Nucleon form factors from Nf=2+1+1 twisted mass fermions at the physical point

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in collaboration with ETM Collaboration:

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- ★ **K. Hadjiyiannakou,** The Cyprus Institute
- ★ **K. Jansen,** DESY, Zeuthen
- ★ **G. Koutsou,** The Cyprus Institute
- ★ **A. Vaquero,** University of Utah

Lattice Conference 2018

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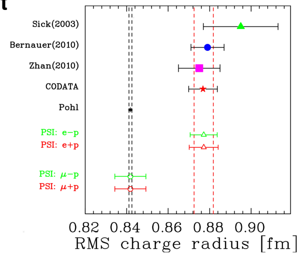
OUTLINE OF TALK

1. Introduction
2. Lattice Evaluation
3. E/M form factors (connected & disconnected)
4. Axial form factors (connected & disconnected)
5. Summary - Future prospects

Introduction

FFs have been studied for decades as a tool to understand nucleon structure

- **Nucleon electric & magnetic radii, magnetic moment extracted from Electromagnetic FFs**
- **Intrinsic quark spin obtained from g_A ($G_A(Q^2=0)$)**
- **Axial FFs relevant to experiments searching neutrino oscillations**



[E. J. Downie, EPJ Conf. 113 (2016) 05021]

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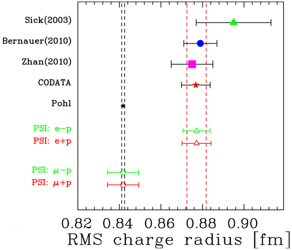

[E. J. Downie, EPJ Conf. 113 (2016) 05021]

Information from experiments not without ambiguities

- ★ **Discrepancy of $\langle r_p^2 \rangle$ between electron scattering and muonic hydrogen Lamb shifts**
- ★ **Large uncertainties in cross section of quasielastic neutrino-nucleon scattering**
  $\Rightarrow$ not well-constrained Axial FFs
- ★ **Strange E/M FFs are compatible with zero (HAPPEX collaboration, A4 exper., SAMPLE exper.)**

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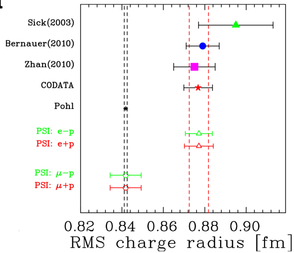

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⇓

Lattice QCD ideal *ab initio* formulation to study nucleon form factors

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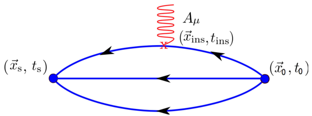
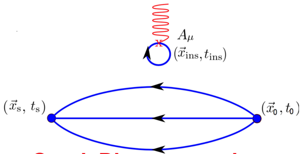
Nucleon Structure from Lattice QCD

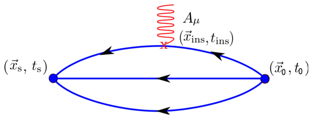


Connected



Quark Disconnected

Nucleon Structure from Lattice QCD



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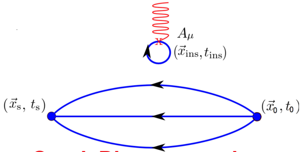

Quark Disconnected

★ **Calculation of 2pt- and 3pt-functions**

$$G^{2pt} : \langle N(p', s') | N(p, s) \rangle \qquad G^{3pt}_{\mathcal{O}} : \langle N(p', s') | \mathcal{O} | N(p, s) \rangle$$

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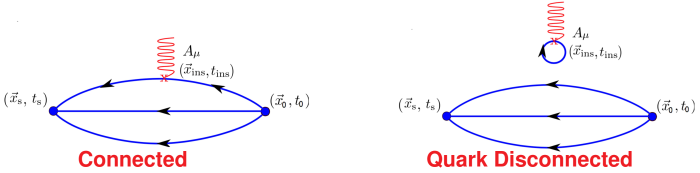

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ultra-local: scalar, vector, axial, tensor, 1-Deriv: vector, axial, tensor

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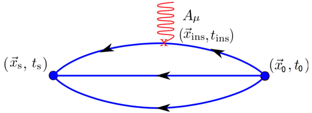

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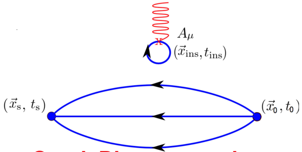

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  ultra-local: scalar, vector, axial, tensor, 1-Deriv: vector, axial, tensor

- ★ **Construction of optimized ratio**

$$R_{\mathcal{O}}(\Gamma, \vec{q}, t) = \frac{G_{\mathcal{O}}(\Gamma, \vec{q}, t)}{G(\vec{0}, t_f)} \times \sqrt{\frac{G(-\vec{q}, t_f - t) G(\vec{0}, t) G(\vec{0}, t_f)}{G(\vec{0}, t_f - t) G(-\vec{q}, t) G(-\vec{q}, t_f)}}$$

Plateau Method: $R_{\mathcal{O}}(\Gamma, \vec{q}, t) \xrightarrow[t - t_i \to \infty]{t_f - t \to \infty} \Pi_{\mathcal{O}}(\Gamma, \vec{q})$

2-state fits: **include first excited state**

Summation Method: $\sum_t R_{\mathcal{O}}(\Gamma, \vec{q}, t) \xrightarrow[t_f \to \infty]{} \mathcal{C} + \Pi_{\mathcal{O}}(\Gamma, \vec{q}) * t_f$

Setup of Calculation

★ **Renormalization:**

For most quantities multiplicative: $\Pi^R(\Gamma, \vec{q}) = Z_{\mathcal{O}}\, \Pi(\Gamma, \vec{q})$

- **non-singlet: connected**
- **singlet: both connected and disconnected**

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★ Extraction of form factors

Electromagnetic:

$$\langle N(p',s')|\mathcal{V}^q_\mu|N(p,s)\rangle \propto \bar{u}_N(p',s')\left(\gamma_\mu F^q_1(Q^2) + \frac{i\sigma_{\mu\nu}Q^\nu}{2m_N}F^q_2(Q^2)\right)u_N(p,s)$$

Axial:

$$\langle N(p',s')|\mathcal{A}^q_\mu|N(p,s)\rangle \propto \bar{u}_N(p',s')\left(\gamma_\mu\gamma_5 G^q_A(Q^2) + \frac{\gamma_5 Q_\mu}{2m_N}G^q_p(Q^2)\right)u_N(p,s)$$

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$G_E(Q^2) = F_1(Q^2) - \frac{Q^2}{4m_N^2} F_2(Q^2)$

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In this work:

- **Isovector combination ($u-d$): only connected**
- **Flavor decompositions: both connected and disconnected**
- **Strange $\&$ charm contributions purely disconnected (for nucleon)**

Setup of Calculation

- **Fermion part:** **Twisted Mass including a clover term (ETMC)**
  - ***Maximally twisted fermions:***
    - ▷ ***Automatic $\mathcal{O}(a)$-improvement***
    - ▷ ***No operator improvement needed, simplifies renormalization***
  - ***Addition of clover term reduces isospin symmetry breaking***
    $\Rightarrow$ ***simulation at physical pion masses became feasible***
- **Gluon action:** **Iwasaki**

Setup of Calculation

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★ **Gluon action:** Iwasaki

	Ensemble	N_f	$L^3 \times T$	$a(\text{fm})$	m_π (MeV)	Lm_π
	Nf2.48c	2	$48^3 \times 96$	0.094	135	2.98
NEW!	Nf2.64c	2	$64^3 \times 128$	0.094	132	3.97
NEW!	Nf211.64c	2+1+1	$64^3 \times 128$	0.081	135	3.55

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Nf211.64c (conn.)						Nf211.64c (disc.)		
$T_{\rm sink}$	$N_{\rm conf}$	$N_{\rm src}$	$T_{\rm sink}$	$N_{\rm conf}$	$N_{\rm src}$	Flavor	$N_{\rm conf}$	N_{src}
—	—	—	16a	625	16	u	750	200
12a	625	2	18a	625	32	d	750	200
14a	625	6	20a	625	32	s	750	200

Results PRELIMINARY: • More statistics to collect • finalize analyses

Quark Disconnected diagram

- Great progress in computing disconnected diagram
- Computer architecture (GPUs) and special techniques allow calculations at the physical point
- In this work:
  - Hierarchical probing (HP)
  - One-end trick for Twisted Mass Fermions (OET)
  - Spin-color dilution (SCD)
  - Deflation (D)

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In particular:

light quarks:

- **HP:** 2^3 **distance (512 Hadamard vectors)**
- **OET: 1 stochastic source**
- **D: 200 low eigenvectors**

strange quark:

- **HP:** 2^2 **distance (32 Hadamard vectors)**
- **OET: 12 stochastic source**

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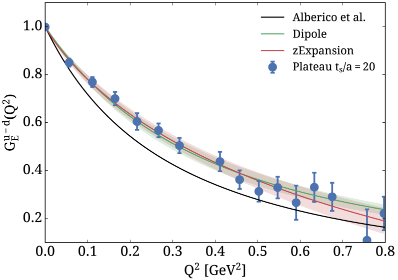
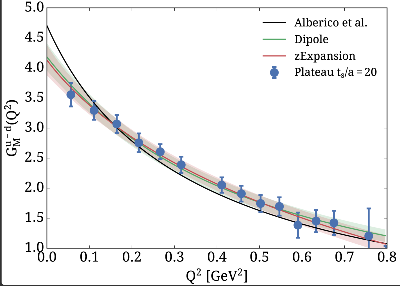
Nucleon EM Form Factors





- G_E: **slope of lattice data diff from experiments**
- G_M: **small-Q^2 has improved slope ($T_{\text{sink}}=1.6\text{fm}$)**

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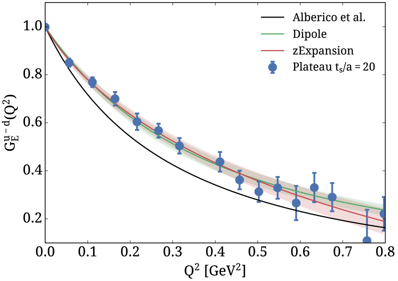
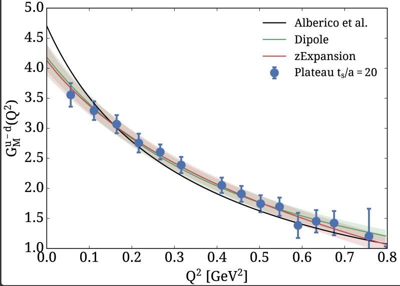
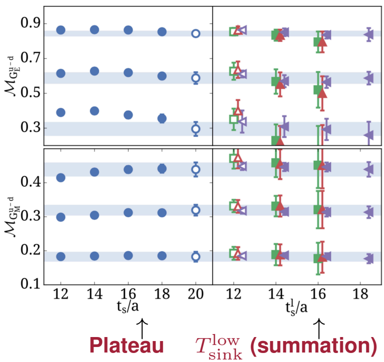




- ★ G_E: **slope of lattice data diff from experiments**
- ★ G_M: **small-Q^2 has improved slope ($T_{\rm sink}=1.6$fm)**
- ★ **excited states investigations:** $T_{\rm sink}=1-1.6$fm

$(Q)^2 = 0.06$**GeV**



Nucleon charged radii

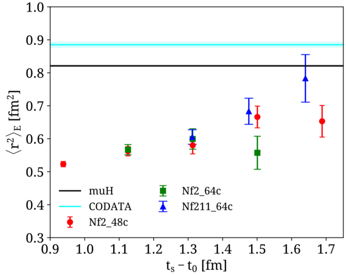
Dipole fit: **motivated by vector-meson pole contributions to FFs**

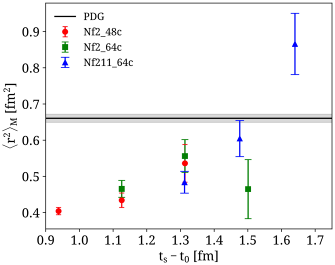
$$G_E(Q^2) = \frac{1}{\left(1+\frac{Q^2}{m_E^2}\right)^2}, \quad G_M(Q^2) = \frac{G_M(0)}{\left(1+\frac{Q^2}{m_M^2}\right)^2}, \quad \langle r^2_{E,M}\rangle = \frac{12}{m^2_{E,M}}$$

z-expansion: **model-independent, expected to model better the low-Q^2**

$$G_i(Q^2) = \sum_k a_k z(Q^2)^k, \; z(Q^2) = \frac{\sqrt{t_{cut}+Q^2}-\sqrt{t_{cut}}}{\sqrt{t_{cut}+Q^2}+\sqrt{t_{cut}}}, \; t_{cut}=4m_\pi^2, \; \langle r^2_{E,M}\rangle = -\frac{6a_1^{E,M}}{4t_{cut}a_0^{E,M}}$$

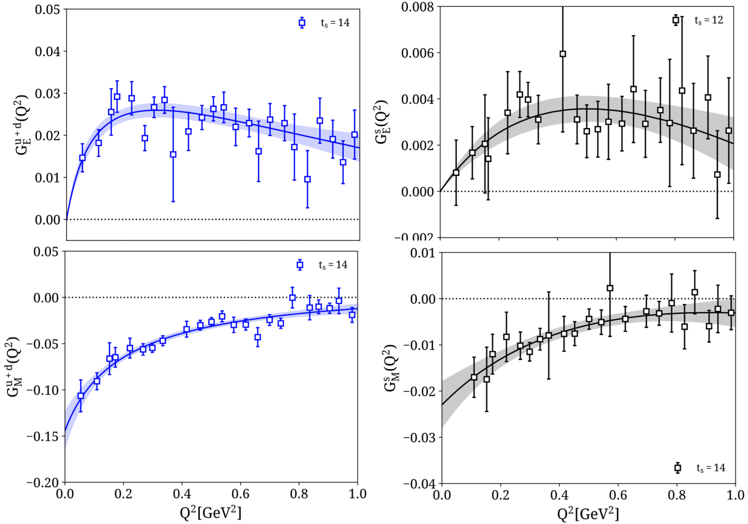
- ★ **Estimation of radii strongly depends on small Q^2**
- ★ **Large volume: access to Q^2 close to zero**





- ★ **Delicate to extract quantity, must examine fit methods**

Nucleon EM Form Factors (disconnected)



- ★ **Clear signal due to algorithmic advances**
- ★ T_{sink} **chosen based on excited states and quality of fits**

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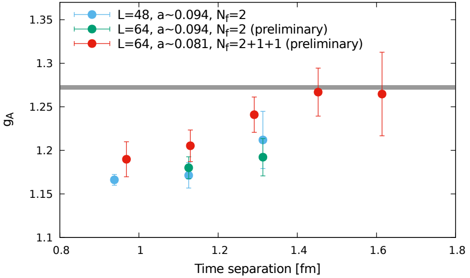
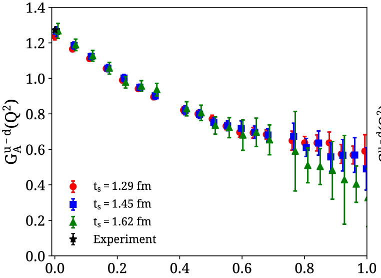
Nucleon Axial charge

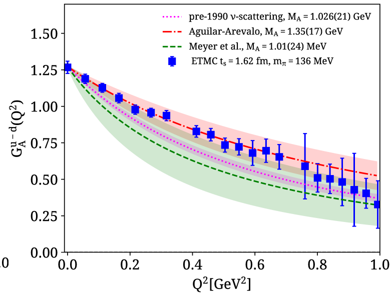
- ★ **Determined directly from lattice data**
- ★ **forward limit of matrix element of axial current**
- ★ **We study volume and quenching effects**



- ★ **Need of $T_{\rm sink}>1.3{\rm fm}$ to find agreement with experiment**
- ★ **Volume effects within statistical uncertainties**
- ★ **Currently increasing statistics for $T_{\rm sink}=1.5, 1.7{\rm fm}$**

Nucleon axial form factors



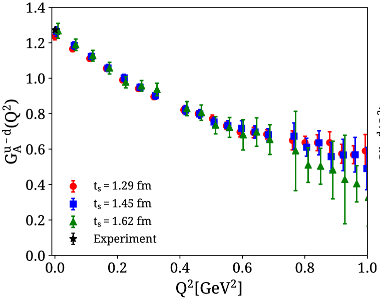


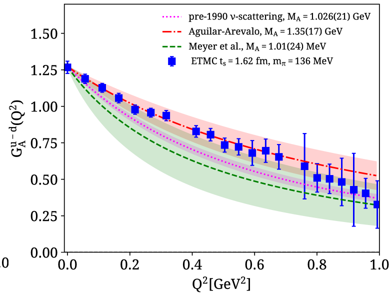
Left: Data for $T_{\mathrm{sink}} > 1.2\mathrm{fm}$ compatible, but slope different

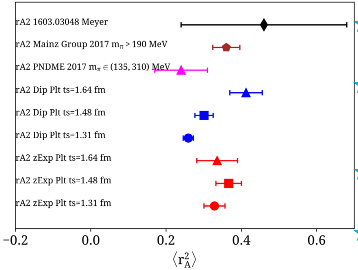
Right: Lattice data compatible with upper range of neutrino-nucleus cross sections (green band) and with MiniBooNE (red band)

$\langle r_A^2 \rangle$

Nucleon axial form factors

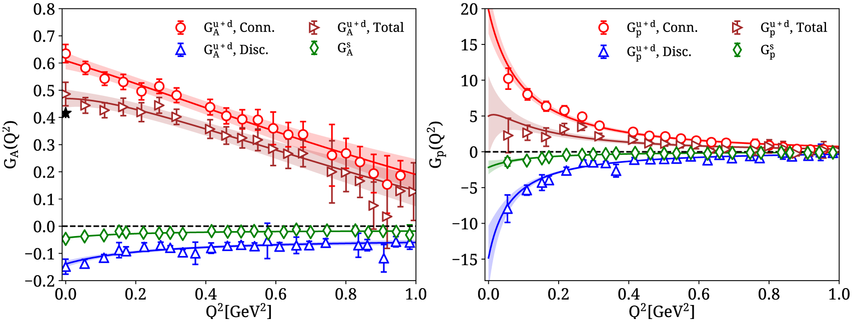






- **Left: Data for $T_{\mathrm{sink}} > 1.2\mathrm{fm}$ compatible, but slope different**
- **Right: Lattice data compatible with upper range of neutrino-nucleus cross sections (green band) and with MiniBooNE (red band)**
- **Parametrization of lattice data:**
  - **Dipole fit, z-expansion**
- **Deviations on $\langle r_A \rangle$ from different methods**

Nucleon Axial Form Factors (isoscalar)



- ★ **Light quark disconnected contributions are sizable**
- ★ G_A^s **small but necessary in order to bring** G_A^{total} **close to experimental value**
- ★ $G_p^{u+d,DI}$ **cancels (within uncertainties)** $G_p^{u+d,CI}$
- ★ G_p^s **suppressed compared to light quark contribution**

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- ★ Dedicated study of nucleon structure quantities using 3 ensembles and multiple source-sink separations

- ★ Investigation of systematic uncertainties: excited states, volume effects, quenching effects
  - g_A agreement with experiment for $T_{\rm sink}\gtrsim 1.5\text{fm}$
  - Slope of Axial $\&$ E/M form factors sensitive to $T_{\rm sink}$

- ★ Disconnected contributions to FFs have been computed for u, d, s:
  - necessary to bring g_A^{u+d} in agreement with experiment
  - large contributions to G_p^{u+d} that partly cancels connected part
  - strange contributions to FFs non-negligible

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Future work

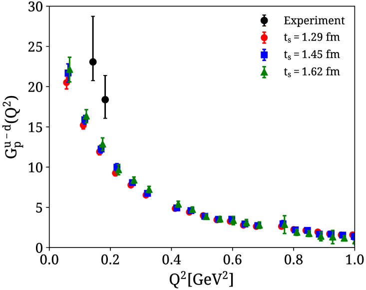
- ★ Increase statistics and addition of separations larger than 1.5fm
- ★ Connected and disconnected contributions to charged radii
- ★ 2 additional 2+1+1 ensembles @ physical point and same physical volume as $64^3 \times 128$: $80^3 \times 160 (a \sim 0.065{\rm fm})$, $96^3 \times 192 (a \sim 0.055{\rm fm})$

THANK YOU

Grant No. PHY-1714407

BACKUP

Nucleon Axial Form Factors



- **Excited states suppressed for** $T_{\mathrm{sink}} > 1.3\mathrm{fm}$
- **Limited data from experiment**

  [S. Choi et al. Phys. Rev. Lett. 71 3927 (1993)]
- **Pion pole fit:**

$$G_p(Q^2) = G_A(Q^2)\frac{4m_N^2}{(Q^2 + m_\pi^2)}$$

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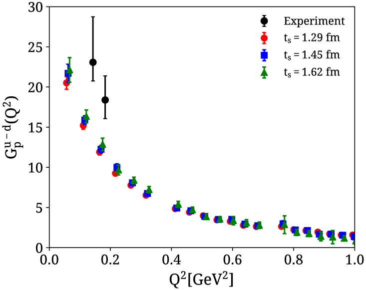

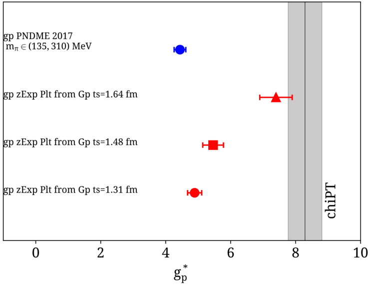

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- ★ **Comparison with HBχPT**

$$g_p^\star = \frac{m_\mu}{2m_N} G_p(Q^2 = 0.88 m_\mu^2)$$

- ★ $g_p^\star$ **moves towards HBχPT estimate as** T_{sink} **increases**
- ★ **Sensitive quantity to extract**