# Towards models with an unified dynamical mechanism for elementary particle masses

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# Introduction

Talk based on

- R. Frezzotti and G.C. Rossi, Phys. Rev. D92 (2015) 054505
  and work in preparation
- R. Frezzotti, M. Garofalo and G.C. Rossi, Phys. Rev. D93 (2016) 105030

& on the results of recent lattice work (talk by M. Garofalo) in collaboration with

- P. Dimopoulos, G.C. Rossi (Univ. of Roma Tor Vergata and Centro Fermi)
  and M. Garofalo (INFN of Roma Tor Vergata)
- B. Kostrzewa, F. Pittler, C. Urbach (HISKP - University of Bonn)

Outline:

- Elementary fermion mass mechanism as a non-perturbative "anomaly"
- Mass mechanism in a Toy Model with weak interactions ($g_W > 0$)
- Towards a realistic BSM model: mechanism & all interactions ?
- Outlook & Conclusions

# Fermion mass as a non-perturbative (NP) "anomaly"

Results in the talk by M. Garofalo $\Longleftrightarrow$ unnoticed NP feature in QFT

1. Consider a renormalizable UV regulated gauge model (RGI scale $\Lambda_S$ ) with fermionic chiral symmetries ($\tilde{\chi}_{L,R}$) broken in hard way at the UV cutoff scale, $\Lambda_{UV}$
2. Include fundamental scalar doublet ($\phi$) in order to have an exact chiral symmetry involving fermions and scalars: prerequisite for EW theory & no $\Lambda_{UV}\bar{Q}Q$ term
3. Interplay of strong gauge interactions and $\tilde{\chi}_{L,R}$ breaking $\Rightarrow$ NP "anomaly" in the Nambu-Goldstone phase ($\langle\phi\rangle = v$): seen in $\tilde{\chi}_{L,R}$ Schwinger Dyson Eq.s (SDE)
4. In the critical model, $\eta = \eta_{cr}(\rho)$, where $\tilde{\chi}_{L,R}$ breaking is minimized we have

   bare $\tilde{\chi}_L$ SDE: $\partial \tilde{J}^{Li}(x) = \eta\, \tilde{\delta}^{L\,i} O_4^Y(x) + \Lambda_{UV}^{-2}\, \rho\, \tilde{\delta}^{L\,i} O_6^{Wil}(x)$

   but for insertion in correlators at momenta $p$ s.t. $\Lambda_S \ll p \ll \Lambda_{UV}$

   a) renormalized $\tilde{\chi}_L$ SDE in Wigner phase: $Z_{\tilde{J}}\, \partial \tilde{J}^{L\,i}(x) = \Lambda_{UV}^{-2} O_{n\geq 6}^{L\,i}(x)$

   b) renormalized $\tilde{\chi}_L$ SDE in NG phase: $Z_{\tilde{J}}\, \partial \tilde{J}^{L\,i}(x) = O_4^{L\,i}(x) + \Lambda_{UV}^{-2} O_{n\geq 6}^{L\,i}(x)$

# NP fermion mass "anomaly" and LE effective action

What's $O_4^{L\,i}(x)$ ? In what context is it defined? Is it UV finite & RG invariant?

1. If non-zero, it should be finite & RG invariant: @ $\eta = \eta_{cr}(\rho)$ the current $Z_{\tilde{J}}\tilde{J}^{L\,i}$ is conserved up to O($\Lambda_{UV}^{-2}$) in Wigner phase $\Rightarrow$ SDE's r.h.s. RG invariant in NG phase
2. No exact WTI for $\tilde{\chi}_{L,R}$ fermionic transformations (even @ $\eta_{cr}$) $\Rightarrow$ at NP level the renormalized form of $\tilde{\chi}_{L,R}$ SDE at low energy *may* differ in Wigner and NG phases *[ NP renormalization ultimately based on "common wisdom" + numerical evidence]*
3. NP term in r.h.s. of the renormalized $\tilde{\chi}_{L,R}$ SDE $\Longleftrightarrow$ NP $\tilde{\chi}_{L,R}$ breaking terms in the LE effective action, $\Gamma_{LE}$, for correlators at momenta $p$ s.t. $\Lambda_S \ll p \ll \Lambda_{UV}$
4. @ $\eta = \eta_{cr}(\rho)$ no $\tilde{\chi}_{L,R}$ breaking term is allowed in $\Gamma_{LE}^{Wigner}$ as $\Lambda_{UV}^{-1} \to 0$ ; but $\Gamma_{LE}^{NG}$ may include $\Lambda_S C_1(\bar{Q}_L U Q_R + \mathrm{h.c.})$ , ... $\frac{\Lambda_S^2}{v^4} C_{-2}((\bar{Q}_L U Q_R)(\bar{Q}_L U Q_R) + \mathrm{h.c.})$, ...

$\rho$ generic, effective fields in $\Gamma_{LE}^{NG}$ : $U = \exp(\frac{i}{v}\sum_{j=1,2,3}\tau^j\zeta^j)$ , R = v + $\zeta^0$

$O_4^{L\,i}(x) = \tilde{\delta}^{L\,i}[\Lambda_S C_1(\bar{Q}_L U Q_R + \mathrm{h.c.}) + \ldots]$ describes NP $\tilde{\chi}_{L,R}$ breaking in NG phase

# NP mass in LE eff. action: universality, naturalness

Implications of RG invar. $\Lambda_S C_1(\bar{Q}_L U Q_R + \mathrm{h.c.}) + \ldots$ in $\Gamma_{LE}^{NG}$ for predictive power?

1. The whole $\tilde{\chi}_{L,R}$ breaking NP term in $\Gamma_{LE}^{NG}$, or in the r.h.s. of renormalized SDEs
   a) is irreducibly different as an operator from Yukawa's $R(\bar{Q}_L U Q_R + \mathrm{h.c.})$
   b) cannot & needs not be renormalized by adding any Lagrangian counterterm
   c) involves coefficients ( $C_1$ ) depending on the $\tilde{\chi}_{L,R}$ breaking UV parameters ( $\rho$ )

2. LE physics determined by gauge coupling & $\tilde{\chi}_{L,R}$ parameters at the UV-scale:
   "universality" $\Longleftrightarrow$ equivalence classes of different UV complete models with identical $\tilde{\chi}_{L,R}$ effects at LE $\leftrightarrow$ take the representative with simplest $\tilde{\chi}_{L,R}$ term;

3. $m_Q^{eff} = C_1\text{"}\Lambda_S\text{"}$ with $C_1 = O(\alpha_S^2)$ and $\text{"}\Lambda_S\text{"} \sim \frac{\Lambda_S v^2}{\Lambda_S^2 + v^2}$ or so, vanishing as $v \to 0$
   is the elementary fermion mass in $\Gamma_{LE}^{NG}$, normalized at same scale as $\bar{Q}_L U Q_R$
   is natural à la 't Hooft: RGI scale $\Lambda_S$, no Yukawa mass due to minimization of $\tilde{\chi}_{L,R}$

4. Massless Goldstones $\zeta^{1,2,3}$ : ... can be eaten up by massive $W$ gauge bosons

# Mechanism in Toy Model at $g_W > 0$

Mechanism in Toy Model with weak interactions ($g_W > 0$)

UV regularization left unspecified: hard UV cutoff $\Lambda_{UV} \sim \frac{1}{b}$

# Simplest toy model with weak interactions

Introducing weak gauge interactions for L-handed fermions, what about

- ⋆ definition and restoring of $\tilde{\chi}_{L,R}$ (fermionic chiral) symmetries ?
- ⋆ $W$–boson mass: from strong interactions & same mechanism as for fermions ?

Extended toy model with minimal fermion content (avoiding global SU(2) anomaly) :

gauge: $SU(3)_s \times SU(2)_L$ $(g_Y = 0)$ & Dirac fermions: $Q \in 3$ & $N \in 1$ of $SU(3)_s$

$$\mathcal{L}_{\rm toy}(Q,N,A,\Phi,W) = \mathcal{L}_{kin}(Q,N,A,\Phi,W) + \mathcal{V}(\Phi) + \mathcal{L}_{Wil}(Q,N,A,\Phi,W) + \mathcal{L}_{Yuk}(Q,N,\Phi)$$

$$\mathcal{L}_{kin} = \frac{1}{4} F^{A;a}_{\mu\nu} F^{A;a}_{\mu\nu} + \frac{1}{4} F^{W;i}_{\mu\nu} F^{W;i}_{\mu\nu} + \bar{Q}_L \gamma_\mu \mathcal{D}^{A,W}_\mu Q_L + \bar{Q}_R \gamma_\mu \mathcal{D}^{A}_\mu Q_R$$
$$+ \bar{N}_L \gamma_\mu \mathcal{D}^{W}_\mu N_L + \bar{N}_R \gamma_\mu \partial_\mu N_R + \frac{1}{2} \mathrm{Tr}\left[\Phi^\dagger \overleftarrow{\mathcal{D}}^{W}_\mu \mathcal{D}^{W}_\mu \Phi\right]$$

$$\mathcal{L}_{Wil} = \frac{b^2}{2} \rho \left( \bar{Q}_L \overleftarrow{\mathcal{D}}^{A,W}_\mu \Phi \mathcal{D}^{A}_\mu Q_R + \bar{Q}_R \overleftarrow{\mathcal{D}}^{A}_\mu \Phi^\dagger \mathcal{D}^{A,W}_\mu Q_L + N_L \overleftarrow{\mathcal{D}}^{W}_\mu \Phi \partial_\mu N_R + \bar{N}_R \overleftarrow{\partial}_\mu \Phi^\dagger \mathcal{D}^{W}_\mu N_L \right)$$

$\phi$ : $SU(2)_L$ doublet, $SU(3)_s$ singlet  matrix notation: $\Phi = [\tilde{\varphi}|\varphi]$ , $\tilde{\varphi} = -i\tau^2 \varphi^*$

# Toy model with weak interactions: symmetries

Usual quartic scalar potential ($\hat{m}_\Phi^2 = m_0^2 - m_{cr}^2$) and Yukawa term (coupling $\eta$ ):

$$\mathcal{V}(\Phi) = \frac{m_0^2}{2}\mathrm{Tr}\left[\Phi^\dagger\Phi\right] + \frac{\lambda_0}{4}\left(\mathrm{Tr}\left[\Phi^\dagger\Phi\right]\right)^2 \qquad \mathcal{L}_{Yuk}(Q,\Phi) = \eta\left(\bar{Q}_L\Phi Q_R + \bar{Q}_R\Phi^\dagger Q_L\right)$$

SU(2)$_L$ gauge symmetry: $W_\mu^{1,2,3}$ bosons & covariant derivatives on $f = Q,\ N$, e.g.

$$\mathcal{D}_\mu^{A,W} f_L = (\partial_\mu - i\delta_{f,Q} g_s\lambda^a A_\mu^a - ig_w \tfrac{\tau^i}{2} W_\mu^i) f_L \qquad \bar{f}_L \overleftarrow{\mathcal{D}}_\mu^{A,W} = \bar{f}_L(\overleftarrow{\partial}_\mu + i\delta_{f,Q} g_s\lambda^a A_\mu^a + ig_w \tfrac{\tau^i}{2} W_\mu^i)$$

Global SU(2)$_L\times$ SU(2)$_R$ invariance, if $W$'s transform (as $\in$ su(2)$_L$) under $\tilde{\chi}_L$ :

$$\chi_L \ \equiv\ \tilde{\chi}_L \otimes \chi_L^\Phi \qquad \text{and} \qquad \chi_R \ \equiv\ \tilde{\chi}_R \otimes \chi_R^\Phi \qquad \text{with}$$

$$\tilde{\chi}_L:\ Q[N]_L \to \Omega_L\, Q[N]_L\,,\ \bar{Q}[N]_L \to \bar{Q}[N]_L\,\Omega_L^\dagger\,, \qquad \chi_L^\Phi:\ \Phi \to \Omega_L\Phi\,, \qquad \Omega_L \in \mathrm{SU(2)}_L\,,$$

$$W_\mu \to \Omega_L W_\mu \Omega_L^\dagger\,,$$

$$\tilde{\chi}_R:\ Q[N]_R \to \Omega_R Q[N]_R\,,\ \bar{Q}[N]_R \to \bar{Q}[N]_R\Omega_R^\dagger\,, \qquad \chi_R^\Phi:\ \Phi \to \Phi\Omega_R^\dagger\,, \qquad \Omega_R \in \mathrm{SU(2)}_R\,,$$

$\tilde{\chi}_{L,R}$ and $\chi_{L,R}^\Phi$ transf.s are no symmetries ... term $\mathrm{Tr}\left[\Phi^\dagger \overleftarrow{\mathcal{D}}_\mu^W \mathcal{D}_\mu^W \Phi\right]$ breaks $\tilde{\chi}_L$ , too

# $g_W > 0$ : maximal $\tilde\chi$–symmetry restoring at $(\rho_{cr}\,,\eta_{cr})$

$$\partial_\mu\Big\langle \tilde J^{L,\,i}_\mu(x)\,\hat O(0)\Big\rangle = \Big\langle \tilde\Delta^i_L \hat O(0)\Big\rangle\delta(x) - \eta\Big\langle \textstyle\sum_{f=Q,N}\Big(\bar f_L\frac{\tau^i}{2}\Phi f_R - \bar f_R\Phi^\dagger\frac{\tau^i}{2}f_L\Big)(x)\,\hat O(0)\Big\rangle +$$
$$-\frac{b^2}{2}\rho\Big\langle \textstyle\sum_{f=Q,N}\Big(\bar f_L\overleftarrow{\mathcal D}{}^{A,W}_\mu\frac{\tau^i}{2}\Phi\mathcal D^A_\mu f_R - \bar f_R\overleftarrow{\mathcal D}{}^A_\mu\Phi^\dagger\frac{\tau^i}{2}\mathcal D^{A,W}_\mu f_L\Big)(x)\,\hat O(0)\Big\rangle +$$
$$+\frac{i}{2}g_W\Big\langle \mathrm{Tr}\,\Big(\Phi^\dagger[\tfrac{\tau^i}{2},W_\mu]\mathcal D^W_\mu\Phi + \Phi^\dagger\overleftarrow{\mathcal D}{}^W_\mu[W_\mu,\tfrac{\tau^i}{2}]\Phi\Big)(x)\hat O(0)\Big\rangle$$

$$\tilde J^{L,\,i}_\mu = \textstyle\sum_{f=Q,N}\Big\{\bar f_L\gamma_\mu\frac{\tau^i}{2}f_L - \frac{b^2}{2}\rho\Big(\bar f_L\frac{\tau^i}{2}\Phi\mathcal D^A_\mu f_R - \bar f_R\overleftarrow{\mathcal D}{}^A_\mu\Phi^\dagger\frac{\tau^i}{2}f_L\Big)\Big\} + g_W\,\mathrm{Tr}\,\Big([W_\nu, F^W_{\mu\nu}]\frac{\tau^i}{2}\Big)$$

upon renormalization one expects the $\tilde\chi_L$–SDE's of the LE effective theory to read

$$Z_{\tilde J}\partial_\mu\Big\langle \tilde J^{L,\,i}_\mu(x)\hat O(0)\Big\rangle = \Big\langle \tilde\Delta^i_L\hat O(0)\Big\rangle\delta(x) - (\eta-\bar\eta_L)\Big\langle \textstyle\sum_{f=Q,N}(\bar f_L\frac{\tau^i}{2}\Phi f_R - h.c.)(x)\,\hat O(0)\Big\rangle +$$
$$+(1-\bar\gamma)\,\frac{i}{2}g_W\Big\langle \mathrm{Tr}\,\Big(\Phi^\dagger[\tfrac{\tau^i}{2},W_\mu]\mathcal D^W_\mu\Phi + \Phi^\dagger\overleftarrow{\mathcal D}{}^W_\mu[W_\mu,\tfrac{\tau^i}{2}]\Phi\Big)(x)\hat O(0)\Big\rangle + \ldots + \mathrm{O}(b^2)$$

Critical model: $\eta_{cr} = \bar\eta_L(g_s, g_W; \mu_0, \lambda_0; \eta_{cr}, \rho_{cr})\,,\quad 1 = \bar\gamma(g_s, g_W; \mu_0, \lambda_0; \eta_{cr}, \rho_{cr})$

here $\tilde\chi_L$ and $\tilde\chi_R$ restored at low momenta, up to possible NP terms $\sim \Lambda_s^n$, $n = 2, 1, \ldots$

# $\hat{m}^2_\Phi > 0$ : full restoring of $\tilde{\chi}$–symmetry @ $(\rho_{cr}, \eta_{cr})$

Wigner phase at $(\rho_{cr}, \eta_{cr})$ : $\mathcal{V}(\Phi)$ has a single minimum $(\hat{m}^2_\Phi > 0)$ $\Rightarrow$ renormalized

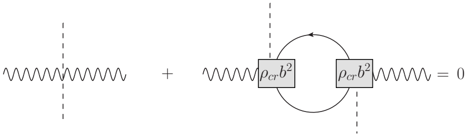
SDE's of $\tilde{\chi}_{L(R)}$ : $\quad Z_{\tilde{J}} \partial_\mu \left\langle \tilde{J}^{L(R),\,i}_\mu(x) \hat{O}(0) \right\rangle = \left\langle \tilde{\Delta}^i_{L(R)} \hat{O}(0) \right\rangle \delta(x) + \mathrm{O}(b^2)$

$\star$ requiring full $\Phi$ decoupling at low energy determines $(\rho_{cr}, \eta_{cr})$ : at LO see figure

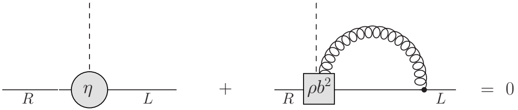
$\star$ no NP operators $\sim \Lambda_s$ occur in r.h.s. $\Longleftrightarrow$ due to $v_\Phi = 0$ here $U$-field is undefined

$\Phi$ is fully decoupled & auxiliary in the effective action valid for $\Lambda_{UV} \gg p \gg \Lambda_s$

$$\Gamma^{Wig}_{\rm loc} \equiv \Gamma^{\hat{m}^2_\Phi > 0}_{\rm loc} = \tfrac{1}{4}[(F^A F^A) + (F^W F^W)] + \textstyle\sum_{f=Q,N} [\bar{f}_L \not{\mathcal{D}}^{A,W} f_L + \bar{f}_R \not{\mathcal{D}}^A f_R] + \mathcal{V}^{\hat{m}^2_\Phi > 0}_{eff}[\Phi]$$



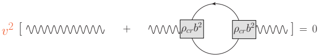

$\rho_{cr} \sim \mathrm{O}(1/\sqrt{N_F^{\rm tot}})$



# $\hat{m}^2_\Phi < 0$ : dynamical $\tilde{\chi}$SB & NP masses @ $(\rho_{cr}, \eta_{cr})$

NG phase at $(\rho_{cr}, \eta_{cr})$: $\mathcal{V}(\Phi)$ has many degenerate minima $(\hat{m}^2_\Phi < 0)$ $\Rightarrow$

- ⋆ by def. of $(\rho_{cr}, \eta_{cr})$ no O($v$) $W$-mass and O($v$) $Q(N)$-mass terms: e.g. at LO




- ⋆ under dynamical $\tilde{\chi}$SB vacuum polarized by residual O($b^2 v$) $\tilde{\chi}$-breaking terms
- ⋆ interplay of O($b^2$) $\not{\tilde{\chi}}_{L,R}$ and $\tilde{\chi}$SB dynamics $\Longrightarrow$ O($b^0$) $\not{\tilde{\chi}}_{L,R}$ terms in

$$\Gamma^{NG}_{\rm loc} = \Gamma^{\hat{m}^2_\Phi<0}_{\rm loc} + C_{1,Q}\Lambda_s[\bar{Q}_L U Q_R + \bar{Q}_R U^\dagger Q_L] + C_2\Lambda_s^2 \,\mathrm{Tr}\,[U^\dagger \overleftarrow{\mathcal{D}}^W_\mu \mathcal{D}^W_\mu U]$$

$\Leftrightarrow$ $(M_W^{eff})^2 = g_W^2 C_2 \Lambda_s^2$ and $m_Q^{eff} = C_{1,Q}\Lambda_s$ from common mechanism; $m_N^{eff} = 0$

short distance NP vertex corrections


NP mass @ low energy

# NP masses in $\Gamma^{NG}_{\rm loc}$ @ $(\rho_{cr}, \eta_{cr})$ : remarks

1. Effective NP masses modulated by gauge couplings & loop suppression factors:

$$C_2 = \mathrm{O}(g_s^4 \rho_{cr}^6 N_f; \sim 3 \text{ loop suppr.}) , \qquad C_{1,Q} = \mathrm{O}(g_s^4 \rho_{cr}^3 N_f; \sim 2 \text{ loop suppr.})$$

$C_{1,N} = 0$ owing to a symmetry of $\mathcal{L}_{\rm toy}$ valid in case of sterile R-handed fermions

$N_R(x) \to N_R(x) + c\,, \quad \bar N_R(x) \to \bar N_R(x) + \bar c\,, \quad c, \bar c$ constant [Goltermann & Petcher, 1990]

2. Absence in $\Gamma^{NG}_{\rm loc}$ of term $\tilde C \Lambda_s R\, \mathrm{Tr}\,[U^\dagger \overleftarrow{\mathcal{D}}^{\mathcal{W}}_\mu \, {}_{\mathcal{D}}{}^W_\mu U]$ is peculiar of the critical model:

as $\rho_{cr}^2 - \rho^2 \to 0^+$ & $\eta \to \eta_{cr}$ we have $1 - \bar\gamma \to 0^+$, $m^2_{\zeta^0} \sim |\hat m^2_\Phi|/(1-\bar\gamma) \to +\infty$

$\Rightarrow$ decoupling of $\zeta^0$. For $\Gamma^{NG}_{\rm loc}$ to describe the decoupling of $\zeta^0$, e.g. in

$WW \to WW$ amplitudes, $|\tilde C| \leq O((1-\bar\gamma)^{1/2}) \xrightarrow{\rho \to \rho_{cr}, \eta \to \eta_{cr}} 0$ is necessary

3. NP kinetic term for GB's: canonical normalization $\Rightarrow$ $U = \exp(i\vec\zeta \vec\tau / \sqrt{C_2}\Lambda_s)$

basic GBs NP-ly coupled so as to provide longitudinal d.o.f.s for massive $W$s

4. Further $\tilde\chi$-breaking terms in $\Gamma^{NG}$: $\frac{1}{\Lambda_s^2}[(\bar Q_L U Q_R)(\bar Q_R U^\dagger Q_L)]$, $[\mathrm{Tr}\,(D^W_\mu U^\dagger D^W_\mu U)]^2$, ...

# Towards a realistic BSM model

Towards a realistic BSM model: mechanism & all interactions ?

A possible explanation of why nothing new is seen close to the EW scale

Hints at $\Lambda_T \sim 5$ TeV: calling for experimental search in that energy range

# Dynamical NP mass: towards a realistic model

Hypothesis: effective mass of elementary particles stemming from our mechanism

Consistency of hypothesis with experimentally observed masses implies (hints at)

- new Tera-strong SU($N_T$) interaction with RGI scale $\Lambda_T > M_W \gg \Lambda_{QCD}$
- new set of Tera-fermions subjected to the new force (besides to SM interactions)
  - $Q_L \in (N_T, 3, 2; Y_Q^L)$ , $\quad L_L \in (N_T, 1, 2; Y_L^L)$
  - $Q_R^u \in (N_T, 3, 1; Y_R^u)$ , $\quad L_R^u \in (N_T, 1, 1; Y_L^u)$
  - $Q_R^d \in (N_T, 3, 1; Y_R^d)$ , $\quad L_R^d \in (N_T, 1, 1; Y_L^d)$

  with (irrep. of SU($N_T$), SU(3)$_c$, SU(2)$_L$; $Y = Q_{em} - T_3$) ; besides SM fermions, e.g.
  - $q_L \in (1, 3, 2; 1/6)$ , $\quad \ell_L \in (1, 3, 2; -1/2)$
  - $t_R \in (1, 3, 1; 2/3)$ , $\quad \nu_R \in (1, 1, 1; 0)$
  - $b_R \in (1, 3, 1; -1/3)$ , $\quad \tau_R \in (1, 1, 1; -1)$
- composite higgs: a bound state in $WW$, $ZZ$, $Q\bar{Q}$, $L\bar{L}$ ... channel; Tera-fermions & -strong force crucial for binding; needed for unitarity of $WW \to WW$ scattering at LE

# UV complete Lagrangian: towards a realistic model

$$\mathcal{L}^{BSMM} = \frac{1}{4}\left(F^B F^B + F^W F^W + F^A F^A + F^G F^G\right) + \frac{1}{2}\mathrm{Tr}\big(\Phi^\dagger \overleftarrow{\mathcal{D}}_\mu^{B,W} \mathcal{D}_\mu^{B,W}\Phi\big) + \mathcal{V}(\Phi) +$$

$$+ \sum_{\text{3 families}} \Big[\bar{q}_L D^{BWA} q_L + \bar{q}_R^u D^{BA} q_R^u + \bar{q}_R^d D^{BA} q_R^d + \ell_L D^{BW} \ell_L + \bar{\ell}_R^u D^B \ell_R^u + \bar{\ell}_R^d D^B \ell_R^d\Big] +$$

$$+ \Big[\bar{Q}_L D^{BWAG} Q_L + \bar{Q}_R^u D^{BAG} Q_R^u + \bar{Q}_R^d D^{BAG} Q_R^d + \bar{L}_L D^{BWG} L_L + \bar{L}_R^u D^{BG} L_R^u + \bar{L}_R^d D^{BG} L_R^d\Big] +$$

$$+ \sum_{f=q,\ell,Q,L} \Big[\eta_{f,cr}(\bar{f}_L\,\tilde{\phi}\,f_R^u + \text{h.c.}) + \frac{1}{2}b^2\rho_f O_{f,\tilde{\chi}}^u + \eta_{f,cr}(\bar{f}_L\,\phi\,f_R^d + \text{h.c.}) + \frac{1}{2}b^2\rho_f O_{f,\tilde{\chi}}^d\Big]$$

$f$ runs over doublets $q^{(3)}$, $\ell^{(3)}$, $q^{(2)}$, $\ell^{(2)}$, $q^{(1)}$, $\ell^{(1)}$, $Q$ and $L$: in principle different $\rho_f$'s, but

$\tilde{\chi}$-symmetry restoring $\Rightarrow$ $\sum_{f=1}^{N_{ferm}^{tot}} \rho_f^2(1 + \mathrm{O}(\rho_f^2)) = O(1)$ and $\eta_f = \eta_{f,cr}(\{\rho\})$

- In NG phase: dynamical mechanism yields $m_{Q,L}^{eff} \sim \Lambda_T$ (as $\alpha_T(\Lambda_T) = \mathrm{O}(1)$) $\Longrightarrow$

  $m^{eff}$ for $W^\pm$, $Z$, $q$ and $\ell$ (not for $\nu$'s) [$\sim$ Bardeen, Hill & Lindner, 1990 + "naturalness"]

- Assuming $\rho_{Q,L} \simeq \rho_{q^{(3)}} \simeq \rho_{\ell^{(3)}}$: $m_W^{eff}$ and $m_t^{eff}$ may be $(0.10 \div 0.01)m_{Q,L}^{eff}$ due to

  loop suppression of $m^{eff}$'s (lattice test) & gauge coupling dependence: $g_T^4|_{\Lambda_T} \gg g_3^4|_{\Lambda_T}$

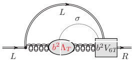
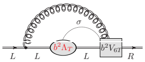
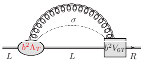
# Masses of EW bosons, Tera-fermions and top quark

- $(M_Z^{eff})^2 = [(g_W^2 + g_Y^2)/g_W^2](M_W^{eff})^2$ , while $M_\gamma^{eff} = 0$

$\Gamma_{LE}{}^{NG} \supset C_2 \Lambda_T^2 \frac{1}{2} \mathrm{Tr}\,[D_\mu^{W,B} U^\dagger D_\mu^{W,B} U] \supset C_2 \Lambda_T^2 [g_W^2 \sum_{j=1^3} (W^j \cdot W^j) + g_Y^2 B \cdot B + 2 g_W g_Y W^3 \cdot B]$

$\Rightarrow$ diagonalization in $W^3$–$B$ sector gives massless $\gamma$ and $M_Z^2 = (g_W^2 + g_Y^2) C_2 \Lambda_T^2$

owing to the custodial SU(2)$_L \times$ SU(2)$_R$ symmetry of $\mathcal{L}^{BSMM}$ in the $g_Y \to 0$ limit

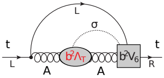




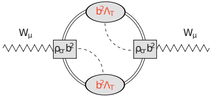
- Tera-hadron resonances with $E_{CoM} \simeq (2 \div 3) m_{Q(L)}^{eff} \sim 5 \div 10$ TeV to be observed

$m_{Q(L)}^{eff} = C_{1,Q(L)} \Lambda_T \qquad C_{1,Q(L)}|_{\bar\mu} = \mathrm{O}(g_T^4|_{\bar\mu} \rho_{Q,L}^3 N_{Q+L};\ \sim 2\text{ loop suppr.}) \simeq \mathrm{O}(1)$

$m_t = C_{1,t} \Lambda_T \qquad C_{1,t}|_{\bar\mu} = \mathrm{O}(2 g_3^4|_{\bar\mu} \rho_t^3 N_{Q+L};\ \sim 2\text{ loop suppr.}) = \mathrm{O}(0.05)$

$(M_W^{eff})^2 = g_W^2 C_2 \Lambda_T^2 \qquad C_2|_{\bar\mu} = \mathrm{O}(g_T^4|_{\bar\mu} \rho_{Q,L}^6 N_{Q+L};\ \sim 3\text{ loop suppr.}) \simeq \mathrm{O}((0.03)^2)$

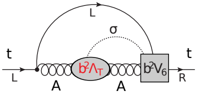


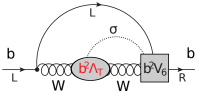



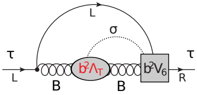
# 3rd family SM fermions – unitarity & $h$ boson

- Ratio $M_{\rm T-meson}/m_t^{\rm pole}$: key info for experiment, computable with controlled errors via lattice simulations with T-strong & QCD interactions, $q$ and (unquenched) $Q$
- Order of magnitude of $m_\tau^{eff}/m_t^{eff} = \mathrm{O}(\alpha_Y^2/\alpha_3^2)$ , $m_b^{eff}/m_t^{eff} = \mathrm{O}(\alpha_W^2/\alpha_3^2)$ and $m_{\nu_\tau}^{eff} = 0$ (due to a shift symmetry of sterile $\nu_R$) can be understood

$$O^t_{q,\tilde\chi} = \bar q_L \overleftarrow{\mathcal{D}}^{BWA}_\mu \tilde\phi \mathcal{D}^{BA}_\mu t_R \qquad O^b_{q,\tilde\chi} = \bar q_L (\mathcal{D}\mathcal{D}^{BW}\phi) b_R \qquad O^\tau_{\ell,\tilde\chi} = \bar\ell_L \overleftarrow{\mathcal{D}}^{BW}_\mu \phi \mathcal{D}^B_\mu \tau_R \quad [\text{+h.c.}]$$



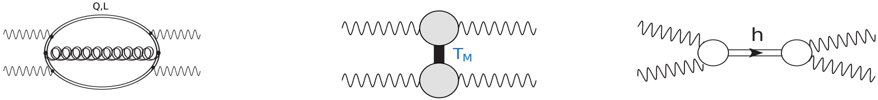

more accuracy & other families require info/assumptions on $\tilde\chi_{L,R}$ terms in $\mathcal{L}^{BSMM}$

- Model $\mathcal{L}^{BSMM}$ unitary with $M_W^{eff} \ll M_{\rm T-meson}$, $\alpha_W \ll 1$, no fundamental higgs:

interactions must produce (among other bound states) one scalar state with SM-like couplings to W/Z or several scalars with tuned couplings for $W_L W_L \to W_L W_L$ to be unitary at energies $\ll \Lambda_T$ [Cornwall, Levin & Tiktopoulos 1974, Lee, Quigg & Thacker 1977]

# 125 GeV Higgs boson ($h$) as $WW + ZZ$ bound state

WW channel: assume binding force from T-hadron exchange to form $h$, $M_h < 2M_W$




$$G(p) = \int d^4x e^{-i\vec{p}\vec{x} - ip_0x_0} V_3^{-1} \int d^3z \left\langle W(\vec{x}, x_0) W(\vec{z}+\vec{x}, x_0) W^\dagger(\vec{0}, 0) W^\dagger(\vec{0}, 0) \right\rangle$$

in free theory has a cut starting at $p^2 = -4M_W^2$; in interaction due to weak and T-strong forces

$$G(p) = \frac{g_{analyt}(E_p^2, \vec{p}^2)}{p^2 + 4M_W^2} \left\{ 1 + \frac{\Delta_0^2}{p^2 + 4M_W^2} + \dots \right\} = \frac{g_{analyt}(E_p^2 \vec{p}^2)}{p^2 + 4M_W^2 - \Delta_0^2} \qquad \text{with}$$

$\Delta_0^2 = \mathrm{O}(1)\, g_W^4\, 4M_W^2$ LE quantity, computable $\Rightarrow$ a pole is expected to appear at

$$p^2 = -M_h^2 = -4M_W^2 + \Delta_0^2 \quad \leftrightarrow \quad M_h = 2M_W(1 - \mathrm{O}(1) g_W^4)^{1/2}$$

Likely just one bound state: in non-relativistic approximation binding would be given by a potential well of height $V_0 \sim M_W$ and width $a \sim \Lambda_T^{-1}$ , with $8m_{red} V_0 a^2 \ll 1$

# Effective Lagrangian for $E < 1$ TeV, $h$-couplings, ...

A model like $\mathcal{L}^{BSMM}$ should be described for $E \ll \Lambda_T$ by the effective Lagrangian

$$\Gamma_{LE} = \frac{1}{4}F_A F_A + \frac{1}{4}(F_W F_W + F_B F_B) + \sum_f (\bar{f}_R \not{D}_f^{A,B} f_R + \bar{f}_L \not{D}_f^{A,W,B} f_L) +$$
$$+ \frac{1}{2}\partial_\mu h \partial_\mu h + V_{eff}(h) + [\underline{C_2 \Lambda_T^2} + C_2' \Lambda_T h + C_2'' h^2] \frac{1}{2} \mathrm{Tr}\,[D_\mu^{W,B} U^\dagger D_\mu^{W,B} U] +$$
$$+ \sum_{f \neq \nu} [\underline{m_f} + y_f h](\bar{f}_L\, u_{Y_f}\, f_R + \bar{f}_R\, u_{Y_f}^\dagger\, f_L) + \mathrm{O}\left(\frac{1}{\Lambda_T}\right), \qquad \Lambda_T = \text{a few TeV}$$

where $\quad U = \exp\left[\frac{i\vec{\zeta}\cdot\vec{\tau}}{\sqrt{c}\Lambda_T}\right] = \left[\tilde{u}_{Y=-\frac{1}{2}} | u_{Y=\frac{1}{2}}\right] \equiv \left[\tilde{u} = -i\tau_2 u^\star \,|\, u\right] \quad$ and

- $m_h < \Lambda_T$ and $h$-couplings as (to be) measured in experiments, for instance

  $2C_2\Lambda_T / C_2' \simeq m_f / y_f$ (unitarity) and $m_f \simeq y_f \sqrt{C_2} \Lambda_T$ (plausible, non-trivial)

- flavour changing currents much like in the SM: here no tree level FCNC

- loop effects consistent with precise EW data: contribution of non-SM $T$-fermions to $S$-parameter $\sim \frac{1}{M^2_{\mathrm{T-meson}}} < \frac{1}{100 v^2_{SM}}$, likely $> 12$ $T$-fermion doublets allowed in $\mathcal{L}^{BSMM}$

# Conclusions

A mass mechanism for fermions and EW gauge bosons: based on strong interactions, fermion chirality ($\tilde{\chi}$) broken at the UV cutoff scale & an exact symmetry ($\chi$) that once gauged describes EW interactions; changing "universality" paradigm: irrelevant terms control LE physics.

Once combined with experimental info it has interesting implications:

- new strong interaction with RGI scale $\Lambda_T > v_{SM}$ and $\sim$ a few TeV
- new fermions with mass O($\Lambda_T$) confined in detectable resonances
- solving the naturalness problem: EW & top mass scale derived from $\Lambda_T$
- composite Higgs boson: a bound state in the WW+ZZ channel
- a low energy ($p < 1$ TeV) effective action quite similar to the SM (composite Higgs couplings may show small deviations from SM)
- insights on fermion mass hierarchy pattern: $\frac{m_\tau}{m_t}$, $\frac{m_b}{m_t}$, $m_\nu \simeq 0$, ...

# Backup

Backup slides

# Predictivity in models with $\tilde{\chi}$-breaking & NP mass

Renormalizable model: low energy $\tilde{\chi}_L$ SDE at the point of maximal $\tilde{\chi}$ symmetry:

$$\partial_\mu \tilde{J}^{L,i}_\mu = 0\,, \qquad \text{(Wigner phase)}$$

$$\partial_\mu \tilde{J}^{L,i}_\mu = \sum_f C_{1,f}\Lambda_T \mathcal{D}^{L,i}_f + \frac{ig_W}{2} C_2 \Lambda_T^2 \mathrm{tr}\Big(U^\dagger [\tfrac{\tau^i}{2}, W_\mu] D^{WB}_\mu U - \mathrm{h.c.}\Big)\,, \qquad \text{(NG phase)}$$

⋆ RGI of l.h.s. $\Rightarrow$ RGI (& UV-finite) NP $\tilde{\chi}$-breaking terms on the r.h.s.

with $\quad \mathcal{D}^{L,i}_f = [\bar{f}_L \frac{\tau^i}{2} U f_R - \mathrm{h.c.}]$ and $\quad C_{1,f} = \mathrm{O}(\rho^2_{f,cr})\, \alpha^{n(f)}_{\mathrm{coup}(f)} [1 + \mathrm{O}(\alpha\ldots)]$

⋆ effective masses: $\mathrm{C}_{1,f}\Lambda_T \leftrightarrow m^{eff}_f$, $\quad C_2 g_W^2 \Lambda_T^2 \leftrightarrow (m^{eff}_W)^2$

UV cutoff $b^{-1} \to \infty$ at fixed $M_{\mathrm{Tglueball}}$, $M_{\mathrm{proton}}$, $G_F$, $\sin^2\theta_W \leftrightarrow \hat{\alpha}_{T,S,W,Y}$

$\tilde{\chi}$-symmetry $\Rightarrow \sum_{f=1}^{N^{tot}_{ferm}} \rho^2_{f,cr}(1 + \mathrm{O}(\rho^2_{f,cr})) = O(1)$ entails bounds for the $\rho_{f,cr}$'s

$\to \quad \rho_{Q,cr}, \rho_{L,cr}$ control $m^{eff}_Q$, $m^{eff}_L$, as well as $m^{eff}_W$, $m^{eff}_Z$

$\to \quad \rho_{t,cr}$ controls $m^{eff}_t$, ... $\rho_{\tau,cr}$ controls $m^{eff}_\tau$, ...

Are the $\rho_{f,cr}$'s of the most massive fermions ($Q$, $L$, 3rd SM family) of similar size ?

Plausible, if all fermions $\in$ some GUT multiplet & peculiar $\tilde{\chi}$ terms for 2nd/1st family