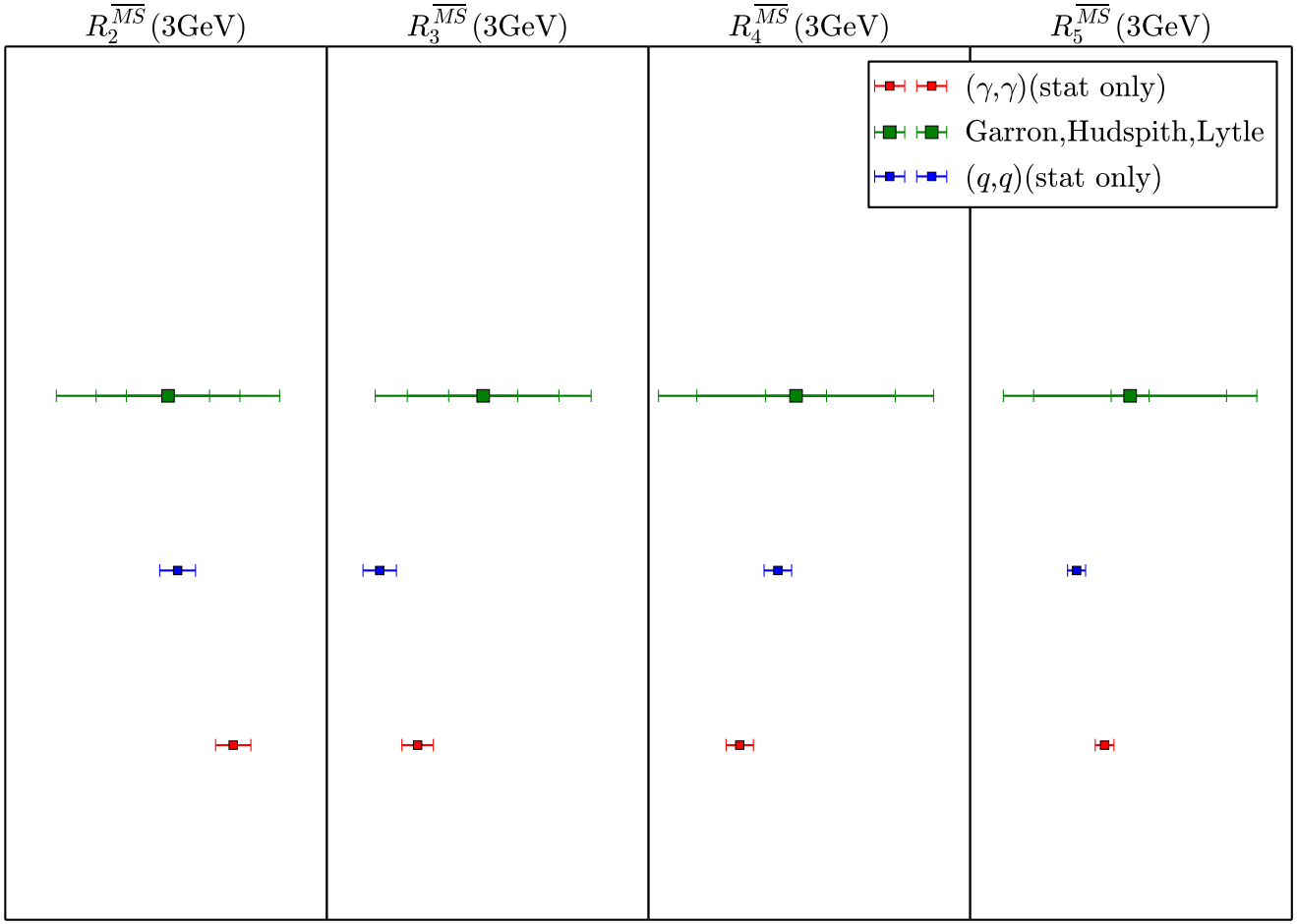
Physical point continuum limit results - with stat error only

		-	R_2	R_3	R_4	R_5
SMOM	(γ,γ)	-	-18.50(18)	5.529(64)	38.59(37)	10.9(10)
	$(\not{q},\not{q})$	-	-19.83(19)	5.416(71)	41.12(39)	10.427(96)
$\overline{\text{MS}}$	(γ,γ)	-	-18.79(19)	5.795(68)	41.46(40)	10.720(99)
	$(\not{q},\not{q})$	-	-19.38(19)	5.630(73)	42.58(40)	10.424(95)
		B_1	B_2	B_3	B_4	B_5
SMOM	(γ,γ)	0.5156(12)	0.5125(20)	0.7509(49)	0.9088(33)	0.7789(28)
	$(\not{q},\not{q})$	0.5337(12)	0.5115(17)	0.6667(54)	0.9002(29)	0.6968(23)
$\overline{\text{MS}}$	(γ,γ)	0.5177(12)	0.4699(18)	0.7089(46)	0.8820(32)	0.6954(23)
	$(\not{q},\not{q})$	0.5289(12)	0.4741(16)	0.6555(55)	0.8839(27)	0.6610(21)

Values of χ^2/dof

		-	R_2	R_3	R_4	R_5
MOM	(γ,γ)	-	0.05	0.07	0.97	0.40
	$(\not{q},\not{q})$	-	0.01	0.15	0.73	0.38
ms	(γ,γ)	-	0.09	0.14	0.96	0.40
	$(\not{q},\not{q})$	-	0.01	0.16	0.75	0.38
		B_1	B_2	B_3	B_4	B_5
MOM	(γ,γ)	1.86	1.36	2.37	0.52	0.92
	$(\not{q},\not{q})$	1.34	1.59	5.74	0.43	1.63
ms	(γ,γ)	1.84	0.63	1.50	1.20	1.87
	$(\not{q},\not{q})$	1.32	1.43	6.08	0.57	2.17

Comparison with Previous Results



PT error $\approx 2\%$

Comparison with Previous Results

Consistency within errors with previous results.

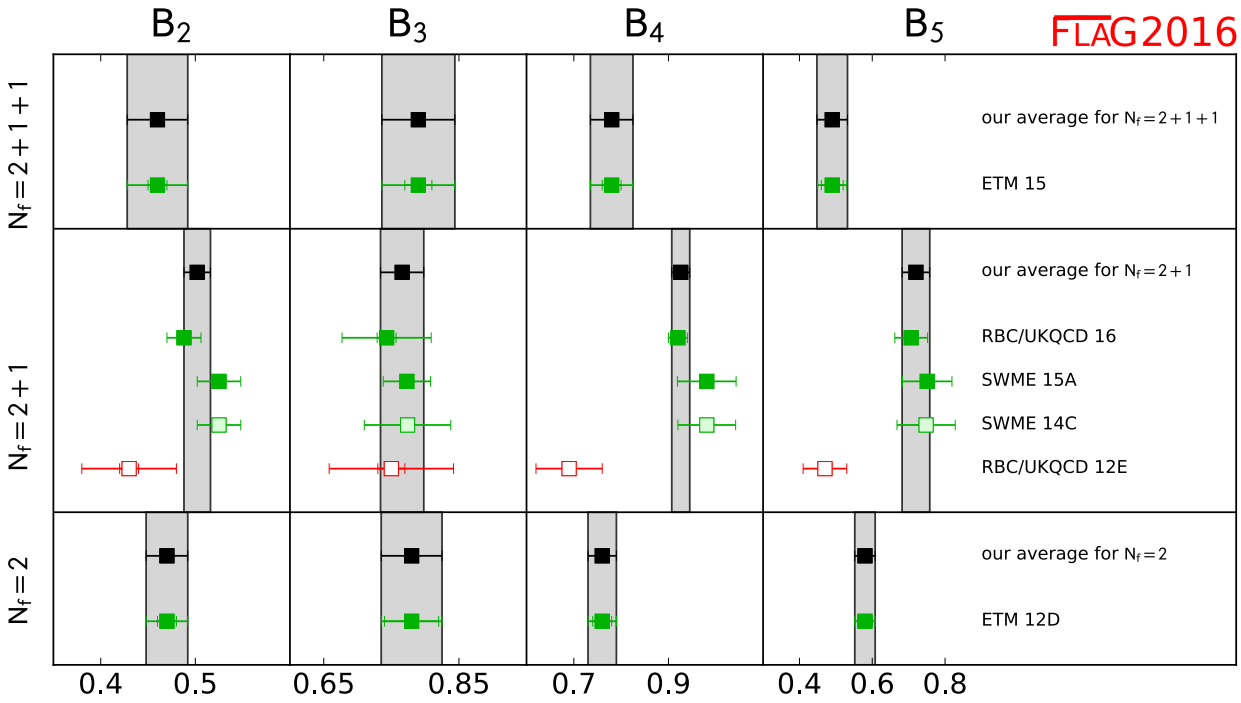
	this work	RBC-UKQCD14[4]	RBC-UKQCD16[5]
$B_K^{(\not{q},\not{q})}(\overline{\mathrm{MS}}, 3\mathrm{GeV})$	0.5289(12)	0.5293(17)(106)	0.536(9)(6)(11)
$R_2^{(\gamma,\gamma)}(\overline{\mathrm{MS}}, 3\mathrm{GeV})$	-18.79(19)	-	-19.48(44)(32)(42)
$R_3^{(\gamma,\gamma)}(\overline{\mathrm{MS}}, 3\mathrm{GeV})$	5.795(68)	-	6.08(15)(18)(14)
$R_4^{(\gamma,\gamma)}(\overline{\mathrm{MS}}, 3\mathrm{GeV})$	41.5(40)	-	43.11(89)(201)(112)
$R_5^{(\gamma,\gamma)}(\overline{\mathrm{MS}}, 3\mathrm{GeV})$	10.720(99)	-	10.99(20)(82)(32)

Stat error improved significantly.

[4] Blum et al, 2014, arXiv:1411.7017

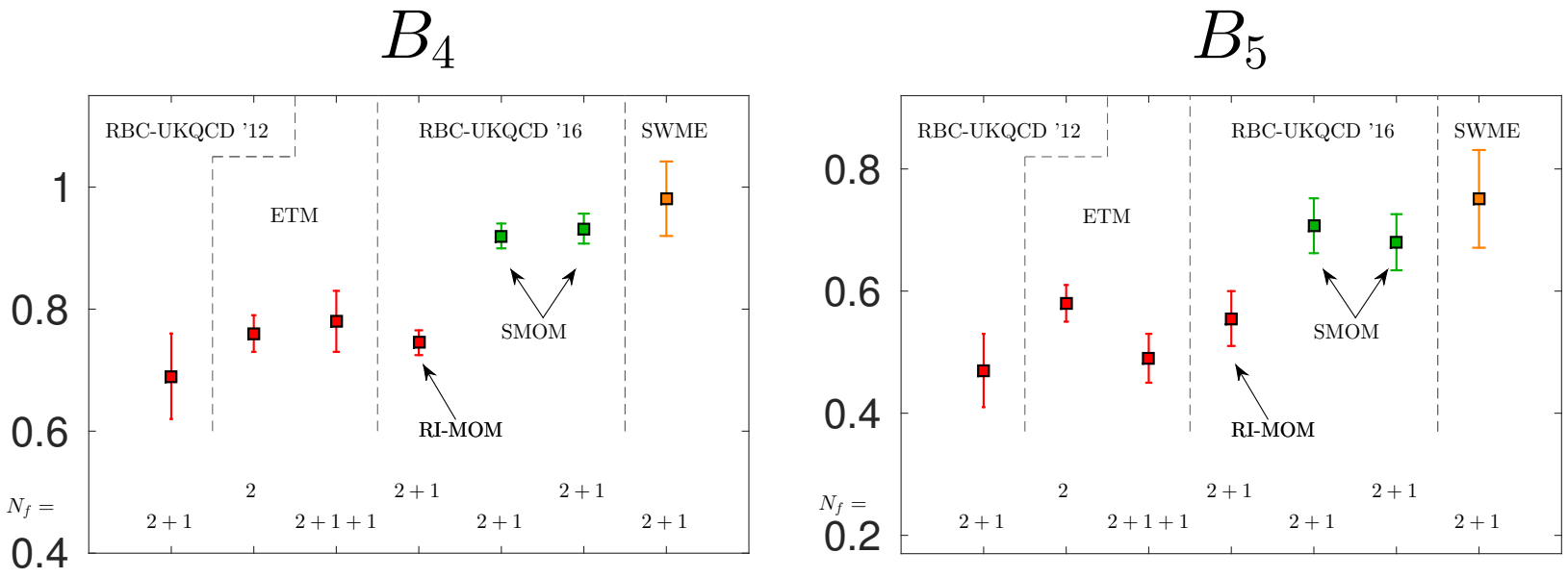
[5] Garron,Hudspith, Lytle, 2016, arXiv:1609.03334

Tensions in previous results



Tensions in previous results

In Garron et al [6] it was proposed that source of tension come from choice of kinematics in renormalisation.



[6] Garron et al, RBC-UKQCD, 16 arXiv:1609.03334

Summary

- Two times smaller statistical error
- Third lattice spacing in fit
- Data directly at physical point for first time
- Systematic error will be dominated by perturbation theory
- Quantify truncation in perturbation theory by difference between schemes
- Consistent with previous RBC-UKQCD results with NE schemes
- Exceptional schemes remain in disagreement (pion pole model suspect)

Acknowledgements

Thanks to all members of RBC-UKQCD for their weekly discussions and comments.

Backup slides

Figure: Preliminary results for R_2 and R_3 in $\overline{MS}$ at 3 GeV RI-SMOM$^{(\gamma,\gamma)}$ intermediate scheme.

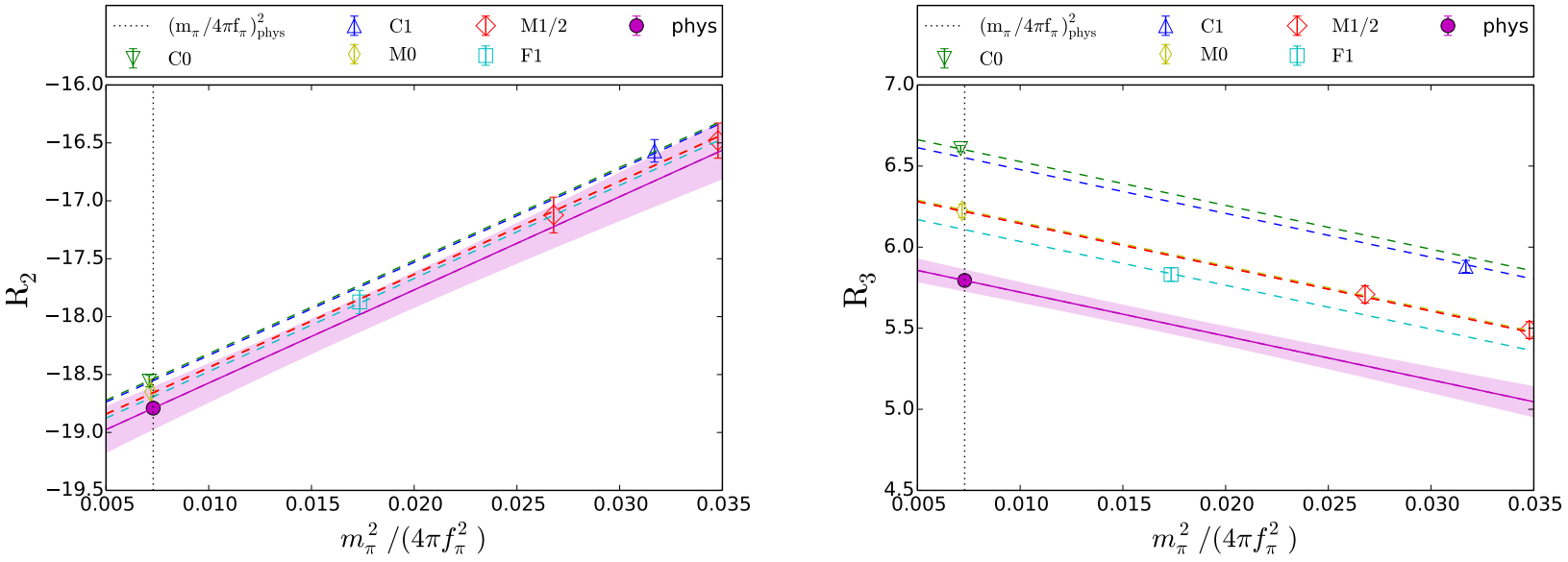
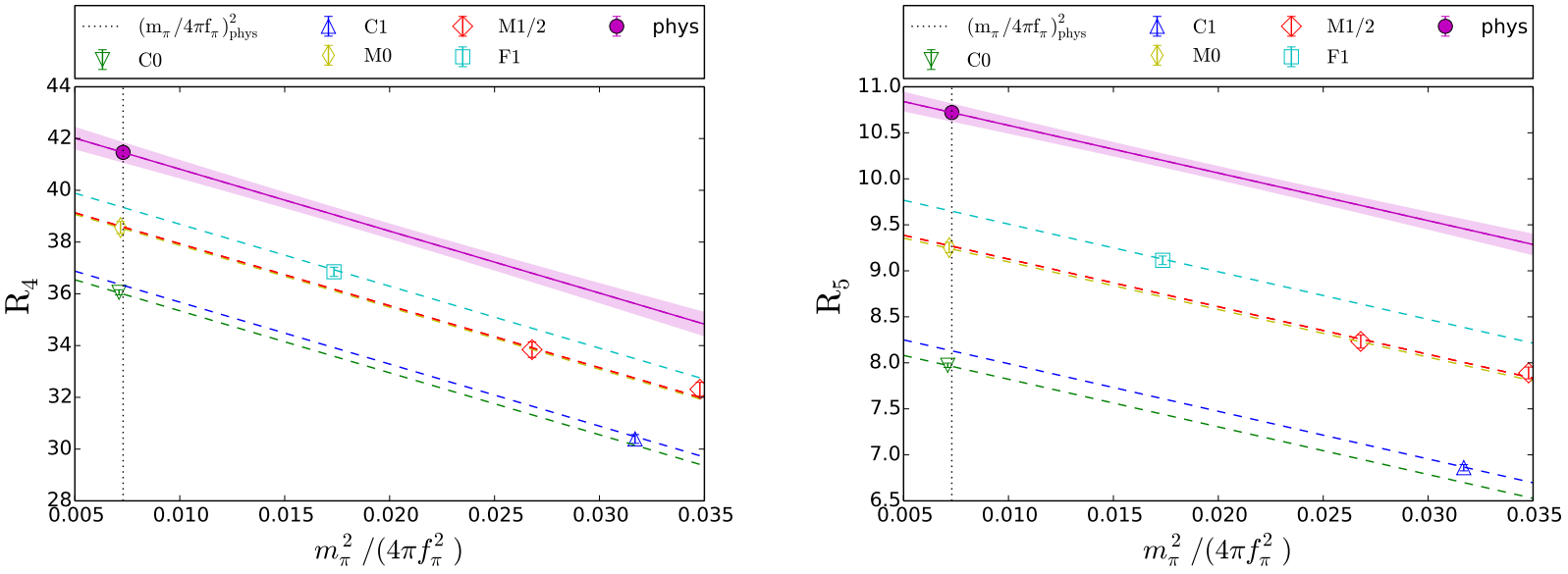


Figure: Preliminary results for R_4 and R_5 in $\overline{MS}$ at 3 GeV RI-SMOM$^{(\gamma,\gamma)}$ intermediate scheme.



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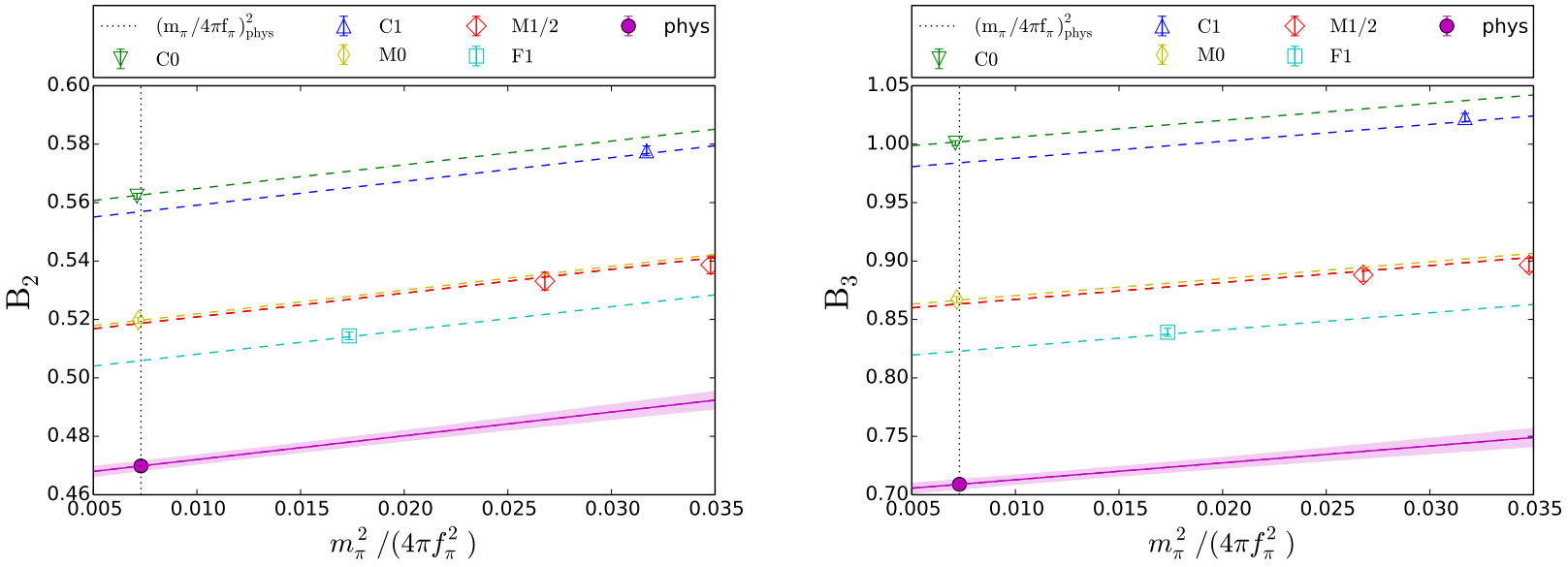

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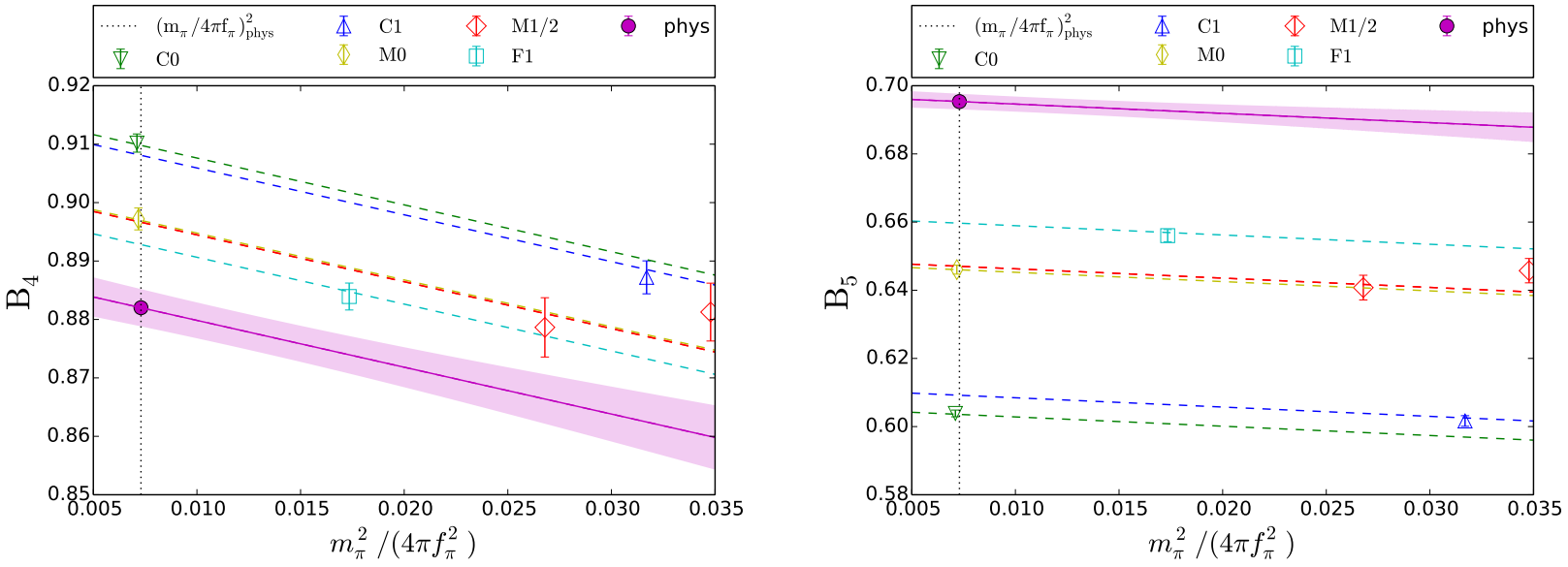
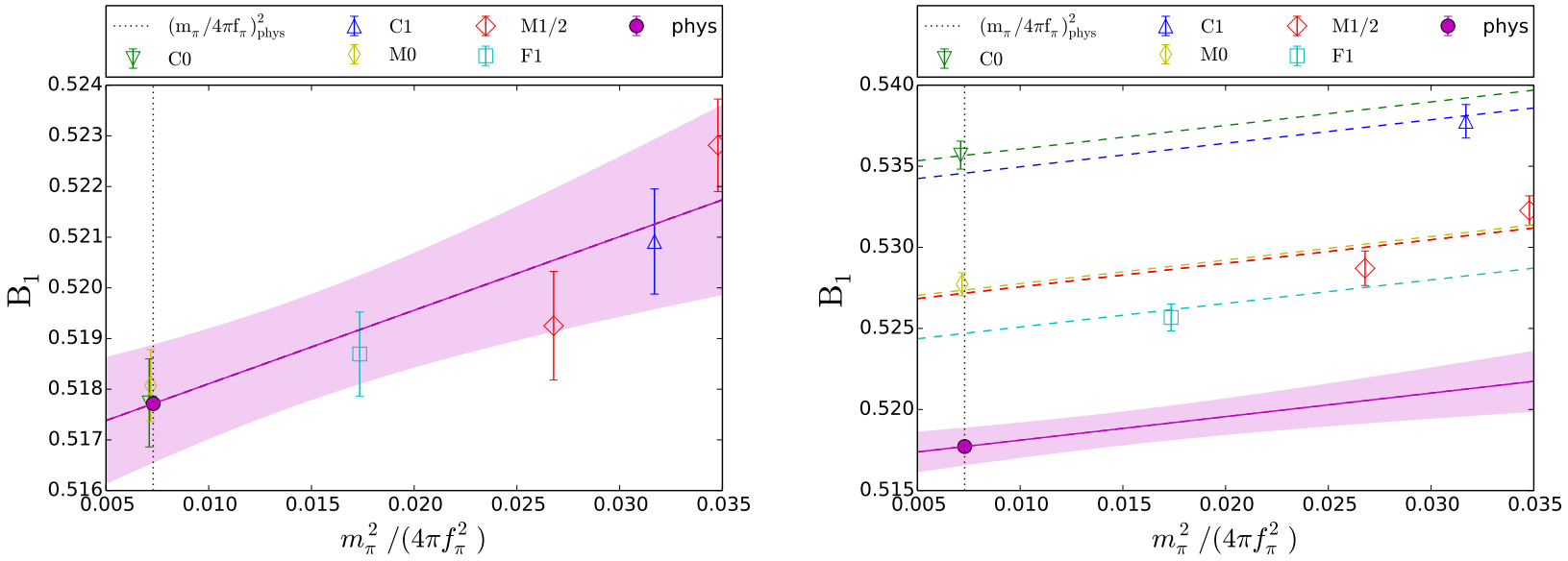


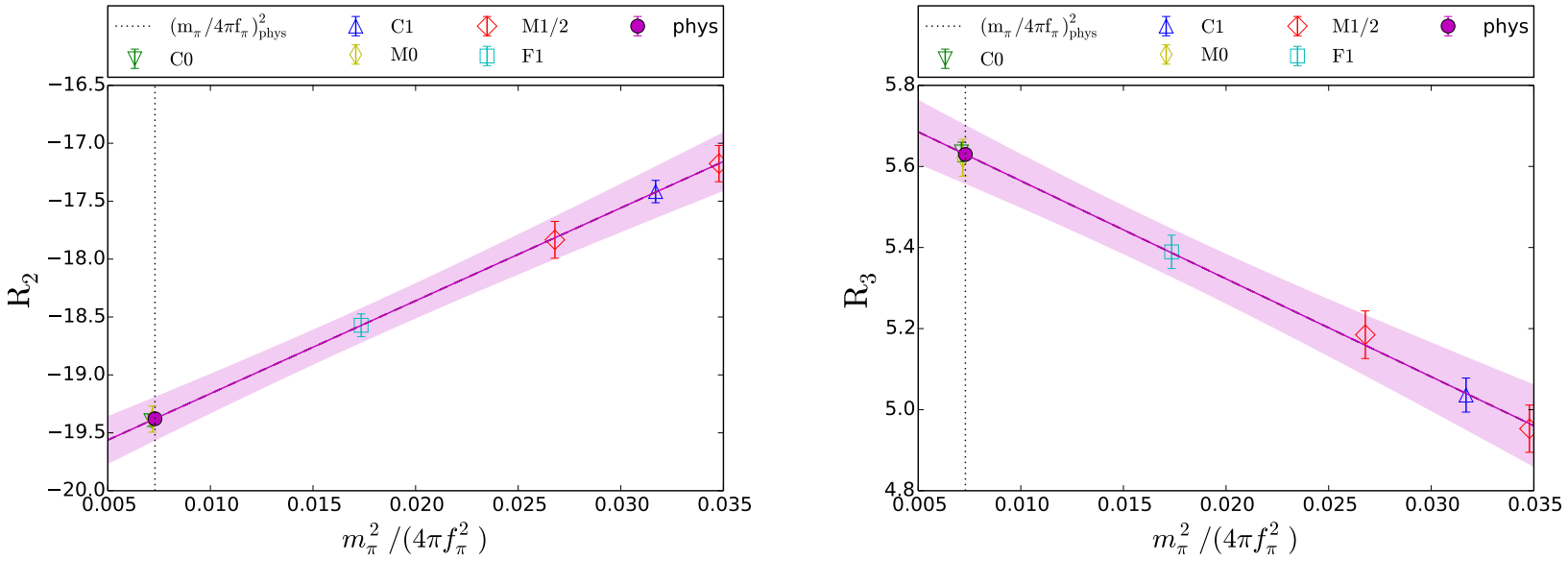
SM Bag parameter

Figure: Preliminary results for B_K in $\overline{MS}$ at 3 GeV RI-SMOM$^{(\gamma,\gamma)}$ intermediate scheme.



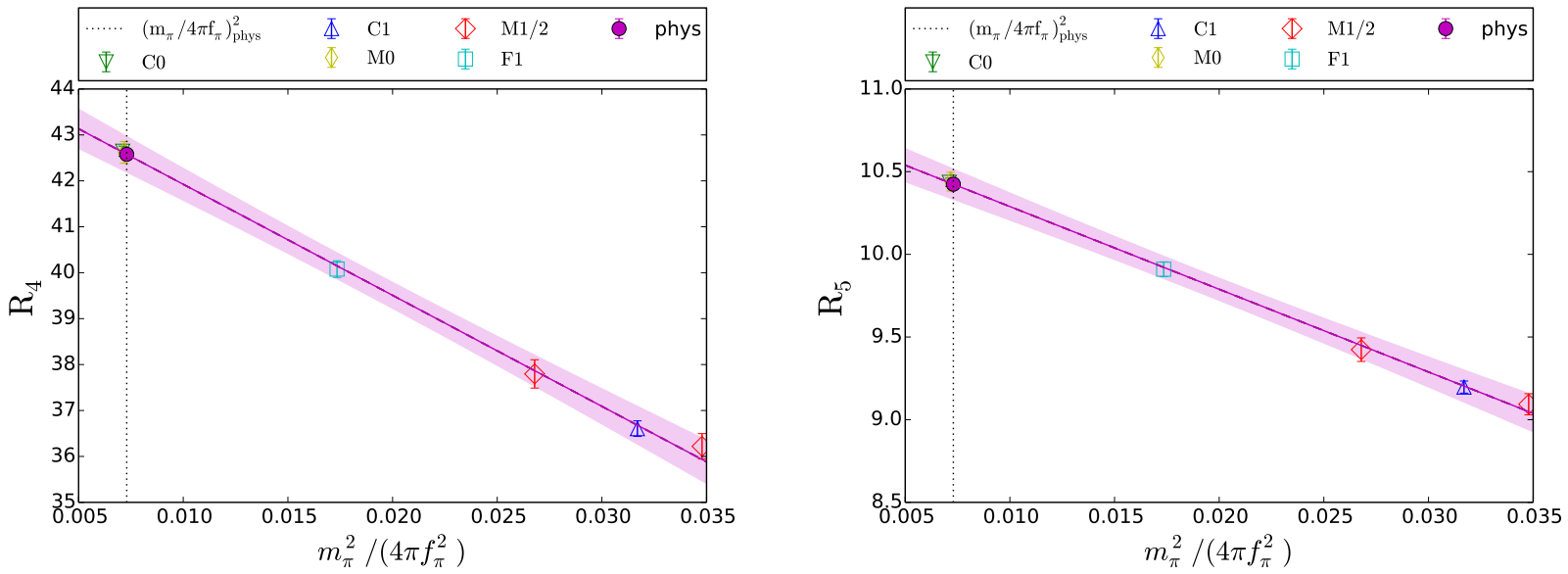
Results - $(\not{q}, \not{q})$

Figure: Preliminary results for R_2 and R_3 in $\overline{MS}$ at 3 GeV RI-SMOM$^{(\not{q},\not{q})}$ intermediate scheme.



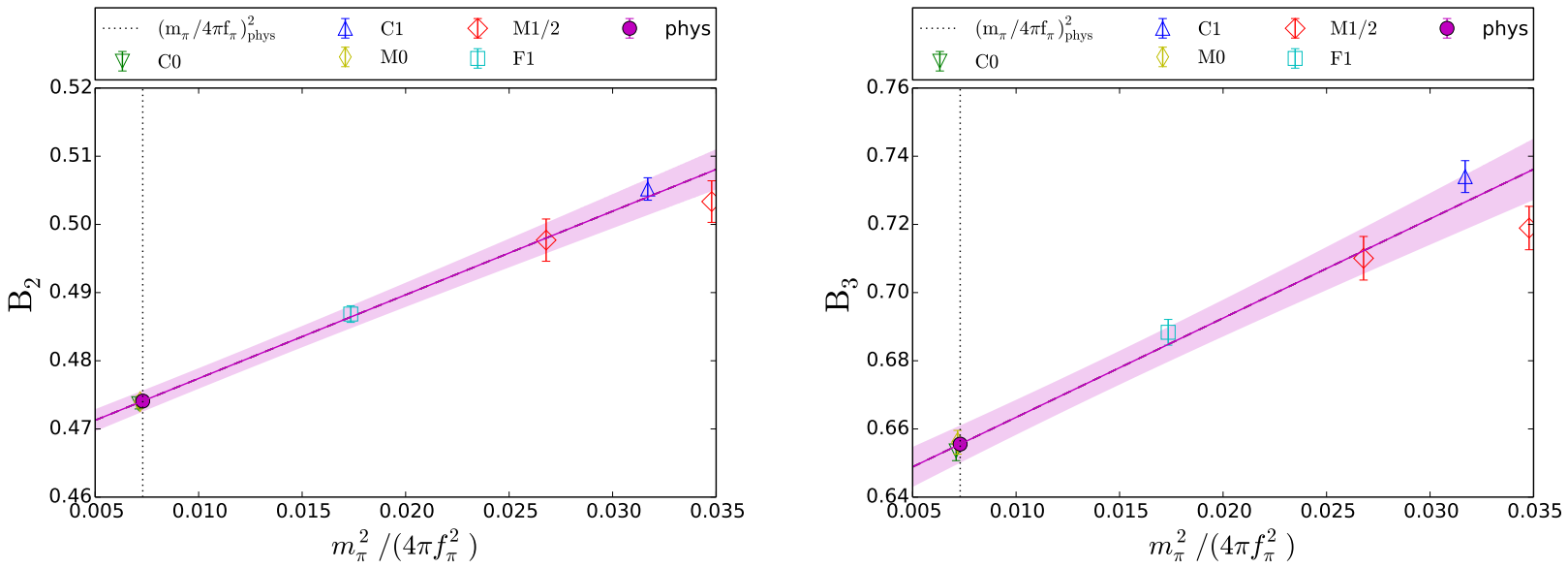
Results - $(\not{q}, \not{q})$

Figure: Preliminary results for R_4 and R_5 in $\overline{MS}$ at 3 GeV RI-SMOM$^{\not{q},\not{q}}$ intermediate scheme.



Results - $(\not{q}, \not{q})$

Figure: Preliminary results for B_2 and B_3 in $\overline{MS}$ at 3 GeV RI-SMOM$^{\not{q},\not{q}}$ intermediate scheme.



Results - $(\not{q}, \not{q})$

Figure: Preliminary results for B_4 and B_5 in $\overline{MS}$ at 3 GeV RI-SMOM$^{\not{q},\not{q}}$ intermediate scheme.