Test of factorization for the long-distance effects from charmonium in $B \to K l^+ l^-$

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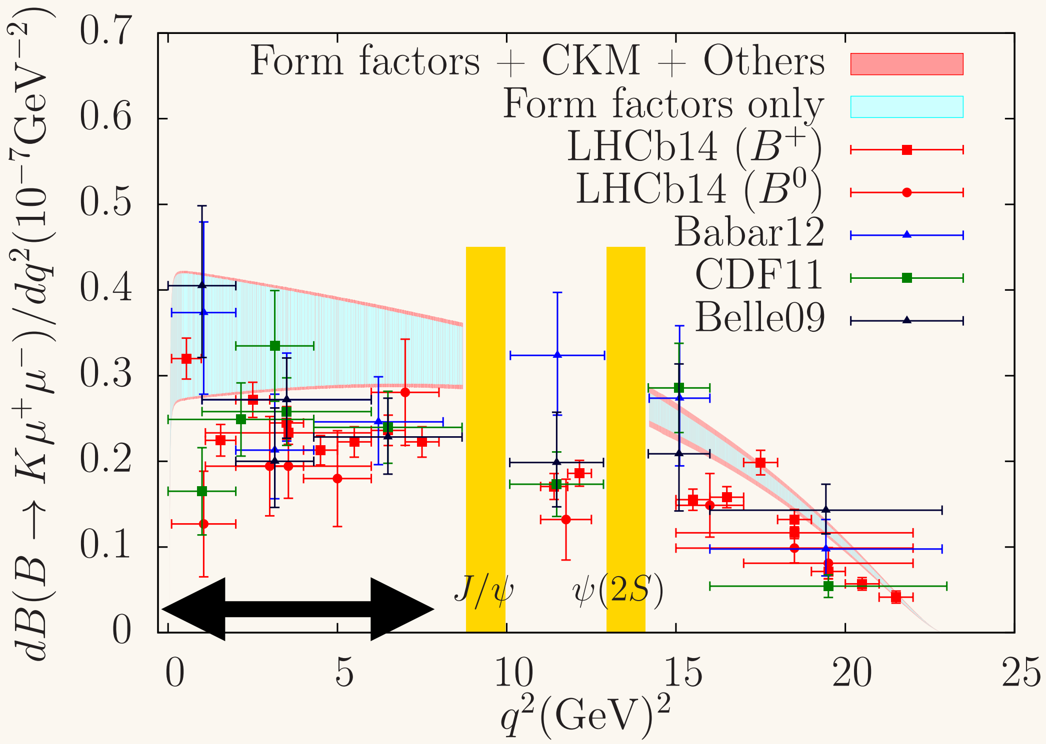
Photo: Granada,, Spain, 2017

Motivation

(1): $B \to K l^+ l^-$ is a penguin-induced FCNC process.
(GIM and loop suppressed)



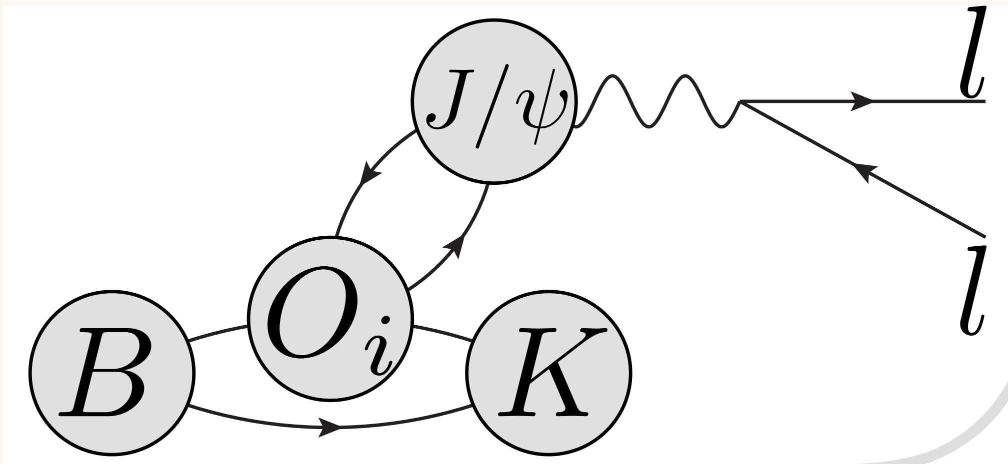
[D. Du et al. (Fermilab, MILC) 1510.02349]

(2): Anomaly in experiments:

$$q^2 < m_{J/\psi}^2$$

Question: Are the long distance contributions understood?

→We calculate the corresponding amplitude by lattice formulation.

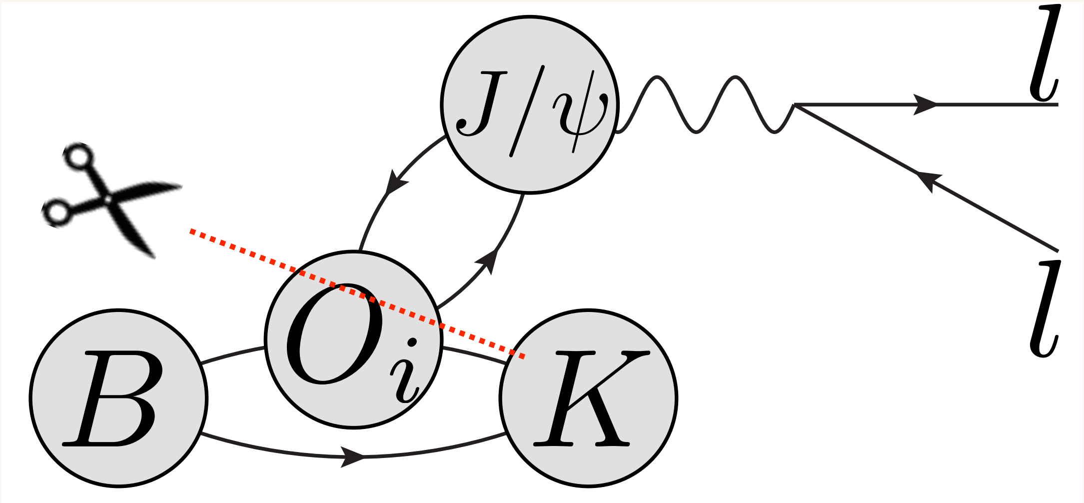


Charmonium resonance contributions

◇ We focus on the charmonium resonance contribution,

$$H_{\mathrm{eff}} = \frac{G_F}{\sqrt{2}} \left(\sum_{i=1}^{2} (V_{us}^* V_{ub} C_i O_i^u + V_{cs}^* V_{cb} C_i O_i^c) - V_{ts}^* V_{tb} \sum_{i=3}^{10} C_i O_i \right)$$

$$O_1^c = (\bar{s}_i \gamma_\mu P_- c_j)(\bar{c}_j \gamma_\mu P_- b_i)$$
$$O_2^c = (\bar{s}_i \gamma_\mu P_- c_i)(\bar{c}_j \gamma_\mu P_- b_j)$$



→ O_1^c and O_2^c induce the long-distance contribution, which is often estimated by the factorization approximation. How good is that?

$K \to \pi$ decay amplitudes

◇ We'd like to calculate the B decay amplitudes on the lattice.
Formulation is analogous to the $K \to \pi ll$ decay amplitudes.

[N.H. Christ et al. (RBC, UKQCD) 1507.03094]

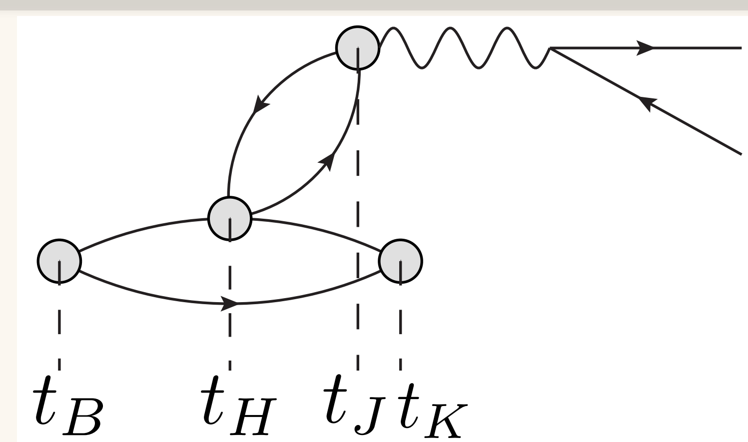
$$\mathcal{A}_\mu\left(q^2\right) = \int \mathrm{d}^4x \left\langle \pi(\mathbf{p}) \left| \mathrm{T}\left[J_\mu(0) H_{\mathrm{eff}}(x)\right] \right| K(\mathbf{k}) \right\rangle$$

$$\downarrow$$

$$\mathcal{A}_\mu\left(q^2\right) = \int \mathrm{d}^4x \left\langle K(\mathbf{k}) \left| \mathrm{T}\left[J_\mu(0) H_{\mathrm{eff}}(x)\right] \right| B(\mathbf{p}) \right\rangle$$

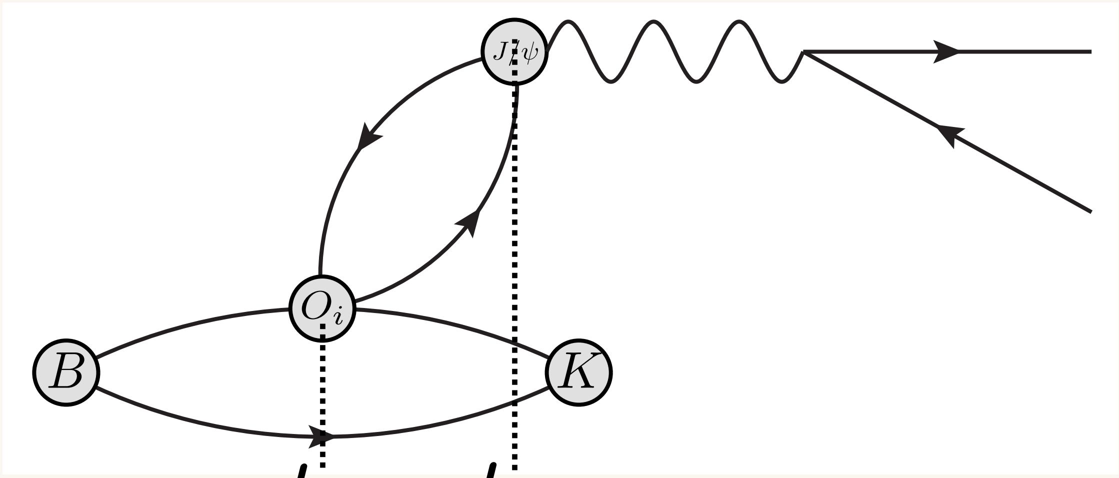
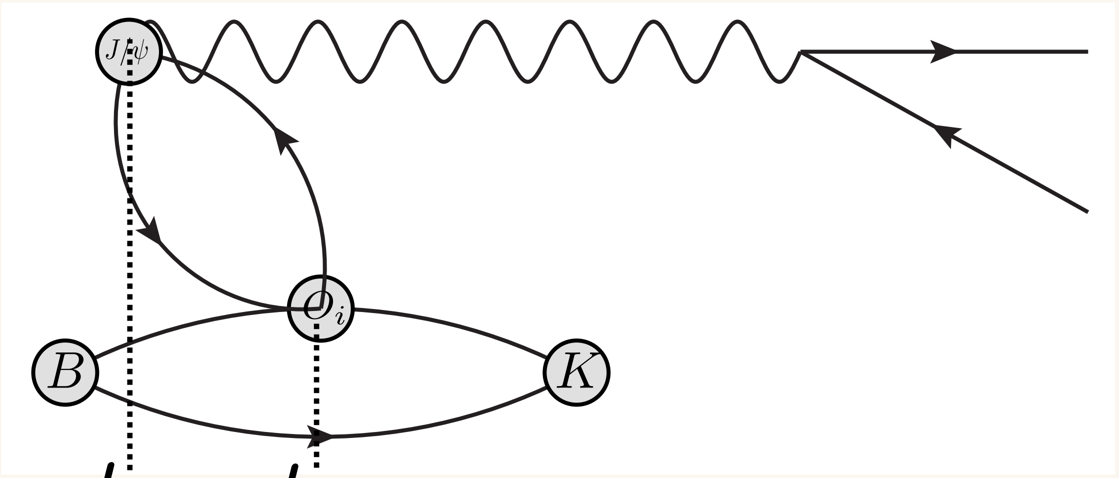
◇ The amplitude is calculable from the integration of 4pt-func.

$$I_\mu\left(T_a, T_b, \mathbf{p}, \mathbf{k}\right) \simeq \int_{t_J - T_a}^{t_J + T_b} \mathrm{d}t_H \left[\quad \right]$$



$\left(0 \ll t_J - T_a \le t_J + T_b \ll t_K\right)$

$B \to K l^+ l^-$ decay amplitudes

$I_\mu =$  $+$ 

$t_H \quad t_J$

$$\int_{t_J - t_a}^{t_J} dt_H$$

$t_J \quad t_H$

$$\int_{t_J}^{t_J + t_b} dt_H$$

◇ Charm loop would produce contributions like

$$\int_0^\infty dt e^{\omega t} e^{-E_{J/\psi} t} = \frac{1}{\omega - E_{J/\psi}} \qquad \int_{-\infty}^0 dt e^{\omega t} e^{E_{J/\psi} t} = \frac{1}{\omega + E_{J/\psi}}$$

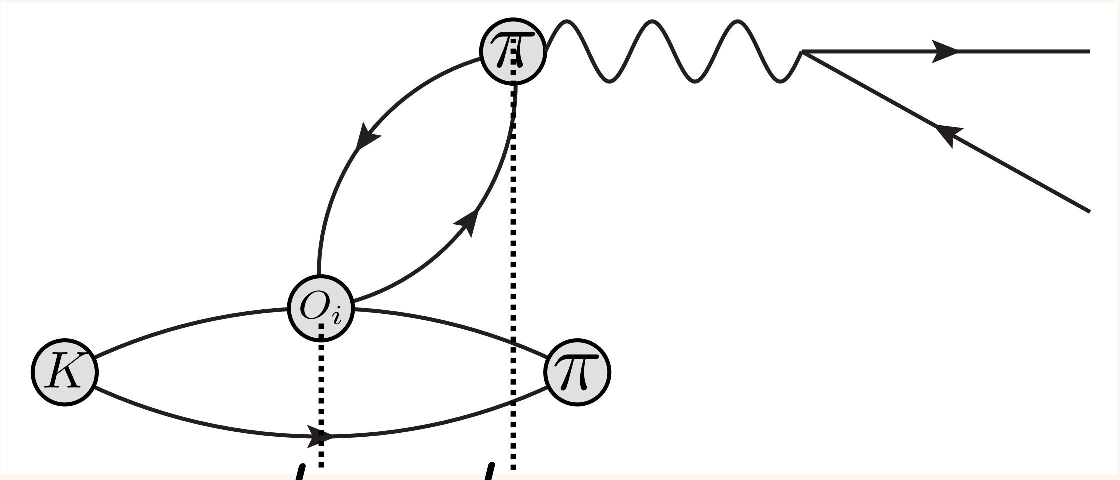
◇ If we focus on $\omega = \sqrt{m_{J/\psi}^2 - \mathbf{q}^2} \sim E_{J/\psi}$, $t_H < t_J$ part is dominant.

Divergence of the amplitude?

◇ $K \to \pi l^+ l^-$ case

[N.H. Christ et al. (RBC, UKQCD) 1507.03094]

$I_\mu =$



$+(t_J < t_H)$

$t_H \quad t_J$

$$I_\mu(T_a, T_b, \mathbf{p}, \mathbf{k}) = -\int_0^\infty \mathrm{d}E \frac{\rho(E)}{2E} \frac{\langle \pi(\mathbf{k}) | J_\mu(0) | E, \mathbf{p}\rangle \langle E, \mathbf{p} | H_{\mathrm{eff}}(0) | K(\mathbf{p})\rangle}{E_K(\mathbf{p}) - E} \left(1 - e^{[E_K(\mathbf{p}) - E]T_a}\right)$$

$+(t_J < t_H)$

◇ Some intermediate states have E lower than E_K

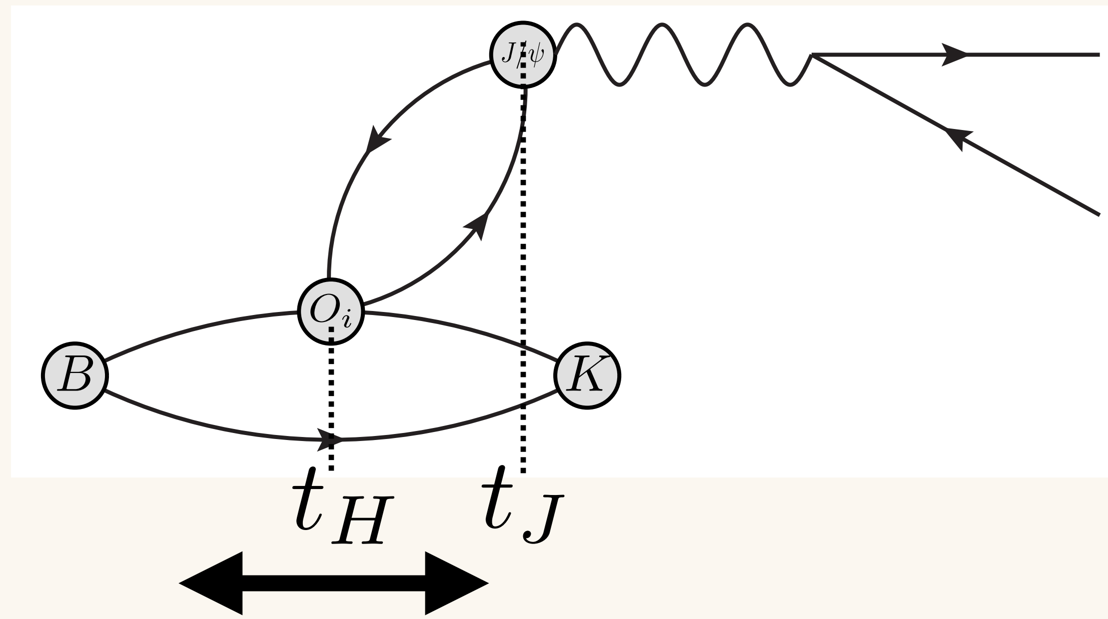
→ Since $T_a \to \infty$, they must be subtracted.

(e.g. $K \to \pi, \pi\pi, \pi\pi\pi$)

Divergence of the amplitude?

◇ $B \to K l^+ l^-$ case

$I_\mu =$



$+(t_J < t_H)$

$$I_\mu\left(T_a, T_b, \mathbf{p}, \mathbf{k}\right) = -\int_0^\infty \mathrm{d}E \frac{\rho_S(E)}{2E} \frac{\langle K(\mathbf{k})\,| J_\mu(0)|\, E, \mathbf{p}\rangle \,\langle E, \mathbf{p}|H_{\text{eff}}(0)|B(\mathbf{p})\rangle}{E_B(\mathbf{p}) - E} \left(1 - e^{[E_B(\mathbf{p})-E]T_a}\right)$$

$+(t_J < t_H)$

◇ We restrict ourselves in the setup of

$$m_B < m_{J/\psi} + m_K$$

→ No divergence.

Amplitude of $B \to K l^+ l^-$.

◇ From the integration of the 4-point correlators, we can extract the amplitude after taking $T_{a,b} \to \infty$ limit.

$$I_\mu\left(T_a, T_b, \mathbf{p}, \mathbf{k}\right) = -\int_0^\infty \mathrm{d}E \frac{\rho_S(E)}{2E} \frac{\langle K(\mathbf{k})\left|J_\mu(0)\right| E, \mathbf{p}\rangle \langle E, \mathbf{p}|H_{\text{eff}}(0)|B(\mathbf{p})\rangle}{E_B(\mathbf{p}) - E} \left(1 - e^{[E_B(\mathbf{p})-E]T_a}\right)$$

$$+(t_J < t_H)$$

$$\mathcal{A}_\mu\left(q^2\right) = \int \mathrm{d}^4x \left\langle K(\mathbf{k})\left|\mathrm{T}\left[J_\mu(0) H_{\text{eff}}(x)\right]\right| B(\mathbf{p})\right\rangle$$

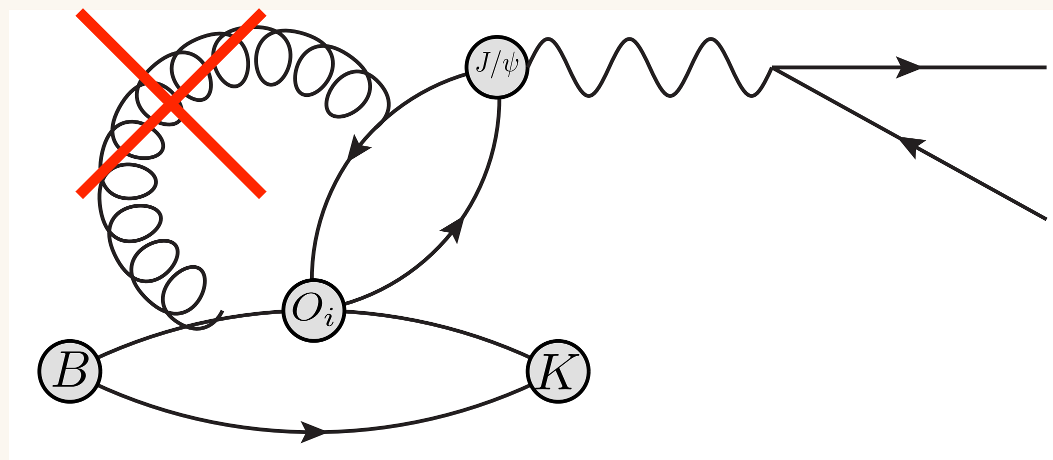
$$\Downarrow$$

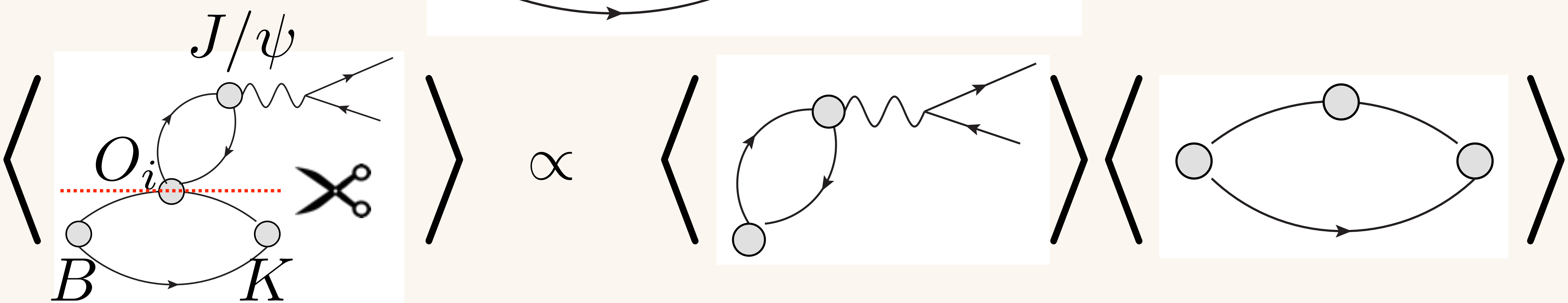
$$\boxed{A_\mu(q^2) = -i \lim_{T_{a,b}\to\infty} I_\mu(T_a, T_b, \mathbf{k}, \mathbf{p})}$$

Factorization method for $B \to K l^+ l^-$ decay

◉ Factorization

◇ Assume that long-range gluon exchange can be ignored





$$\langle P_K | J_\nu^{\bar{c}c} (\bar{c}_i \gamma_\mu P_- c_i)(\bar{s}_j \gamma_\mu P_- b_j) | P_B \rangle = \frac{1}{(\mathrm{Vol.})} \langle 0 | J_\nu^{\bar{c}c} J_\mu^{\bar{c}c} | 0 \rangle \langle P_K | V_\mu | P_B \rangle$$

→ We test this assumption by the lattice calculation.

Factorization

◇ Factorizable operator O_F and non-factorizable operator O_{NF}

Fierz trans.

$$O_1^c = (\bar{s}_i\gamma_\mu P_- c_j)(\bar{c}_j\gamma_\mu P_- b_i)$$
$$O_2^c = (\bar{s}_i\gamma_\mu P_- c_i)(\bar{c}_j\gamma_\mu P_- b_j)$$

$\longrightarrow$

$$O_F^{(1)} = (\bar{c}_i\gamma_\mu P_- c_i)(\bar{s}_j\gamma_\mu P_- b_j)$$
$$O_{NF}^{(8)} = \left(\bar{c}_i\,[T^a]_{ij}\,\gamma_\mu P_- c_j\right)(\bar{s}_k\,[T^a]_{kl}\,\gamma_\mu P_- b_l)$$

$$O_1^c = O_F^{(1)} \qquad O_2^c = \frac{1}{3}O_F^{(1)} + 2O_{NF}^{(8)}$$

◇ Assume non-factrizable operator $O_{NF}^{(8)}$ could be ignored

→ We test this assumption $O_2^c = \frac{1}{3}O_1^c$.

$K \to \pi\pi$ case, Lattice. $O_2^l \simeq -0.7 O_1^l$

[P.A. Boyle et al. (RBC, UKQCD) 1212.1474]

Preliminary result for the test of factorization

Current status

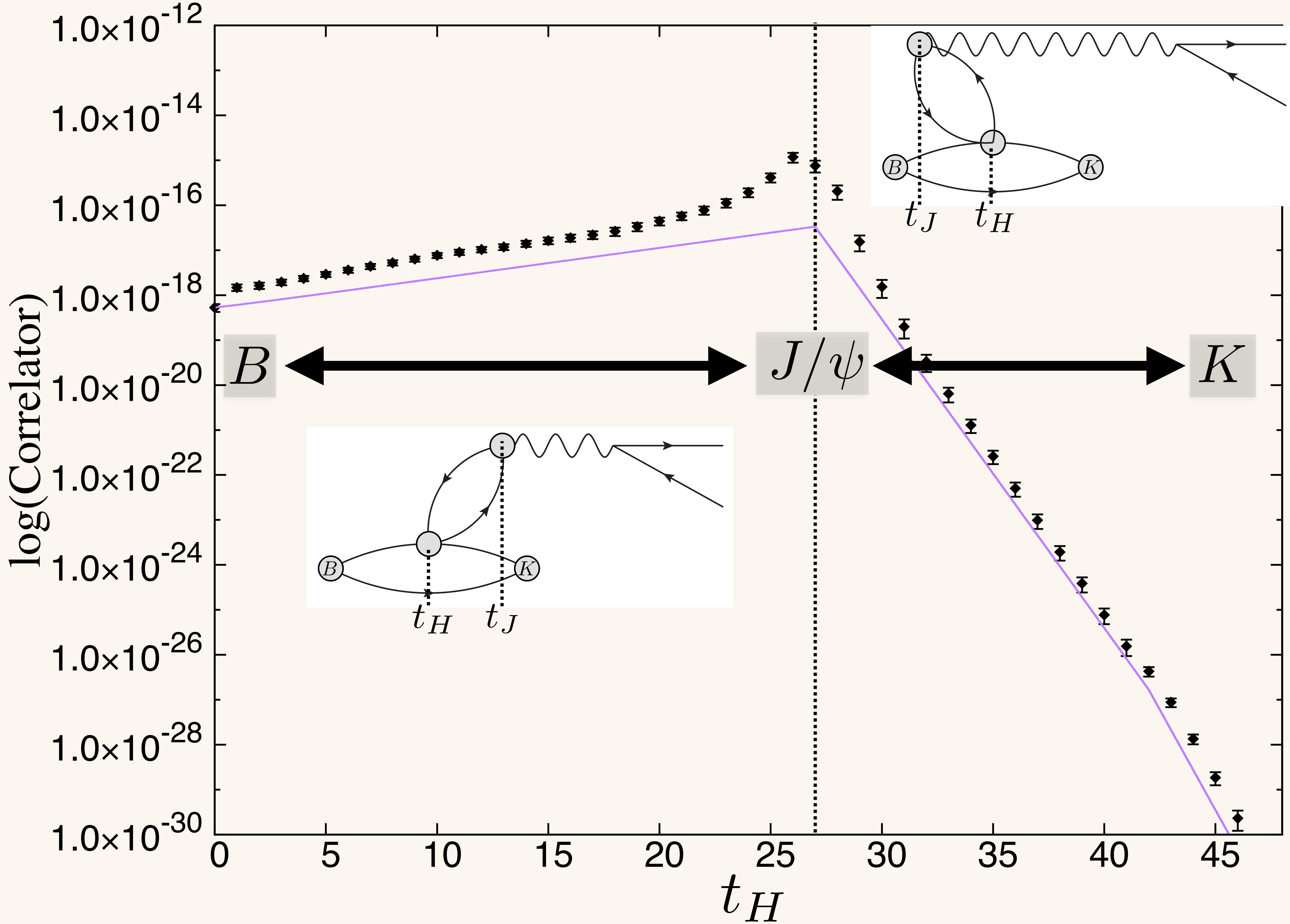
β	a^{-1}	$L^3 \times T(\times L_s)$	meas	am_{uds}	am_c	am_b	am_π	aE_K	$am_{J/\psi}$	am_B
4.17	2.453(4)	$32^3 \times 64(\times 12)$	100	0.04	0.44037	0.68808	0.2904(5)	0.3513(16)	1.2809(6)	1.063(11)
4.35	3.610(9)	$48^3 \times 96(\times 8)$	90	0.025	0.27287	0.66619	0.1986(3)	0.2378(9)	0.8701(3)	0.9543(19)
	GeV						$\simeq$ 714 MeV	$\simeq$ 855 MeV	$\simeq$ 3.14 GeV	$\simeq$ 3.44 GeV

Mobius domain-wall fermion with 2+1 flavor.

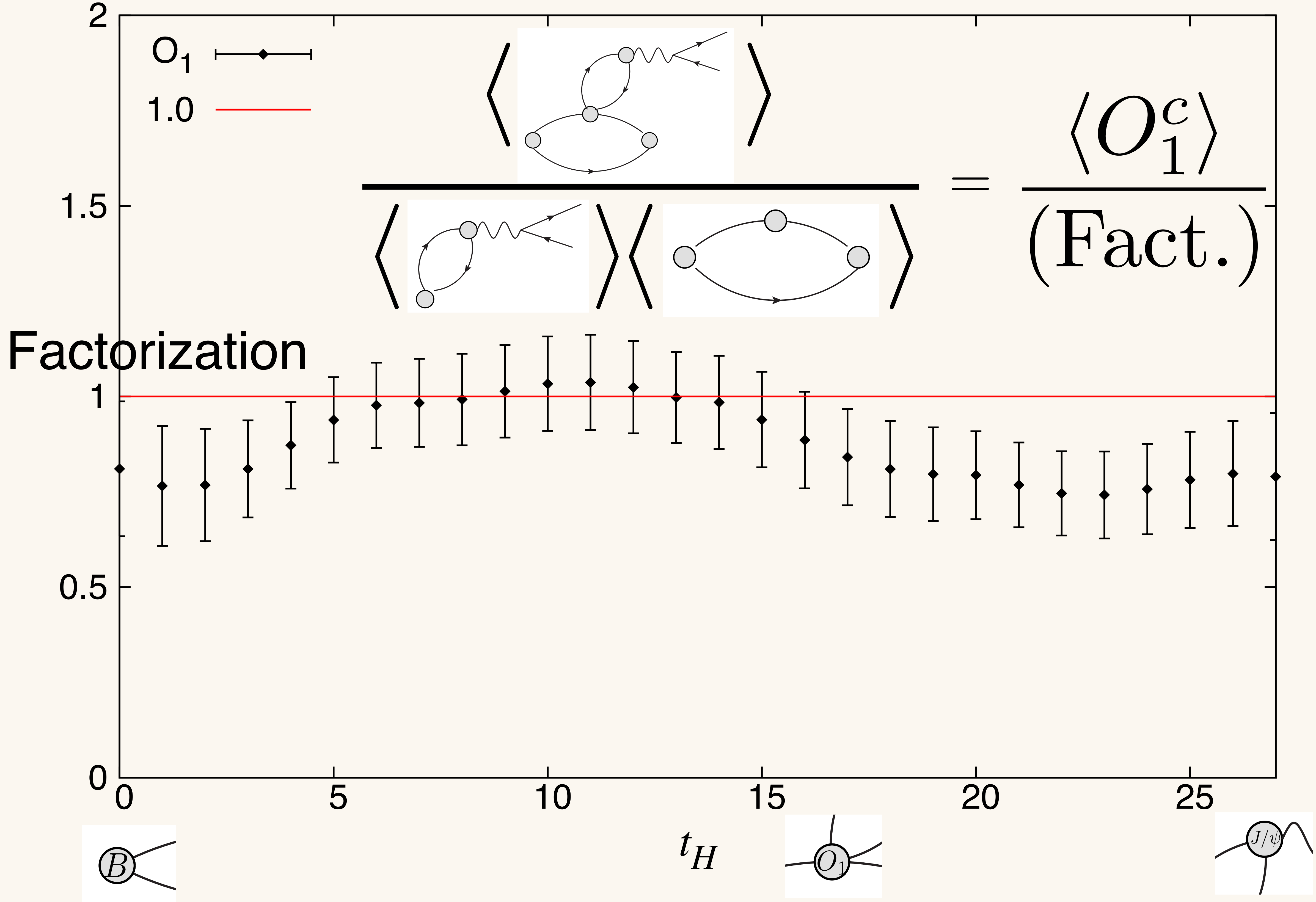
- up and down mass same as strange.
- "bottom" mass: $m_b = \{(1.25^2)m_c, (1.25^4)m_c\}$
- Finite momentum in the final state $\mathbf{k} = \frac{2\pi}{L}(1,0,0)$

4-point functions

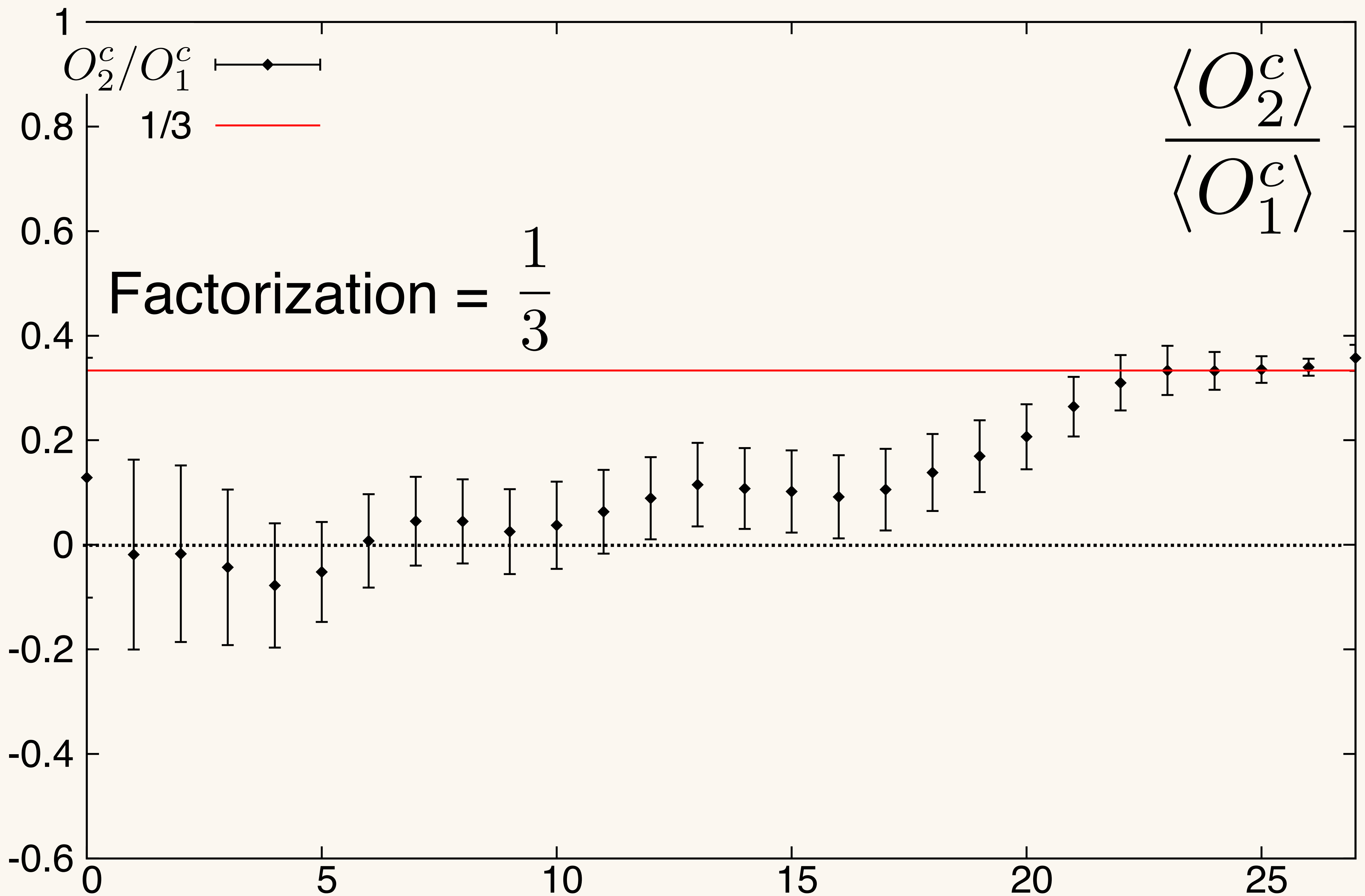
$$\Gamma_{\mu}^{(4)}\left(t_H, t_J, \mathbf{p}, \mathbf{k}\right) = \int \mathrm{d}^3\mathbf{x}\mathrm{d}^3\mathbf{y} e^{-i\mathbf{q}\cdot\mathbf{y}} \left\langle \phi_K\left(t_K, \mathbf{k}\right) \mathrm{T}\left[J_\mu\left(t_J, \mathbf{y}\right) H_{\mathrm{eff}}\left(t_H, \mathbf{x}\right)\right] \phi_B^\dagger(0, \mathbf{p}) \right\rangle$$

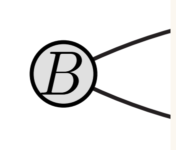


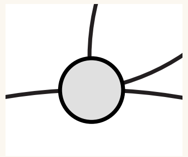
◉ 4-point functions

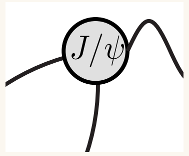


4-point functions









TO DO LIST

We have to…

(1):determine the lattice renormalization constants.

(2):Input more realistic momentum.

$$E_B(0) = E_{J/\psi}(\mathbf{k}) + E_K(\mathbf{k}) \longrightarrow \quad \mathbf{k} \geq \frac{2\pi}{L}(2,2,2)$$

(3):Input or extrapolate to physical quark masses.

(4):complete integration and taking limit to extract amplitude.

Summary

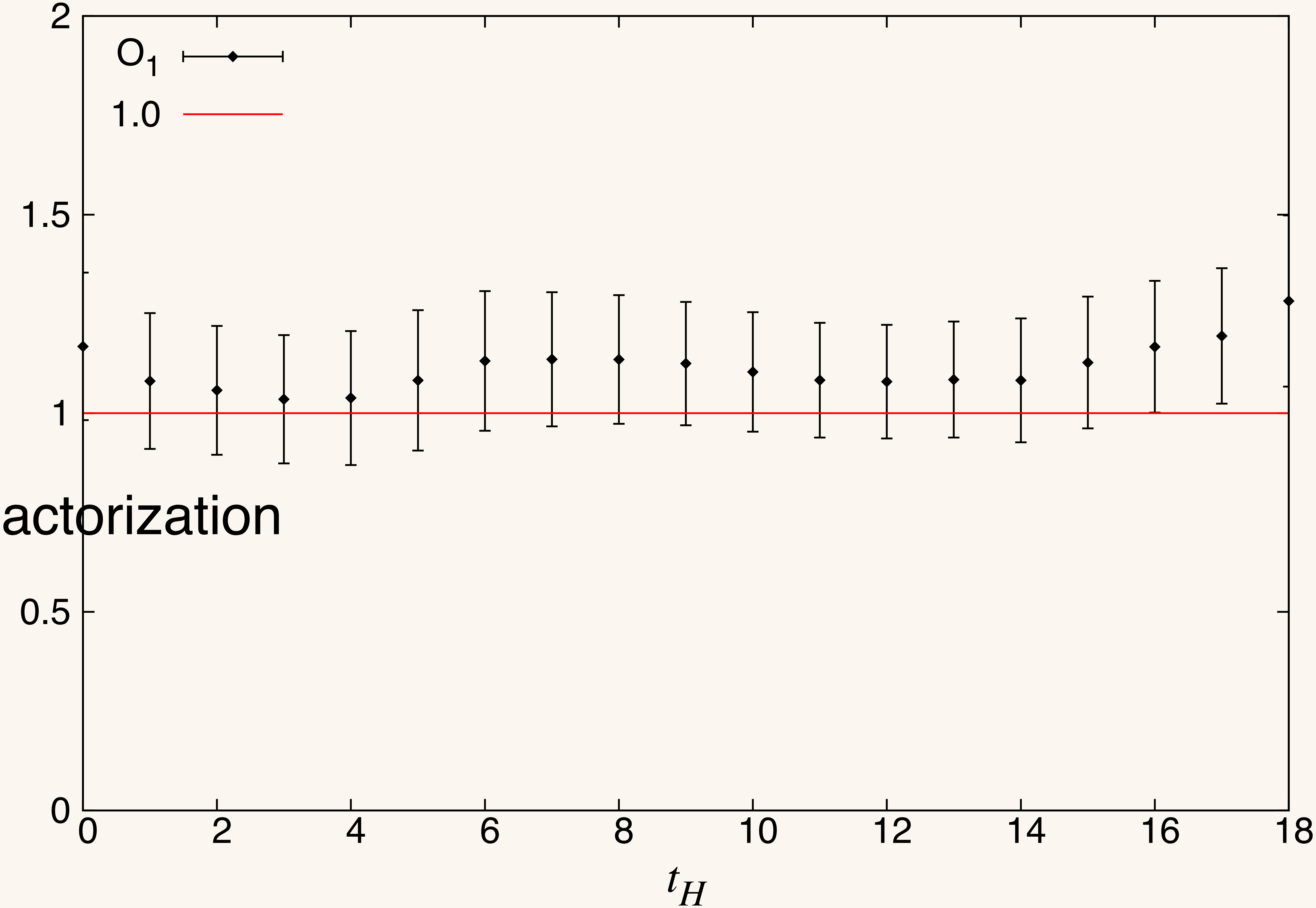
- We study the charmonium contribution to $B \to K l^+ l^-$ by the lattice calculation.

- $B \to K l^+ l^-$ is calculable analogously to $K \to \pi l^+ l^-$ for lighter bottom quark masses.

- As a first step, we study the accuracy of the factorization approximation.

- Sizable non-factorizable contribution is observed in the long-distance region.

◉ 4 point functions $a = 2.45\ \mathrm{GeV}$



Factorization

◉ 4 point functions $a = 2.45$ GeV