

ZD and Savage, PRD 86, 054505 (2012).

ZD, Will Detmold, Mike Endres, Andrew Pochinsky and Phiala Shanahan, work in progress.

THE 36TH INTERNATIONAL SYMPOSIUM ON LATTICE FIELD THEORY
MICHIGAN STATE UNIVERSITY, JULY 2018

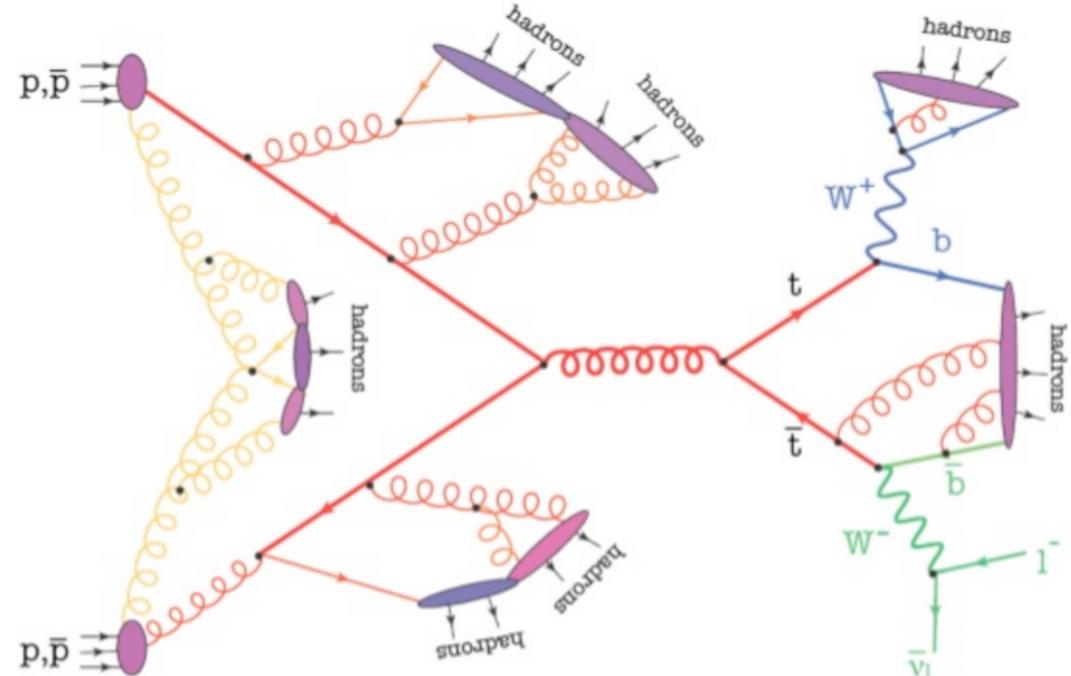
HIGHER MOMENTS OF PARTON DISTRIBUTION FUNCTIONS FROM LATTICE QCD

ZOHREH DAVOUDI
UNIVERSITY OF MARYLAND, RIKEN FELLOW

CONSTRUCTING PDFS FROM MOMENTS:

$$\langle x^n \rangle_{q,\mu^2} = \int dx x^n q(x; \mu^2)$$

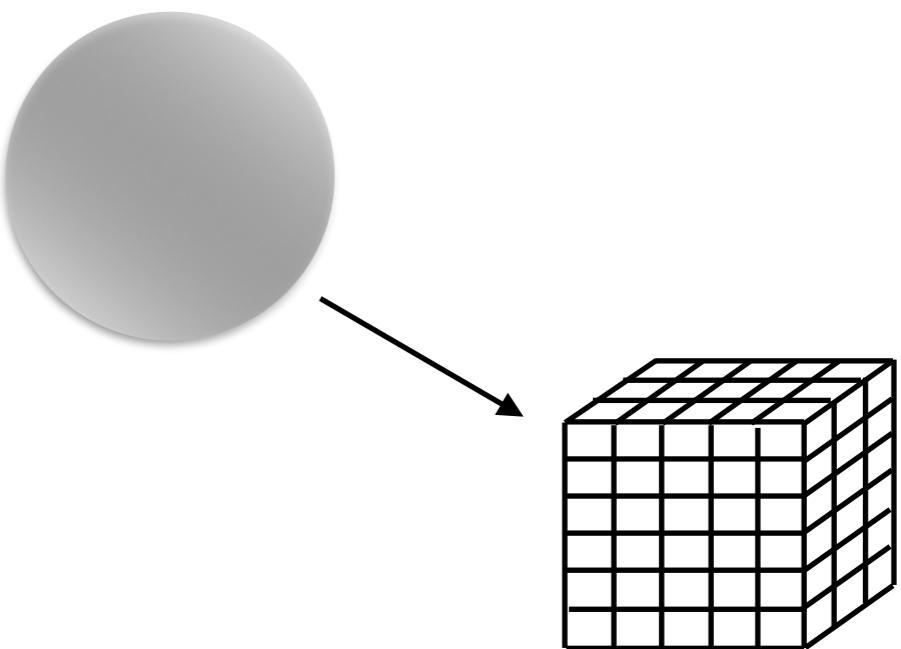
$$\langle p, s | \mathcal{O}_{\mu_1 \mu_2 \dots \mu_n} | p, s \rangle|_{\mu^2} = 2 \langle x^n \rangle_{q,\mu^2} p^{\{\mu_1} p^{\mu_2} \dots p^{\mu_n\}}$$



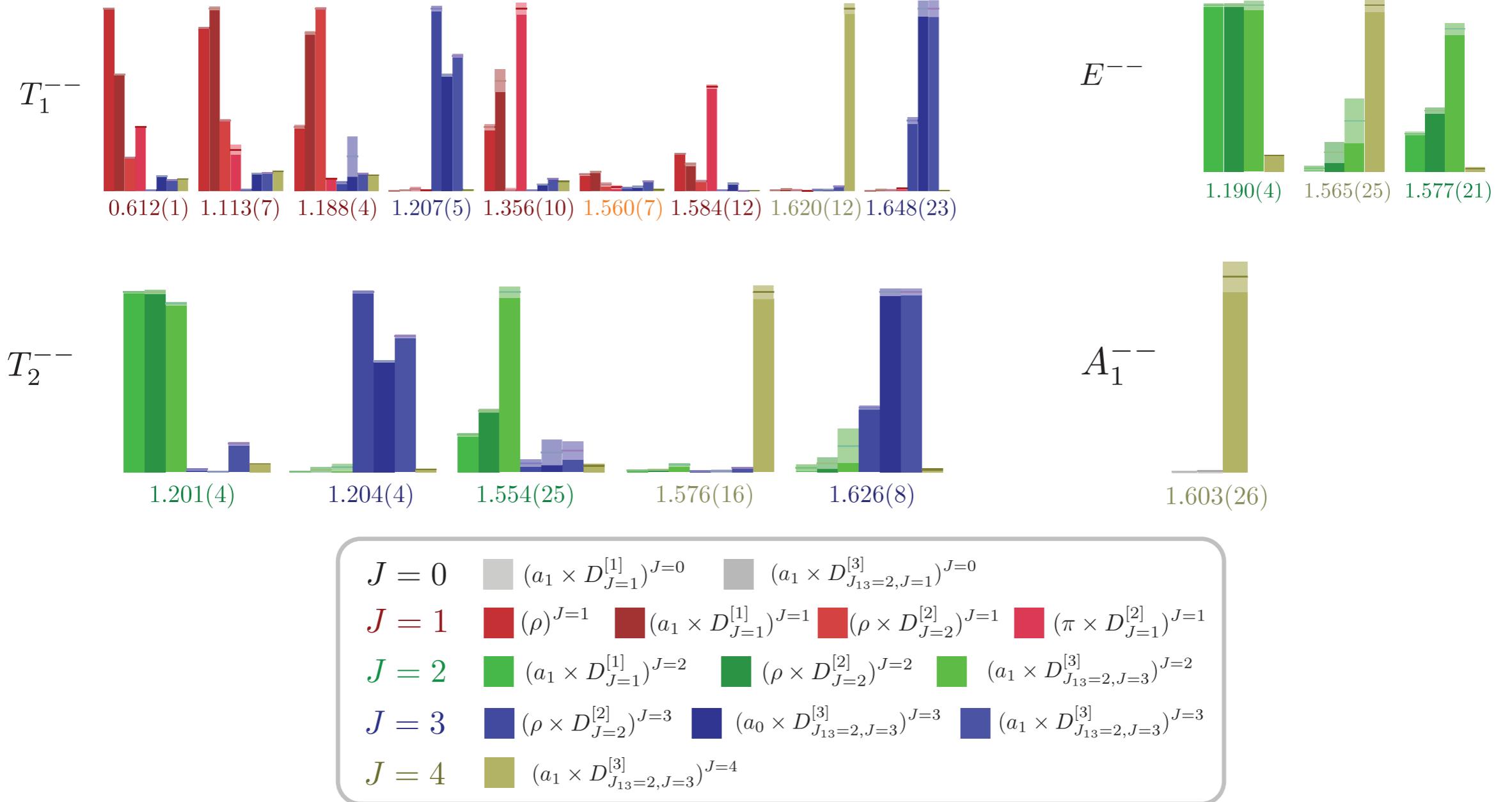
LQCD IS IDEAL FOR EVALUATING SUCH MES.

PHENOMENOLOGICALLY 6-8 MOMENTS APPEAR TO BE SUFFICIENT.

HOWEVER, ONLY UP TO THE FIRST THREE MOMENTS HAVE BEEN ACCESSIBLE WITH LQCD DUE TO A POWER-DIVERGENCE MIXING WITH LOWER DIMENSIONAL OPERATORS.



LESSON FROM MODERN LQCD SPECTROSCOPY STUDIES:



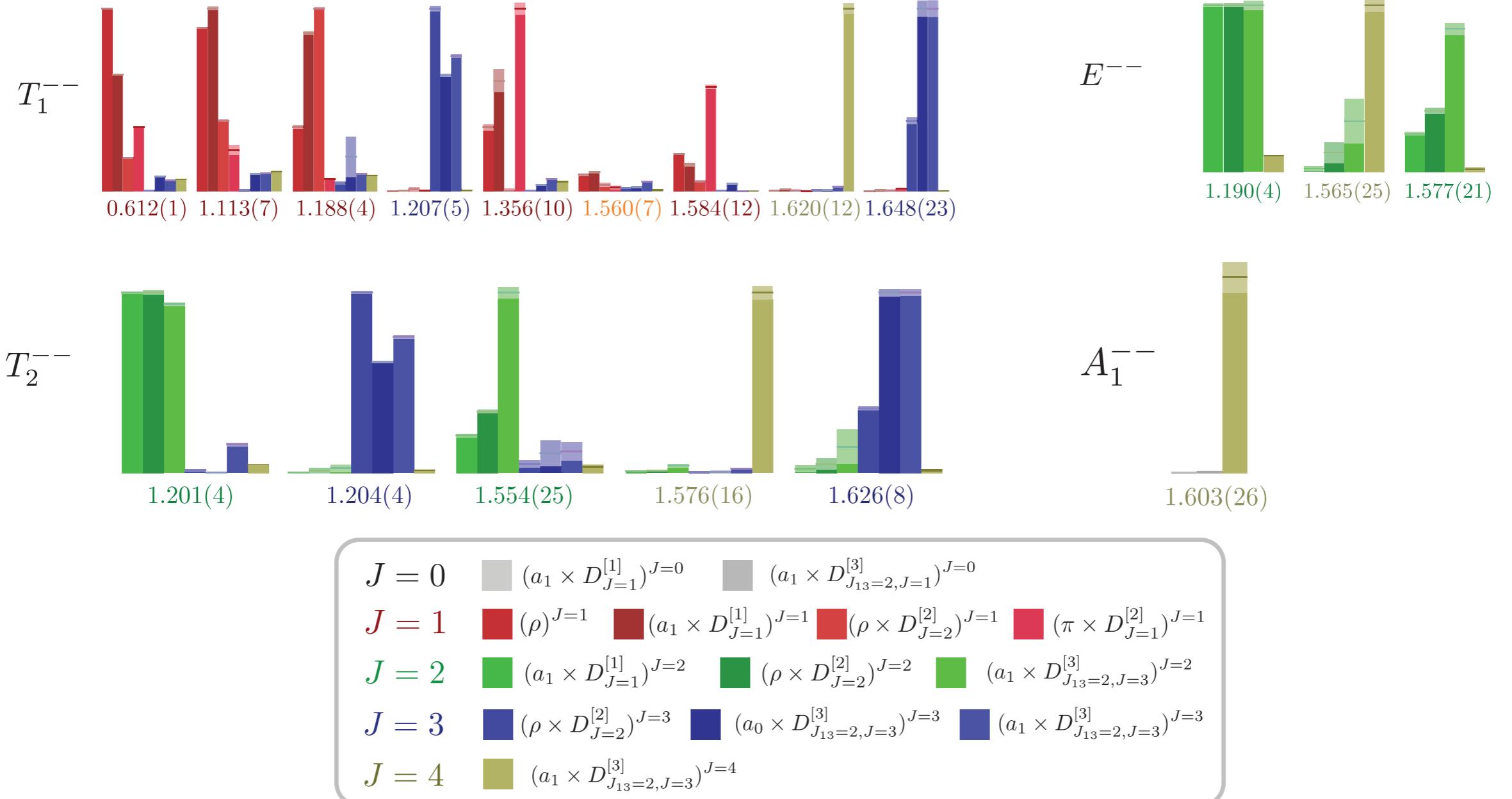
SMEARED OPERATORS FROM A CONTINUUM
OPERATOR WITH A GIVEN J

$$\mathcal{O}_{\Lambda,\lambda}^{[\mathbf{J}]} \equiv \sum_M \mathcal{S}_{\Lambda,\lambda}^{\mathbf{J},M} \mathcal{O}^{\mathbf{J},M}$$

$$\mathcal{S}_{\Lambda,\lambda}^{J,M} = \langle \Lambda, \lambda | J, M \rangle$$

$$\mathcal{O}^{\mathbf{J},M} \equiv (\Gamma \times D^{n_D})^{\mathbf{J},M}$$

LESSON FROM MODERN LQCD SPECTROSCOPY STUDIES:



RELATED IDEAS:

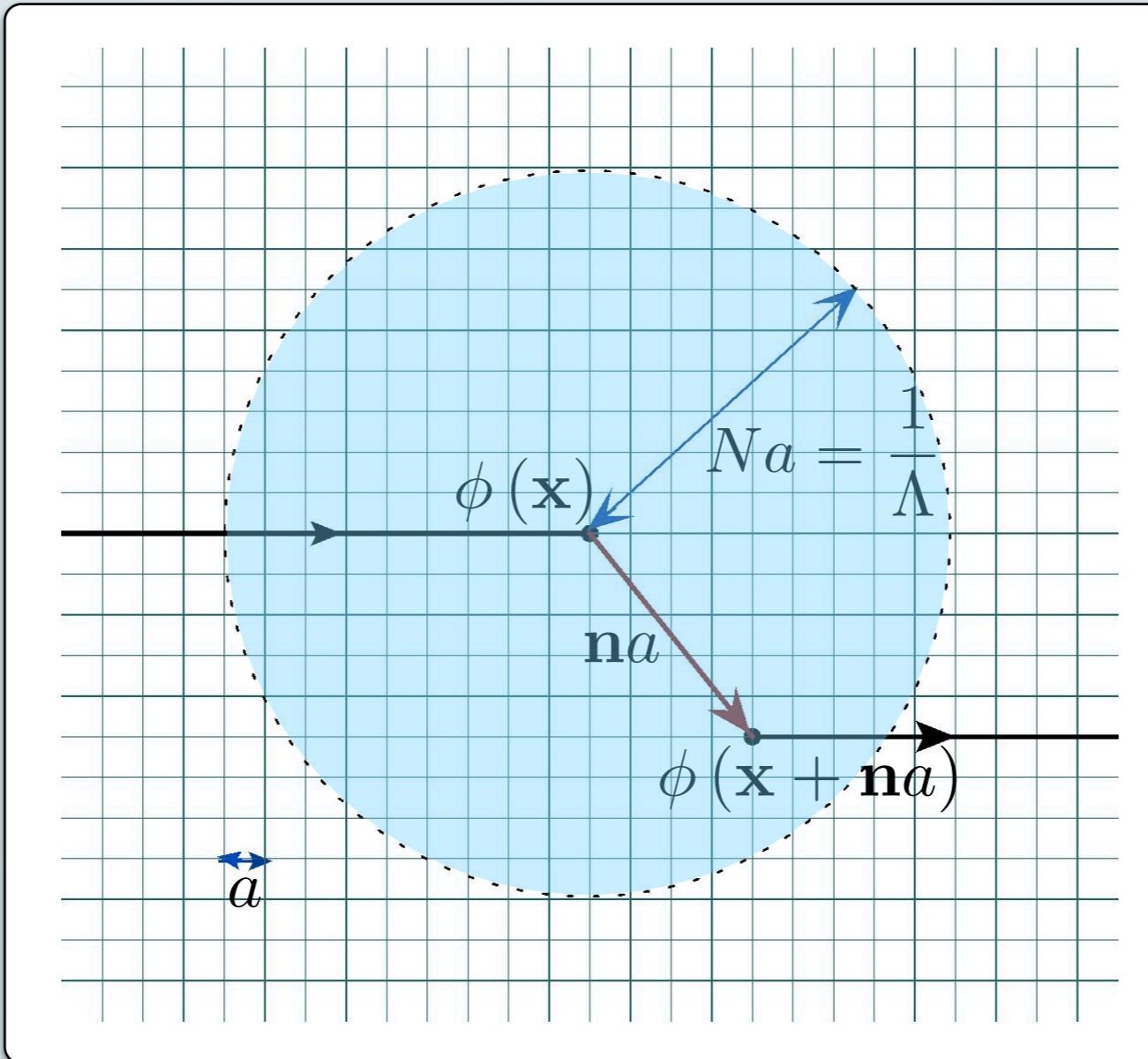
Detmold and Lin, Phys. Rev. D73, 014501 (2006).

Dawson, Martinelli, Rossi, Sachrajda, Sharpe, Talevi, and Testa, Nucl. Phys. B514, 313 (1998).

Monahan and Orginos, Phys. Rev. D91, 074513 (2015).

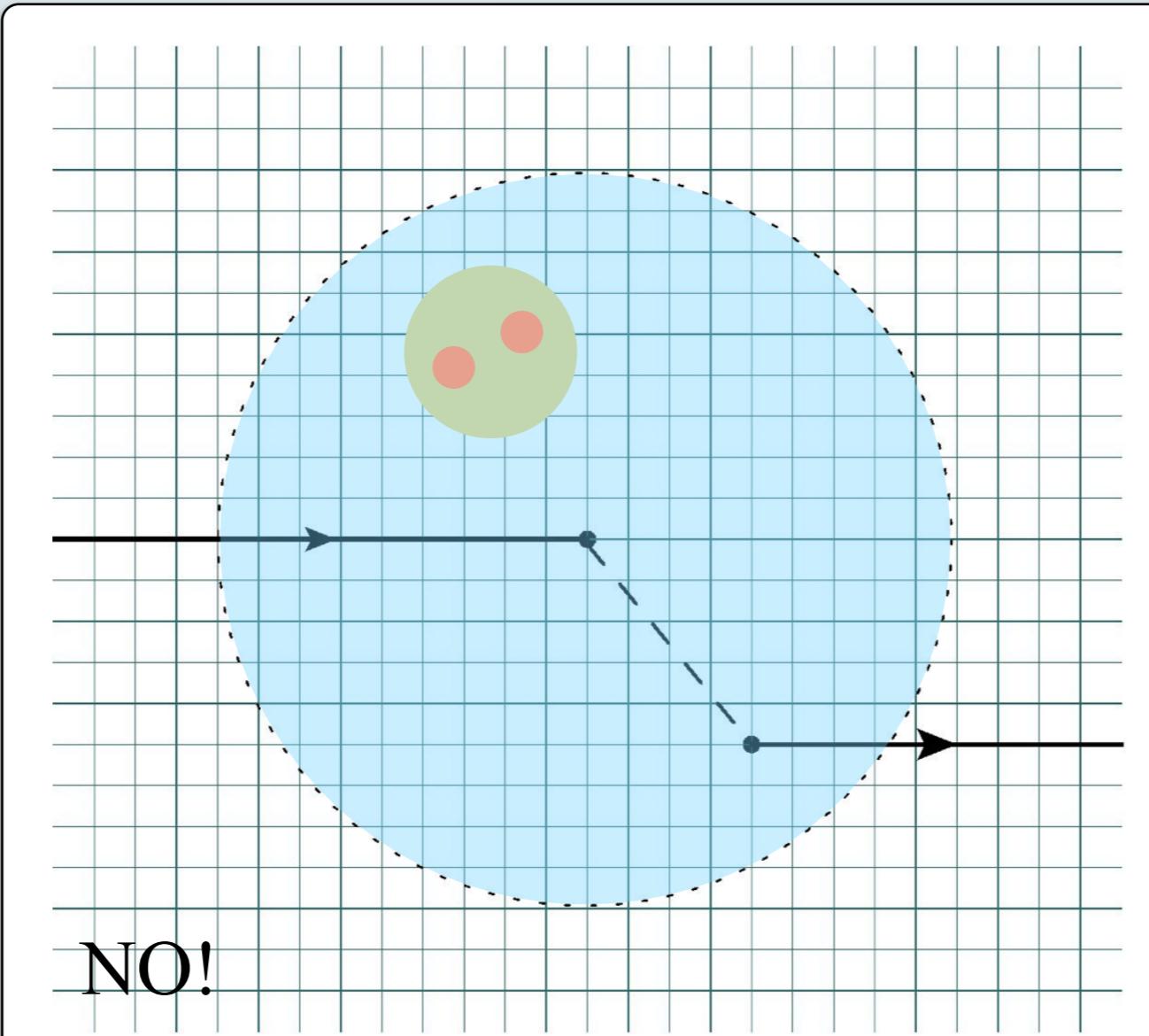
IN THAT SPIRIT, WE CONSIDERED A SIMPLE OPERATOR:

$$\hat{\theta}_{L,M}(\mathbf{x}; \mathbf{a}, N) = \frac{3}{4\pi N^3} \sum_{\mathbf{n}}^{|\mathbf{n}| \leq N} \phi(\mathbf{x}) \phi(\mathbf{x} + \mathbf{n}\mathbf{a}) Y_{L,M}(\hat{\mathbf{n}})$$



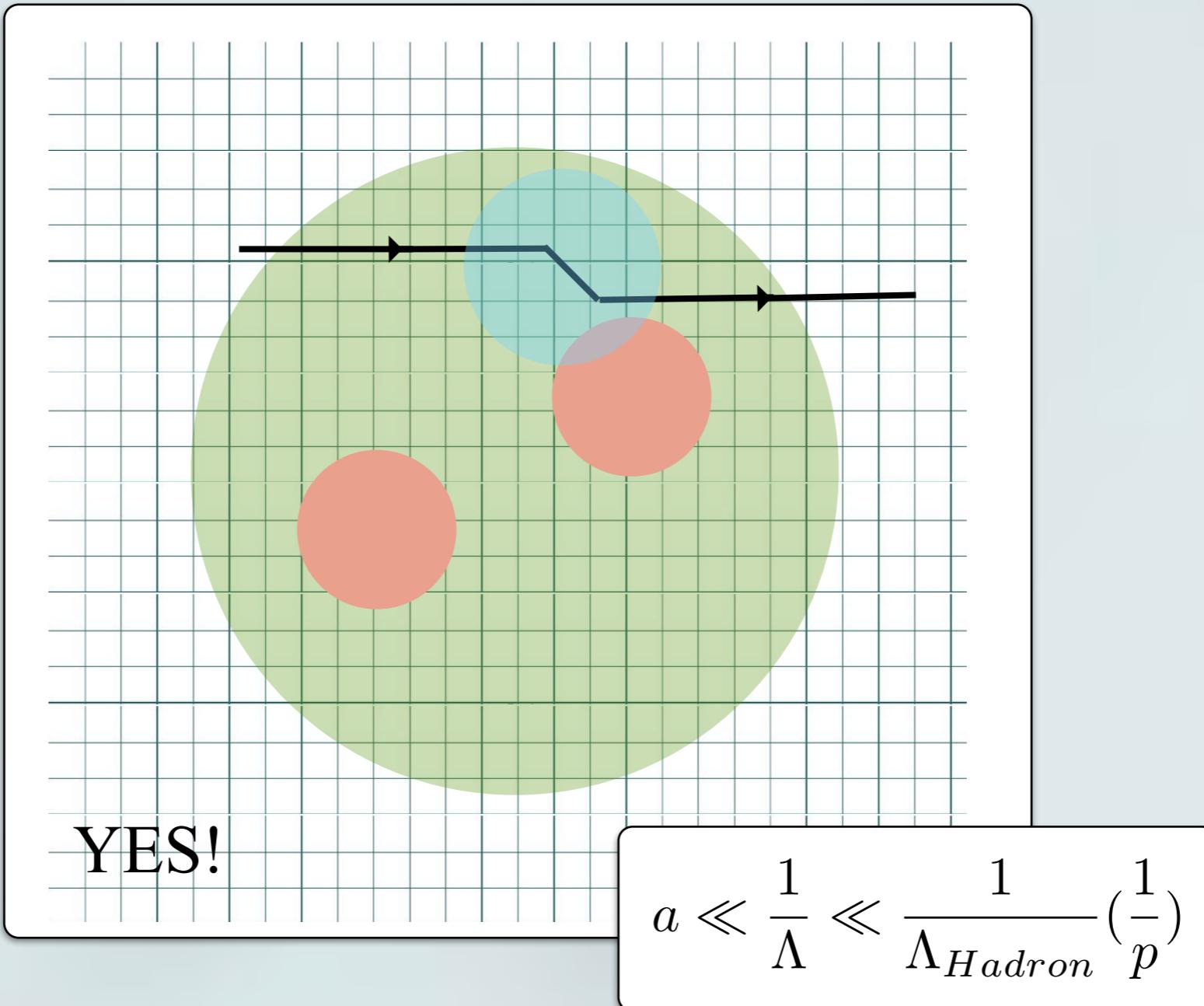
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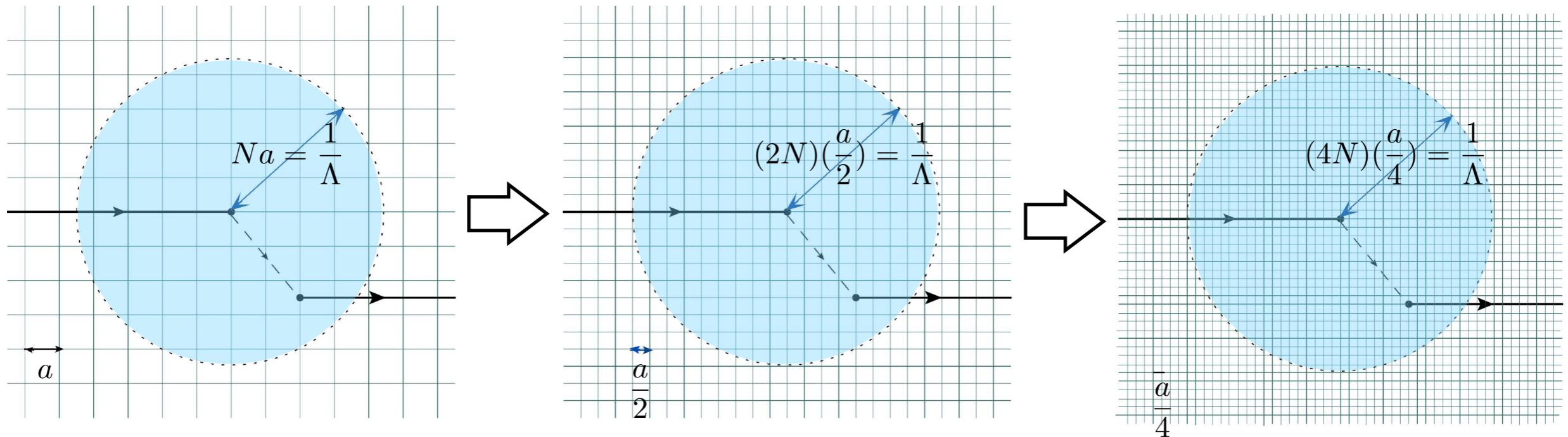


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CORRECT PROCEDURE:
KEEP THE PHYSICAL SIZE OF THE OPERATOR FIXED, THEN TAKE THE CONTINUUM LIMIT:



AN EXAMPLE:

$$\hat{\theta}_{L,M}(\mathbf{x}; \mathbf{a}, N) = \frac{3}{4\pi N^3} \sum_{\mathbf{n}}^{|n| \leq N} \phi(\mathbf{x}) \phi(\mathbf{x} + \mathbf{n}\mathbf{a}) Y_{L,M}(\hat{\mathbf{n}})$$

$$\begin{aligned} \hat{\theta}_{3,0}(\mathbf{x}; \mathbf{a}, N) &= \frac{C_{30;10}^{(1)}(N)}{\Lambda} \mathcal{O}_z^{(1)}(\mathbf{x}; \mathbf{a}) + \frac{C_{30;10}^{(3)}(N)}{\Lambda^3} \mathcal{O}_z^{(3)}(\mathbf{x}; \mathbf{a}) + \frac{C_{30;10}^{(5)}(N)}{\Lambda^5} \mathcal{O}_z^{(5)}(\mathbf{x}; \mathbf{a}) + \\ &\quad \frac{C_{30;10}^{(5;RV)}(N)}{\Lambda^5} \mathcal{O}_z^{(5;RV)}(\mathbf{x}; \mathbf{a}) + \frac{C_{30;30}^{(3)}(N)}{\Lambda^3} \mathcal{O}_{zzz}^{(3)}(\mathbf{x}; \mathbf{a}) + \frac{C_{30;30}^{(5)}(N)}{\Lambda^5} \mathcal{O}_{zzz}^{(5)}(\mathbf{x}; \mathbf{a}) + \\ &\quad \frac{C_{30;50}^{(5)}(N)}{\Lambda^5} \mathcal{O}_{zzzzz}^{(5)}(\mathbf{x}; \mathbf{a}) + \mathcal{O}\left(\frac{\nabla_z^7}{\Lambda^7}\right) \end{aligned}$$

DESIRED $L = 3$ OPERATOR

HOW DO THE COEFFICIENTS SCALE WITH $N(a)$? BETTER HAVE:

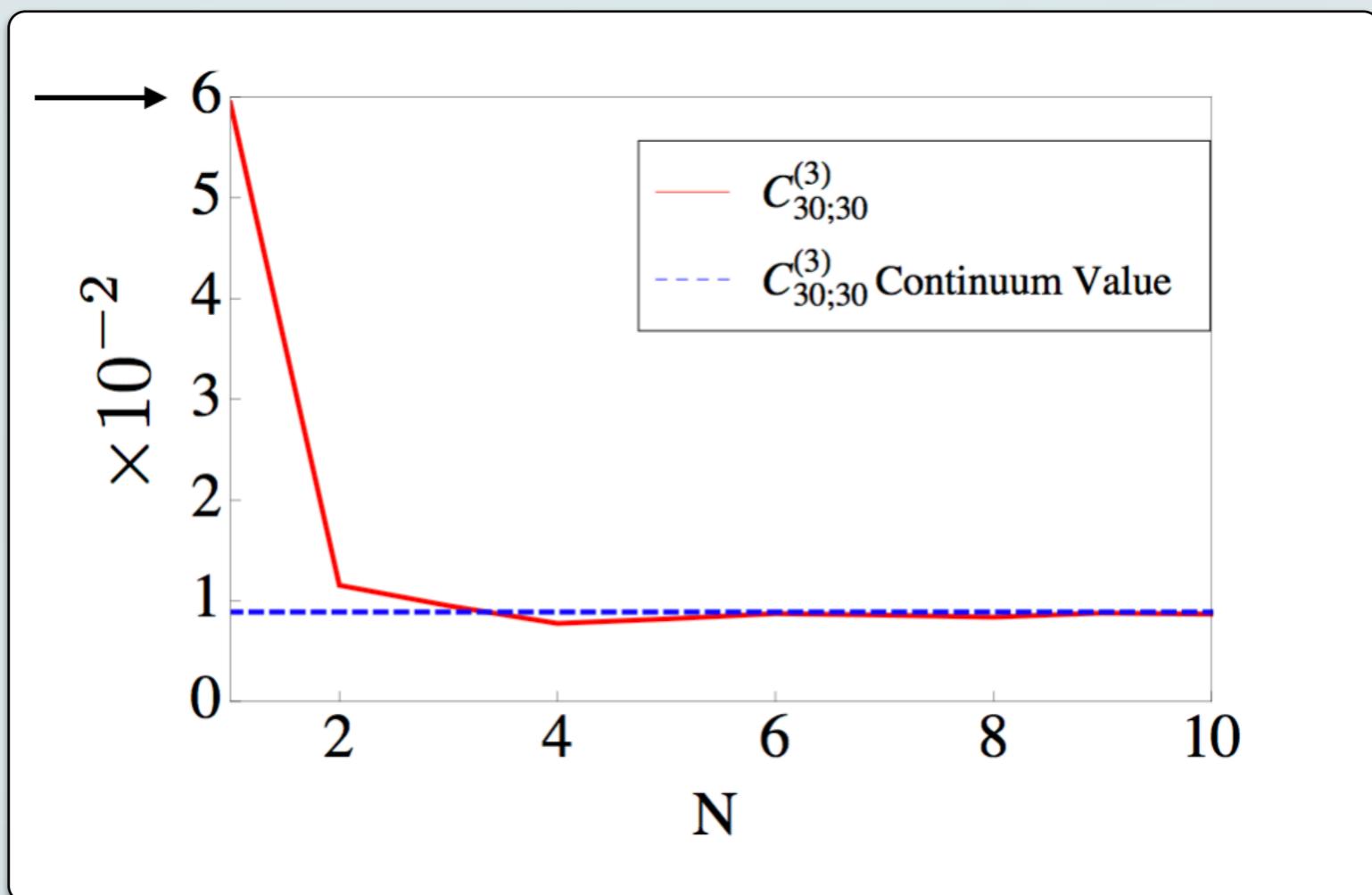
$C_{30;L'0}^{(d)}(N)$ IS FINITE FOR $L' = 3$

$C_{30;L'0}^{(d)}(N) \rightarrow 0$ FOR $L' \neq 3$

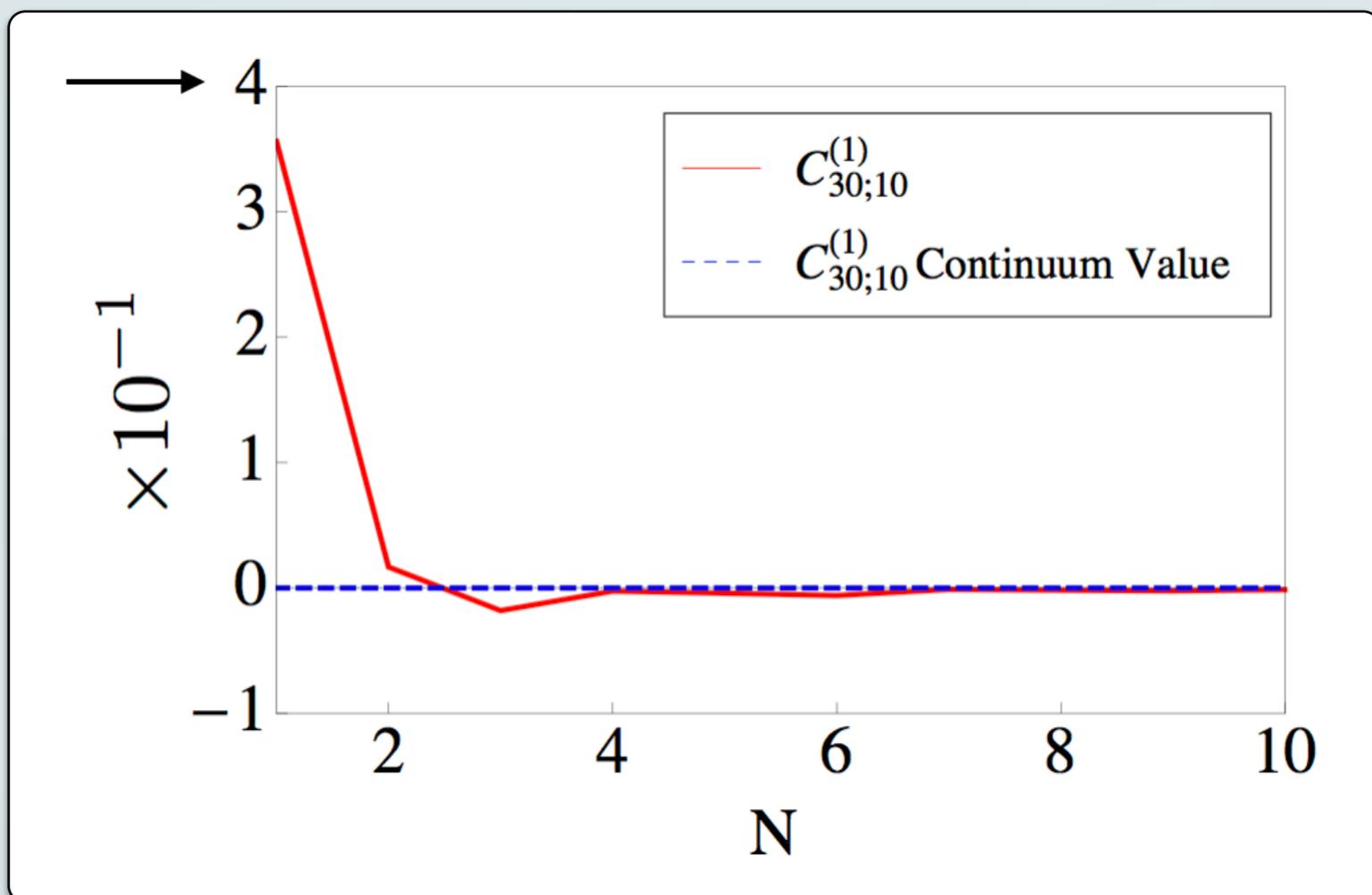
$C_{30;L'0}^{(d;RV)}(N) \rightarrow 0$

SEE ANALYTICAL RESULTS IN THE PAPER...

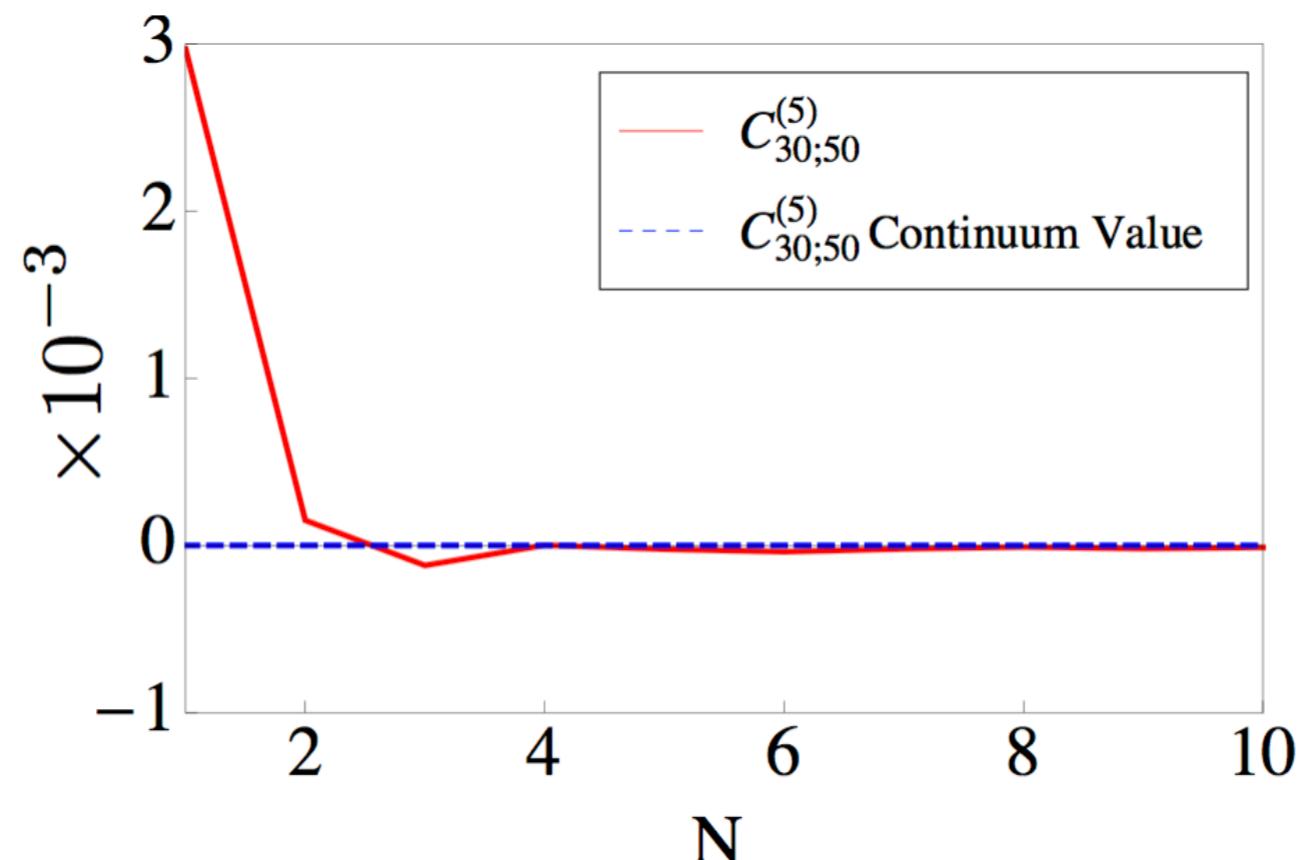
THE COEFFICIENT OF DESIRED OPERATOR:



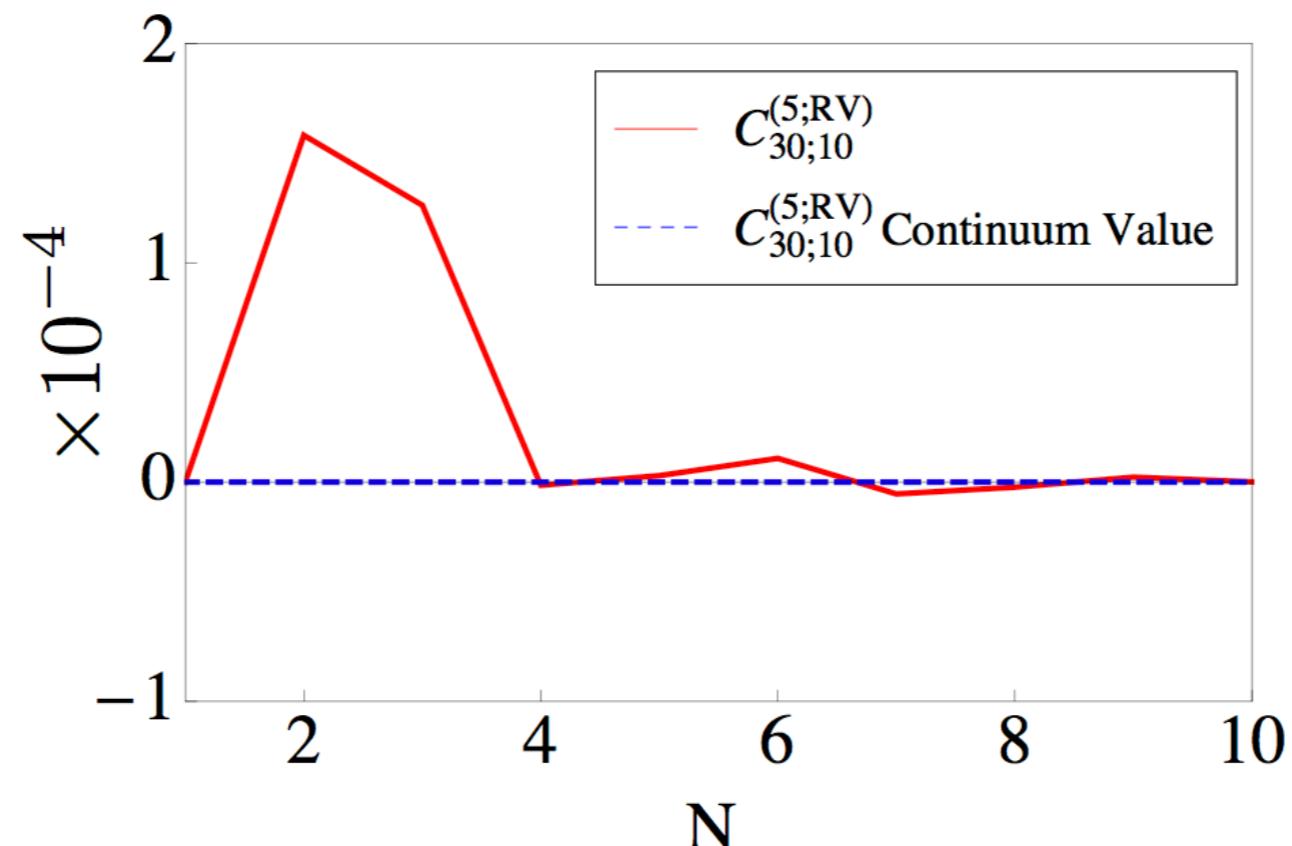
THE COEFFICIENT OF LOWER-DIMENSIONAL OPERATOR:



THE COEFFICIENT OF HIGHER-DIMENSIONAL OPERATOR:



THE COEFFICIENT OF LORENTZ-BREAKING OPERATOR:



RECALLING THE EXPANSION OF OUR CHOSEN OPERATOR...

$$\begin{aligned}
\hat{\theta}_{3,0}(\mathbf{x}; \mathbf{a}, N) = & \frac{C_{30;10}^{(1)}(N)}{\Lambda} \mathcal{O}_z^{(1)}(\mathbf{x}; \mathbf{a}) + \frac{C_{30;10}^{(3)}(N)}{\Lambda^3} \mathcal{O}_z^{(3)}(\mathbf{x}; \mathbf{a}) + \frac{C_{30;10}^{(5)}(N)}{\Lambda^5} \mathcal{O}_z^{(5)}(\mathbf{x}; \mathbf{a}) + \\
& \frac{C_{30;10}^{(5;RV)}(N)}{\Lambda^5} \mathcal{O}_z^{(5;RV)}(\mathbf{x}; \mathbf{a}) + \frac{C_{30;30}^{(3)}(N)}{\Lambda^3} \mathcal{O}_{zzz}^{(3)}(\mathbf{x}; \mathbf{a}) + \frac{C_{30;30}^{(5)}(N)}{\Lambda^5} \mathcal{O}_{zzz}^{(5)}(\mathbf{x}; \mathbf{a}) + \\
& \frac{C_{30;50}^{(5)}(N)}{\Lambda^5} \mathcal{O}_{zzzzz}^{(5)}(\mathbf{x}; \mathbf{a}) + \mathcal{O}\left(\frac{\nabla_z^7}{\Lambda^7}\right)
\end{aligned}$$

...WE HAVE NOW FOUND THAT THE OPERATOR IS DOMINATED BY THE DESIRED TERM:

$$\begin{aligned}
\Lambda^3 \hat{\theta}_{3,0}(\mathbf{x}; \textcolor{brown}{a}, N) = & \alpha_1 \frac{\Lambda^2}{N^2} \mathcal{O}_z^{(1)}(\mathbf{x}) + \alpha_2 \frac{1}{N^2} \mathcal{O}_z^{(3)}(\mathbf{x}) + \alpha_3 \frac{1}{\Lambda^2 N^2} \mathcal{O}_z^{(5)}(\mathbf{x}) + \\
& \alpha_4 \frac{1}{\Lambda^2 N^2} \mathcal{O}_z^{(5;RV)}(\mathbf{x}) + \alpha_5 \mathcal{O}_{zzz}^{(3)}(\mathbf{x}) + \alpha_6 \frac{1}{\Lambda^2} \mathcal{O}_{zzz}^{(5)}(\mathbf{x}) + \\
& \alpha_7 \frac{1}{\Lambda^2 N^2} \mathcal{O}_{zzzzz}^{(5)}(\mathbf{x}) + \mathcal{O}\left(\frac{\nabla_z^7}{\Lambda^4}\right)
\end{aligned}$$



POWER DIVERGENCE OF THE NAIVE OPERATOR EVIDENT:

$$\begin{aligned}
& \alpha_1 \frac{1}{a^2} \mathcal{O}_z^{(1)} + \alpha_2 \mathcal{O}_z^{(3)} + \alpha_3 \textcolor{brown}{a}^2 \mathcal{O}_z^{(5)} + \alpha_4 \textcolor{brown}{a}^2 \mathcal{O}_z^{(5;RV)} + \\
& \alpha_5 \mathcal{O}_{zzz}^{(3)} + \alpha_6 \textcolor{brown}{a}^2 \mathcal{O}_{zzz}^{(5)} + \alpha_7 \textcolor{brown}{a}^2 \mathcal{O}_{zzzzz}^{(5)} + \mathcal{O}(\textcolor{brown}{a}^4 \nabla_z^7)
\end{aligned}$$

$N = 1$

SEE THE PAPER FOR CAREFUL TREATMENT OF THESE FEATURES IN LATTICE PERTURBATION THEORY. THE CONCLUSION IS THAT:

SCALING OF ROTATIONAL-INVARIANT CONTRIBUTIONS AT 1-LOOP LPT WITH WILSON FERMIONS:

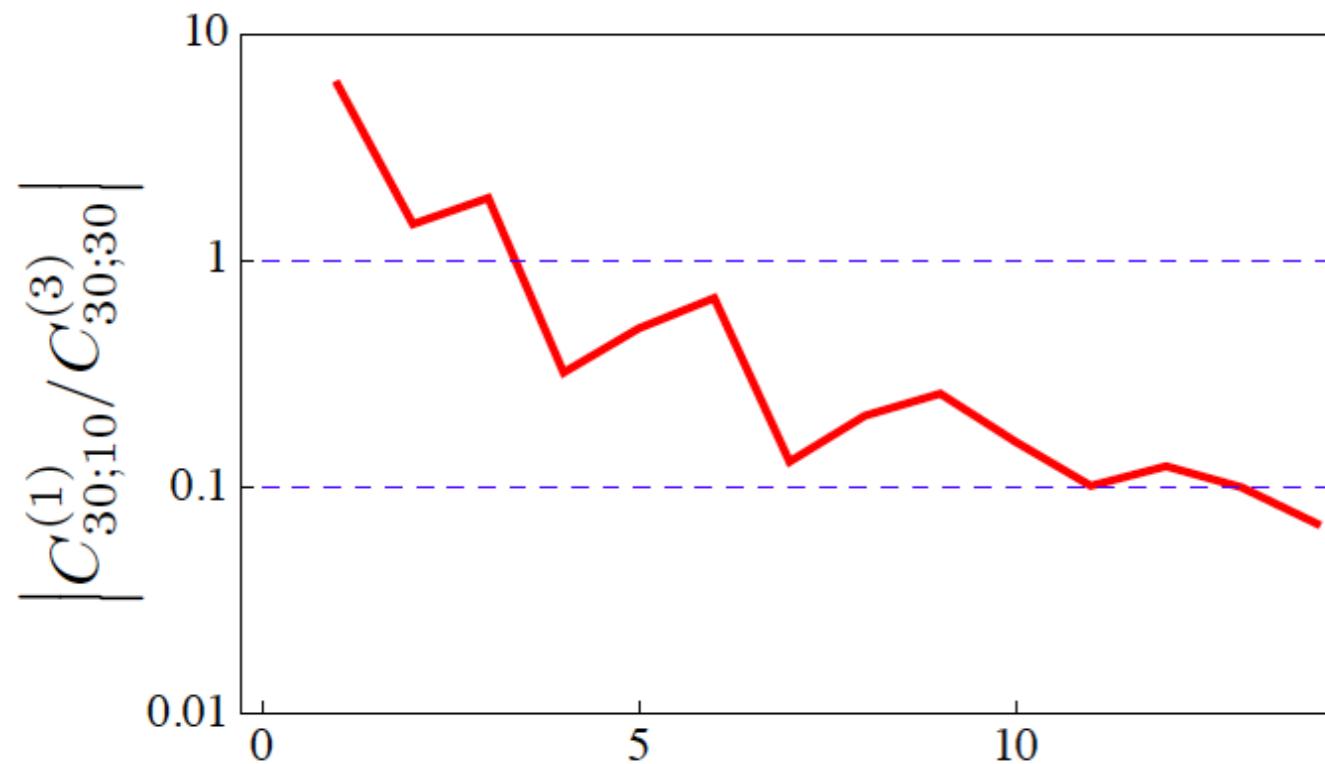
$$\sim \alpha_s / N$$

SCALING OF NON-ROTATIONAL-INVARIANT CONTRIBUTIONS AT 1-LOOP LPT WITH WILSON FERMIONS:

$$\sim \alpha_s a^2 \Lambda_g^2 \sim \frac{\alpha_s}{N_g^2}$$

DOES THIS WORK NON-PERTURBATIVELY?

EVEN A SMALL SHELL LARGELY ELIMINATES THE
CONTAMINATION:

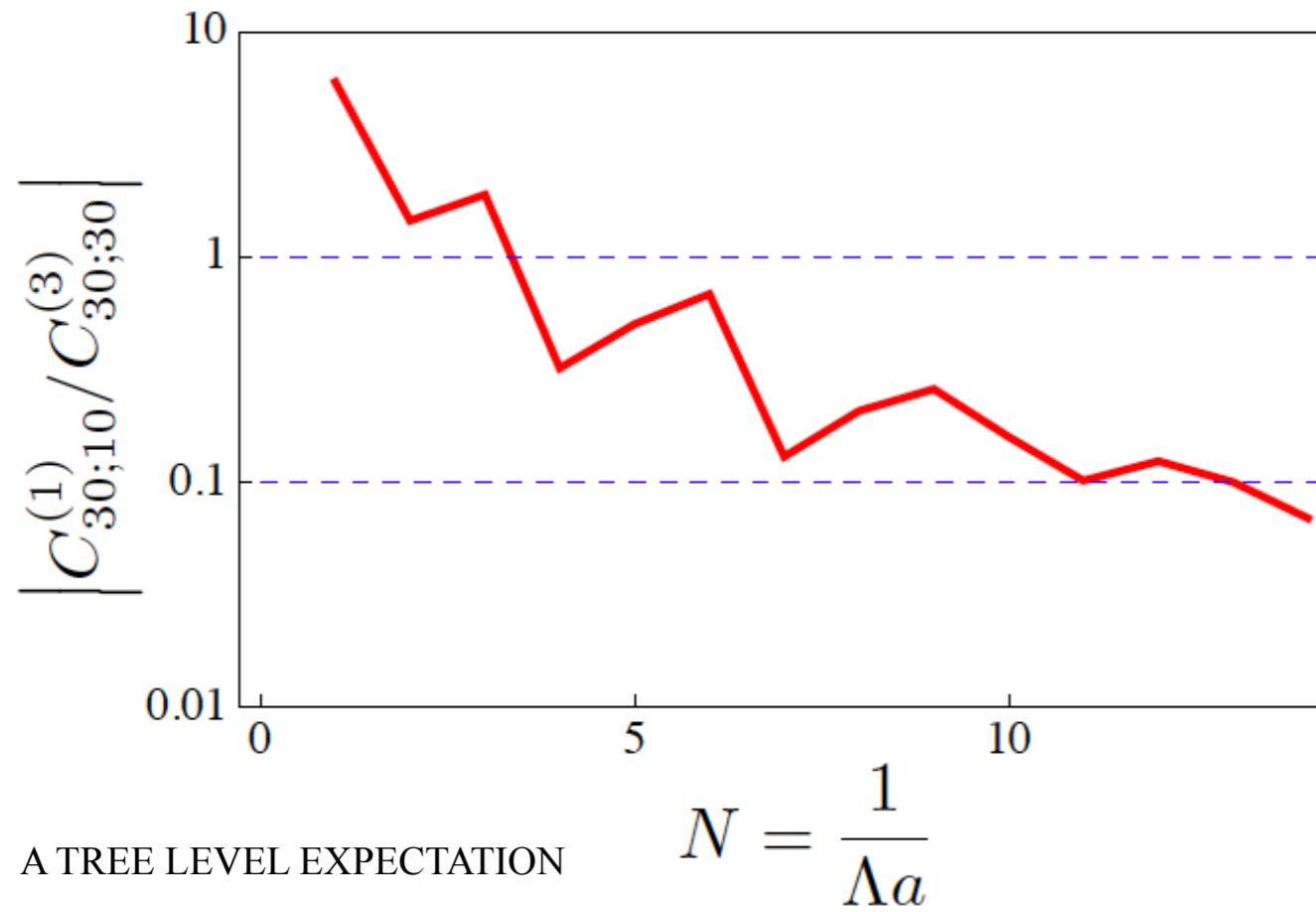


A TREE LEVEL EXPECTATION

$$N = \frac{1}{\Lambda a}$$

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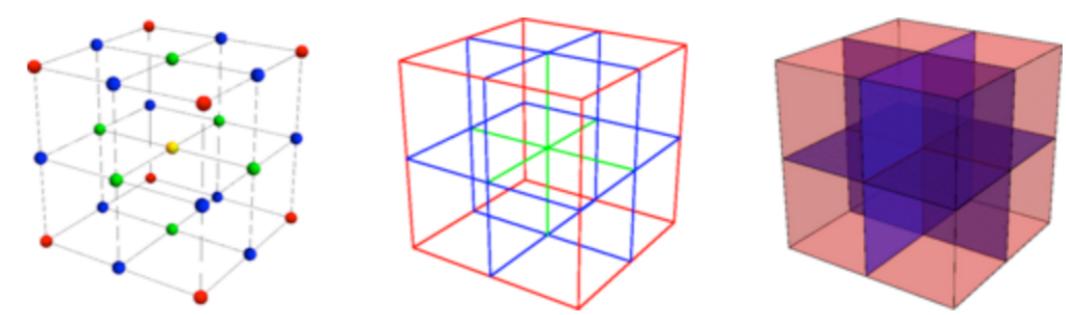


ZD and Savage, PRD 86, 054505 (2012).

Endres, Brower, Detmold, Orginos, and Pochinsky, Phys. Rev. D 92, 114516,
Endres and Detmold, Phys. Rev. D 94, 114502 (2016), and arXiv:1801.06132 [hep-lat].

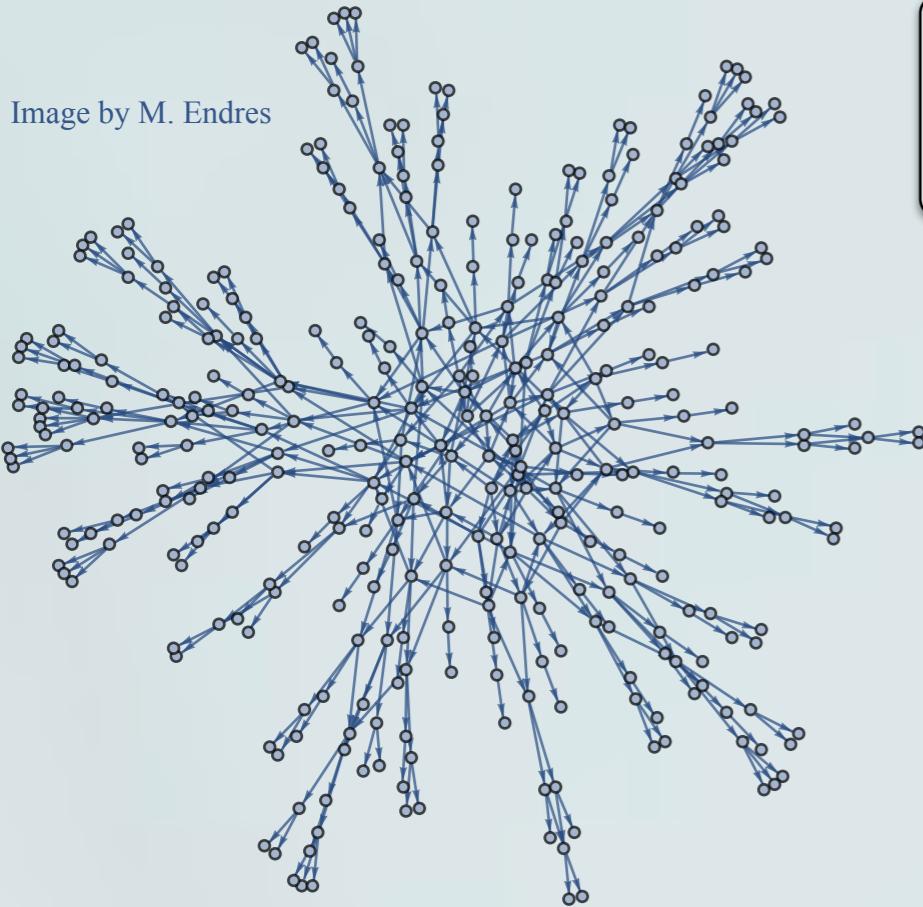
IN PRACTICE, HOW LARGE CAN THE OPERATOR BE?

a	μ	N^2
0.08 fm	— ~ 2 GeV	— 1
0.06 fm	— ~ 2 GeV	— 2
0.05 fm	— ~ 2 GeV	— 4
0.04 fm	~ 5 GeV ~ 2 GeV	1 6
0.03 fm	~ 5 GeV ~ 2 GeV	2 11

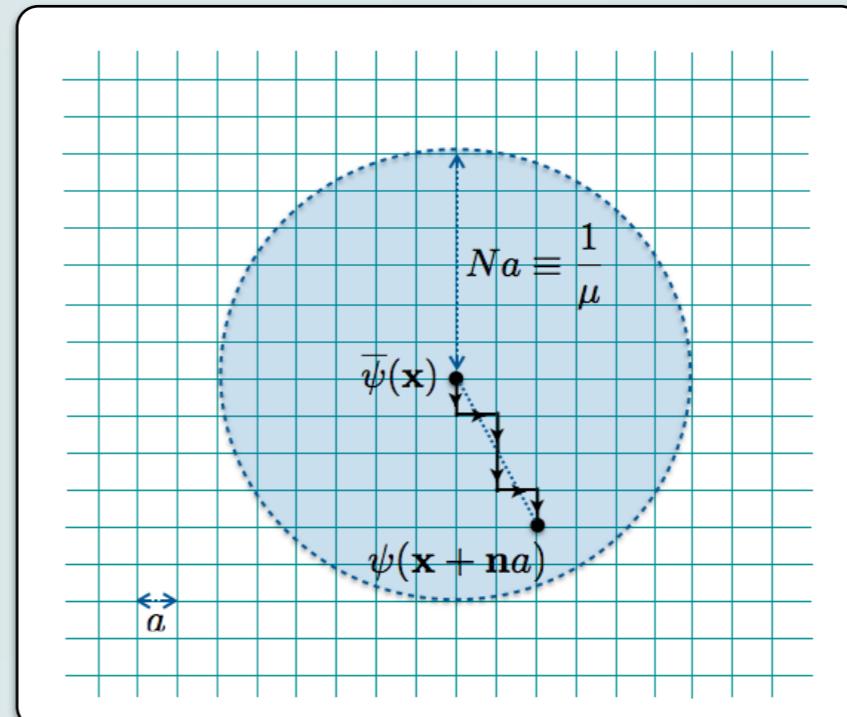


GENERATING GAUGE CONFIGURATIONS WITH A MULTI-SCALE ALGORITHM METHOD

Image by M. Endres



$$\hat{\theta}_{n,l,m}(x_\mu; \mu) = \frac{2}{\pi^2 N^4} \sum_{n_\mu}^{|n_\mu| \leq N} \bar{\psi}(x_\mu) U_{x_\mu, x_\mu + n_\mu a}^{(\mathcal{C})} \psi(x_\mu + n_\mu a) \mathcal{Y}_{n,l,m}(\hat{n}_\mu)$$

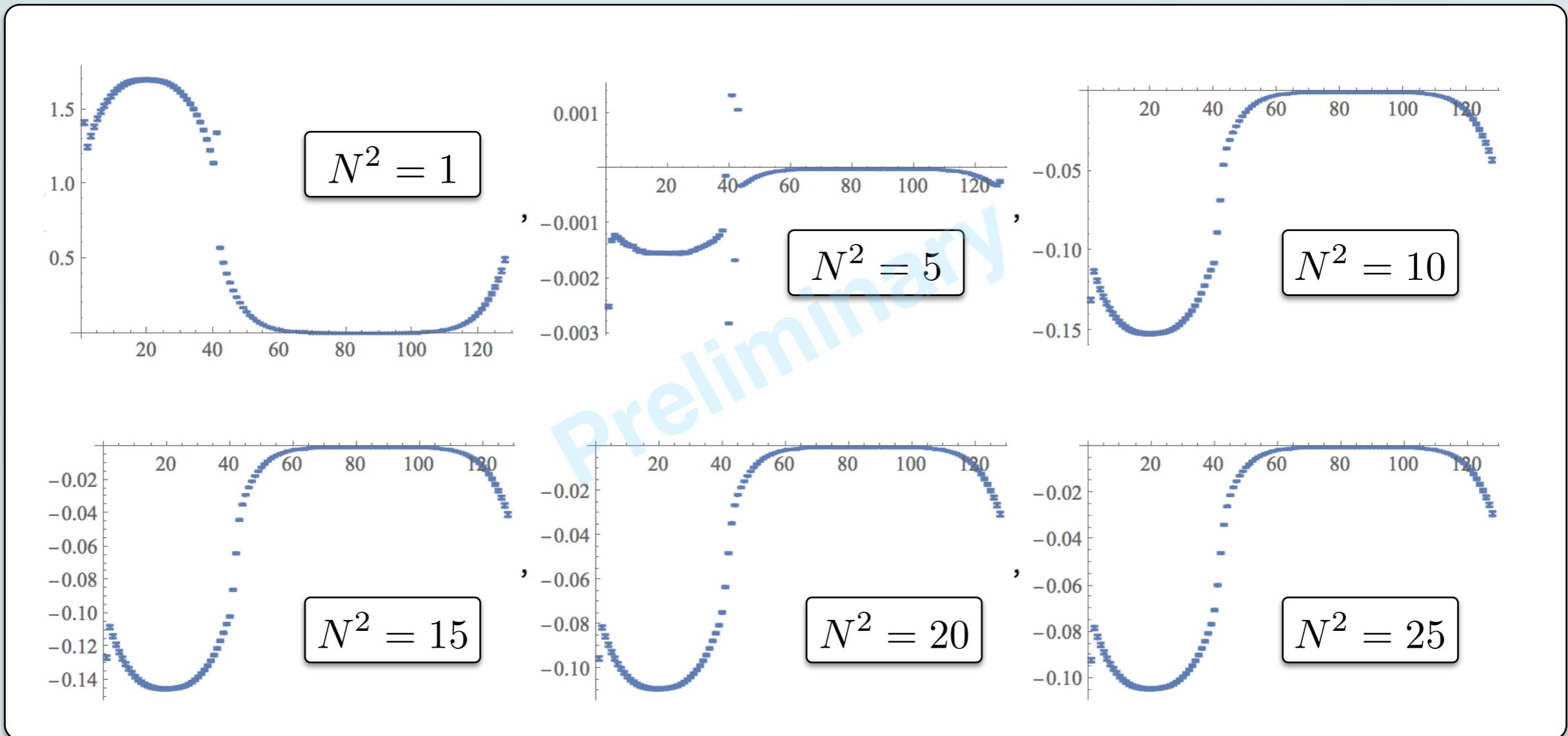


$$C_{3p}(t_f, t) = \sum_{\mathbf{x}_f, \mathbf{x}} \sum_{\mathbf{n}, n_t}^{|n_\mu| \leq N} (0, 0) \times \mathcal{Y}_{n,l,m}(\hat{n}_\mu)$$

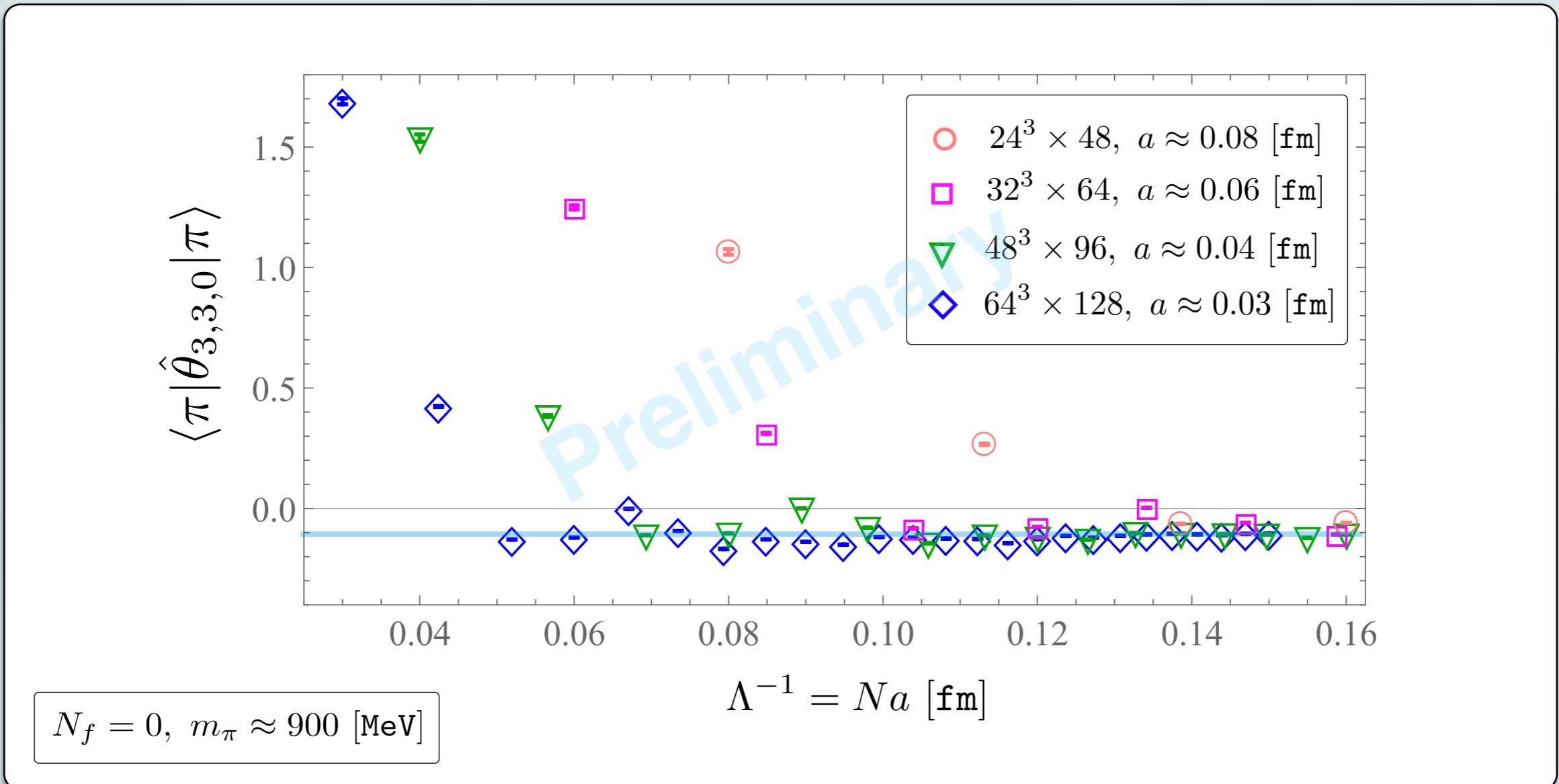
$(\mathbf{x} + \mathbf{n}a, t + n_t a)$

$$C_{3p}(t_f, t) = \sum_{\mathbf{x}_f, \mathbf{x}} \langle 0 | \chi_\pi(\mathbf{x}_f, t_f) \hat{\theta}_{n,l,m}(\mathbf{x}, t; \mu) \chi_\pi^\dagger(0, 0) | 0 \rangle$$

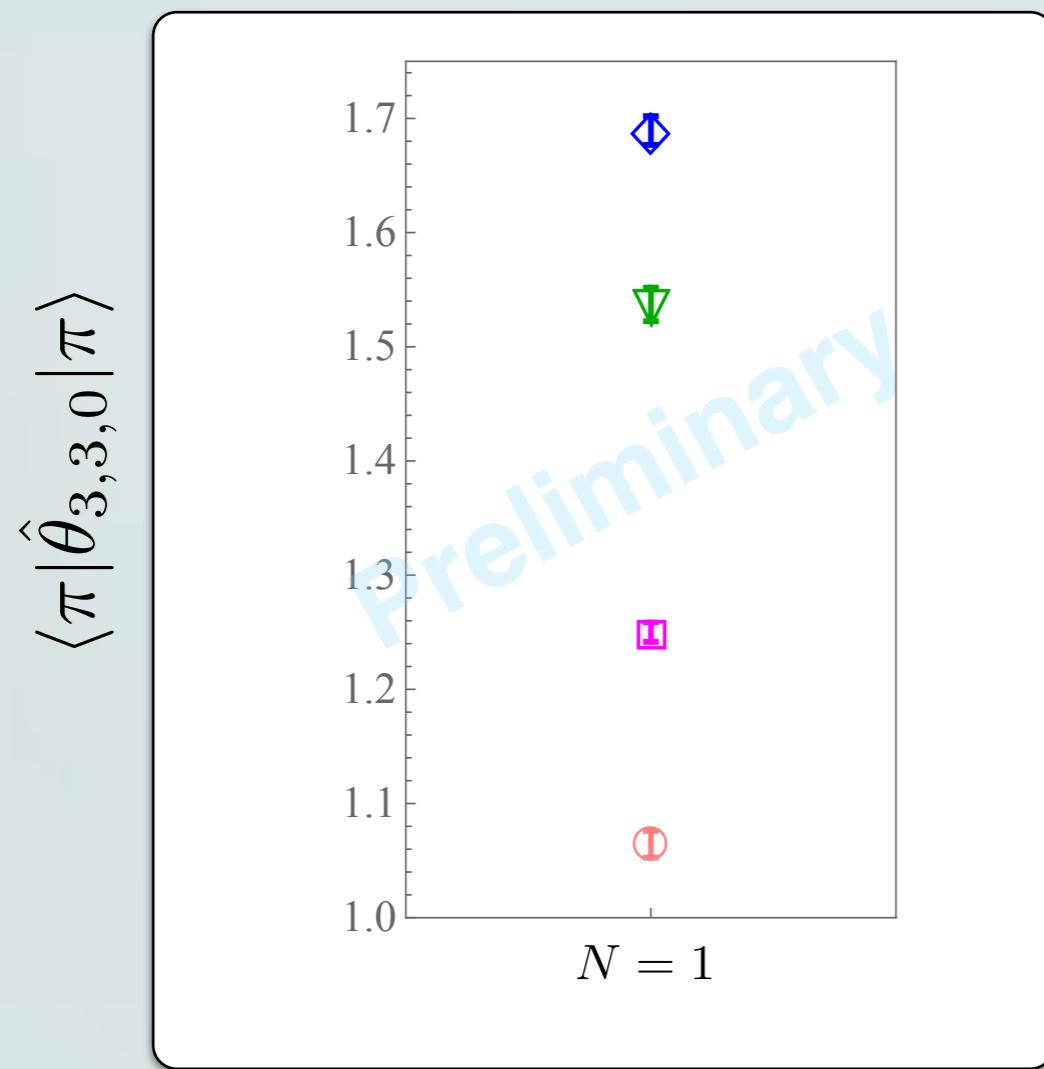
$C_{3pt}(t_f; t)/C_{2pt}(t)$ AS A FUNCTION OF t AT A FIXED t_f



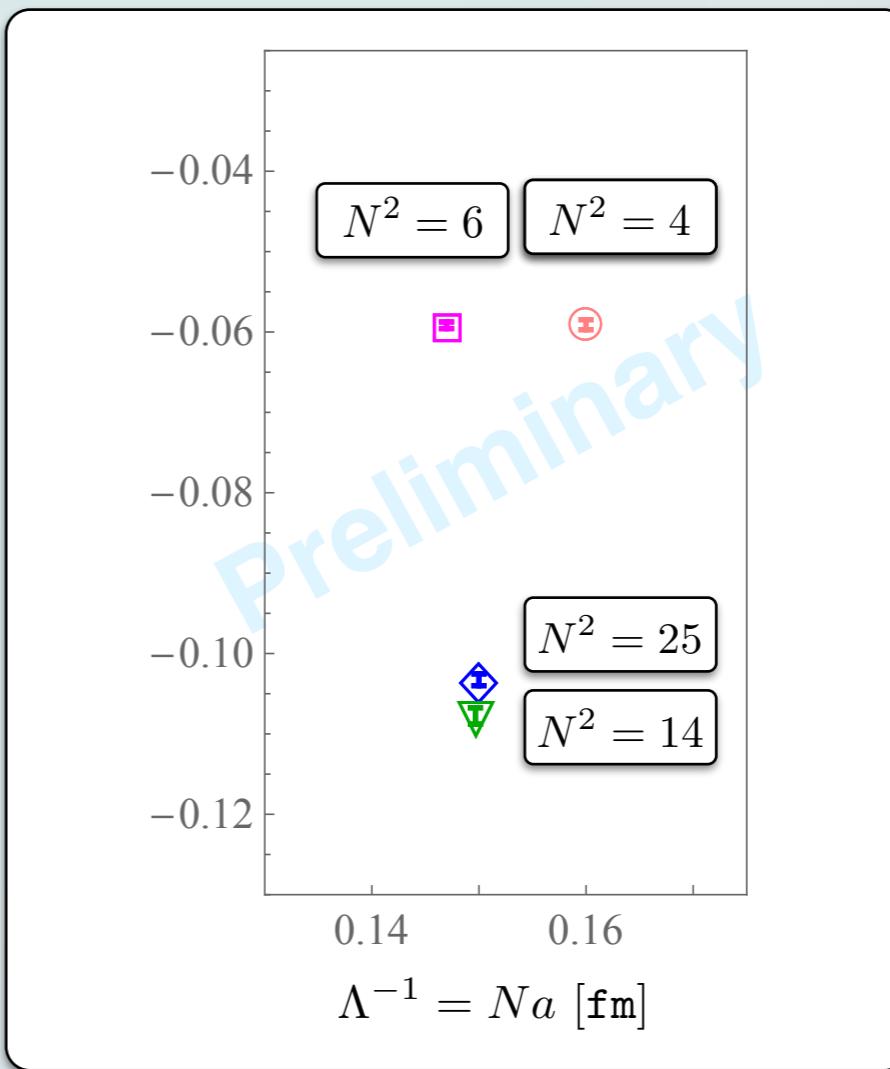
THE MATRIX ELEMENT OF A HIGH “ANGULAR MOMENTUM” QUARK BILINEAR OPERATOR IN PION AT REST AS A FUNCTION OF THE OPERATOR SIZE:



CONTINUUM LIMIT OF THE NAIVE
OPERATOR



EXTENDED OPERATOR WITH A
FIXED SIZE



$N_f = 0, m_\pi \approx 900$ [MeV]

IN SUMMARY

- THE PROPOSED OPERATOR ON THE LATTICE APPROACHES THE CONTINUUM OPERATOR IN A SMOOTH WAY WITH CORRECTIONS THAT SCALE AT MOST BY a^2 . TADPOLE IMPROVEMENT AND GAUGE-FIELD SMEARING ARE ESSENTIAL FOR RECOVERING ROTATIONAL INVARIANCE IN LATTICE GAUGE THEORIES.
- NO POWER DIVERGENCE! THE SPECTRUM OF EXCITED STATES AND HIGHER MOMENTS OF HADRON DISTRIBUTION FUNCTIONS ARE CALCULABLE FROM LATTICE QCD.

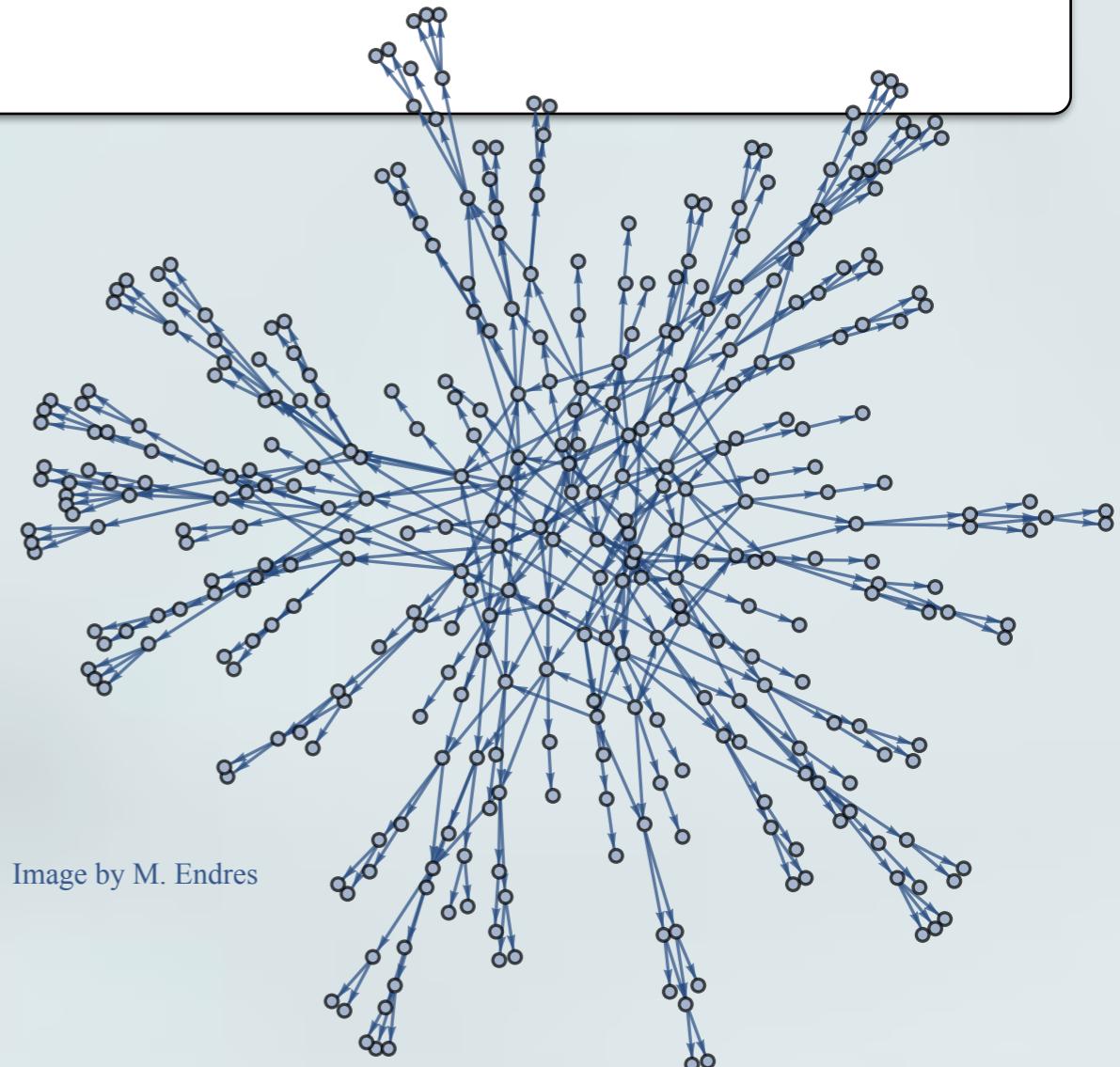
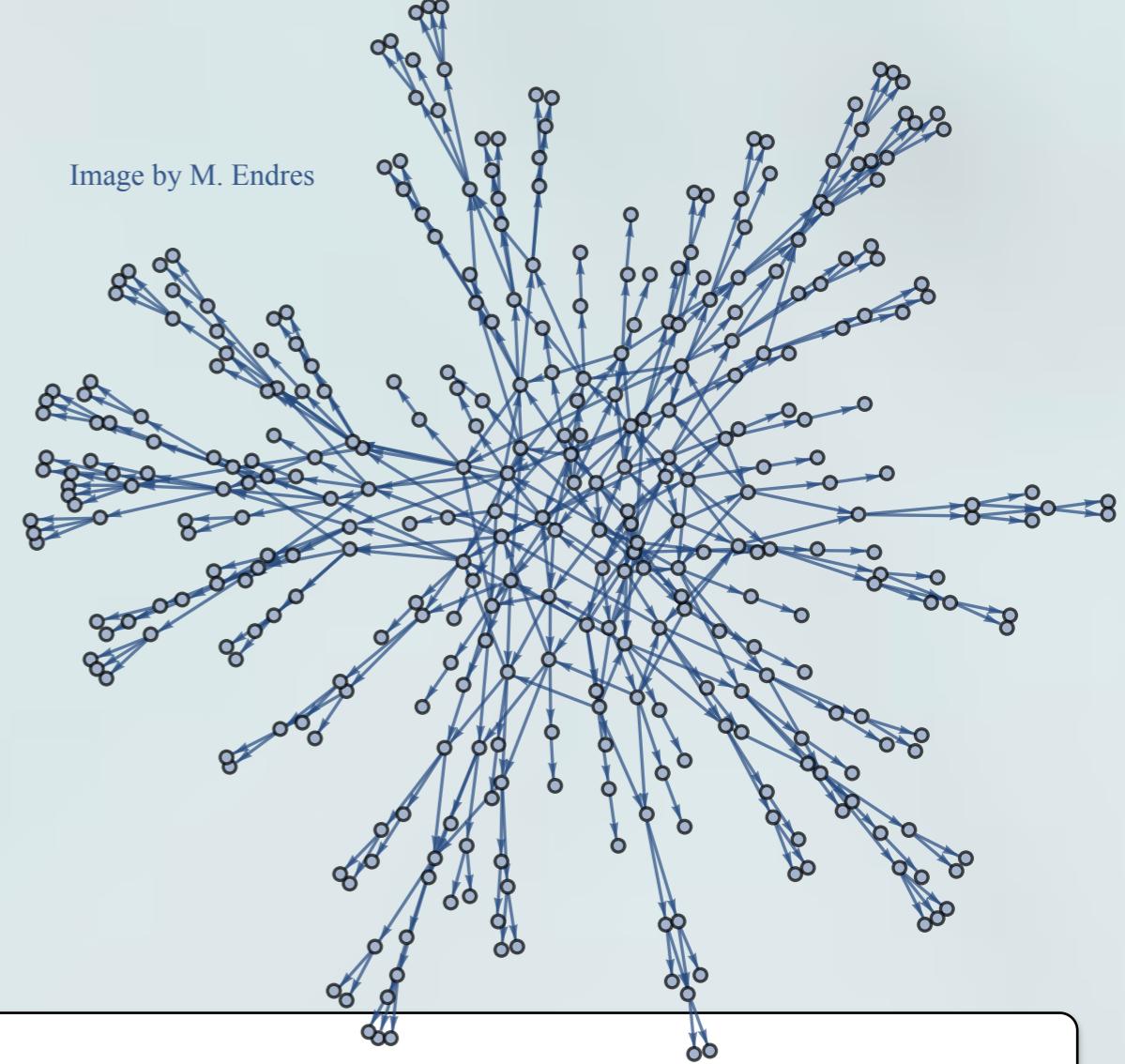


Image by M. Endres

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OUTLOOK

- CAN THE OPERATOR BE FURTHER IMPROVED TOWARDS THE CONTINUUM LIMIT?
- RENORMALIZATION OF THE OPERATOR AND MATCHING.
- ARE OTHER SMEARING PROFILES POTENTIALLY MORE USEFUL?
- COMPARISON WITH OTHER METHODS AND PROPOSALS, e.g., DETMOLD AND LIN, JI, MONAHAN AND ORGINOS.

SUPPLEMENTARY SLIDES

AN EXAMPLE OF OPERATOR BASIS: L=1, m=0

$$\mathcal{O}_z^{(1)}(\mathbf{x}) = \phi(\mathbf{x}) \nabla_z \phi(\mathbf{x})$$

$$\mathcal{O}_z^{(3)}(\mathbf{x}) = \phi(\mathbf{x}) \nabla^2 \nabla_z \phi(\mathbf{x})$$

$$\mathcal{O}_z^{(5)}(\mathbf{x}) = \phi(\mathbf{x}) (\nabla^2)^2 \nabla_z \phi(\mathbf{x})$$

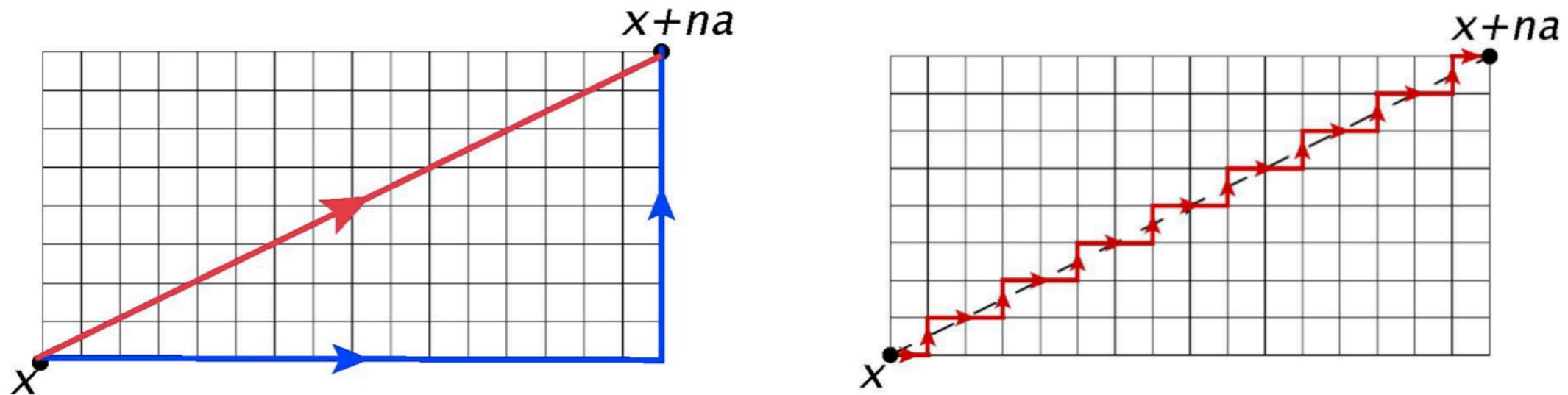
$$\mathcal{O}_z^{(5, RV)}(\mathbf{x}) = \phi(\mathbf{x}) \sum_j \nabla_j^4 \nabla_z \phi(\mathbf{x})$$

LORENTZ-VIOLATING OPERATOR

HOW ABOUT QCD AND BEYOND CLASSICAL EFFECTS?

$$\hat{\theta}_{L,M}(\mathbf{x}; \mathbf{a}, N) = \frac{3}{4\pi N^3} \sum_{\mathbf{n}}^{|\mathbf{n}| \leq N} \bar{\psi}(\mathbf{x}) U(\mathbf{x}, \mathbf{x} + \mathbf{n}\mathbf{a}) \psi(\mathbf{x} + \mathbf{n}\mathbf{a}) Y_{L,M}(\hat{\mathbf{n}})$$

FEATURE 1: SOME EXTENDED LINKS MAXIMALLY BREAK ROTATIONAL SYMMETRY



FEATURE 2: NONVANISHING TADPOLES WITH LATTICE REGULARIZATION

