

Calculating the ρ radiative decay width with lattice QCD

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in collaboration with:

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MOTIVATION

Photoproduction:

- Study the properties of the strong interaction
- Experimentally very precise
- Some experimental facilities where photoproduction is utilized:
 - CEBAF (Jlab)
 - ELSA (Bonn)
 - MAMI (Mainz)
 - ...

ρ :

- QCD-unstable
- Couples to $\pi\pi$ in P-wave with 99.9% Branching ratio [PDG]
- Elastic scattering below $\bar{K}K$ threshold
- Well understood and broadly studied in lattice QCD

[PACS, BMW*, Feng et al., Lang et al., Pelissier et al., HadSpec, RQCD, Bulava et al., Hu et al., Guo et al., ...]

Simplest light hadron process where photoproduction can be studied is:

$$\pi\gamma \rightarrow \rho \rightarrow \pi\pi$$

QED – perturbative

QCD – nonperturbative (use lattice QCD)

[HadSpec]



OUTLINE

- About the photoproduction process
- What are the relevant quantities
- Our setup and the workflow
- The transition amplitude parameterization
- ... and how it looks ...
- Analytical continuation
- Briefly on systematics
- The Radiative decay width

ABOUT THE $\pi\gamma \rightarrow \pi\pi$ PROCESS

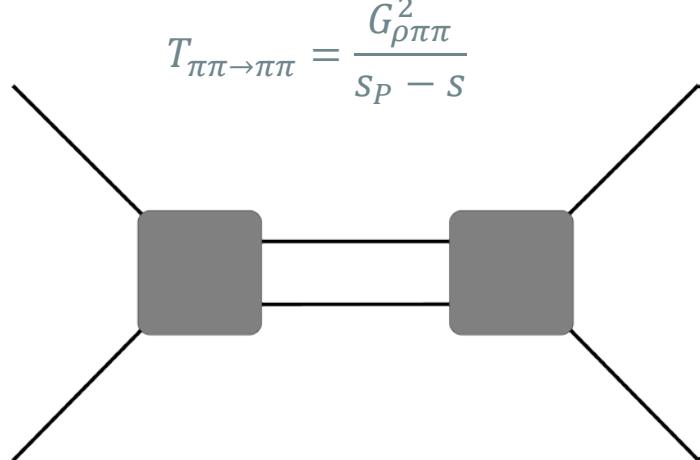
multi-hadron transition matrix element:

$$\langle \pi \pi | J_\mu(0) | \pi \rangle = \frac{2 i V_{\pi\gamma \rightarrow \pi\pi}(q^2, s)}{m_\pi} \epsilon^{\nu\mu\alpha\beta} \epsilon_\nu(P, m) (p_\pi)_\alpha P_\beta$$

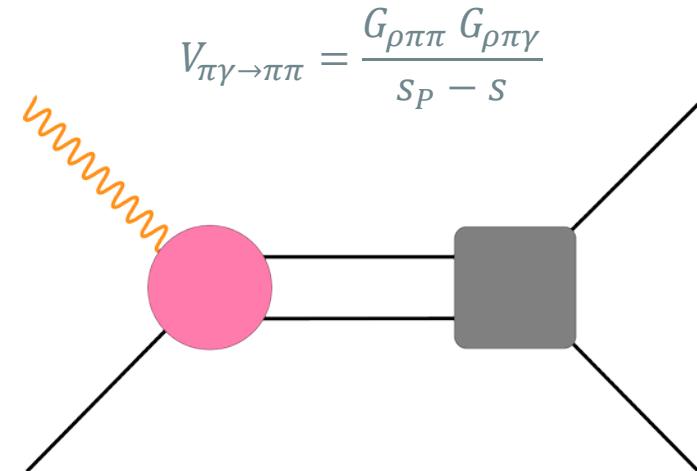
Momentum transfer: $q^2 = (p_\pi - P)^2$

Invariant mass: $\sqrt{s} = 2 \sqrt{m_\pi^2 + k^2}$

$\pi\pi \rightarrow \pi\pi$ scattering in vicinity of s_P :



$\pi\gamma \rightarrow \pi\pi$ scattering in vicinity of s_P :



[Briceno and Hansen, Hofferichter et al.]

ABOUT THE $\pi\gamma \rightarrow \pi\pi$ PROCESS

Scattering amplitude:

$$T_{\pi\pi \rightarrow \pi\pi} = \frac{16\pi \sqrt{s}}{k} \frac{1}{\cot \delta - i}$$

Breit-Wigner: $\cot \delta = \frac{m_R^2 - s}{\sqrt{s} \Gamma}$

BW I : $\Gamma_I = \frac{g_{\rho\pi\pi}^2}{6\pi} \frac{k^3}{s}$

BW II : $\Gamma_{II} = \frac{g_{\rho\pi\pi}^2}{6\pi} \frac{k^3}{s} \frac{1 + (k_R r_0)^2}{1 + (k r_0)^2}$

(Blatt-Weisskopf barrier factor)

ρ pole position in complex s plane:

$$s_P = m_R^2 + i m_R \Gamma_R$$

Photoproduction amplitude (Watson):

$$V_{\pi\gamma \rightarrow \pi\pi} = \sqrt{\frac{16\pi}{k \Gamma}} \frac{F(q^2, s)}{\cot \delta - i}$$

Resonant form factor (analytic cont.):

$$F_{\pi\gamma \rightarrow \rho}(q^2) = F(q^2, s = s_P)$$

photocoupling:

$$G_{\rho\pi\gamma} = F_{\pi\gamma \rightarrow \rho}(0)$$

Radiative decay width: [PDG]

$$\Gamma(\rho \rightarrow \pi\gamma) = \frac{2\alpha}{3} \left(\frac{m_\rho^2 - m_\pi^2}{2m_\rho} \right)^3 \frac{|G_{\rho\pi\gamma}|^2}{m_\pi^2}$$

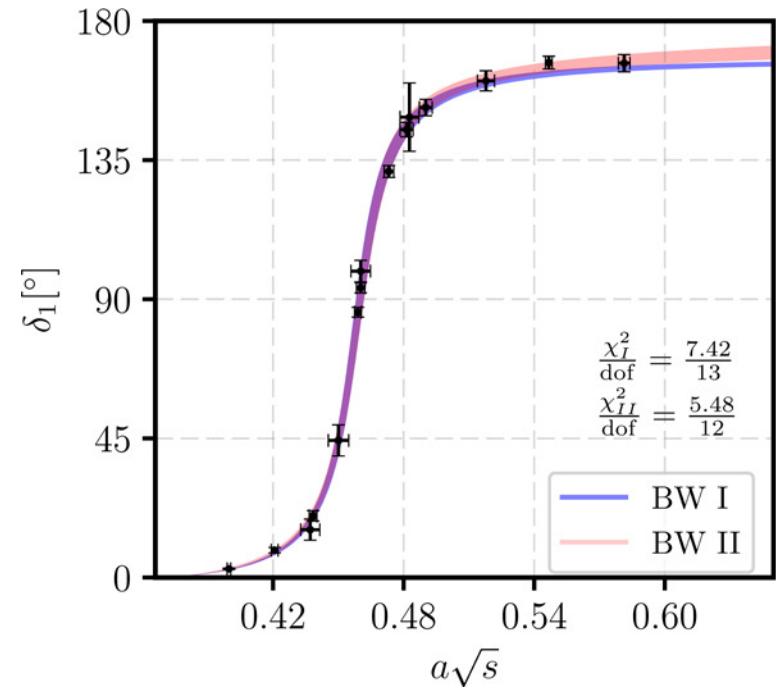
LATTICE GAUGE ENSEMBLE

- $N_f = 2 + 1$ clover-Wilson fermions
- Isotropic lattice by K. Orginos et al.
- $m_\pi L \approx 5.8$
- ρ unstable

Label	$N_s^3 \times N_t$	a [fm]	L [fm]	m_π [MeV]	N_{config}
C13	$32^3 \times 96$	0.11403(77)	3.649(25)	317	1041

Determined ρ parameters
 [Alexandrou et al. PRD96 2017]

Quantity	BW I	BW II
m_R	0.4609(16)(14)	0.4603(16)(14)
$g_{\rho\pi\pi}$	5.69(13)(16)	5.77(13)(13)
r_0^2	/	9.6(5.9)(3.7)



ON THE WORKFLOW

1. Determine finite volume matrix elements at well defined (q^2, s)
2. Map the finite volume matrix elements to infinite volume matrix elements
3. Determine infinite volume transition amplitude $V_{\pi\gamma \rightarrow \pi\pi}$ at discrete (q^2, s)
4. Parametrize $V_{\pi\gamma \rightarrow \pi\pi}$ and fit
5. Analytically continue to pole in s
6. Determine the radiative decay width

OPTIMIZED THREE-POINT FUNCTIONS

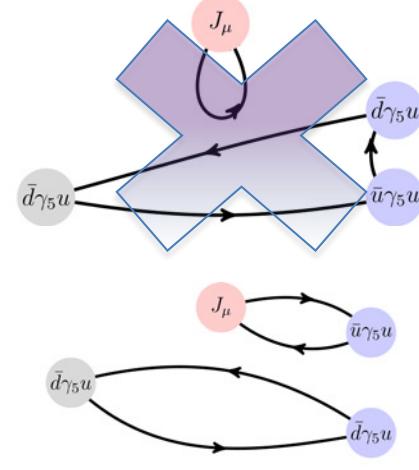
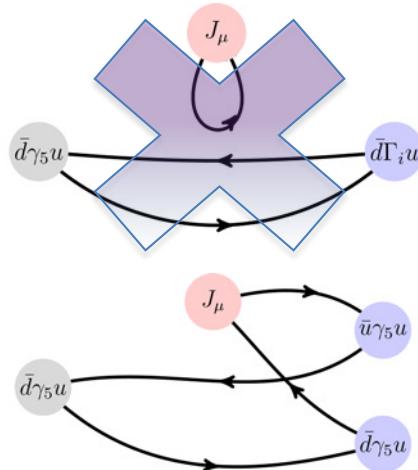
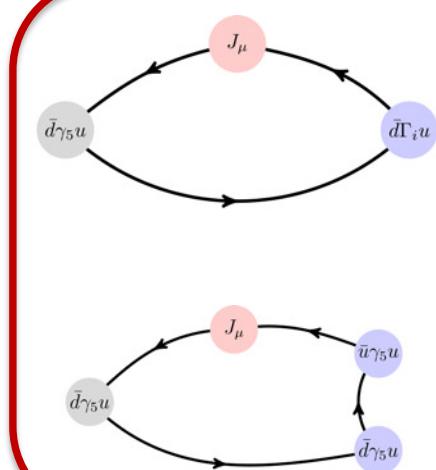
$$\Omega_{3,\mu,n}^{\vec{p}_\pi, \vec{P}, \Lambda, r}(t_\pi, t_J, t_{\pi\pi}, t_0) = v_i^{n, \vec{P}, \Lambda}$$

$$C_{3,\mu,i}^{\vec{p}_\pi, \vec{P}, \Lambda, r}(t_\pi, t_J, t_{\pi\pi})$$

$$C_{ij}^{\vec{P}, \Lambda, r}(t) v_j^{n, \vec{P}, \Lambda} = \lambda_n^{\vec{P}, \Lambda}(t, t_0) C_{ij}^{\vec{P}, \Lambda, r}(t_0) v_j^{n, \vec{P}, \Lambda}$$

$$v_i^{n, \vec{P}, \Lambda} + C_{ij}^{\vec{P}, \Lambda, r}(t_0) v_i^{m, \vec{P}, \Lambda} = \delta_{nm}$$

[Dudek et al., Becirevic et al., Shultz et al.]



$$\left| \left\langle \pi, \vec{p}_\pi \middle| J_\mu(0, \vec{q}) \middle| n, \vec{P}, \Lambda, r \right\rangle_{FV} \right|^2 = 4 E_n^{\vec{P}, \Lambda} E_\pi^{\vec{p}_\pi} \frac{\Omega_{3,\mu,n}^{\vec{p}_\pi, \vec{P}, \Lambda, r}(t_\pi, t_J, t_{\pi\pi}, t_0) \Omega_{3,\mu,n}^{\vec{p}_\pi, \vec{P}, \Lambda, r} + (t_\pi, t_{\pi\pi} + t_\pi - t_J, t_{\pi\pi}, t_0)}{C_\pi^{\vec{p}_\pi}(\Delta t) \lambda_n^{\vec{P}, \Lambda}(\Delta t, t_0)}$$

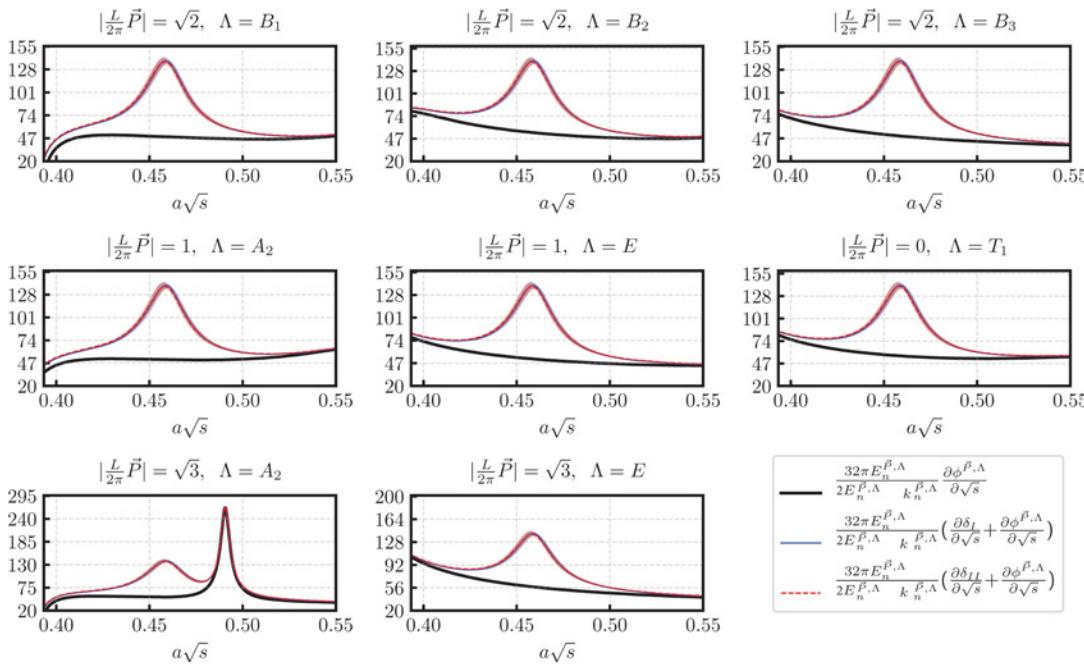
FROM FINITE VOLUME TO INFINITE VOLUME

$$\left| \frac{\langle \pi, \vec{p}_\pi | J_\mu(0) | s, q^2; \vec{P}, \Lambda, r \rangle_{IV}}{\langle \pi, \vec{p}_\pi | J_\mu(0, \vec{q}) | n, \vec{P}, \Lambda, r \rangle_{FV}} \right|^2 = \frac{1}{2 E_n^{\vec{P}, \Lambda}} \frac{16 \pi \sqrt{s_n^{\vec{P}, \Lambda}}}{k_n^{\vec{P}, \Lambda}} \left(\frac{\partial \delta}{\partial E} + \frac{\partial \phi^{\vec{P}, \Lambda}}{\partial E} \right) \Big|_{E=E_n^{\vec{P}, \Lambda}}$$

$$\cot \delta = \cot \phi^{\vec{P}, \Lambda}$$

[Lellouch and Lüscher,
Lin et al., Briceno et
al., Briceno and
Hansen, Meyer,
Bernard et al., Christ
et al., Agadjanov et
al., Detmold et al.]

$$\cot \delta = \frac{m_R^2 - s}{\sqrt{s} \Gamma}$$



At each kinematic point (s, q^2)
determine

$$V_{\pi\gamma \rightarrow \pi\pi}$$

from

$$\left| \langle \pi, \vec{p}_\pi | J_\mu(0) | s, q^2; \vec{P}, \Lambda, r \rangle_{IV} \right|$$

and associated

Lorentz Decomposition of the
vector $\langle PS | V | V \rangle$ matrix element.

THE TRANSITION AMPLITUDE: parameterization

Transition amplitude with $\pi\pi$ Breit-Wigner:

$$V_{\pi\gamma \rightarrow \pi\pi}(q^2, s) = \sqrt{\frac{16\pi s\Gamma}{k}} \frac{F(q^2, s)}{m_R^2 - s - i\sqrt{s}\Gamma}$$

Expand F in :

- $S = \frac{s-m_R^2}{m_R^2}$
- $z = \frac{\sqrt{t_+-q^2} - \sqrt{t_+-t_0}}{\sqrt{t_+-q^2} + \sqrt{t_+-t_0}}$

[Boyd et al.,
Bourelly et al.]

Factor out the pole, located at m_P , in q^2 :

$$F(q^2, s) = \frac{\sum_{n,m} A_{nm} z^n S^m}{1 - \frac{q^2}{m_P^2}}$$

Systematic truncation in sum:

- F1: $n + m \leq K$
- F2: $n \leq N, m \leq K - n$
- F3: $n \leq N, m \leq M$

Fit $V_{\pi\gamma \rightarrow \pi\pi}$ with systematic
truncations in F and investigate the
results

THE TRANSITION AMPLITUDE: the fit result

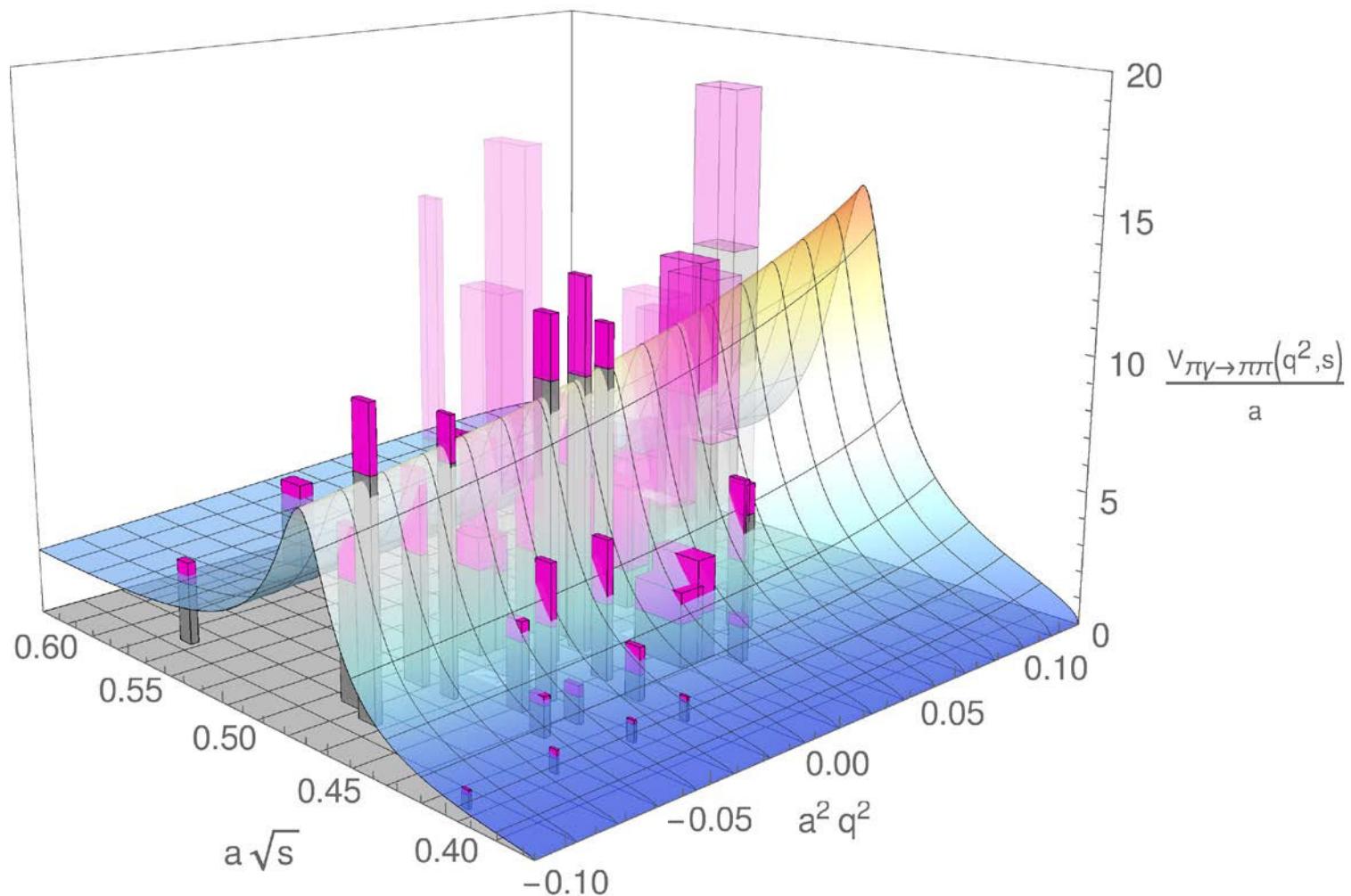
parameterization
BWI F1 K2
BWI F1 K3
BWI F2 N1 K2
BWI F2 N1 K3
BWI F3 N1 M1
BWI F3 N1 M2
BWI F3 N2 M2
BWI F3 N2 M3
BWII F1 K2
BWII F2 N1 K2
BWII F2 N1 K3
BWII F3 N1 M2
BWII F3 N1 M3
BWII F3 N2 M3

- Investigated all parameterizations with $K, N, M \leq 5$
- Fitted to 48 kinematic points
- Keep only good parametrizations
 - $\frac{\chi^2}{dof} \leq 1.1$
 - σ_p is of sane value
(i.e. we are not probing a flat direction)
- 8 parameterizations with **BW I**
- 6 parameterizations with **BW II**

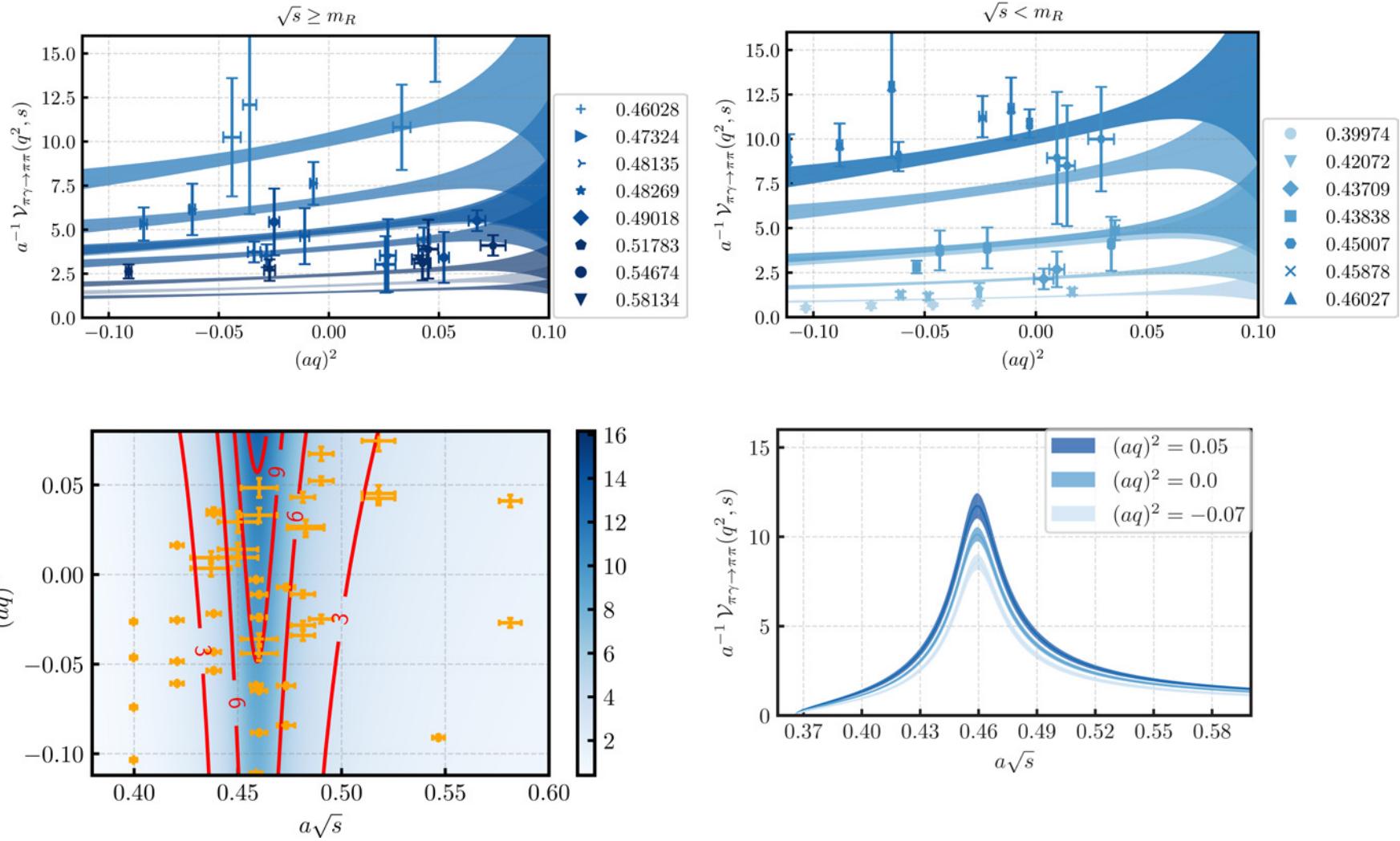
Choose BWII F1 K2 as central value

Use the others for systematical check of consistency.

THE TRANSITION AMPLITUDE: BWII F1 K2



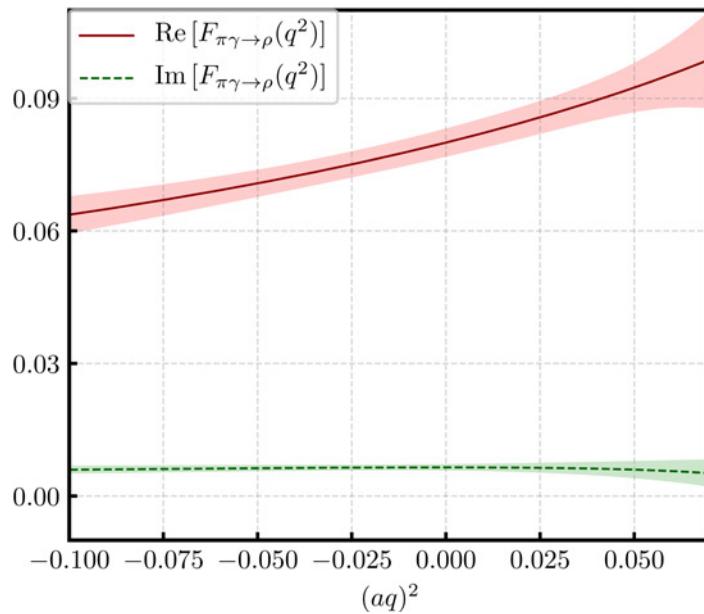
THE TRANSITION AMPLITUDE: BWII F1 K2



ANALYTIC CONTINUATION

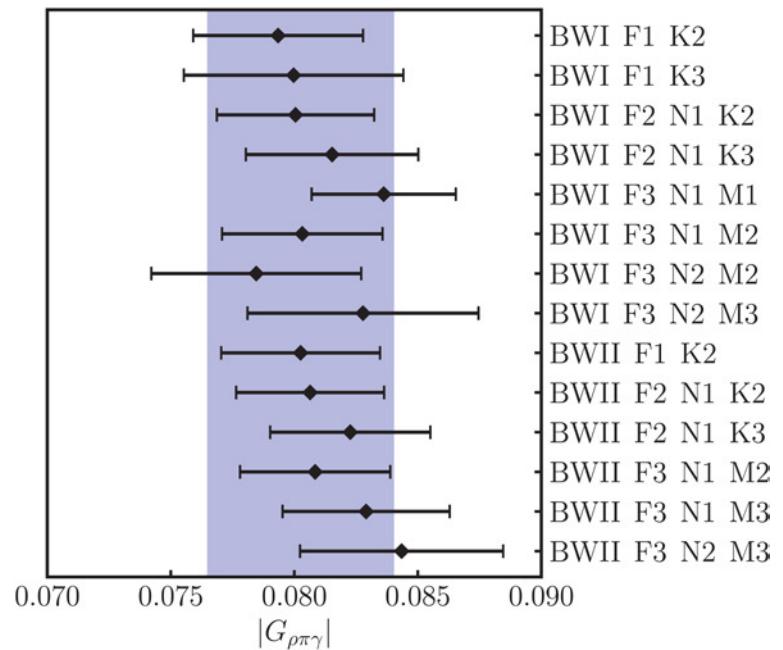
Evaluate the transition form factor at the ρ pole:

$$F_{\pi\gamma \rightarrow \rho}(q^2) = F(q^2, s = s_p)$$



Photocoupling

$$G_{\rho\pi\gamma} = F_{\pi\gamma \rightarrow \rho}(0)$$

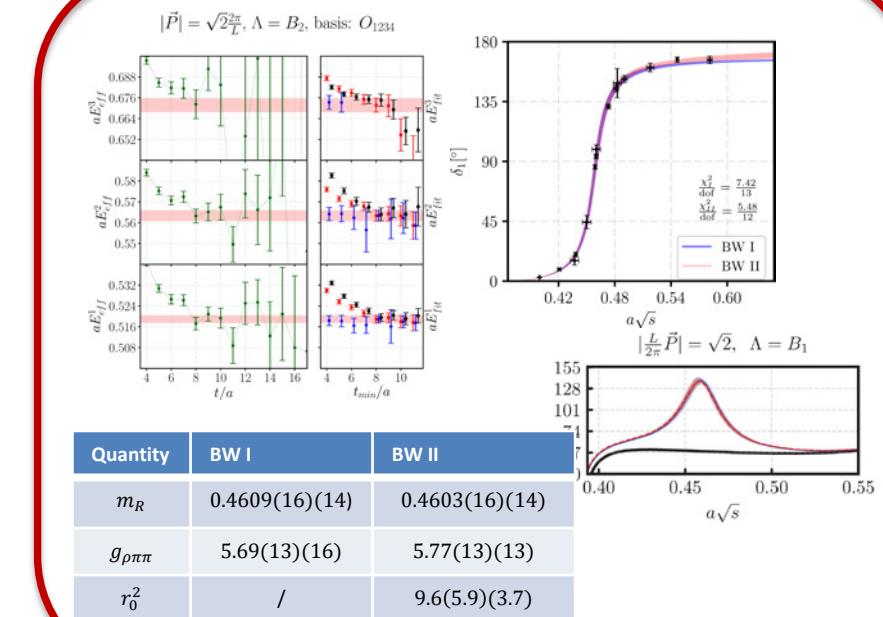
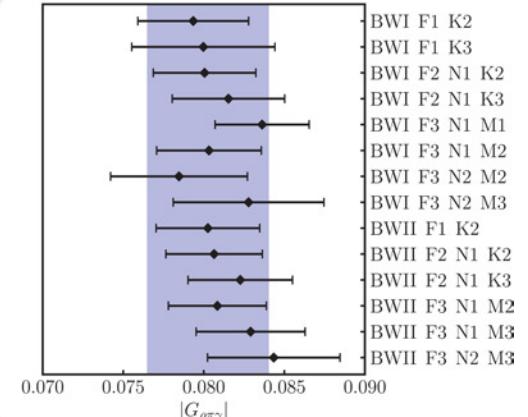
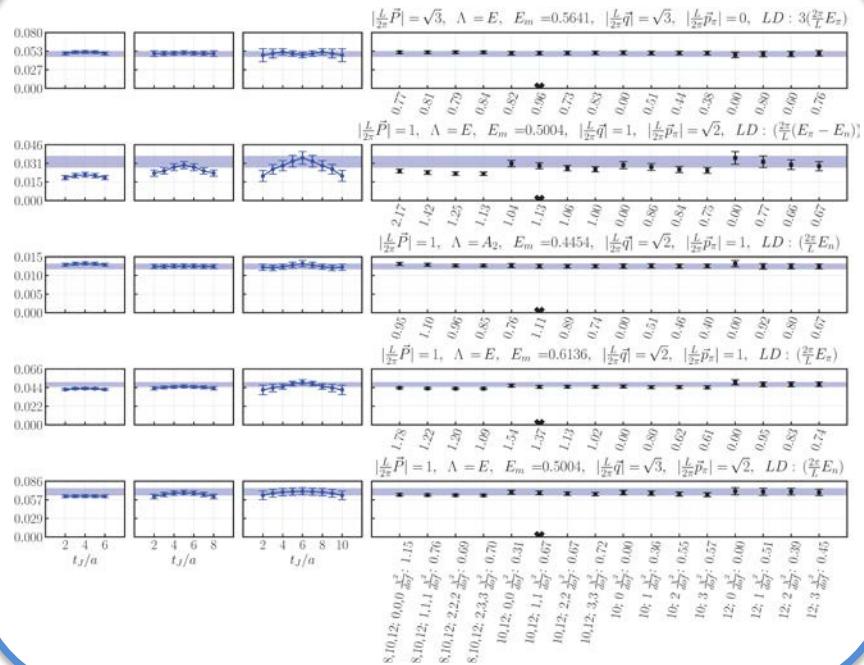


Parameterization uncertainty

$$\sigma_{par} = \sqrt{\frac{\sum_{i=1}^{N=14} (x_i - x_{chosen})^2}{N-1}}$$

PHOTOCOUPING: systematical uncertainties

$$G_{\rho\pi\gamma} = 0.802 \quad (32) \quad (20)$$



OBSERVABLE: radiative decay width

$$\Gamma(\rho \rightarrow \pi\gamma) = \frac{2 \alpha}{3} \left(\frac{m_\rho^2 - m_\pi^2}{2 m_\rho} \right)^3 \frac{|G_{\rho\pi\gamma}|^2}{m_\pi^2}$$

In our lattice calculation light quark masses were higher than physical:

- $m_\pi \approx 316.6$ MeV
- $m_\rho \approx 797.6$ MeV

Which leads to unphysical kinematics.

Assume:

- Photocoupling $|G_{\rho\pi\gamma}|$ m_π independent
(a strong assumption)

And use physical values for:

- $m_\pi \approx 139.6$ MeV
- $m_\rho \approx 775.1$ MeV

$$\Gamma(\rho \rightarrow \pi\gamma) = 84.2 (6.7) (4.3) \text{ keV}$$

$$\Gamma(\rho \rightarrow \pi\gamma)_{EXP} = 68 (7) \text{ keV} \quad [\text{PDG}]$$

CONCLUSIONS

- Lattice QCD calculation at $m_\pi \approx 320$ MeV and $L \approx 3.6$ fm
- Utilized cutting edge methods to determine $\pi\gamma \rightarrow \pi\pi$ transition amplitude in $\pi\pi$ P-wave $I = 1$ channel around the ρ resonance and at positive and negative photon virtuality
- Performed analytic continuation to the ρ pole and determined the photocoupling $G_{\rho\pi\gamma}$
- Determined the ρ radiative decay width

Future

- Apply formalism to heavy meson decays
- Investigate the baryon sector
- Investigate pion mass dependence
- Continuum and chiral extrapolations

$K\pi$: G. Rendon @Thursday 9:30

$N\pi$: G. Silvi @Friday 16:30

$N\pi$: S. Paul @Friday 16:50

THANK YOU
😊



BACKUP

