Investigations of $\mathcal{N} = 1$ supersymmetric SU(3) Yang-Mills theory

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Münster-DESY(-Regensburg-Jena) collaboration

Lattice 2018 - East Lansing

Westfälische Wilhelms-Universität Münster

Graduiertenkolleg 2149 Research Training Group

- Standard model is incomplete
  - SUSY could cure some of the problems (hierarchy problem, dark matter, etc.)
  - SUSY is not yet very well understood
    - (nonperturbative) SUSY-breaking
- Extended symmetry allows better theoretical control of the theory
  - Tool to understand nonperturbative phenomena of QFT's
- Gauge part of supersymmetric QCD (SQCD, talks by G. Bergner, B. Wellegehausen)

Outline

Why SUSY?

$\mathcal{N} = 1$ $SU(3)$ SUSY Yang-Mills Theory

Simulation

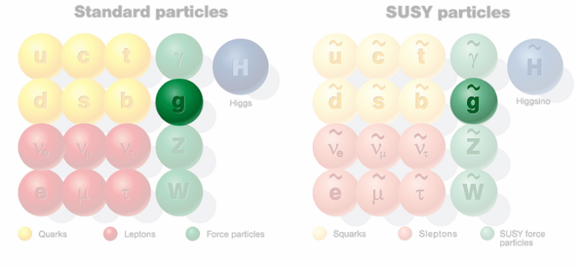
- Lattice formulation
- Sign problem
- Finite size analysis
- Topological freezing
- Ensembles

Chiral extrapolation & Ward Identities

Spectrum

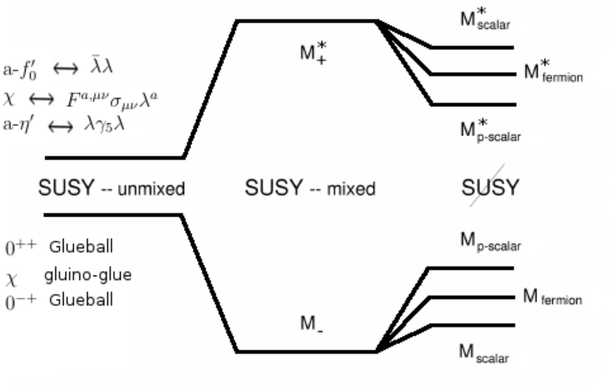
Summary & Outlook

$\mathcal{N} = 1$ $SU(3)$ SUSY Yang-Mills Theory



$$\boxed{\mathcal{L} = \mathrm{Tr}\left(-\tfrac{1}{2}F_{\mu\nu}F^{\mu\nu} + \mathrm{i}\bar{\lambda}\not{D}\lambda - m_0\bar{\lambda}\lambda\right)}$$

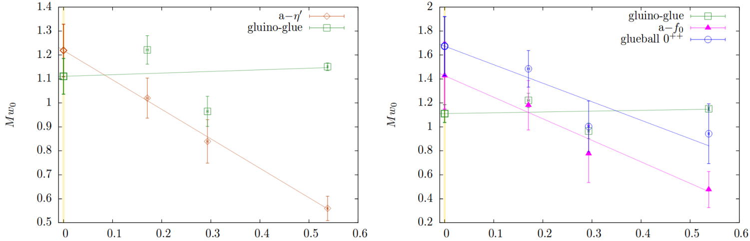
- $A_\mu^a(x)$: gauge fields
- $\lambda(x)$: gluino fields, Majorana fermions in adjoint representation
  - $(D_\mu\lambda)^a = \partial_\mu\lambda^a + gf_{abc}A_\mu^b\lambda^c$
- $m_0\bar{\lambda}\lambda$: soft SUSY-breaking term



G. R. Farrar, G. Gabadadze, M. Schwetz, 1999

Results from SU(2)

Indeed formulation of chiral multiplet of 0^-, 0^+, spin-1/2 groundstates.



Extrapolation of the lightest supermultiplet to the continuum limit. Desy-Münster collaboration arXiv:1512.07014

- best signal from gluino-glue
- mesonic states noisier but can be handled
  - 0^+-glueball challenging, but consistent with a-f_0
  - 0^--glueball operator seems to have no overlap with 0^- groundstate.
  - Analysis of excited multiplets ongoing
  - Started analysing baryonic states (Talk by S. Ali, canceled)

Lattice formulation

- Wilson fermions
- Clover term to supress O(a) effects

$$-\frac{c_{sw}}{4}\bar{\lambda}\sigma_{\mu\nu}F^{\mu\nu}\lambda$$

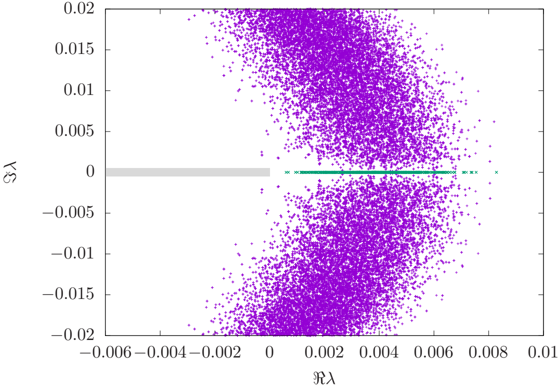
- Sign problem in SYM: Pfaffian (instead of determinant)

$$\int [d\lambda]\, e^{-\frac{1}{2}\bar{\lambda}D_w\lambda} = \mathrm{Pf}(CD_w) = \pm\sqrt{\det D_w}, \qquad C \hat{=} \text{charge conj. operator}$$

- SUSY is explicitely broken by the lattice
- Wilson fermions: breaking of chiral symmetry
- Veneziano, Curci [1987]: SUSY restaration in the chiral and continuum limit:
  - limit of vanishing renormalized gluino mass $m_\lambda \rightarrow 0$

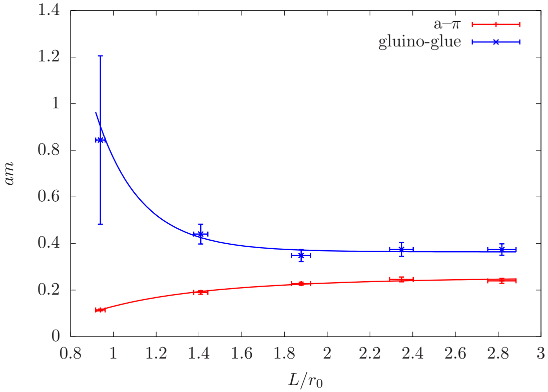
Sign problem

- RHMC, negative sign rare when simulation not too close to the chiral limit



Eigenvalues of the Dirac-Wilson operator, $\beta = 5.5$, $\kappa = 0.1683$. Green: $\langle v|\, \gamma^5\, |v\rangle > 0.001$

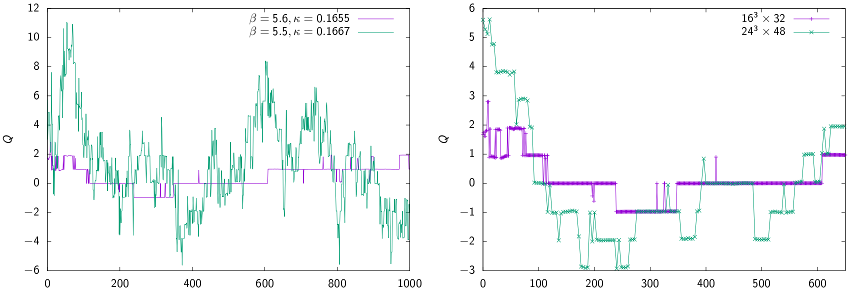
Finite size effects



$\beta = 5.6, \quad \kappa = 0.1660,$
$8^3 \times 32, 12^3 \times 32, 16 \times 32,\ 16^3 \times 32,\ 24^3 \times 48$
$f(x) = exp(-\alpha L)/L$

- finite size effects vanish for $L/r_0 > 2.4$
- also observed in SU(2)

Topological freezing



Left: Topological charge history for different β, Right: Topological charge history for different lattice sizes

Ensembles

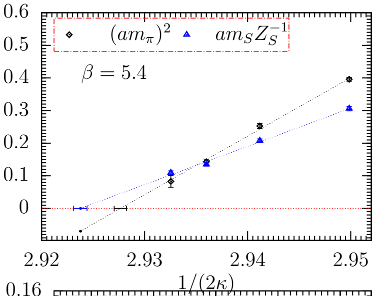
β	largest Volume	lattice spacing [fm]	lattice size [fm]
5.4	$12^3 \times 24$	0.067	0.8
5.45	$16^3 \times 32$		
5.5	$16^3 \times 32$	0.057	0.912
5.6	$24^3 \times 48$	0.047	1.12
5.8	$16^3 \times 32$	not used (finite size effects)	

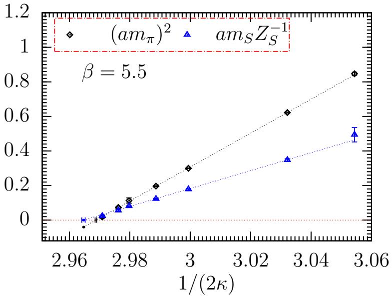
- for each β: ~ 4 κ , ~ 4000 configs

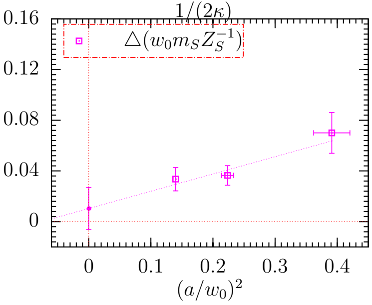
Chiral extrapolation

1. SYM $\hat{=}$ partially quenched Yang-Mills theory with two Majorana flavours
   $\Rightarrow$ Adjoint pion a-π is pseudo-Nambu-Goldstone particle from spontaneous chiral symmetry breaking
   - tune $m_\pi^2 \to 0$ for chiral limit
2. Supersymmetric Ward Identities (arXiv:1711.05504, Lattice 2017 talk by S. Ali)
3. Change of the gluino condensate at zero temperature at the chiral phase transition (C. Lopez' talk)

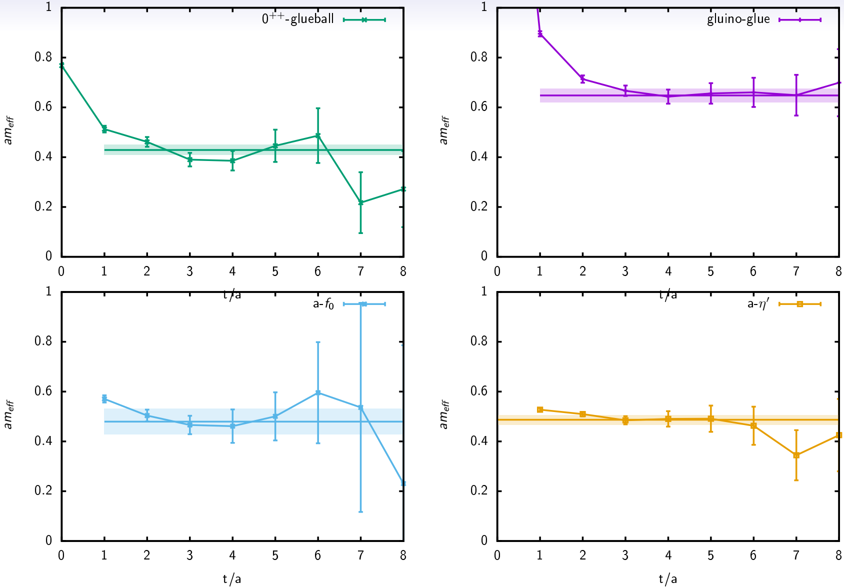
Ward Identities and (unphysical adjoint pion)



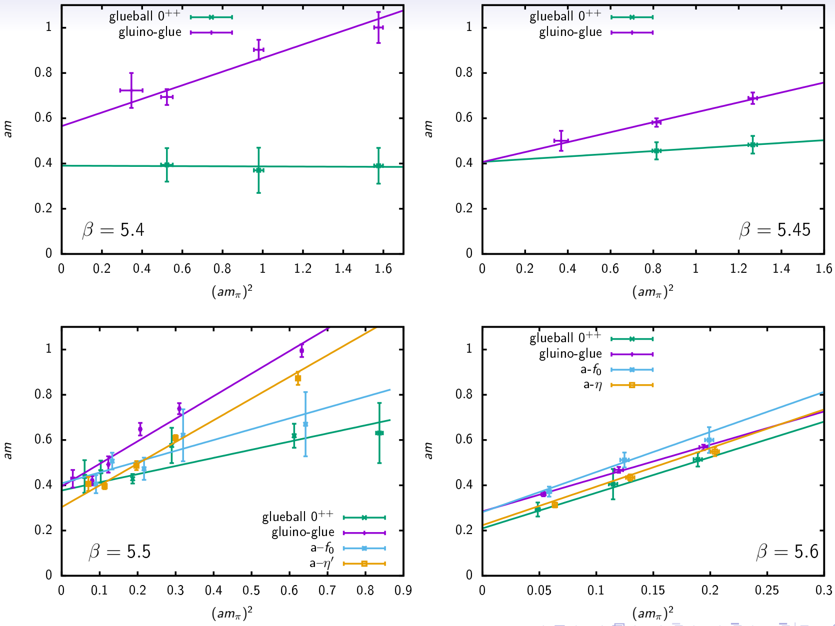




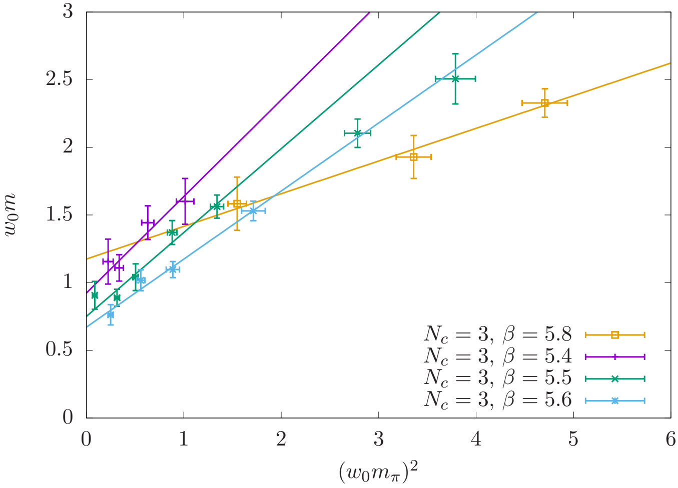
- κ_c from $m_{a-\pi}$ and from SUSY Ward Identities quite close
- small discrepancy due to lattice artifacts, vanish in the continuum limit



$\beta = \mathbf{5.5}, \ \kappa = \mathbf{0.1673}$



glueball 0^{++}
gluino-glue
$\beta = 5.4$
$\beta = 5.45$
$\beta = 5.5$
$\beta = 5.6$
a–f_0
a–η'
a-η
am
$(am_\pi)^2$



gluino-glue mass in units of w_0 for different values of β

Summary & Outlook (arxiv:1802.07067)

- We investigate SU(3) SYM with Wilson fermions and clover improvement
- Sign problem, topological freezing, chiral extrapolations are under control
- Ensembles: 4 different β, ~ 4 different κ each
- SUSY Ward Identities hint to supersymmetric continuum limit
- Spectrum compatible with supermultiplet formation in the continuum limit, however errorbars still large
- Final analysis with full statistics is currently being carried out
- Continuum extrapolation to be done
- For SQCD see talks by G. Bergner, B. Wellegehausen

Thank you for your attention!!

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- Calculate the lowest eigenvalues of Q and corresponding eigenvectors
  - using Arnoldi (ARPACK)
  - Chebyshev Polynomials of order 11
  - Even/Odd-Preconditioning
- Stochastic estimator technique for space orthogonal to the previously calculated eigenvectors:
  - $\frac{1}{N_S} \sum_i^{N_S} \left|\eta^i\right\rangle \left\langle\eta^i\right| = \mathbb{1} + \mathcal{O}\left(\sqrt{N_S}\right)$
    - use $\mathbb{Z}_4$-noise
  - $Q\left|s^i\right\rangle = \left|\eta^i\right\rangle$
  - $Q^{-1} = \frac{1}{N_S} \sum_i^{N_S} \left|s^i\right\rangle \left\langle\eta^i\right|$
  - Conjugate gradient
  - $N_S = 40$ for $\beta = 1.9$, $32^3 \times 64$

Infinitesimal supersymmetry transformations:

$$\delta A_\mu(x) = -2g\bar{\lambda}(x)\gamma_\mu\epsilon(x)$$
$$\delta\lambda(x) = -\frac{\mathrm{i}}{g}\sigma_{\mu\nu}F_{\mu\nu}(x)\epsilon(x)$$
$$\delta\bar{\lambda}(x) = +\frac{\mathrm{i}}{g}\bar{\epsilon}(x)\sigma_{\mu\nu}F_{\mu\nu}(x),$$

$\epsilon(x)$: Grassmann valued parameter.
Noether's theorem yields a supercurrent
$S_\mu(x) = -\frac{2\mathrm{i}}{g}\mathrm{Tr}\left[F_{\rho\nu}(x)\sigma_{\rho\nu}\gamma_\mu\lambda(x)\right]$
with $\partial^\mu S_\mu(x) = m_{\tilde{g}}\chi(x)$, $\chi(x) = \frac{2\mathrm{i}}{g}\mathrm{Tr}\left[F_{\rho\nu}(x)\sigma_{\rho\nu}\lambda(x)\right]$
In the quantised theory the corresponding unrenormalised SUSY Ward identities are

$$\left\langle \partial^\mu S_\mu(x)Q(y)\right\rangle = m_0\left\langle\chi(x)Q(y)\right\rangle - \left\langle \frac{\delta Q(y)}{\delta\bar{\epsilon}(x)}\right\rangle.$$

- $Q(y)$ is any suitable insertion operator
- last term represents a contact term
  - vanishes if $Q(y)$ is localised at space-time points different from x

Renormalisation

- gluino mass receives an additive renormalisation
  - the supercurrent mixes with another dimension 7/2 current: $T_\mu(x) = \frac{2i}{g}\mathrm{Trs}\big[F_{\mu\nu}(x)\gamma_\nu\lambda(x)\big]$

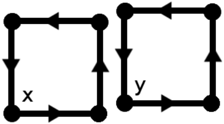
Resulting SUSY Ward identity, omitting contact terms:

$$\Big\langle\big(Z_S\partial^\mu S_\mu(x) + Z_T\,\partial^\mu T_\mu(x)\big)Q(y)\Big\rangle = m_S\big\langle\chi(x)Q(y)\big\rangle$$

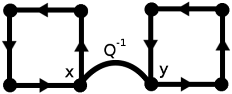
m_S is the subtracted gluino mass, and Z_S and Z_T are renormalisation coefficients.

A renormalised supercurrent can then be defined through $S^R_\mu = Z_S S_\mu + Z_T T_\mu$.

- Glueballs: $O(x) = \sum_{i<j} P_{ij}(x)$



- Gluino-glue: $O^{\alpha}(x) = \sum_{i<j} \sigma_{ij}^{\alpha\beta} Tr_c \left[P_{ij}(x)\tilde{g}^{\beta}(x)\right]$



- Mesons: $O(x) = \bar{\tilde{g}}(x)\Gamma\tilde{g}(x)$