QED corrections to pion and kaon leptonic decay rates

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Outline

- Introduction
- Work in progress: QED corrections to leptonic decay processes.
- Progress towards an All-to-All set up and creation of meson fields.
- How we plan to use domain wall fermion meson fields to calculate the leptonic decay correlators at the physical point.
- Outlook

Motivation

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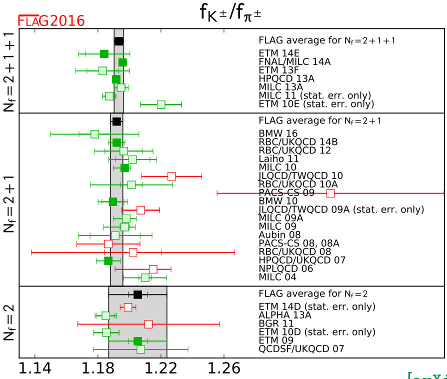

[arXiv:1607.00299v1]

- Dominant contributions to error budgets are from isospin breaking effects.

Leptonic decays

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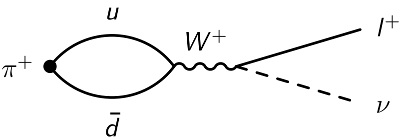

- The leading order (α^0) decay rate is:

$$\Gamma(\pi^+ \to l^+\nu) = \frac{m_\pi}{8\pi} G_F^2 |f_\pi|^2 |V_{ud}|^2 m_l^2 \left(1 - \frac{m_l^2}{m_\pi^2}\right)^2$$

$$\langle 0 | \bar{d}\gamma_\mu\gamma_5 u | \pi^+(p)\rangle = i p_\mu f_{\pi^+}$$

- IR finite order α decay rate:

$$\Gamma_\alpha = \Gamma_0 + \Gamma_1$$

[Bloch and Nordsieck (1937)] [N.Carrasco et al, Phys.Rev.D91(2015)no.7,07450]

  - Γ_0: Order alpha corrections without a final state photon.
  - Γ_1: Order alpha corrections with a final state photon.

QED Corrections

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- The order alpha correction can be calculated by using a perturbative approach: [G.M.de Divitiis et al.Phys.Rev.D87(2013)114505]

$$\langle O \rangle = \langle O \rangle_0 + \frac{e^2}{2}\frac{\partial^2}{\partial e^2}\langle O \rangle \bigg|_{e=0} + O(\alpha^2)$$

- If the operator is α independent then the correction has the form:

$$\langle O \rangle = \langle O \rangle_0 - \frac{e^2 q_f q_{f'}}{2}\langle O V^c_\mu(x) V^c_\nu(y) \rangle_0 \Delta_{\mu\nu}(x-y) - \frac{(eq_f)^2}{2}\langle O T_\mu(x) \rangle_0 \Delta_{\mu\mu}$$

[G.M.de Divitiis et al.Phys.Rev.D87(2013)114505]

- We use QEDL with photons in the Feynman gauge:

$$\Delta_{\mu\nu}(x-y) = \delta_{\mu\nu}\frac{1}{N}\sum_{k,\vec{k}\neq 0}\frac{e^{ik.(x-y)}}{\hat{k}^2}$$

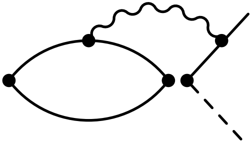
- Photon propagator generated by inserting stochastic photons:

$$\Delta_{\mu\nu}(x-y) = \langle A_\mu(x) A_\nu(y) \rangle$$

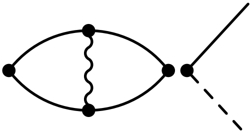
[D.Guisti et al Phys.Rev.Lett.120(2018)072001]

Γ_0 Connected Diagrams

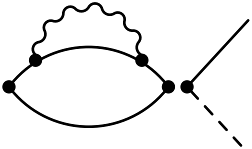
- The connected diagrams that contribute to the order α QED correction to leptonic decays:



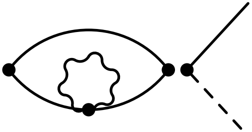
Lepton coupling diagram



Exchange diagram



Self-energy diagram



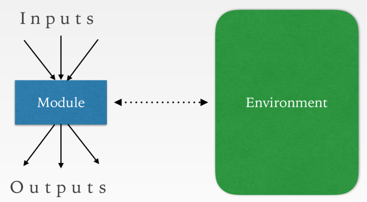
Tadpole diagram

Implementation and Production Plan

Implementation:

- Hadrons:
  - Grid-powered Workflow Management System for lattice calculations
  - High modularity
  - Automatic Scheduling
- GRID: www.github.com/paboyle/Grid

[Antonin Portelli]



[Peter Boyle et al]

Plan:

- Solve for the correlators at physical point (physical pion mass).
- $48^3 \times 96$ Lattice, $a^{-1} = 1.73$ GeV [T. Blum et al.Phys.Rev.D93 (2016) no.7,074505]
- Use 2000 eigenvectors we have generated to deflate the solves.

Deflation

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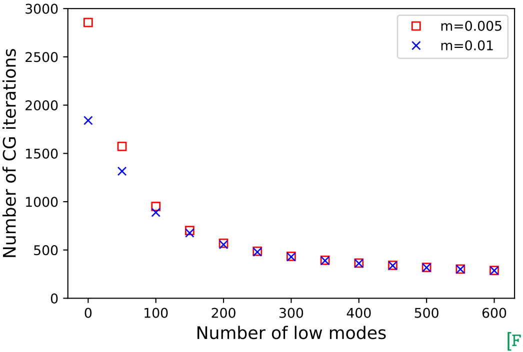
- Using deflation we are seeing a factor 10 decrease CG iterations required per solve on the unphysical 24^3 ensembles.



[Fionn Ó hÓgáin]

- We see a ≈ 20 speedup at physical quark masses with 2000 eigenvectors on the $48^{\times}96$ lattice using z-Möbius DWF.

Lepton coupling diagram test results

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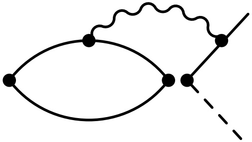
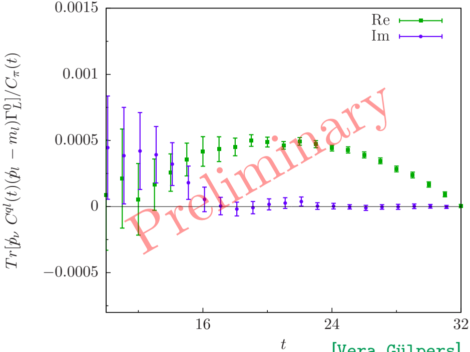
- Test run: $M_\pi = 340$ MeV; $a^{-1} = 1.78$ GeV; $2+1$ flavour ; Electro-quenched.
- Self energy, exchange & tadpole diagram calculated in previous project. [P.Boyle et al 10.1007/JHEP09(2017)153]
- First look at lepton coupling decay correlator:



Lepton coupling diagram



[Vera Gülpers]

All to All Propagator and the meson field

- All to all propagator:

$$D_{A2A}^{-1}(x,y) = \sum_{i=0}^{N_l+N_h} v_i(x) w_i^\dagger(y) = \sum_{l=0}^{N_l} v_l(x) w_l^\dagger(y) + \sum_{h=N_l}^{N_l+N_h} v_h(x) w_h^\dagger(y)$$

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- Low modes (from eigenvectors):

$$v_l(x) = \phi_l(x)$$
$$w_l(x) = \phi_l(x)/\lambda_l$$

- High modes (from stochastic solves):

$$v_h(x) = D^{-1}\eta_h(x)$$
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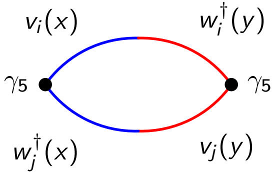
- Two point function:



- Meson Field:

$$\Pi_{ji}(t_x;\gamma_5) = \sum_{\vec{x}} w_j^\dagger(x)\gamma_5 v_i(x)$$

[J.Foley et al, CPC 172 (2005)0010-4655]

[M.Peardon et al,Phys.Rev.D.80.054506(2009)]

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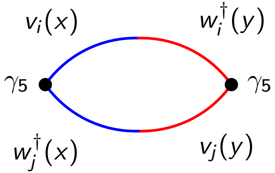

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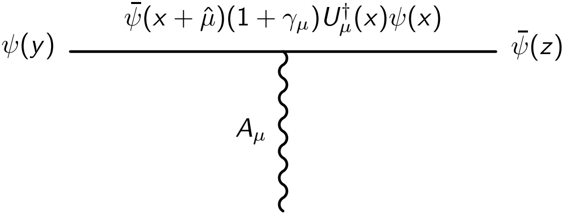
- 3,4,... pt functions can be made contracting the relevant meson fields with the correct gamma structure.

All to All conserved current meson field

- Take the Wilson Fermion vector current:

$$V^c_\mu(x) = \frac{1}{2}\left[\bar{\psi}(x+\hat{\mu})(1+\gamma_\mu)U^\dagger_\mu(x)\psi(x) - \bar{\psi}(x)(1-\gamma_\mu)U_\mu(x)\psi(x+\hat{\mu})\right]$$



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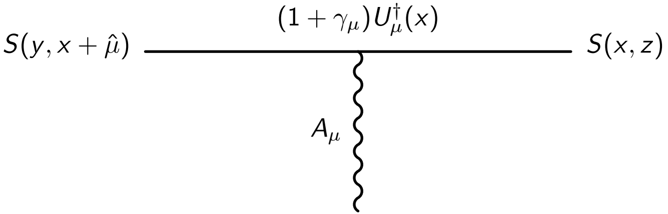

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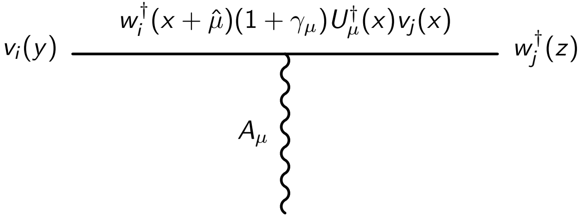

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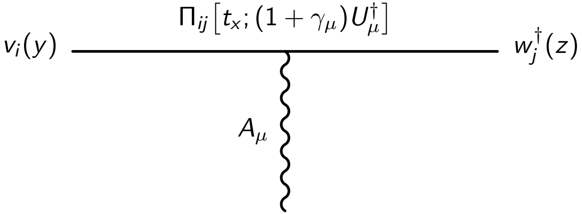

- Wilson fermion vector current meson field:

$$\Pi_{ij}(t_x) = \sum_{\vec{x},\mu}\frac{1}{2}\left[w^\dagger_i(x+\hat{\mu})(1+\gamma_\mu)U^\dagger_\mu(x)v_j(x) - w^\dagger_i(x)(1-\gamma_\mu)U_\mu(x)v_j(x+\hat{\mu})\right]$$

- Generalisation to 5D DWF current (work in progress)

All to All: Connected leptonic decay diagrams

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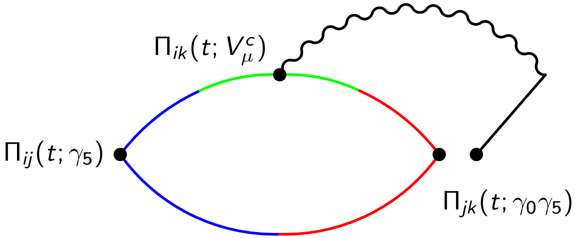
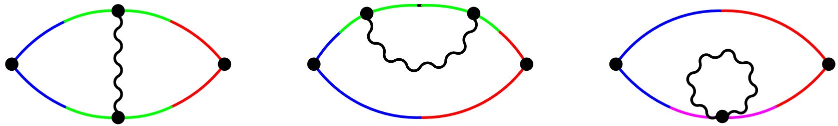
- Leptonic coupling correlator using meson fields:

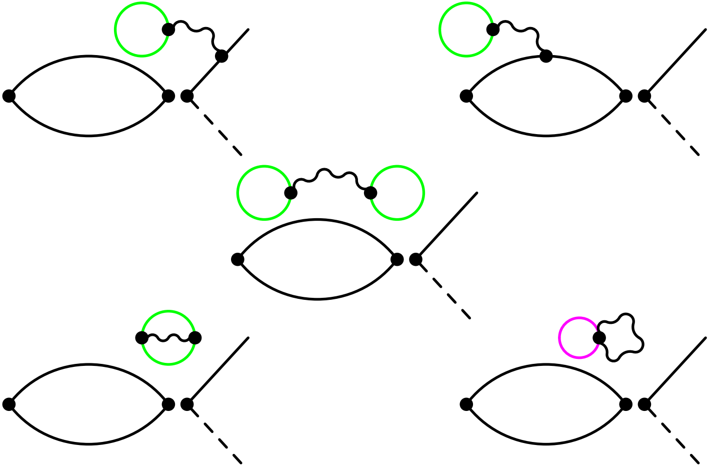


- Similarly the other diagrams that contribute to the decay rate can be split into meson fields:



All to All: Disconnected diagrams

- We can also construct the disconnected diagrams from the same meson fields:



- No extra inversions are required to calculate the disconnected diagrams!

Current status & Outlook

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Status:

- Testing implementation and analysis on $24^3 \times 64$ lattice.
- All to All set up into Hadrons in the process of testing.
- 2000 eigenvectors for physical mass ensemble generated and on disc.
- Optimizing $48^3 \times 96$ set up with an aim to start generating meson fields and form correlators offline.

Outlook:

- Calculate the isospin breaking correction to leptonic kaon/pion decays at the physical point. Using meson fields.
- Isospin breaking corrections to Semi-leptonic decay rates.
- Γ_1 leptonic decays with final state photons.
- Use stored meson fields to form correlators for other processes.