Probing the composite light scalar of the sextet model for dilaton fingerprints

Chik Him (Ricky) Wong

Lattice Higgs Collaboration (LatHC):
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Introduction

- Sextet model: SU(3) $N_f = 2$ fermions in two-index symmetric representation
- Near conformal window $\Rightarrow$ candidate of walking theory
- Analysis of M_π and F_π in the chiral limit using χPT has become difficult as m decreases in the dataset
- Previous attempts have limited success but still difficult (e.g. NLO chiral log fits, Rooted staggered χ PT)
- Possible cause: Near conformal window, emergent 0^{++} scalar can be as light as NGBs $\Rightarrow$ χPT needs modification
- Dilaton hypothesis: The scalar acts as a dilaton from scale symmetry breaking (details in Julius Kuti's talk)
- Tree-level predictions of Dilaton Effective Lagrangian
  - Scaling formulae: $M_\pi^2 \, F_\pi^{2-y} = C\, m,$ where $C = 2B_\pi f_\pi^{2-y}, \;\; y = 3-\gamma, \;\; f_\pi = F_\pi(m\to 0)$
  - Asymptotic dilaton potential $V(\chi) \sim \chi^p$: $M_\pi^2 \, F_\pi^{2-p} = B$

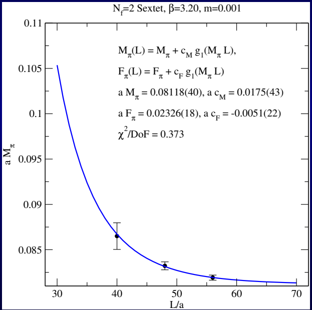
Introduction

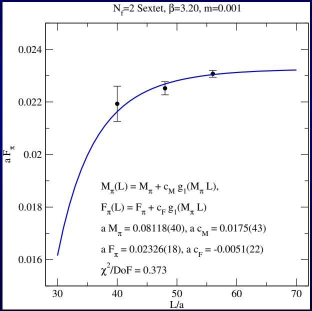
- A recent work (Appelquist et al, 2018) attempted to do these tests on dataset extracted from our published plots and claimed success
- The study was limited
  - The dataset used was outdated. New data are available and the estimates of some data entries are improved.
  - Error estimates were ball-park estimates obtained only graphically instead of using the actual data
  - Only $\beta = 3.20$ was analyzed. Cutoff effects was not studied.
- $\Rightarrow$ The analysis is not conclusive. We need a more comprehensive study.

Challenges of probing the dilaton hypothesis

- Finite Size Scaling
- M_π and F_π values are defined in infinite volume limit, while simulations are done in finite volumes $\Rightarrow$ Extrapolation into infinite volume limit is required
- Ansatz:

$$M\pi(L) = M\pi(L \to \infty) + c_M\ g_1(M\pi L, \eta = N_t/N_L), \quad F\pi(L) = F\pi(L \to \infty) + c_F\ g_1(M\pi L, \eta = N_t/N_L)$$





- In the presence of light dilaton $M_d \sim M_\pi$, the particle being exchanged can also be dilaton. However, the combined correction term would still be g_1
- Note: $Q = 0$ effect is ignored

Challenges of probing the dilaton hypothesis

- Dilaton hypothesis
  - Scaling formulae (primary test): $M_\pi^2\, F_\pi^{2-y} = C\, m$,
  - Asymptotic dilaton potential $V(\chi) \sim \chi^p$: $M_\pi^2\, F_\pi^{2-p} = B$
- $M_\pi^2\, F_\pi^{2-y}$ and $M_\pi^2\, F_\pi^{2-p}$ are nonlinear functions of (M_π,F_π, y) and (M_π, F_π, p) respectively
- The distribution of the error of a function $f(M_\pi, F_\pi)$ can be approximated by a normal distribution
$$f(M_\pi, F_\pi) - f(\langle M_\pi\rangle, \langle F_\pi\rangle) \quad \sim \quad N(\ 0,\ \nabla f^\dagger\, \Sigma\, \nabla f\ ),$$
  in which Σ is the covariance matrix between M_π and F_π from the FSS fit
- $\partial_{F_\pi}(M_\pi^2\, F_\pi^{2-y}) = (2-y) F_\pi^{1-y}\, M_\pi^2$, a function of y (same case for p)
- Problem: How to properly propagate errors in M_π and F_π into these quantities, in the presence of p and y?

Challenges of probing the dilaton hypothesis

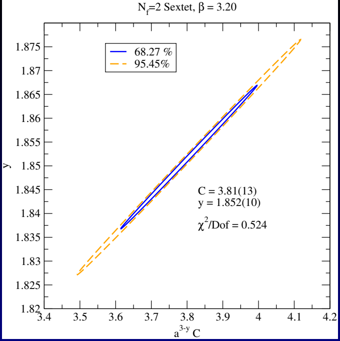
- Generalized Maximum Likelihood Estimates
- Consider testing a hypothesis $Y = f_\theta(X)$ parameterized by θ with observable Y at X:
  - Each Y_i is measured at X_i's with Gaussian uncertainties described by $N(0, \sigma^2_{Y,i})$
  - Measurements at each X_i are assumed to be independent
- The likelihood $L(\theta;Y) \propto \exp(-\sum_i \frac{(Y_i - f_\theta(X_i))^2}{2\sigma^2_{Y,i}})$ can be maximized by minimizing

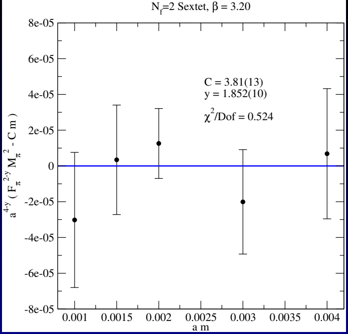
$$\chi^2 = \sum_i \left(\frac{Y_i - f_\theta(X_i)}{\sigma_{Y,i}} \right)^2 ,$$

- If there are more than one parameters, e.g. $\theta = (\theta_0, \theta_1)$, such minimization can be done alternatively holding one of them fixed until convergence.
- $\Rightarrow$ $\sigma^2_{Y_i}$ can be functions of θ as long as the uncertainty of Y_i can be described by $N(0, \sigma^2_{Y,i}(\theta))$ at all values of θ

Challenges of probing the dilaton hypothesis

- Assuming the uncertainty of $M_\pi^2\, F_\pi^{2-y}$ is described by $N\left(0, \nabla(M_\pi^2\, F_\pi^{2-y})^\dagger\, \Sigma\, \nabla(M_\pi^2\, F_\pi^{2-y})\right)$ at all values of y, (and similar for $M_\pi^2\, F_\pi^{2-p}$), one obtains the generalized ML estimates for (C, y) and (B, p) by minimizing χ^2 as in ordinary fitting procedures


N_f=2 Sextet, β = 3.20
68.27 %
95.45%
C = 3.81(13)
y = 1.852(10)
χ^2/Dof = 0.524
y
a^{3-y} C


N_f=2 Sextet, β = 3.20
C = 3.81(13)
y = 1.852(10)
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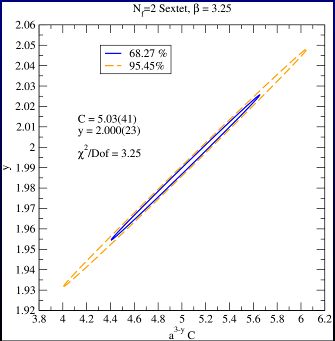
a^{4-y} ($F_\pi^{2-y} M_\pi^2$ - C m)
a m


N_f=2 Sextet, β = 3.25
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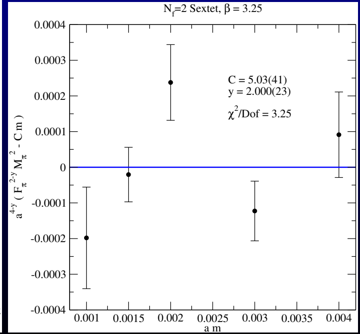
95.45%
C = 5.03(41)
y = 2.000(23)
χ^2/Dof = 3.25
y
a^{3-y} C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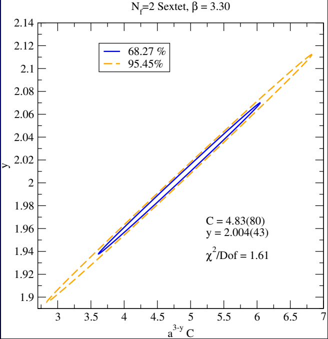
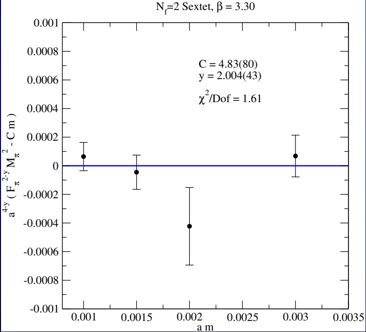
C = 5.03(41)
y = 2.000(23)
χ^2/Dof = 3.25
a^{4-y} ($F_\pi^{2-y} M_\pi^2$ - C m)
a m

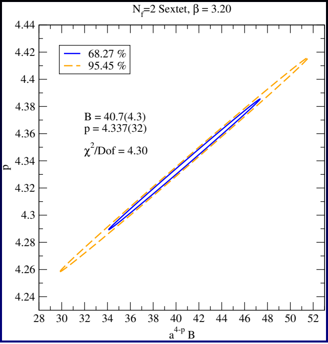
Challenges of probing the dilaton hypothesis

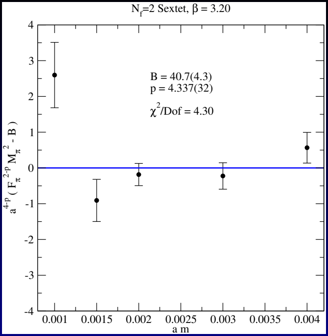
- At $\beta = 3.30$, we do not have enough data for infinite volume extrapolation, but we still attempted to fit using the largest volumes available. ($64^3 \times 96$ for $m = 0.001 - 0.002$ and $48^3 \times 96$ for $m = 0.003$)

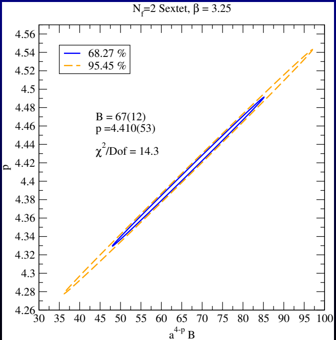


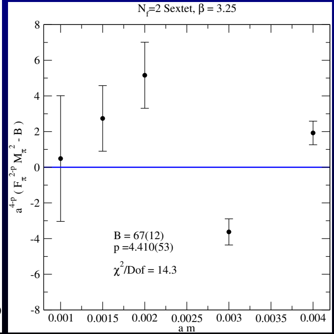


- The (C, y) fits seem to work with acceptable χ^2/Dof at all β's


N_f=2 Sextet, β = 3.20
68.27 %
95.45 %
B = 40.7(4.3)
p = 4.337(32)
χ^2/Dof = 4.30
p
a^{4-p} B


N_f=2 Sextet, β = 3.20
B = 40.7(4.3)
p = 4.337(32)
χ^2/Dof = 4.30
$a^{4-p}(F_\pi^{2-p} M_\pi^2 - B)$
a m


N_f=2 Sextet, β = 3.25
68.27 %
95.45 %
B = 67(12)
p =4.410(53)
χ^2/Dof = 14.3
p
a^{4-p} B


N_f=2 Sextet, β = 3.25
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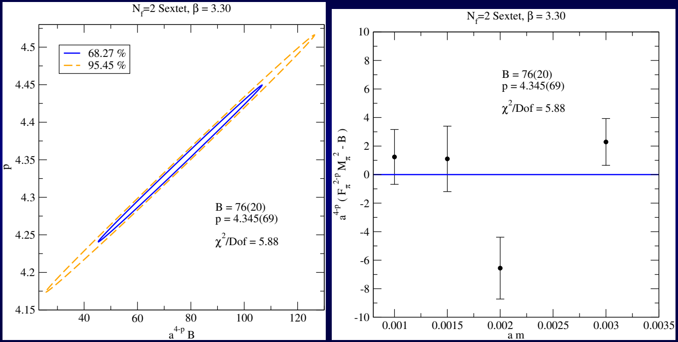
χ^2/Dof = 14.3
$a^{4-p}(F_\pi^{2-p} M_\pi^2 - B)$
a m

Challenges of probing the dilaton hypothesis

- Again, we fitted $\beta = 3.30$ without infinite volume extrapolation



- The (B, p) fits fail, especially at $\beta = 3.25$

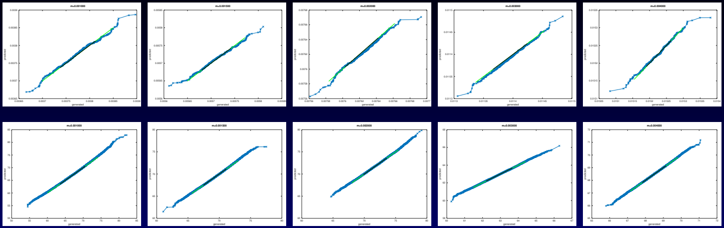
Challenges of probing the dilaton hypothesis

- Our data are consistent with the scaling formulae ((C, y) fits) , but not with the asymptoptic dilaton potential ((B, p) fits)
- Possible explanations:
  - (a) The ansatz is problematic. e.g. The asymptoptic dilaton potential may not be of the form χ^p.
  - (b) Some Normal distribution approximations in our analysis do not hold
- Would the (B, p) fits work if some of the Normal distribution approximations in our analysis are improved? $\Rightarrow$ Monte Carlo and Markove Chain Monte Carlo can help

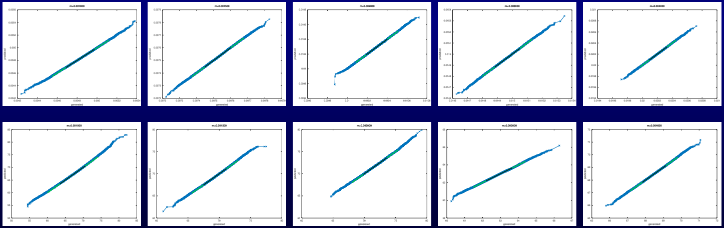
Monte Carlo Analysis

- Assumption: the uncertainty of $M_\pi^2\ F_\pi^{2-y}$ is described by $N\ (\ 0, \nabla(\ M_\pi^2\ F_\pi^{2-y}\ )^\dagger\ \Sigma\ \nabla(\ M_\pi^2\ F_\pi^{2-y}\ )\ )|_m$ at all values of y at each m (and similar for $M_\pi^2\ F_\pi^{2-p}$)
- In order to improve beyond Gaussian by expansion, one needs higher moments than Σ_m
- With only Σ_m, one can still do the following :
  - Approximate the distribution of $\theta_m = \{M_\pi, F_\pi, c_M, c_F\}|_m$ as a multivariant Gaussian distribution $N(\langle\theta\rangle_m, \Sigma_m)$ at each $m \Rightarrow$ Numerically generate such distribution
  - Compute $M_\pi^2\ F_\pi^{2-y}$ for the range of y's suggested by the fitted values (similar for $M_\pi^2\ F_\pi^{2-p}$)
  - Compare the distributions with the ones predicted by $N\ (\ \langle M_\pi\rangle^2\ \langle F_\pi\rangle^{2-y}, \nabla(\ M_\pi^2\ F_\pi^{2-y}\ )^\dagger\ \Sigma\ \nabla(\ M_\pi^2\ F_\pi^{2-y}\ )\ )|_m$ (similar for $M_\pi^2\ F_\pi^{2-p}$)
  - A simple way is to look at Quantile-Quantile plots. The closer to the $y = x$ line the more similar these two distributions are.

$\beta = 3.20$: First row: $M_\pi^2 F_\pi^{2-y_{\text{fit}}}$ Second row: $M_\pi^2 F_\pi^{2-p_{\text{fit}}}$



$\beta = 3.25$: First row: $M_\pi^2 F_\pi^{2-y_{\text{fit}}}$ Second row: $M_\pi^2 F_\pi^{2-p_{\text{fit}}}$



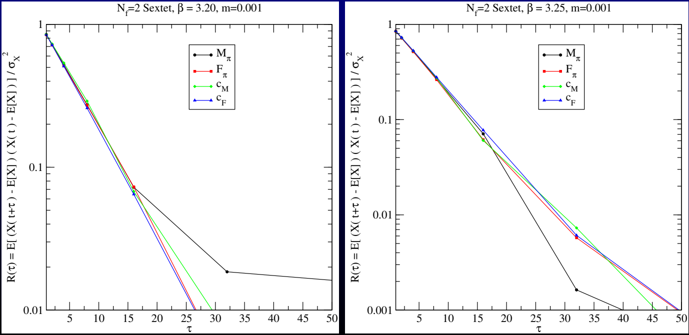
- They are almost identical and normal up to 2 sigma range.
- The uncertainties of fitted p and y are narrow, so only the fitted values are used here for illustration. Yet this level of resemblance holds at broader range of values.

Markov Chain Monte Carlo (MCMC) Analysis

- Assumption of Monte Carlo analysis: At each m, $\theta_m = (\, M_\pi(L \to \infty),\ F_\pi(L \to \infty),\ c_M,\ c_F)|_m$ distributes as Gaussian with covariance matrix Σ_m
- Actual assumption in Maximum Likelihood estimate:
$$(M_\pi(L_i) - [M_\pi(L \to \infty) + c_M\ g_1(M_\pi L, \eta)])|_m$$
$$(F_\pi(L_i) - [F_\pi(L \to \infty) + c_F\ g_1(M_\pi L, \eta)])|_m$$
follows $N(0, \Sigma_m)$ at each m
- In Frequentist picture, it is the likelihood $L(\theta_m; \{M_\pi(L_i), F_\pi(L_i)\}_m)$ In Bayesian picture, this distribution is equivalent to the posterior distribution with uniform prior $P(\theta) \propto 1$
$$P(\theta_m |\ \{M_\pi(L_i), F_\pi(L_i)\}_m\ ) \propto e^{-\chi^2_m/2}\ P(\theta) \propto e^{-\chi^2_m/2}$$
- This distribution can be generated by Markov Chain Monte Carlo (MCMC) method

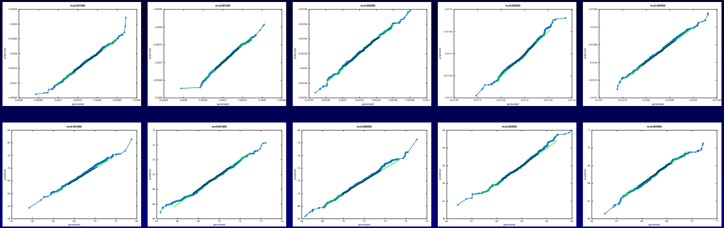
Markov Chain Monte Carlo (MCMC)

- Markov Chain Monte Carlo algorithms sample a desired distribution by constructing a Markov Chain with such distribution as the equilibrium distribution
- A simple implementation is the familiar Metropolis-Hastings algorithm using a Gaussian proposal density
- Once we obtain the distribution, we can check again with $N\left(\langle M_\pi \rangle^2 \langle F_\pi \rangle^{2-y}, \nabla(M_\pi^2 F_\pi^{2-y})^\dagger \Sigma \nabla(M_\pi^2 F_\pi^{2-y}) \right)|_m$ (similar for $M_\pi^2 F_\pi^{2-p}$)
- Autocorrelation is checked and sampling is taken every 64^{th}

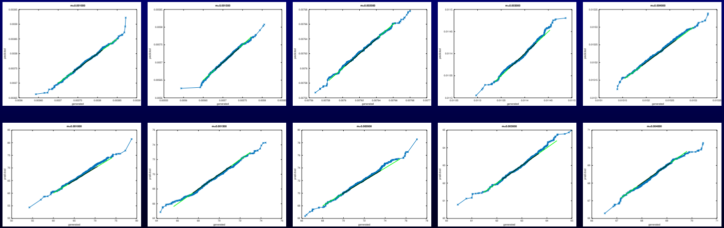


Markov Chain Monte Carlo (MCMC)

$\beta = 3.20$: First row: $M_\pi^2 F_\pi^{2-y_{\rm fit}}$ Second row: $M_\pi^2 F_\pi^{2-p_{\rm fit}}$

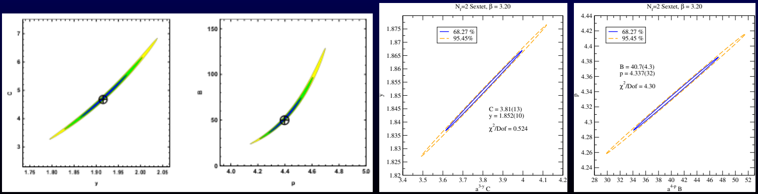


$\beta = 3.25$: First row: $M_\pi^2 F_\pi^{2-y_{\rm fit}}$ Second row: $M_\pi^2 F_\pi^{2-p_{\rm fit}}$



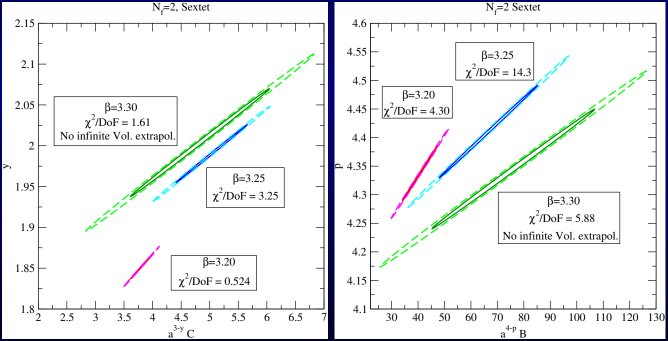
- The distribution is normal up to 2 sigmas again

- All three reproduces one another's results $\Rightarrow$ Our analysis procedures are robust and correct. However,
- Appelquist et al, 2018: Analysis of extracted data $\beta = 3.20$
  - $y = 1.9(1)$, $C = 4.7$, $\chi^2/\mathrm{DoF} = 0.19$
  - $p = 4.4(3)$, $B = 51$, $\chi^2/\mathrm{DoF} = 0.64$
- Our analysis on our data: $\beta = 3.20$
  - $y = 1.852(10)$, $C = 3.81(13)$, $\chi^2/\mathrm{DoF} = 0.524$
  - $p = 4.337(32)$, $B = 40.7(4.3)$, $\chi^2/\mathrm{DoF} = 4.30$
- The two results differ, especially in the (B, p) test. Possible reasons:
  - Their extracted data were outdated and inaccurate. We use our exact up-to-date dataset.
  - Some systematic errors may have been overlooked
  - Further investigation is needed
- Their work did not check $\beta = 3.25$ data in which (B, p) test fails more in our analysis



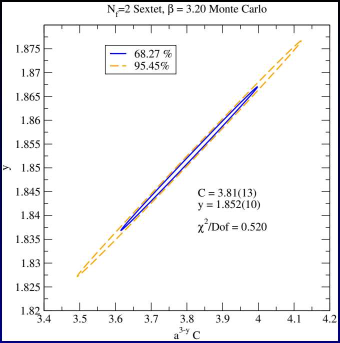
Conclusion

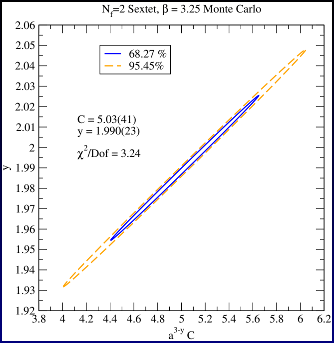
- The dilaton hypothesis is tested in the sextet model using several different comprehensive analysis methods. Three β values are analyzed in order to probe cutoff effects. $\beta = 3.30$ is incomplete due to the lack of infinite volume extrapolation.

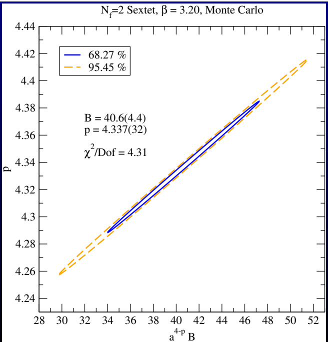


Conclusion

- The scaling formulae $M_\pi^2\, F_\pi^{2-y} = C\, m$ is consistent with our current data. However, $\gamma = 3 - y$ should be cutoff-dependent $\Rightarrow$ what scale does y correspond to ?
- The assumption of asymptoptic dilaton potential form $V(\chi) \sim \chi^p$ does not pass the test of $M_\pi^2\, F_\pi^{2-p} = B$ in our data
  - Possible explanations:
  - It is a tree-level prediction. The significance of 1-loop correction is unknown. In principle it is needed in order to be consistent with the FSS which probes an 1-loop correction effect.
  - Some terms may be missing in the dilaton potential
  - At small m, our data is known to be mostly frozen at $Q = 0$. Its effect is not taken into account in our analysis
- Cutoff effects need to be investigated


N_f=2 Sextet, β = 3.20 Monte Carlo
68.27 %
95.45%
C = 3.81(13)
y = 1.852(10)
χ^2/Dof = 0.520
y
a^{3-y} C


N_f=2 Sextet, β = 3.25 Monte Carlo
68.27 %
95.45%
C = 5.03(41)
y = 1.990(23)
χ^2/Dof = 3.24
y
a^{3-y} C


N_f=2 Sextet, β = 3.20, Monte Carlo
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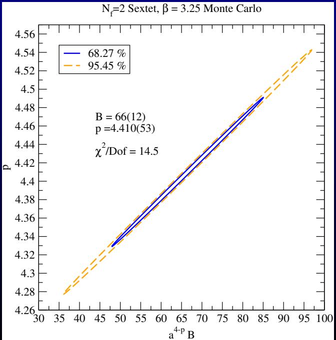
B = 40.6(4.4)
p = 4.337(32)
χ^2/Dof = 4.31
p
a^{4-p} B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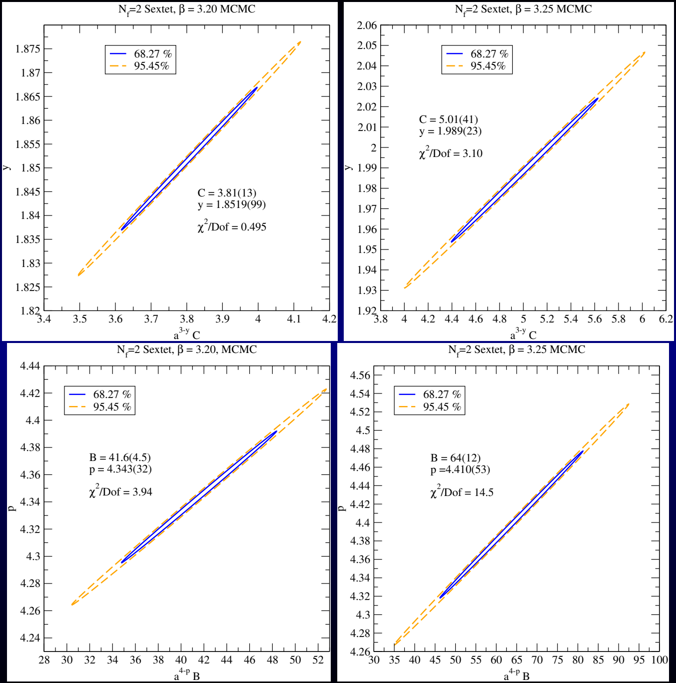
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68.27 %
95.45%
C = 3.81(13)
y = 1.8519(99)
χ^2/Dof = 0.495
y
a^{3-y} C
N_f=2 Sextet, β = 3.25 MCMC
68.27 %
95.45%
C = 5.01(41)
y = 1.989(23)
χ^2/Dof = 3.10
y
a^{3-y} C
N_f=2 Sextet, β = 3.20, MCMC
68.27 %
95.45 %
B = 41.6(4.5)
p = 4.343(32)
χ^2/Dof = 3.94
p
a^{4-p} B
N_f=2 Sextet, β = 3.25 MCMC
68.27 %
95.45 %
B = 64(12)
p =4.410(53)
χ^2/Dof = 14.5
p
a^{4-p} B