

Gluon structure of spin-1 meson as it becomes unstable using variationally optimized operators



Dimitra A Pefkou



Collaborators

- Phiala Shanahan (MIT Center for Theoretical Physics)
- Raul Briceño (Old Dominion University/Jefferson Lab)
- Robert Edwards (Jefferson Lab)

Hadron structure in QCD

- How to correctly describe a QCD state?
e.g. $|n\rangle = c_0 q\bar{q} + c_1 q\bar{q}q\bar{q} + c_2 q\bar{q}g + \dots$
- Which one to choose? Is there a hierarchy?
- Quantifying internal structure might help
- GPDs (Generalized Parton Distributions)
- Encode 3D distribution of partons in hadrons

Gluonic structure

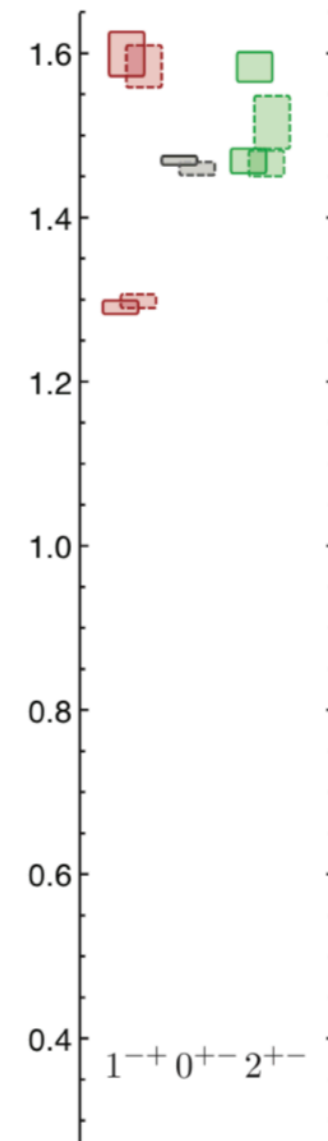
- Gluons are essential in hadron and nuclear structure
- Gluon GPDs harder to measure experimentally than quark
- Better understanding of gluons goal of current and future experiments (GlueX, EIC)

Gluon GPDs in Lattice QCD

- Moments of GPDs (GFFs) directly related to matrix elements calculable on the lattice for **bound states**
- Analogous quark functions (e.g form factors) have been studied extensively in Lattice QCD
- Gluon GFFs less understood. In recent years they have been investigated for pion, phi, nucleon and nuclei
- Goal: understand the gluonic structure of unstable states (resonances)

Exotic mesons

- Spin addition and symmetry transformation of fermion wavefunctions
- Not all J^{PC} combinations can be built from $q\bar{q}$
- However multiple experimental candidates (e.g $\pi_1(1600)$ at COMPASS)
- Possible explanations: Tetraquark states, meson molecules, glueballs, hybrids

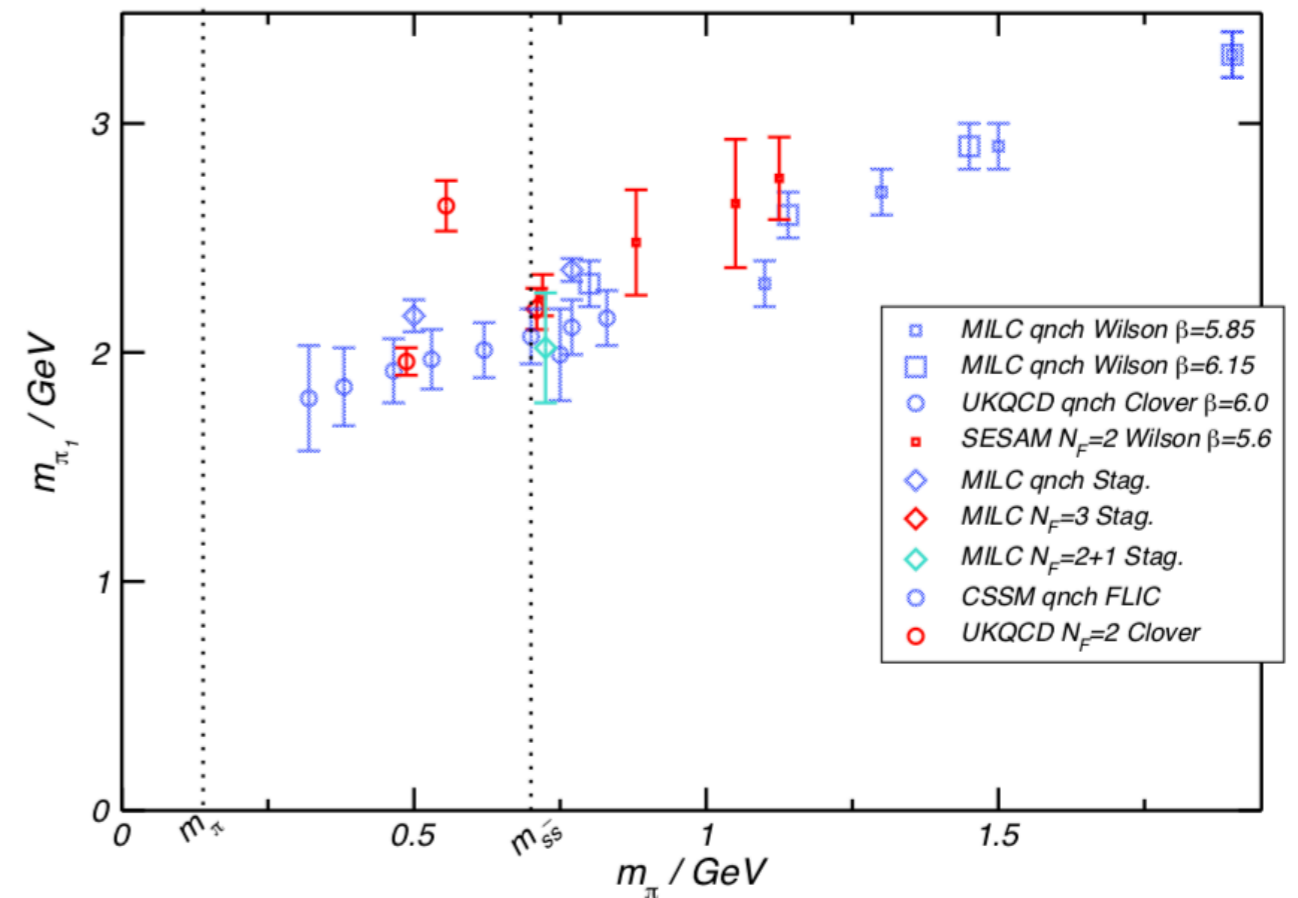


Hybrids $q\bar{q}g$

- Flux tube model (Isgur & Paton PRD 31 (1965), 2910)
→ supported by Lattice (Bali et al PRD 62 (2000) 054503):
- Gluonic flux tube between quark and antiquark
- Excitations of tube results in hybrid states, some exotic
- Model predicts allowed J^{PC} and decay modes

Hybrids $q\bar{q}g$

ζ	L	S	J^{PC}
+	1	0	1^{++}
+	1	1	$(\underline{2}, \underline{1}, \underline{0})^{+-}$
+	2	0	2^{--}
+	2	1	$(\underline{3}, \underline{2}, \underline{1})^{-+}$
-	1	0	1^{--}
-	1	1	$(\underline{2}, \underline{1}, \underline{0})^{-+}$
-	2	0	2^{++}
-	2	1	$(\underline{3}, \underline{2}, \underline{1})^{+-}$



From GlueX Collaboration (presentation to PAC30)

Meyer & Swanson arXiv:1502.07276

GlueX search for light hybrids

First case study: ρ meson

- Mellin moments of GPDs related to gluonic operator matrix elements

$$\bar{O}_{\mu_1 \dots \mu_n} = S[G_{\mu_1 \alpha} i \overleftrightarrow{D}_{\mu_3} \dots i \overleftrightarrow{D}_{\mu_n} G_{\mu_2}^\alpha] \quad \text{Spin-independent}$$

$$\tilde{O}_{\mu_1 \dots \mu_n} = S[\tilde{G}_{\mu_1 \alpha} i \overleftrightarrow{D}_{\mu_3} \dots i \overleftrightarrow{D}_{\mu_n} \tilde{G}_{\mu_2}^\alpha] \quad \text{Helicity}$$

$$O_{\nu_1 \nu_2 \mu_1 \dots \mu_n} = S[G_{\nu_1 \mu_1} i \overleftrightarrow{D}_{\mu_3} \dots i \overleftrightarrow{D}_{\mu_n} G_{\nu_2 \mu_2}] \quad \text{Transversity}$$

- Subduce into appropriate irreps of hypercubic rotation/reflection symmetry
- Lowest twist ($n=2$) decomposition into Lorentz structure for spin-1 meson gives 7 spin-independent, 7 helicity + 8 transversity GFFs
- Each structure function defines a gluonic radius (slope at zero momentum transfer)

Forward limit spin independent GFFs

$$\bar{O}_{\mu_1\mu_2} = G_{\mu_1\alpha} G_{\mu_2}^\alpha - \frac{1}{4} g_{\mu_1\mu_2} G_\alpha^\alpha \longrightarrow \text{Traceless part of gluon energy momentum tensor}$$

$$\langle pE' | S[\bar{O}_{\mu_1\mu_2}] | pE \rangle = S[M^2 E_{\mu_1}^* E_{\mu_2}] B_{2,1} + S[(E \cdot E^* p_{\mu_1} p_{\mu_2})] B_{2,2}$$

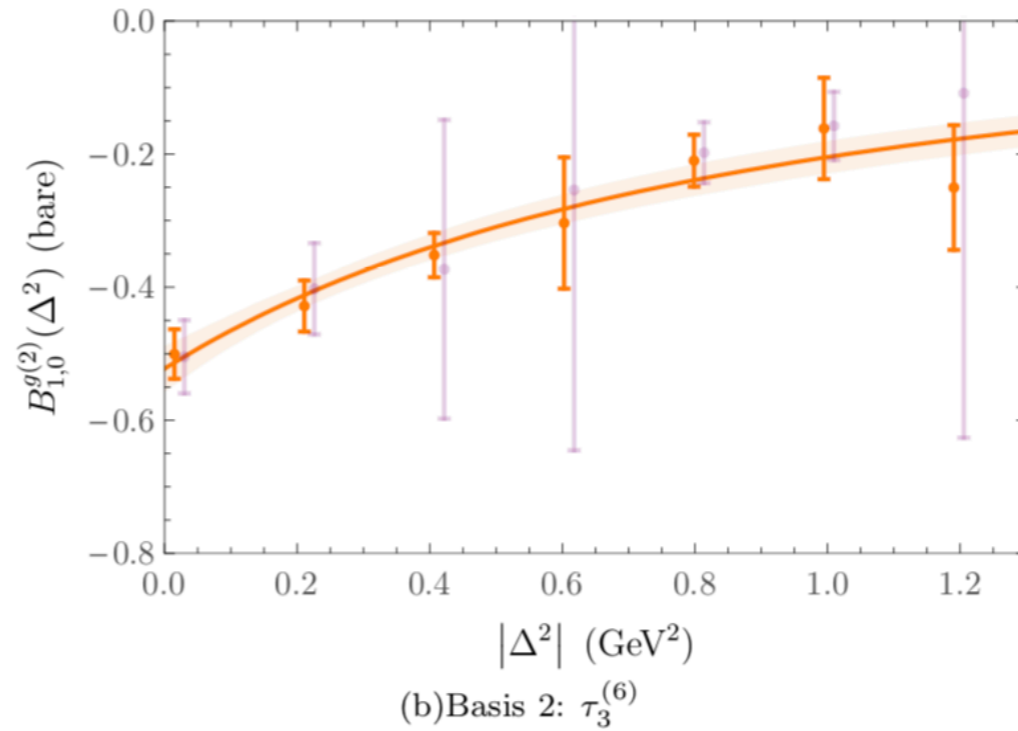
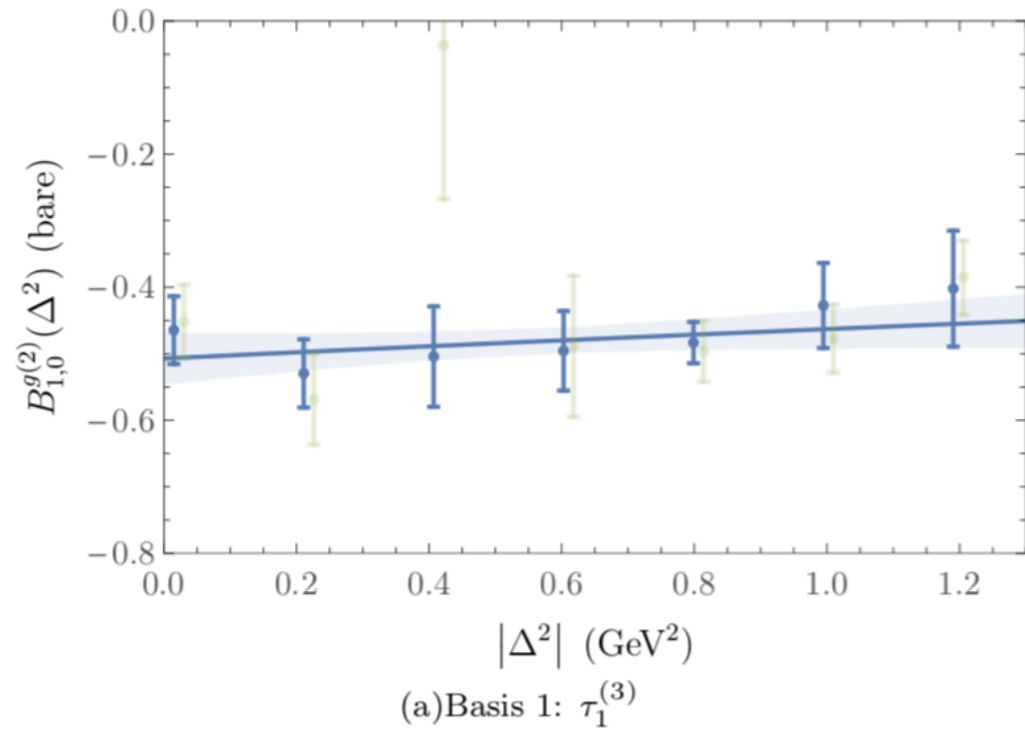
Directly related to gluon momentum fraction in forward limit

On lattice proportional to 3-pt/2-pt function ratio:

$$\frac{\langle O_n \bar{O}_{\mu_1\mu_2}^{E,\Lambda} O_n^\dagger \rangle - \langle O_n O_n^\dagger \rangle \langle \bar{O}_{\mu_1\mu_2}^{E,\Lambda} \rangle}{\langle O_n O_n^\dagger \rangle}$$

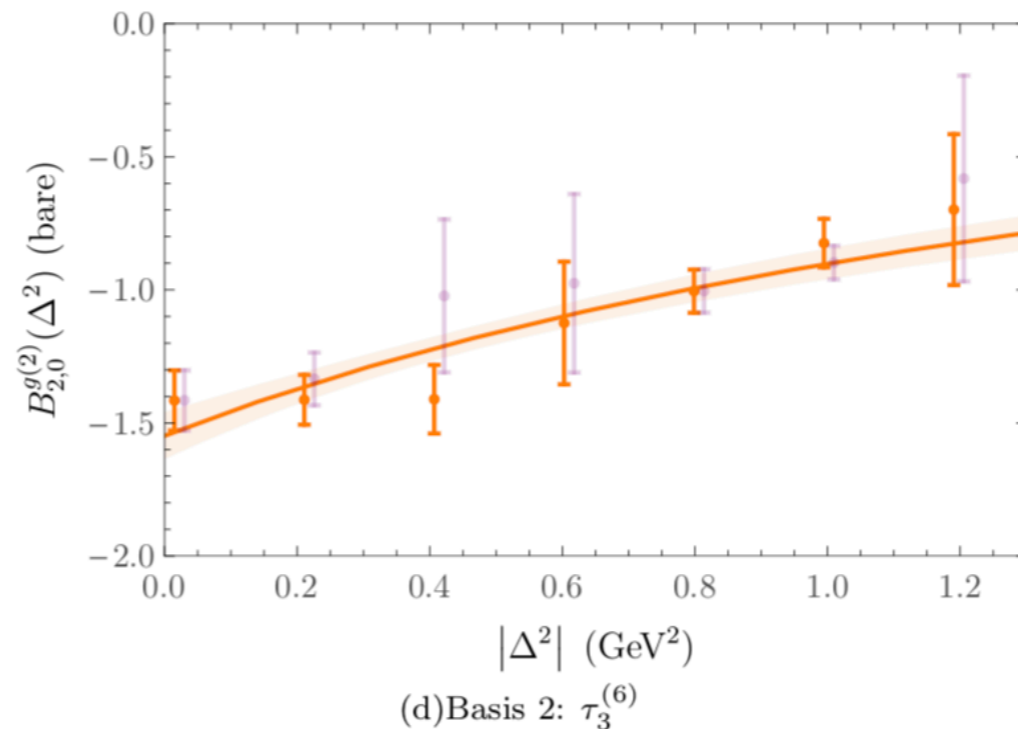
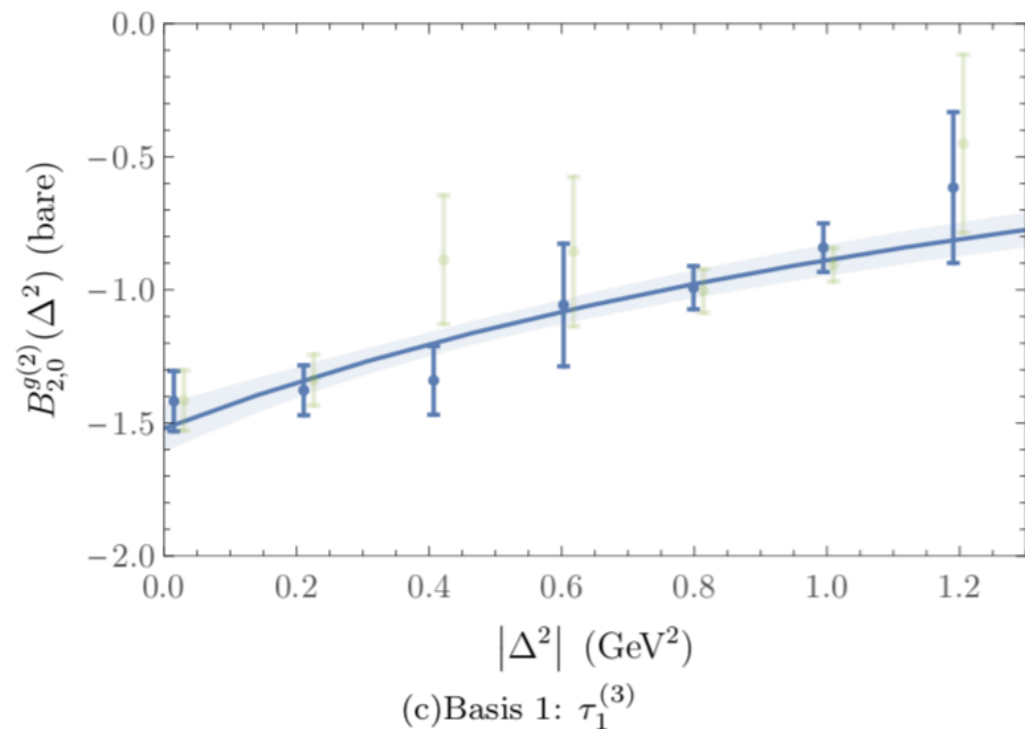
- Λ denotes the hypercubic irrep that the operator has been reduced to. Two convenient choices: $\tau_1^{(3)}$ $\tau_3^{(6)}$ (Goeckeler et al PRD 54 (1996))
- Can mix with quark operator of same dimension. However mixing has been shown to be 10% (Alexandrou et al PRD 96 (2017) 054503)

Stable heavy ρ spin-indep GFFs



$24^3 \times 64$

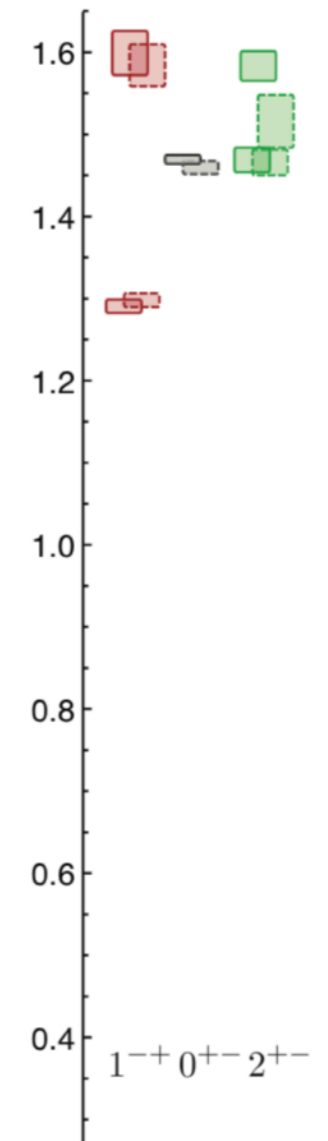
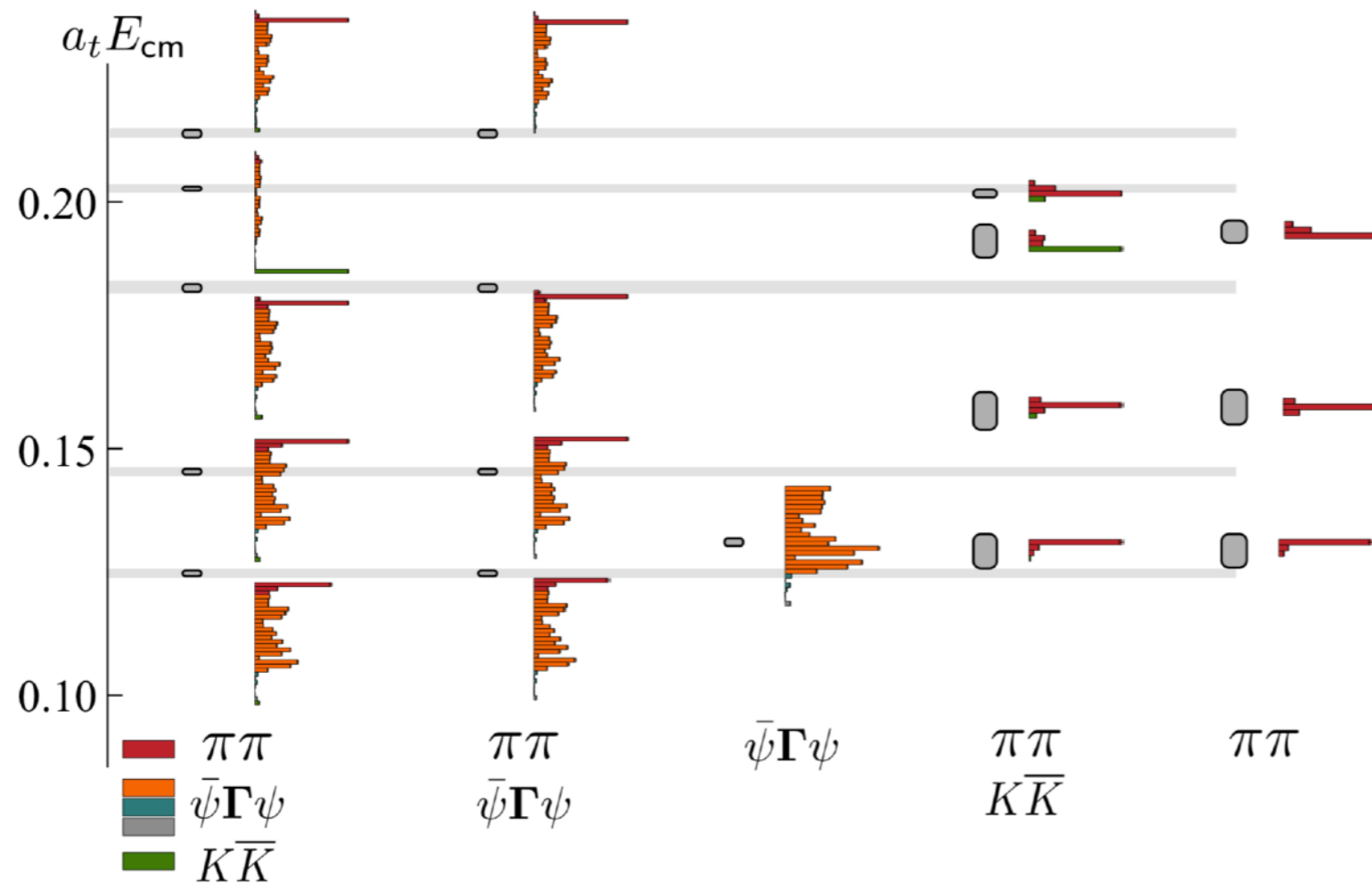
$m_\pi \sim 450 \text{ MeV}$



Variationally optimized interpolators

- Use large basis of definite spin operators of form $\bar{\psi}\Gamma\overleftrightarrow{D}\dots\overleftrightarrow{D}\psi$
- Reduce into cubic irreps if at rest or little group if in flight
- Optimized interpolators look like $\Omega_n^\dagger = \sum_i w_i^n O_i^\dagger$ where the weights are proportional to the eigenstates of the generalized eigenvalue problem $C(t)v^n = \lambda_n(t)C(t_0)v^n$
- Allows to probe in theory any state \rightarrow spectra + structure functions of bound states
- Lüscher: infinite volume spectra of resonances

Results from Hadron Spectrum Collaboration

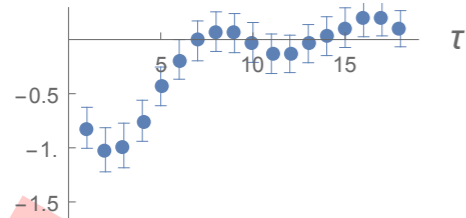


Wilson, Briceño, Dudek, Edwards, Thomas PRD 92, 094502 (2015)

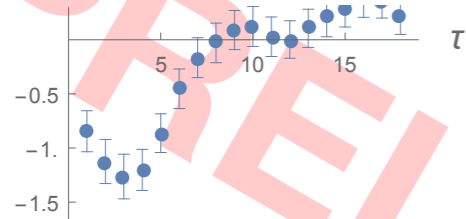
Dudek et al PRD 82, 034508 (2010)

spin-indep ρ matrix element

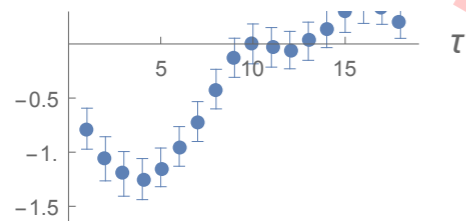
t=4 $\vec{p} = 0$



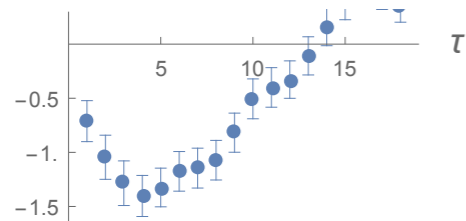
t=6



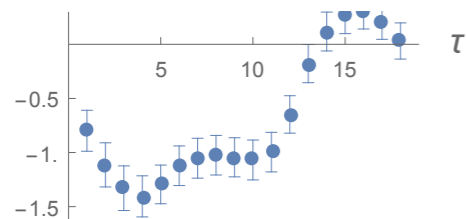
t=8



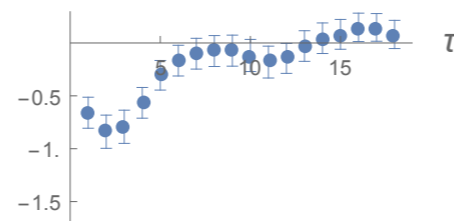
t=10



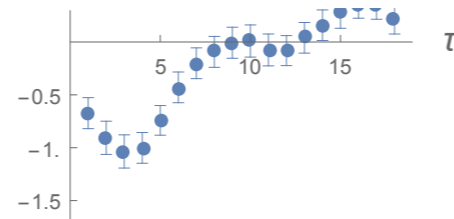
t=12



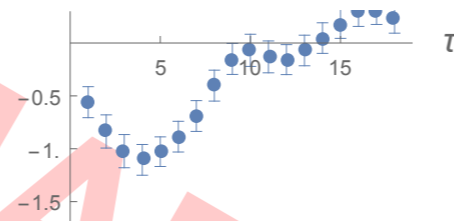
t=4 $\vec{p} = (0,0,1)$



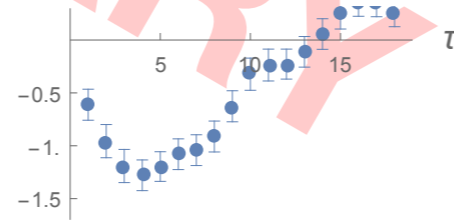
t=6



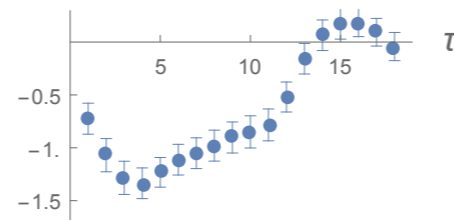
t=8



t=10



t=12



With variationally optimized operators

$$m_\pi \sim 700 \text{ MeV}$$

$$24^3 \times 128$$

$$\frac{a_s}{a_t} \sim 3.5$$

$$\tau_1^{(3)}$$

GFFs of unstable ρ

- Asymptotic state \rightarrow Corresponds to complex valued pole
- No such thing in finite volume
- Several studies have looked into solving that problem
- In particular we will use the Briceno-Hansen formalism (non-perturbatively maps lattice matrix elements to infinite volume amplitudes)
- Necessary elements: π elastic FFs, $\pi\pi$ matrix elements ^{in p wave}
- For more details see Alessandro Baroni's talk

Short term goals

- Calculate the infinite volume matrix element of ρ resonance using BH formalism
- Extract gluonic GFFs and renormalize results
- Parallel efforts on renormalization of gluonic operators by Yang et al (arxiv:1805.00531).
- Compare with stable ρ GFFs to interpret e.g gluon momentum fraction similarities and differences
- What happens to the gluon radius?

Long term goals

- Expand formalism to exotic states (hybrids)
- Predict 3D picture of gluonic structure of states with explicit gluonic degrees of freedom. How does it compare with conventional states?
- Combine with experimental efforts to get better understanding of QCD and nature

THANK YOU