Gauge-fixing with Compact Lattice Gauge Fields

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Intro & Background

- Gauge fixing not necessary for Wilson's standard lattice gauge theory with compact algebra-valued gauge fields

- Problems in Chiral gauge theories with manifestly local lattice fermions

    - These lattice fermions break chiral symmetry explicitly on lattice

  Lattice chiral gauge theories break gauge invariance

    - Functional integration takes place also along the gauge orbit (i.e. in the longitudinal direction or the direction of gauge transformation)

  Longitudinal gauge degrees of freedom (*lgdof*) couple to the physical degrees of freedom in the terms that break gauge invariance

For example $\kappa A_\mu^2 \sim -\kappa \left(U_{x\mu} + U_{x+\mu}^\dagger \right) \rightarrow -\kappa \left(\phi_x^\dagger U_{x\mu} \phi_{x+\mu} + \phi_{x+\mu}^\dagger U_{x\mu}^\dagger \phi_{x\mu} \right)$

where $\phi_x \in G$ are radially frozen (random) scalar fields

Hence a theory with gauge-non-invariant terms

→ Physical fields + *lgdof* (scalar fields)

The new action $S(\phi_x^\dagger U_{x\mu} \phi_{x+\mu})$

is now gauge-invariant under the extended local transformations:

$$U_{x\mu} \to h_x U_{x\mu}\, h^\dagger_{x+\mu}, \quad \phi_x \to h_x \phi_x, \quad h_x \in G$$

$$S_V(U_{x\mu}) \equiv S_H(U_{x\mu},\, \phi_x)$$

The *reduced model* is defined by the action on the trivial orbit: $U_{x\mu} \to \phi_x^\dagger\, \mathbb{I}\, \phi_{x+\mu}$

Reduced model has a global symmetry: $\phi_x \to h\phi_x \text{ with } h \in G$

Early proposals of Lattice Chiral Gauge Theory

- Smit-Swift / Wilson-Yukawa model
- Domain-wall waveguide model

all failed because of interaction of the ***lgdof*s** with fermions.

Remedy: Gauge fix the *lgdof*s to control their dynamics

Gauge invariant observables remains intact provided the following integral

$$\int \mathcal{D}\phi \, \mathcal{D}C \, \mathcal{D}\overline{C} \, \mathcal{D}B \exp\left(-S_{\mathrm{GF}}[U_\mu^\phi, C, \overline{C}, B]\right)$$

over an orbit is a non-zero constant.

A no-go theorem for compact gauge fields

Neuberger 1987

Start with a manifestly gauge invariant lattice gauge theory, the Wilson way:

$$Z = \int \mathcal{D}U \exp(-S_{\mathrm{GI}}[U])$$

One then inserts, *a la* Fadeev-Popov: $Z_{\mathrm{GF}} = \int \mathcal{D}\phi\, \mathcal{D}C\, \mathcal{D}\overline{C}\, \mathcal{D}B \exp\left(-S_{\mathrm{GF}}[U_\mu^\phi, C, \overline{C}, B]\right)$

$$\begin{aligned} S_{\mathrm{GF}} &= \sum_x 2t \left[-i\,\mathrm{tr}\left(BF(U)\right) + \mathrm{tr}\left(\overline{C}\,\delta_{\mathrm{BRST}} F(U)\right)\right] + \sum_x \xi g^2 \mathrm{tr}(B^2) \\ &= \sum_x 2t \left[\delta_{\mathrm{BRST}} \mathrm{tr}\left(\overline{C} F(U)\right)\right] + \sum_x \xi g^2 \mathrm{tr}(B^2) \end{aligned}$$

Z_{GF} is required to be independent of U, so that only a constant was inserted in Z

If this requirement is fulfilled, gauge-invariant correlation functions of the gauge-fixed theory are identical to those of the unfixed (manifestly gauge invariant) theory.

$$\frac{dZ_{\rm GF}}{dt} = \int \mathcal{D}\phi \mathcal{D}C \mathcal{D}\overline{C} \mathcal{D}B \, \delta_{\rm BRST} \left[\sum_x 2\,{\rm tr}(\overline{C} F(U)) \right]$$
$$= 0$$

Indeed $\quad Z_{\rm GF}|_{t=1} = Z_{\rm GF}|_{t=0} \quad$ independent of $\quad U$

But $\quad Z_{\rm GF}|_{t=0} = 0$

because the integrand is then devoid of any Grassmann variables

$$\Rightarrow Z_{\rm GF}|_{t=1} = Z_{\rm GF} = 0$$

Expectation value of any gauge-invariant operator has the indeterminate form

If pure Yang Mills on lattice cannot be non-perturbatively gauge fixed, there would be no hope for lattice Chiral Gauge Theories in the gauge-fixing approach.

Gauge-fixing with compact gauge fields

Cannot maintain BRST

Need to evade Neuberger's no-go theorem

For U(1) lattice gauge theory:

Break BRST explicitly, restore it by tuning counter-terms in the continuum limit and in the process decouple the *lgdof*s

For non-Abelian lattice gauge theory (e.g. SU(2)):

Modify BRST to gauge-fix only the coset space leaving a subgroup (U(1)) gauge invariant —> eBRST formalism

U(1) Lattice gauge theory with HD gauge fixing term

Golterman & Shamir 1997

$$S = S_{\rm W} + S_{\rm GS} + S_{\rm ct}$$

$$S_{\rm W} = \frac{1}{g^2} \sum_{x,\ \mu<\nu} \left(1 - {\rm Re}\, U_{{\rm P}\mu\nu}(x)\right)$$

$$S_{\rm GS} = \tilde{\kappa} \left(\sum_{xyz} \Box_{xy}(U)\, \Box_{yz}(U) - \sum_{x} B_x^2 \right)$$

$$S_{\rm ct} = -\kappa \sum_{x\,\mu} \left(U_{x\mu} + U^{\dagger}_{x\,\mu} \right)$$

$$\Box_{xy}(U) = \sum_{\mu} (\delta_{y,x+\mu} U_{x\mu} + \delta_{y,x-\mu} U^{\dagger}_{x-\mu,\mu} - 2\delta_{xy})$$

$$B_x = \sum_{\mu} (\mathcal{A}_{x-\mu,\mu} + \mathcal{A}_{x\mu})^2/4, \text{ with } \mathcal{A}_{x\mu} = {\rm Im} U_{x\mu}$$

The action has an unique absolute minimum at $U_{x\mu} = 1$

In the naive continuum limit, the HD gauge-fixing term $\longrightarrow$

$$\tilde{\kappa} g^2 \int d^4x (\partial_\mu A_\mu)^2 = (1/2\xi) \int d^4x (\partial_\mu A_\mu)^2$$

where $\xi = 1/(2\tilde{\kappa} g^2)$

➡ **Weak coupling perturbation theory near** $g = 0,\ \tilde{\kappa} \to \infty$ with $\xi \sim 1$

What happens near the WCPT point?

Expand action in powers of g , using constant field approximation:

$$V_{\rm cl}(A_\mu) = \kappa\left(g^2\sum_\mu A_\mu^2 + \ldots\right) + \frac{g^6}{2}\tilde{\kappa}\left\{\left(\sum_\mu A_\mu^2\right)\left(\sum_\mu A_\mu^4\right) + \ldots\right\}$$

→ **Critical surface:** $\kappa \equiv \kappa_{\rm FM-FMD}(g,\tilde{\kappa}) = 0$

where the gauge boson (photon) is rendered massless

$$\langle gA_\mu\rangle = \pm\left(\frac{|\kappa - \kappa_{\rm FM-FMD}|}{6\tilde{\kappa}}\right)^{1/4}, \quad \forall\mu \quad \text{for} \quad \kappa < \kappa_{\rm FM-FMD}$$

$$\langle gA_\mu\rangle = 0, \quad \forall\mu \quad \text{for} \quad \kappa \geq \kappa_{\rm FM-FMD}$$

The phase with the vector condensate is called the FMD phase

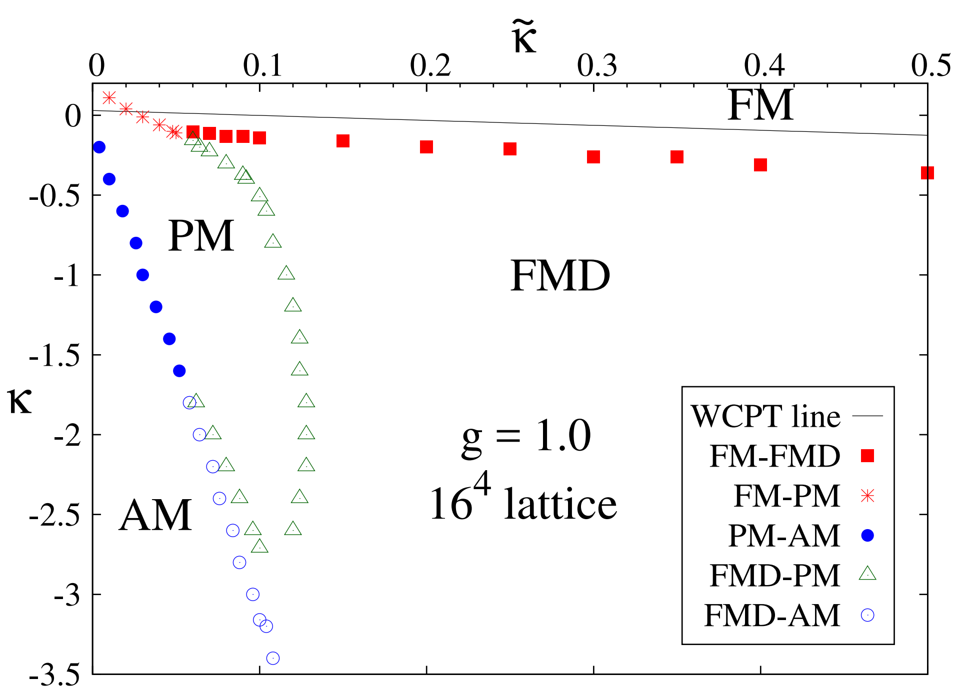
Continuum limit is to be taken at FM-FMD transition from within the FM phase where gauge symmetry is recovered and as a result the *lgdof*s decouple

The above picture is validated very well for weak gauge couplings

Observables for numerical study

$$E_{\mathrm{P}} = \frac{1}{6L^4}\left\langle \sum_{x,\mu<\nu} \mathrm{Re}\, U_{\mathrm{P}\mu\nu}(x) \right\rangle \qquad E_{\kappa} = \frac{1}{4L^4}\left\langle \sum_{x,\mu} \mathrm{Re}\, U_{x\mu} \right\rangle \qquad V = \left\langle \sqrt{\frac{1}{4}\sum_{\mu}\left(\frac{1}{L^4}\sum_{x}\mathrm{Im}\, U_{x\mu}\right)^2} \right\rangle$$

In addition, photon and scalar propagators in momentum space



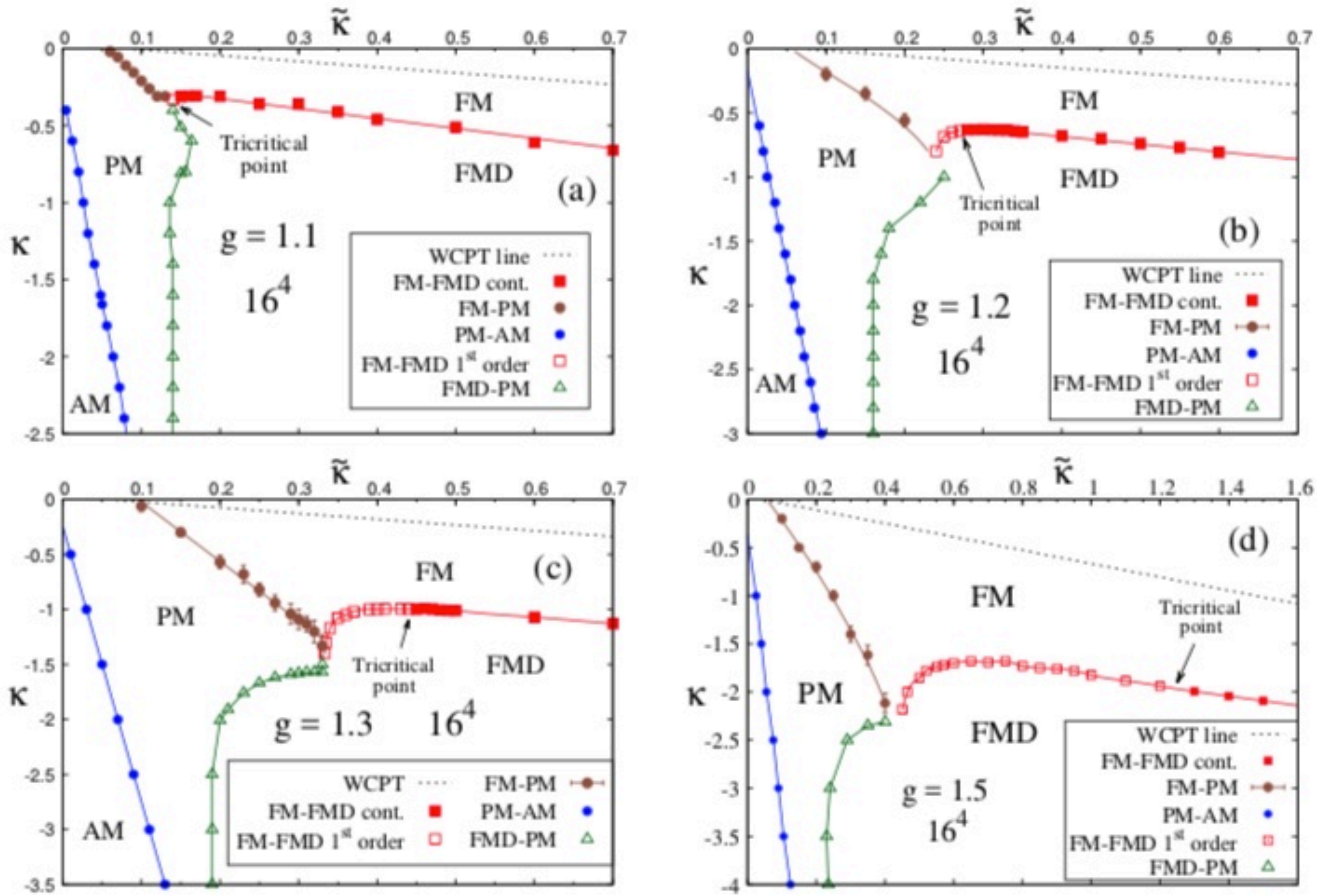
For weak gauge couplings $g \leq 1$
for large enough $\tilde{\kappa}$

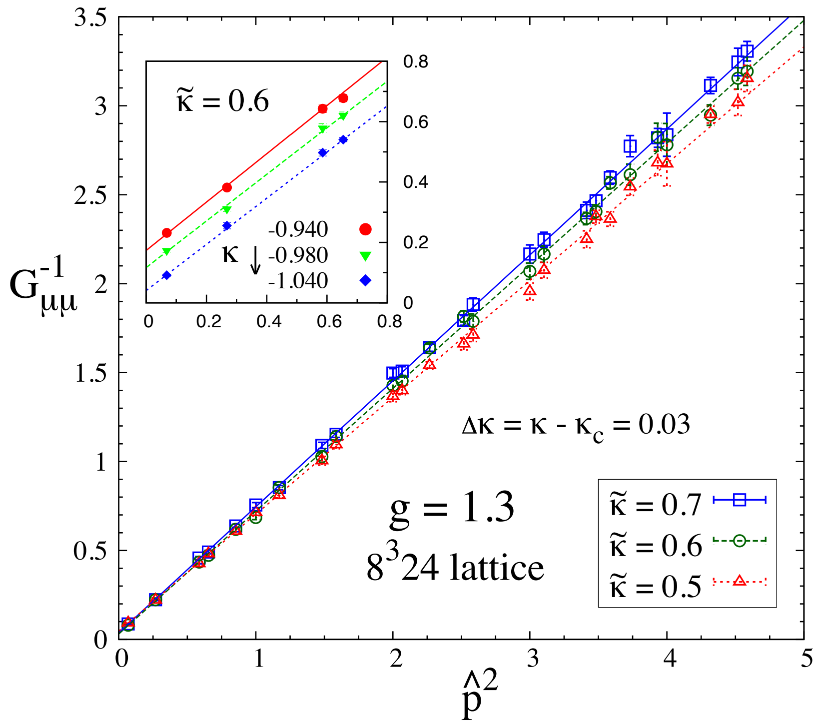
approaching FM-FMD from FM side
reveals free massless photons and
the *lgdof*s appear to be decoupled

Investigations of U(1) Wilson-Yukawa and domain-wall waveguide models in the reduced limit exhibit free chiral fermions

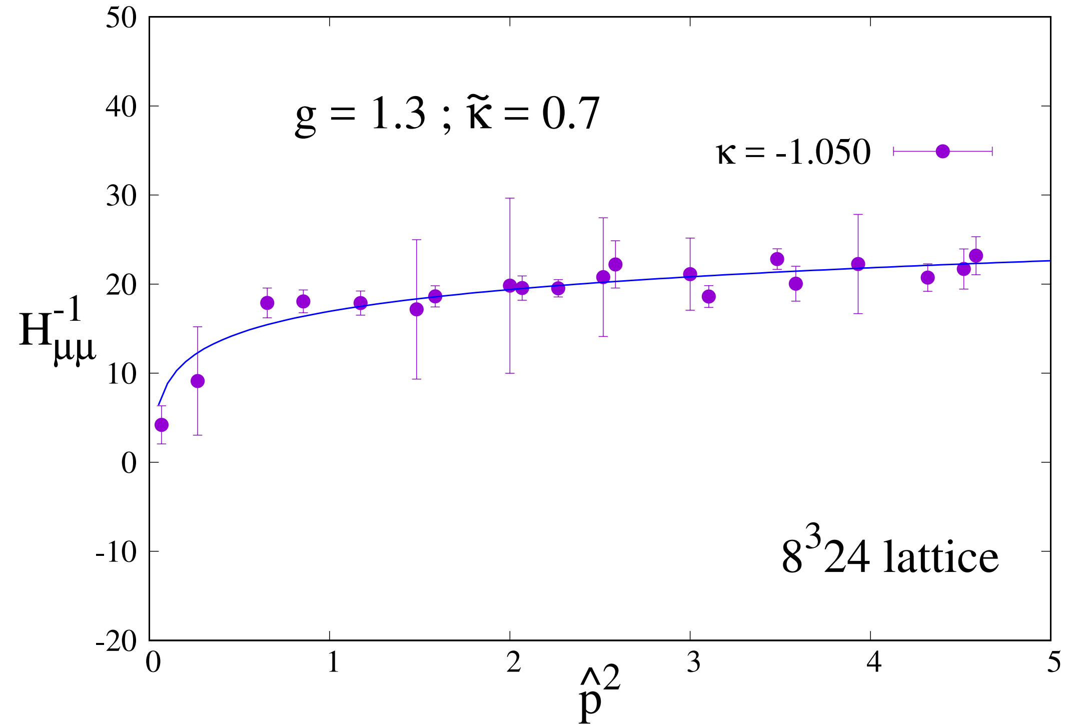
What happens at large gauge coupling?

AKD & Mugdha Sarkar 2016, 2017

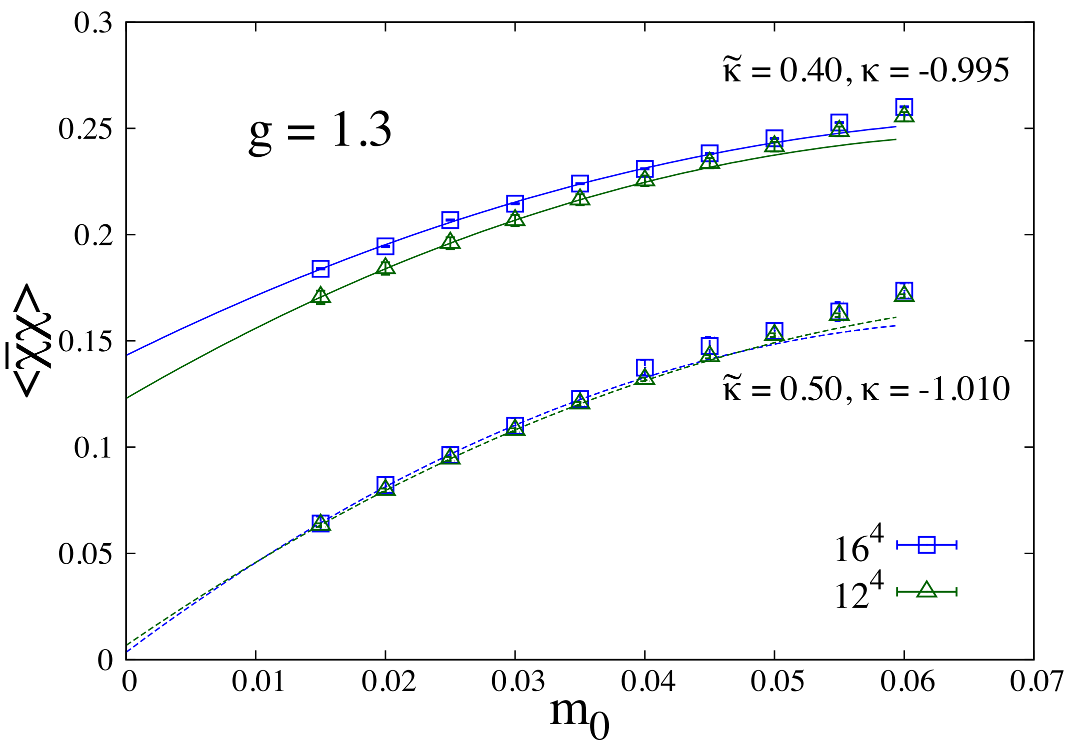




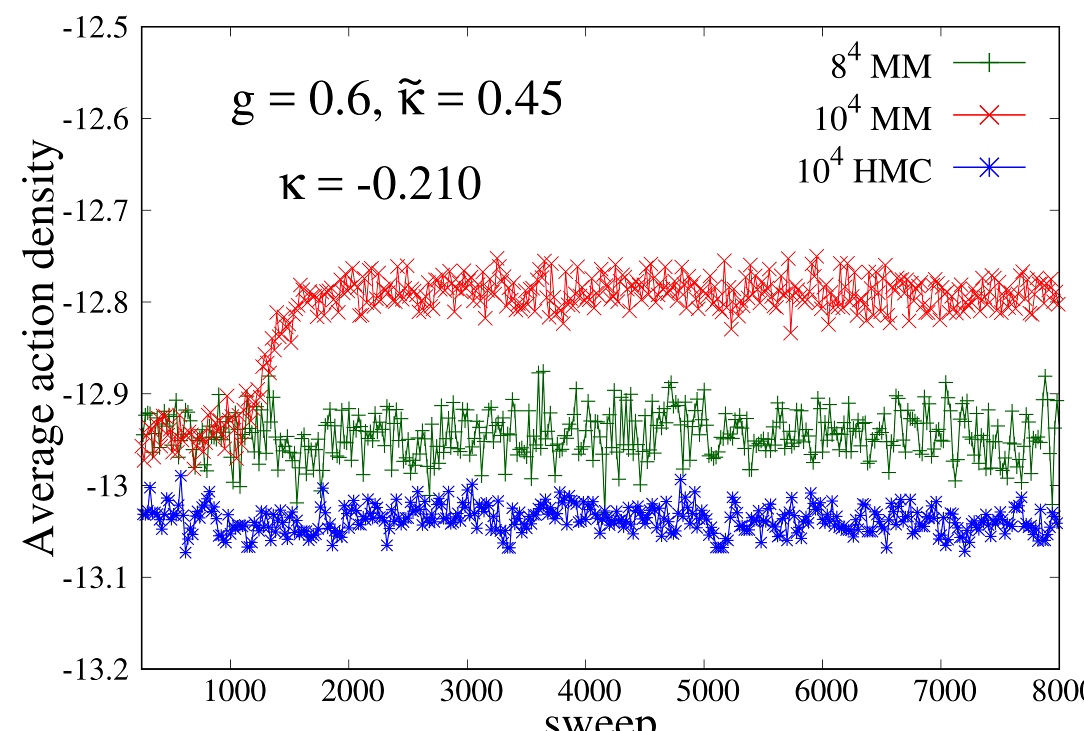
$G^{-1}_{\mu\mu}$
$\hat{p}^2$
$\tilde{\kappa} = 0.6$
κ ↓ -0.940 -0.980 -1.040
Δκ = κ - κ_c = 0.03
g = 1.3
8^3 24 lattice
κ̃ = 0.7
κ̃ = 0.6
κ̃ = 0.5



$H^{-1}_{\mu\mu}$
$\hat{p}^2$
g = 1.3 ; κ̃ = 0.7
κ = -1.050
8^3 24 lattice

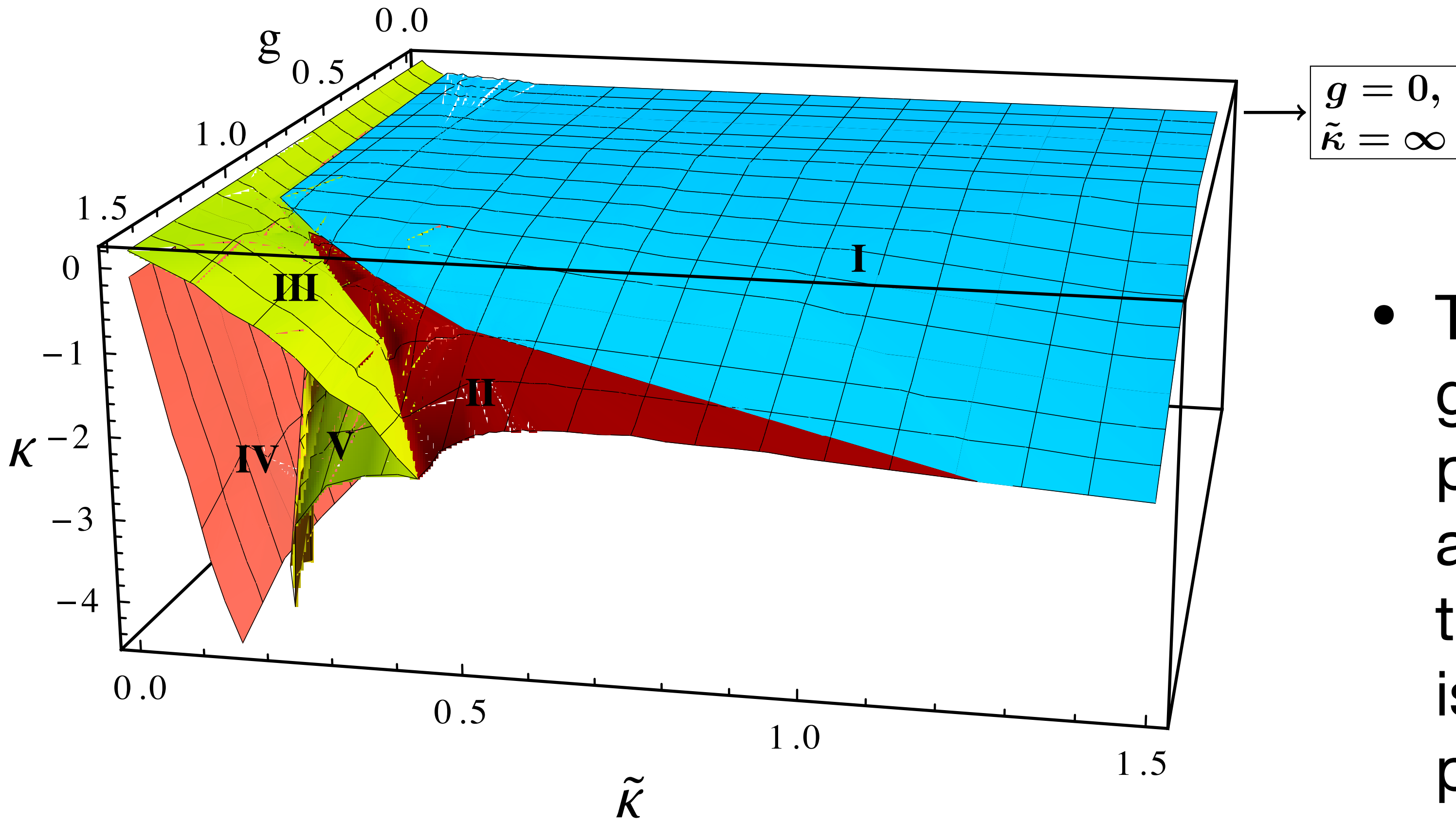


$\langle\bar{\chi}\chi\rangle$
m_0
g = 1.3
κ̃ = 0.40, κ = -0.995
κ̃ = 0.50, κ = -1.010
16^4
12^4



Average action density
sweep
g = 0.6, κ̃ = 0.45
κ = -0.210
8^4 MM
10^4 MM
10^4 HMC

Main Conclusions of the HD Abelian Gauge-fixing at large gauge couplings



- The physics at the large bare gauge couplings (free massless photons with *lgdof*s decoupled) appears to be the same as in the weak gauge couplings and is controlled by the perturbative point $g = 0, \ \tilde{\kappa} \to \infty$

- The tricritical line appears to be the only candidate for non-trivial physics

- Multihit Metropolis (MM) fails to produce faithful field configurations at larger values of $\tilde{\kappa}$, and HMC appears to do much better

Gauge fixed Yang Mills theory on lattice

Schaden 1999; Golterman & Shamir 2004, 2006, 2013, 2014

One way to evade the no-go theorem: Introduce equivariant BRST (**eBRST**)

Gauge fix only the coset space leaving minimally the Cartan subgroup unfixed

For example, for the SU(2) Yang Mills theory, gauge fix SU(2)/U(1) and the U(1) gauge invariance is left intact

Nilpotency of BRST is modified and a 4-ghost term appears

In addition to its importance in regard to

- lattice chiral gauge theory
- an alternate formulation of gauge theories

it is interesting to ask: if non-perturbatively the dynamics of the longitudinal sector can affect the physics of the transverse degrees of freedom, given that $g \text{ and } \tilde{g}$ are both asymptotically free $(\tilde{g}^2 = \xi g^2)$

eBRST: SU(2)/U(1) case

$$U_\mu = \exp(iV_\mu)$$

BRST

$$V_\mu = W_{\mu 1}\tau_1 + W_{\mu 2}\tau_2 + A_\mu \tau_3$$

$$C = C_1\tau_1 + C_2\tau_2 + C_3\tau_3$$

$$s\psi = -iC\psi$$

$$sV_\mu = \mathcal{D}_\mu(V)C$$

$$sC = -iC^2$$

$$s\overline{C} = -iB$$

$$sB = 0$$

$$s^2 = 0$$

eBRST

$$V_\mu = W_{\mu 1}\tau_1 + W_{\mu 2}\tau_2 + A_\mu \tau_3$$

$$C = C_1\tau_1 + C_2\tau_2$$

$$s\psi = -iC\psi$$

$$sW_\mu = \mathcal{D}_\mu(A)C$$

$$sA_\mu = i[W_\mu, C]$$

$$sC = -iC^2|_{SU(2)/U(1)} = 0$$

$$s\overline{C} = -iB$$

$$sB = [iC^2, \overline{C}]$$

$$s^2 = \delta_{U(1)}$$

The eBRST-invariant gauge-fixing action

$$S_{\mathrm{gf}}(U^\phi, C, \overline{C}) = \frac{1}{\xi g^2} \mathrm{tr}\left(F(U^\phi)\right)^2 + 2\mathrm{tr}(\overline{C} M(U^\phi) C) - 2\xi g^2 \,\mathrm{tr}\left(C^2 \overline{C}^2\right)$$

$$F \sim \mathcal{D}_\mu(A) W_\mu \qquad \tilde{\kappa} = \frac{1}{2\xi g^2} = \frac{1}{2\tilde{g}^2} = \frac{\tilde{\beta}}{2}$$

The gauge fixing partition function, a theory on the gauge orbit, does not depend on U and $\tilde{g}^2 = \xi g^2$

Now, because of the presence of the 4-ghost term, it is also not zero, thus evading the no-go

For any $U, \;\; Z_{\mathrm{GF}}(U, \xi g^2) \neq 0$ **defines a TFT**

INVARIANCE THEOREM

$$\langle \mathcal{O}(U) \rangle_{\mathrm{unfixed}} = \langle \mathcal{O}(U) \rangle_{\mathrm{eBRST}}$$

Restricting to expectation values of **gauge-invariant operators**, the eBRST gauge-fixed theory is rigorously equivalent to the unfixed theory

Now go to the trivial orbit ($U = 1$) of the eBRST theory

That is the Reduced Model discussed at the beginning, consisting of the *lgdof*s and the ghost fields, still symmetric under eBRST, local U(1) and now **a global SU(2) symmetry** (more on this in text talk by Mugdha Sarkar)

Can there be SSB in a TFT? $SU(2)_{\text{global}} \to U(1)_{\text{global}}$?

If yes, what is its effect on the full eBRST theory? Is eBRST broken too?

We have implemented the program on the lattice for numerical simulation
- **a very hard problem**

At the very preliminary level, we find evidence for the breaking of the global SU(2) to U(1) in the reduced theory.

What this means for the full eBRST theory is still unclear, but our very preliminary results are so far consistent with the eBRST theory going into a Higgs-like phase with the W and ghost fields appearing to acquire mass.

eBRST is probably left unbroken since this symmetry can allow equal mass terms for coset gauge fields and ghosts