Matching the Quasi Parton Distribution in a Momentum Subtraction Scheme

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I. Stewart and Y.Z., PRD97 (2018), 054512

Outline

- Renormalization of the quasi-PDF
- Matching between RI/MOM quasi-PDF and MSbar PDF
- The “ratio scheme”

Procedure of Systematic Calculation

1. Simulation of the quasi PDF in lattice QCD

3. Subtraction of higher twist corrections

$$\tilde{q}_i(x, P^z, \tilde{\mu}) = \int_{-1}^{+1} \frac{dy}{|y|}\, C_{ij}\left(\frac{x}{y}, \frac{\tilde{\mu}}{P^z}, \frac{\mu}{|y|P^z}\right) q_j(y, \mu) + \mathcal{O}\left(\frac{M^2}{P_z^2}, \frac{\Lambda_{\rm QCD}^2}{P_z^2}\right) ,$$

2. Renormalization of the lattice quasi PDF, and then taking the continuum limit

4. Matching to the MSbar PDF.

Renormalization

- The gauge-invariant quark Wilson line operator can be renormalized multiplicatively in the coordinate space:

$$\tilde{O}_{\Gamma}(z)=\bar{\psi}(z)\Gamma W(z,0)\psi(0)=Z_{\psi,z}e^{-\delta m|z|}\left(\bar{\psi}(z)\Gamma W(z,0)\psi(0)\right)^{R}$$

X. Ji, J.-H. Zhang, and Y.Z., 2017; J. Green, K. Jansen, and F. Steffens, 2017; T. Ishikawa, Y.-Q. Ma, J. Qiu, S. Yoshida, 2017.

- Different renormalization schemes can be converted to each other in coordinate space;

$$\tilde{Q}^{X}(\zeta,z^2\mu_R^2)=\frac{Z_{\overline{MS}}(\varepsilon,\mu)}{Z_X(\varepsilon,z^2\mu_R^2)}\tilde{Q}^{\overline{MS}}(\zeta,z^2\mu^2)=Z'_X(z^2\mu_R^2,\frac{\mu_R}{\mu})\tilde{Q}^{\overline{MS}}(\zeta,z^2\mu^2)$$

Regulator independence

If we apply the same renormalization scheme in both lattice and continuum theories,

$$\begin{aligned}\tilde{O}^R_\Gamma(z,\mu) &= Z_X^{-1}(z,\varepsilon,\mu)\tilde{O}_\Gamma(z,\varepsilon)\\ &= \lim_{a\to 0} Z_X^{-1}(z,a^{-1},\mu)\tilde{O}_\Gamma(z,a^{-1})\end{aligned}$$

- This should apply to all renormalization schemes;
- After renormalization, we just need to calculate the matching coefficient in dimensional regularization;
- However, not all schemes can be implemented nonperturbatively on the lattice.

A momentum subtraction scheme

Martinelli et al., 1994

Regulator-independent momentum subtraction scheme (RI/MOM):

$$Z_{OM}^{-1}(z,a^{-1},p_R^z,\mu_R)\langle p|\tilde{O}_\Gamma(z,a^{-1})|p\rangle\Big|_{\substack{p^2=\mu_R^2\\ p^z=p_R^z}}=\langle p|\tilde{O}_\Gamma(z)|p\rangle_{\text{tree}}$$

$$Z_{OM}(z,a^{-1},p_R^z,\mu_R)=\frac{\langle p|\tilde{O}_\Gamma(z,a^{-1})|p\rangle\Big|_{\substack{p^2=\mu_R^2\\ p^z=p_R^z}}}{\langle p|\tilde{O}_\Gamma(z)|p\rangle_{\text{tree}}}=\frac{\langle p|\tilde{O}_\Gamma(z)|p\rangle\Big|_{\substack{p^2=\mu_R^2\\ p^z=p_R^z}}}{(4p_R^\Gamma\zeta)e^{-ip_R^z*z}}$$

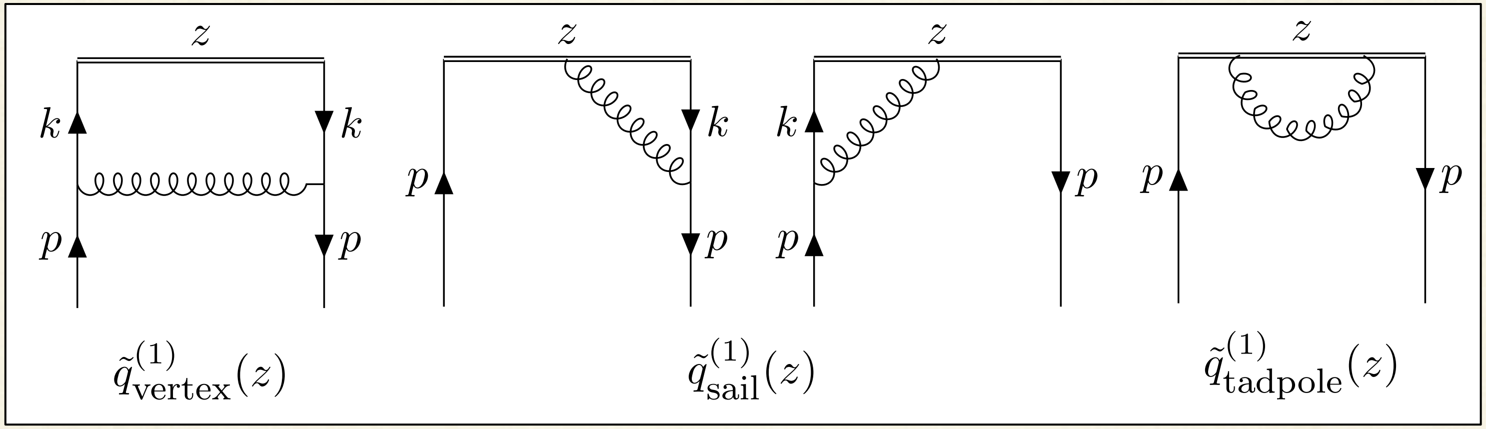
- Can be implemented nonperturbatively on the lattice.
- Scales introduced in renormalization: μ_R, $p_R{}^z$.

Matching coefficient

Strategy:

- Extracting matching coefficient by comparing the quasi-PDF and light-cone PDF in an off-shell quark state;
- Quark off-shellness $p^2<0$ regulates the infrared (IR) and collinear divergences;

One-loop Feynman diagrams



- Dimensional regularization $d=4-2\varepsilon$;
- $\Gamma=\gamma^z$ for discussion in this talk, $\Gamma=\gamma^t$ case calculated in Y.S. Liu et al. (LP[3]), arXiv:1807.06566. External momentum $p^\mu=(p^0,0,0,p^z)$ and $p^2<0$;

One-loop results

I. Stewart and YZ, PRD 2018

- One-loop **bare** matrix element:

$$\tilde{q}^{(1)}(z,p^z,0,-p^2) = \frac{\alpha_s C_F}{2\pi}(4p^z\zeta)\int_{-\infty}^{\infty} dx\,\left(e^{-ixp^z z} - e^{-ip^z z}\right) h(x,\rho)$$

$$\rho \equiv \frac{(-p^2 - i\varepsilon)}{p_z^2},$$

$$h(x,\rho) \equiv \begin{cases} \frac{1}{\sqrt{1-\rho}}\left[\frac{1+x^2}{1-x} - \frac{\rho}{2(1-x)}\right]\ln\frac{2x-1+\sqrt{1-\rho}}{2x-1-\sqrt{1-\rho}} - \frac{\rho}{4x(x-1)+\rho} + 1 & x>1 \\ \frac{1}{\sqrt{1-\rho}}\left[\frac{1+x^2}{1-x} - \frac{\rho}{2(1-x)}\right]\ln\frac{1+\sqrt{1-\rho}}{1-\sqrt{1-\rho}} - \frac{2x}{1-x} & 0<x<1 \\ \frac{1}{\sqrt{1-\rho}}\left[\frac{1+x^2}{1-x} - \frac{\rho}{2(1-x)}\right]\ln\frac{2x-1-\sqrt{1-\rho}}{2x-1+\sqrt{1-\rho}} + \frac{\rho}{4x(x-1)+\rho} - 1 & x<0 \end{cases},$$

- Formally satisfies vector current conservation (v.c.c.), but:

$\lim_{|x|\to\infty} h(x,\rho) \sim -\frac{3}{2|x|}$, $\int_{-\infty}^{\infty} dx\; h(x,\rho)$ is logarithmically divergent needs ε to be regularized!

  - This logarithmic divergence is what needs to be treated carefully for the MSbar scheme; Izubuchi, Ji, Jin, Stewart and Y.Z., 2018
  - **Not a problem for the RI/MOM scheme!**

RI/MOM renormalization

$$\tilde{q}^{(1)}_{\rm CT}(z,p^z,p^z_R,\mu_R) = -Z^{(1)}_{\rm OM}(z,p^z_R,0,\mu_R)\,\tilde{q}^{(0)}(z,p^z)\,.$$

- Renormalization in coordinate space:

$$\tilde{q}^{(1)}_{\rm OM}(z,p^z,p^z_R,-p^2,\mu_R) = \tilde{q}^{(1)}(z,p^z,0,-p^2) + \tilde{q}^{(1)}_{\rm CT}(z,p^z,p^z_R,\mu_R)$$

$$\tilde{q}^{(1)}(z,p^z,0,-p^2) = \frac{\alpha_s C_F}{2\pi}(4p^z\zeta)\int_{-\infty}^{\infty}dx\,\left(e^{-ixp^z z} - e^{-ip^z z}\right)h(x,\rho)$$

$\rho = \dfrac{-p^2}{p_z^2} = \dfrac{p_z^2 - p_0^2}{p_z^2} < 1$ in Minkowski space

$$\tilde{q}^{(1)}_{\rm CT}(z,p^z,p^z_R,\mu_R) = -\frac{\alpha_s C_F}{2\pi}(4p^z\zeta)\int_{-\infty}^{\infty}dx\,\left(e^{i(1-x)p^z_R z - ip^z z} - e^{-ip^z z}\right)h(x,r_R)$$

$r_R = \dfrac{\mu_R^2}{(p_R^z)^2} = \dfrac{(p_R^4)^2 + (p_R^z)^2}{(p_R^z)^2} > 1$ for Euclidean momentum,

analytical continuuation from $\rho < 1$!

- Identify the collinear divergence: onshell limit!

$$\tilde{q}^{(1)}_{\rm OM}(z,p^z,p^z_R,-p^2 << p_z^2,\mu_R) = \tilde{q}^{(1)}(z,p^z,0,-p^2 << p_z^2) + \tilde{q}^{(1)}_{\rm CT}(z,p^z,p^z_R,\mu_R)$$

$$\tilde{q}^{(1)}(z,p^z,0,-p^2 \ll p_z^2) = \frac{\alpha_s C_F}{2\pi}(4p^z\zeta)\int_{-\infty}^{\infty}dx\,\left(e^{-ixp^z z} - e^{-ip^z z}\right)h_0(x,\rho),$$

$$h_0(x,\rho) \equiv \begin{cases} \dfrac{1+x^2}{1-x}\ln\dfrac{x}{x-1} + 1 & x > 1 \\ \dfrac{1+x^2}{1-x}\ln\dfrac{4}{\rho} - \dfrac{2x}{1-x} & 0 < x < 1 \\ \dfrac{1+x^2}{1-x}\ln\dfrac{x-1}{x} - 1 & x < 0 \end{cases},$$

RI/MOM renormalization

- Fourier transform to obtain the *x*-dependent quasi-PDF:

$$\tilde{q}^{(1)}_{\rm OM}(x,p^z,p^z_R,\mu_R) = \int \frac{dz}{2\pi}\, e^{ixzp^z}\, \tilde{q}^{(1)}_{\rm OM}(z,p^z,p^z_R,\mu_R)$$

$$\eta \equiv \frac{p^z}{p^z_R}$$

$$= \frac{\alpha_s C_F}{2\pi}(4\zeta)\Big\{\int dy\,\underbrace{\big[\delta(y-x)-\delta(1-x)\big]\big[h_0(y,\rho)-h(y,r_R)\big]}_{\text{A plus function}} + h(x,r_R) - |\eta|\, h\big(1+\eta(x-1),r_R\big)\Big\},$$

One can explicitly check that the RI/MOM quasi-PDF satisfies v.c.c.:

$$\int_{-\infty}^{\infty} dx\, \tilde{q}^{(1)}_{\rm OM}(x,p^z,p^z_R,-p^2,\mu_R) = \frac{\alpha_S C_F}{2\pi}(4\zeta)\left[\int_{-\infty}^{\infty} dx\, h(x,r_R) - \int_{-\infty}^{\infty} dx\, |\eta| h(1+|\eta|(x-1),r_R)\right] = 0$$

RI/MOM renormalization

- Full result of RI/MOM quasi-PDF: Plus functions with δ-function at x=1

$$\tilde{q}^{(1)}_{\rm OM}(x,p^z,p^z_R,\mu_R) \tag{37}$$
$$= \frac{\alpha_s C_F}{2\pi}(4\zeta)\begin{cases} \left[\frac{1+x^2}{1-x}\ln\frac{x}{x-1} - \frac{2}{\sqrt{r_R-1}}\left[\frac{1+x^2}{1-x}-\frac{r_R}{2(1-x)}\right]\arctan\frac{\sqrt{r_R-1}}{2x-1}+\frac{r_R}{4x(x-1)+r_R}\right]_\oplus & x>1 \\ \left[\frac{1+x^2}{1-x}\ln\frac{4(p^z)^2}{-p^2} - \frac{2}{\sqrt{r_R-1}}\left[\frac{1+x^2}{1-x}-\frac{r_R}{2(1-x)}\right]\arctan\sqrt{r_R-1}\right]_+ & 0<x<1 \\ \left[\frac{1+x^2}{1-x}\ln\frac{x-1}{x} + \frac{2}{\sqrt{r_R-1}}\left[\frac{1+x^2}{1-x}-\frac{r_R}{2(1-x)}\right]\arctan\frac{\sqrt{r_R-1}}{2x-1}-\frac{r_R}{4x(x-1)+r_R}\right]_\ominus & x<0 \end{cases}$$
$$+\frac{\alpha_s C_F}{2\pi}(4\zeta)\left\{h(x,r_R)-|\eta|\,h(1+\eta(x-1),r_R)\right\}.$$

- Unregulated divergence in the $\delta(1\text{-}x)$ part? **No!**

$$\lim_{|x|\to\infty}\tilde{q}^{(1)}_{\rm OM}(x,p^z,p^z_R,-p^2,\mu_R)\sim\frac{1}{x^2},$$ integrable at infinity, no need to regularize!

- MSbar PDF:

$$q^{(1)}(x,\mu)=\frac{\alpha_s C_F}{2\pi}(4\zeta)\begin{cases} 0 & x>1 \\ \left[\frac{1+x^2}{1-x}\ln\frac{\mu^2}{-p^2}-\frac{1+x^2}{1-x}\ln\left[x(1-x)\right]-(2-x)\right]_+ & 0<x<1 \\ 0 & x<0 \end{cases}.$$

Matching coefficient

- Matching coefficient for isovector quasi-PDF in **quark**:

$$C^{\mathrm{OM}}\left(\xi, \frac{\mu_R}{p^z_R}, \frac{\mu}{p^z}, \frac{p^z}{p^z_R}\right) - \delta(1-\xi) \qquad \boxed{\xi = \frac{x}{y}} \tag{40}$$

$$= \frac{\alpha_s C_F}{2\pi}\begin{cases} \left[\frac{1+\xi^2}{1-\xi}\ln\frac{\xi}{\xi-1} - \frac{2(1+\xi^2)-r_R}{(1-\xi)\sqrt{r_R-1}}\arctan\frac{\sqrt{r_R-1}}{2\xi-1} + \frac{r_R}{4\xi(\xi-1)+r_R}\right]_{\oplus} & \xi > 1 \\ \left[\frac{1+\xi^2}{1-\xi}\ln\frac{4(p^z)^2}{\mu^2} + \frac{1+\xi^2}{1-\xi}\ln\left[\xi(1-\xi)\right] + (2-\xi) - \frac{2\arctan\sqrt{r_R-1}}{\sqrt{r_R-1}}\left\{\frac{1+\xi^2}{1-\xi} - \frac{r_R}{2(1-\xi)}\right\}\right]_{+} & 0 < \xi < 1 \\ \left[\frac{1+\xi^2}{1-\xi}\ln\frac{\xi-1}{\xi} + \frac{2}{\sqrt{r_R-1}}\left[\frac{1+\xi^2}{1-\xi} - \frac{r_R}{2(1-\xi)}\right]\arctan\frac{\sqrt{r_R-1}}{2\xi-1} - \frac{r_R}{4\xi(\xi-1)+r_R}\right]_{\ominus} & \xi < 0 \end{cases}$$

$$+ \frac{\alpha_s C_F}{2\pi}\left\{h(\xi, r_R) - |\eta|\, h\big(1+\eta(\xi-1), r_R\big)\right\},$$

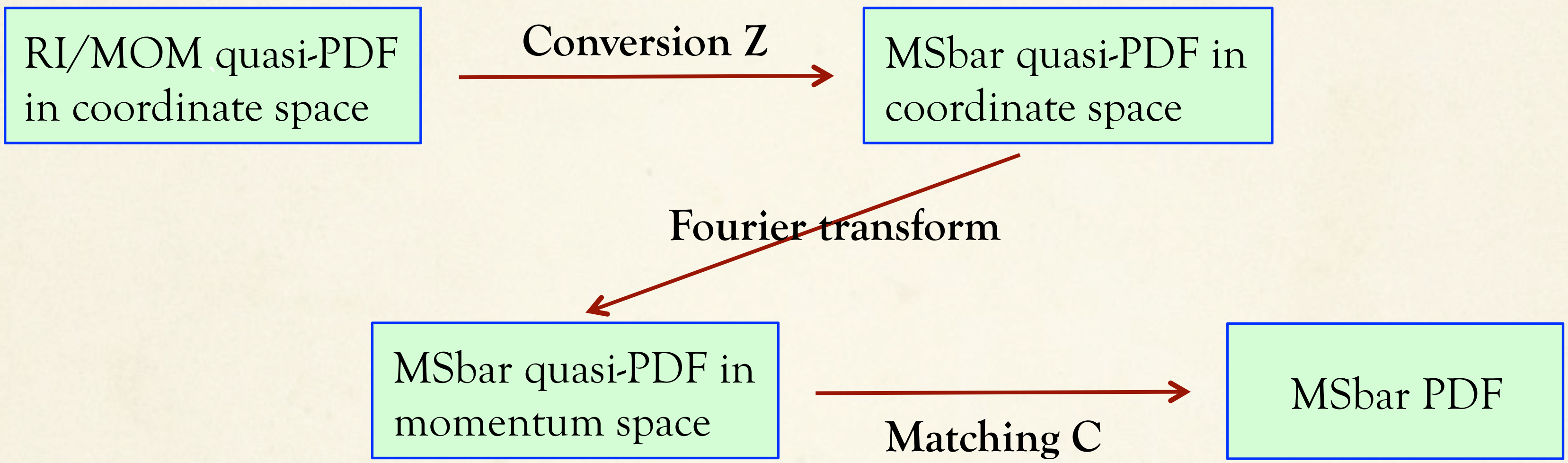
- Matching coefficient for isovector **nucleon** quasi-PDF

$$\boxed{p^z \to yP^z,\ \eta = yP^z / p^z_R}$$

RI/MOM matching also preserves particle number conservation of the nucleon PDF!

Comparison to the matching procedure by ETMC

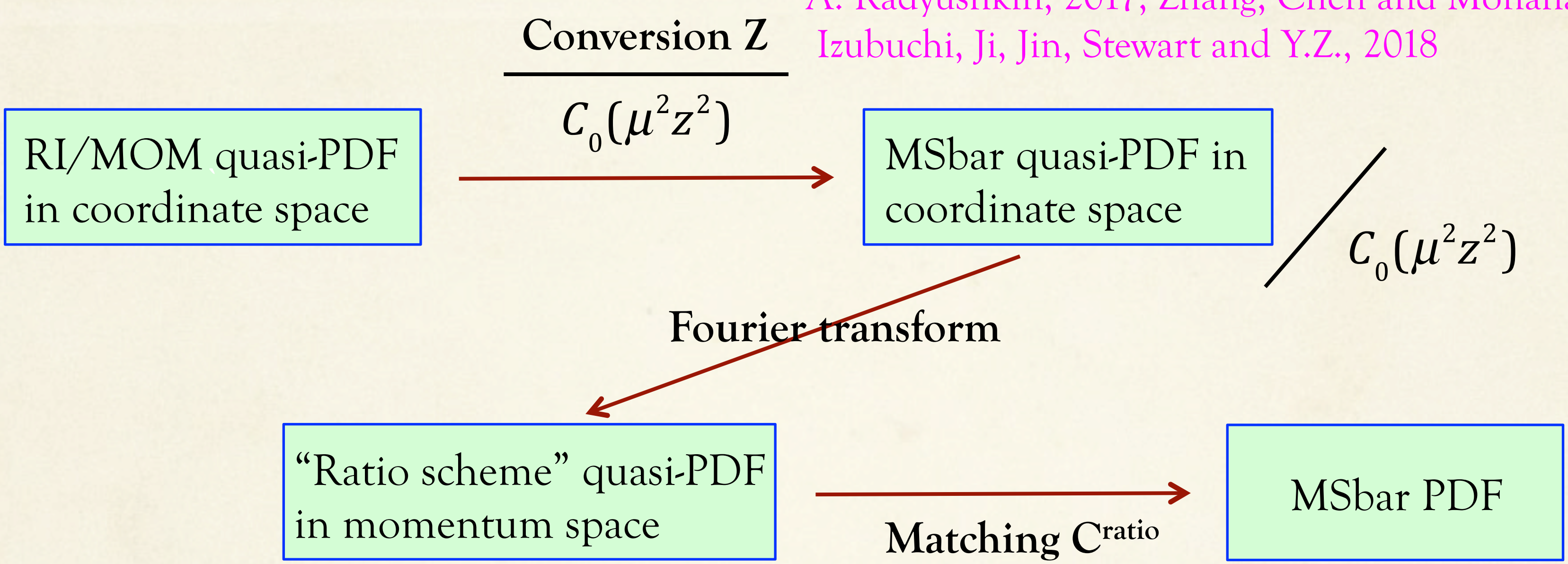
M. Constantinou and H. Panagopoulos, 2017
C. Alexandrou et al. (ETMC), 2017, 2018



- MSbar quasi-PDF in coordinate space is logarithmically divergent as $z^2=0$;
- The matching coefficient C does not preserve v.c.c., but it **cancels out** the above logarithmic divergence to obtain a finite MSbar PDF with v.c.c.;
- Numerically such cancellation of divergences can be tricky!

The "ratio scheme"

A. Radyushkin, 2017; Zhang, Chen and Monahan, 2018; Izubuchi, Ji, Jin, Stewart and Y.Z., 2018



- $C_0(\mu^2 z^2)$ is the lowest order Wilson coefficient in the operator product expansion of the quasi-PDF in coordinate space;
- $C_0(\mu^2 z^2)$ cancels the logarithmic divergence in the MSbar quasi-PDF as z^2=0, and the matching coefficient C^{ratio} preserves v.c.c.;
- Each step is finite, which is friendly to numerical implementation.

The "ratio scheme"

A. Radyushkin, 2017; Zhang, Chen and Monahan, 2018; Izubuchi, Ji, Jin, Stewart and Y.Z., 2018

- For $\Gamma=\gamma^t$ case

$$\boxed{C_0(\mu^2 z^2) = 1 + \frac{\alpha_S C_F}{2\pi}\cdot\left(\frac{3}{2}\ln\frac{\mu^2 z^2 e^{2\gamma_E}}{4} + \frac{5}{2}\right)}$$

Different from the one by ETMC (1803.02685) by a finite term

$$\lim_{|\xi|\to\infty} C^{ratio}(\xi, \frac{\mu}{yP^z}) \sim \frac{1}{\xi^2}\ !$$

$$C^{\rm ratio}\left(\xi, \frac{\mu}{|y|P^z}\right) = \delta\left(1-\xi\right) + \frac{\alpha_s C_F}{2\pi}\begin{cases}\left(\frac{1+\xi^2}{1-\xi}\ln\frac{\xi}{\xi-1} + 1 - \frac{3}{2(1-\xi)}\right)_{+(1)}^{[1,\infty]} & \xi > 1\\ \left(\frac{1+\xi^2}{1-\xi}\left[-\ln\frac{\mu^2}{y^2P_z^2} + \ln\left(4\xi(1-\xi)\right) - 1\right] + 1 + \frac{3}{2(1-\xi)}\right)_{+(1)}^{[0,1]} & 0<\xi<1\\ \left(-\frac{1+\xi^2}{1-\xi}\ln\frac{-\xi}{1-\xi} - 1 + \frac{3}{2(1-\xi)}\right)_{+(1)}^{[-\infty,0]} & \xi < 0\end{cases},$$

To appear in the updated version of Izubuchi, Ji, Jin, Stewart and Y.Z., 2018

- **Moreover, it can be shown that**

$$\boxed{\lim_{p_R^z\to 0} C_{OM}\left(\xi, \frac{\mu_R}{p_R^z}, \frac{\mu}{yP^z}, \frac{yP^z}{p_R^z}\right) = C^{ratio}\left(\xi, \frac{\mu}{yP^z}\right)}$$

Summary

- The matching coefficient for the RI/MOM scheme quasi-PDF and MSbar PDF is derived at one-loop order;
- The matching for RI/MOM quasi-PDF preserves vector current conservation;
- The "ratio scheme" can also preserve the vector current conservation, and is equivalent to a special limit of the RI/MOM scheme.

Nonperturbative renormalization on the lattice

- For $\Gamma=\gamma^z$, we have to choose $p_R{}^z \neq 0$; for $\Gamma=\gamma^t$, we can choose $p_R{}^z = 0$ while $|p^2|>>\Lambda_{QCD}$;

$$\boxed{Z_{OM}(z,a^{-1},p_z^R,\mu_R)=\langle p|\tilde{O}_\Gamma(z)|p\rangle\Big|_{\substack{p^2=\mu_R^2\\ p_z=p_z^R}} / (4p_R^\Gamma \zeta e^{-ip_R^z * z})}$$

- For nonzero $p_R{}^z$, Z_{OM} is a complex number, real part symmetric and imaginary part anti-symmetric;
- Operator mixing on the lattice between O_Γ and O_1 at $O(a^0)$ (for γ^z) and $O(a^1)$ (for γ^t) due to broken chiral symmetry. M. Constantinou and H. Panagopoulos, 2017; T. Ishikawa et al. (LP3), 2017.

MSbar treatment

- Bare quasi-PDF:

$$\tilde{q}^{(1)}(x,p^z,\epsilon) = \frac{\alpha_s C_F}{2\pi}\left\{\frac{3}{2}\left(\frac{1}{\epsilon_{\rm UV}}-\frac{1}{\epsilon_{\rm IR}}\right)\delta(1-x)+\frac{\Gamma(\epsilon+\frac{1}{2})e^{\epsilon\gamma_E}}{\sqrt{\pi}}\frac{\mu^{2\epsilon}}{p_z^{2\epsilon}}\frac{1-\epsilon}{\epsilon_{\rm IR}(1-2\epsilon)}\right.$$
$$\left.\times\left[|x|^{-1-2\epsilon}\left(1+x+\frac{x}{2}(x-1+2\epsilon)\right)-|1-x|^{-1-2\epsilon}\left(x+\frac{1}{2}(1-x)^2\right)+I_3(x)\right]\right\}$$

$$I_3(x)=\theta(x-1)\left(\frac{x^{-1-2\epsilon}}{x-1}\right)^{[1,\infty]}_{+(1)}-\theta(x)\theta(1-x)\left(\frac{x^{-1-2\epsilon}}{1-x}\right)^{[0,1]}_{+(1)}-\delta(1-x)\pi\csc(2\pi\epsilon)+\theta(-x)\frac{|x|^{-1-2\epsilon}}{x-1}$$

- ε expansion:

$$\frac{\theta(x)}{x^{1+\epsilon}}=\left[-\frac{1}{\epsilon_{\rm IR}}\delta(x)+\frac{1}{\epsilon_{\rm UV}}\frac{1}{x^2}\delta^+\left(\frac{1}{x}\right)\right]$$
$$+\left(\frac{1}{x}\right)^{[0,1]}_{+(0)}+\left(\frac{1}{x}\right)^{[1,\infty]}_{+(\infty)}$$
$$-\epsilon\left[\left(\frac{\ln x}{x}\right)^{[0,1]}_{+(0)}+\left(\frac{\ln x}{x}\right)^{[1,\infty]}_{+(\infty)}\right]+O(\epsilon^2)$$

Plus functions with δ-function at $x=\pm\infty$, consistent with DimReg.

$$\int_0^\infty\frac{dx}{x^{1+\epsilon}}=\frac{1}{\epsilon_{\rm UV}}-\frac{1}{\epsilon_{\rm IR}}\,.$$

MSbar treatment

- Renormalized quasi-PDF:

$$\delta\tilde{q}'^{(1)}(x,\mu/|p^z|,\epsilon_{\rm UV}) = \frac{\alpha_s C_F}{2\pi}\frac{3}{2\epsilon_{\rm UV}}\delta(1-x)\,,$$

$$\tilde{q}'^{(1)}(x,\mu/|p^z|,\epsilon_{\rm IR}) = \frac{\alpha_s C_F}{2\pi}\begin{cases} \left(\frac{1+x^2}{1-x}\ln\frac{x}{x-1}+1+\frac{3}{2x}\right)_{+(1)}^{[1,\infty]} - \left(\frac{3}{2x}\right)_{+(\infty)}^{[1,\infty]} & x>1 \\ \left(\frac{1+x^2}{1-x}\left[-\frac{1}{\epsilon_{\rm IR}}-\ln\frac{\mu^2}{4p_z^2}+\ln\left(x(1-x)\right)\right]-\frac{x(1+x)}{1-x}\right)_{+(1)}^{[0,1]} & 0<x<1 \\ \left(-\frac{1+x^2}{1-x}\ln\frac{-x}{1-x}-1+\frac{3}{2(1-x)}\right)_{+(1)}^{[-\infty,0]} - \left(\frac{3}{2(1-x)}\right)_{+(-\infty)}^{[-\infty,0]} & x<0 \end{cases}$$

$$+\frac{\alpha_s C_F}{2\pi}\left[\delta(1-x)\left(\frac{3}{2}\ln\frac{\mu^2}{4p_z^2}+\frac{5}{2}\right)+\frac{3}{2}\gamma_E\left(\frac{1}{(x-1)^2}\delta^+(\frac{1}{x-1})+\frac{1}{(1-x)^2}\delta^+(\frac{1}{1-x})\right)\right]$$

- Plus functions with δ-function at $x=\pm\infty$ needed for V.C.C..

Other schemes

Transverse momentum cut-off scheme (Xiong, Ji, Zhang, and Y.Z., 2014):

Linear divergence

$$C^{\Lambda_T}\left(\xi,\frac{\mu}{p^z},\frac{\Lambda}{P^z}\right)=\delta(1-\xi)+\frac{\alpha_s C_F}{2\pi}\begin{cases}\left[\frac{1+\xi^2}{1-\xi}\ln\frac{\xi}{\xi-1}+1+\frac{1}{(1-\xi)^2}\frac{\Lambda_T}{P^z}\right]_\oplus & \xi>1\\ \left[\frac{1+\xi^2}{1-\xi}\ln\frac{4(p^z)^2}{\mu^2}+\frac{1+\xi^2}{1-\xi}\ln\xi(1-\xi)+1-\frac{2\xi}{1-\xi}+\frac{1}{(1-\xi)^2}\frac{\Lambda_T}{P^z}\right]_+ & 0<\xi<1\\ \left[\frac{1+\xi^2}{1-\xi}\ln\frac{\xi-1}{\xi}-1+\frac{1}{(1-\xi)^2}\frac{\Lambda_T}{P^z}\right]_\ominus & \xi<0\end{cases}$$

MSbar scheme: gives convergent matching integrals (Izubuchi, Ji, Jin, Stewart and Y.Z., 2018)

$$C^{\overline{\rm MS}}\left(\xi,\frac{\mu}{|y|P^z}\right)=\delta(1-\xi)+\frac{\alpha_s C_F}{2\pi}\begin{cases}\left(\frac{1+\xi^2}{1-\xi}\ln\frac{\xi}{\xi-1}+1+\frac{3}{2\xi}\right)_{+(1)}^{[1,\infty]}-\frac{3}{2\xi} & \xi>1\\ \left(\frac{1+\xi^2}{1-\xi}\left[-\ln\frac{\mu^2}{y^2P_z^2}+\ln\left(4\xi(1-\xi)\right)\right]-\frac{\xi(1+\xi)}{1-\xi}\right)_{+(1)}^{[0,1]} & 0<\xi<1\\ \left(-\frac{1+\xi^2}{1-\xi}\ln\frac{-\xi}{1-\xi}-1+\frac{3}{2(1-\xi)}\right)_{+(1)}^{[-\infty,0]}-\frac{3}{2(1-\xi)} & \xi<0\end{cases}$$
$$+\frac{\alpha_s C_F}{2\pi}\delta(1-\xi)\left(\frac{3}{2}\ln\frac{\mu^2}{4y^2P_z^2}+\frac{5}{2}\right).$$

Plus functions with δ-function at x=1

Other schemes

Transverse momentum cut-off scheme (Xiong, Ji, Zhang, and Y.Z., 2014):

Unregulated UV divergence in the plus function

$$\sim -\frac{3}{2|\xi|},\ \xi\to\infty$$

$$C^{\Lambda_T}\left(\xi,\frac{\mu}{p^z},\frac{\Lambda}{P^z}\right)=\delta(1-\xi)$$
$$+\frac{\alpha_s C_F}{2\pi}\begin{cases}\left[\frac{1+\xi^2}{1-\xi}\ln\frac{\xi}{\xi-1}+1+\frac{1}{(1-\xi)^2}\frac{\Lambda_T}{P^z}\right]_{\oplus} & \xi>1\\ \left[\frac{1+\xi^2}{1-\xi}\ln\frac{4(p^z)^2}{\mu^2}+\frac{1+\xi^2}{1-\xi}\ln\xi(1-\xi)+1-\frac{2\xi}{1-\xi}+\frac{1}{(1-\xi)^2}\frac{\Lambda_T}{P^z}\right]_{+} & 0<\xi<1\\ \left[\frac{1+\xi^2}{1-\xi}\ln\frac{\xi-1}{\xi}-1+\frac{1}{(1-\xi)^2}\frac{\Lambda_T}{P^z}\right]_{\ominus} & \xi<0\end{cases}$$

MSbar scheme: gives convergent matching integrals (Izubuchi, Ji, Jin, Stewart and Y.Z., 2018)

$$\sim -\frac{1}{\xi^2},\ \xi\to\infty$$

$$C^{\overline{\mathrm{MS}}}\left(\xi,\frac{\mu}{|y|P^z}\right)=\delta\left(1-\xi\right)+\frac{\alpha_s C_F}{2\pi}\begin{cases}\left(\frac{1+\xi^2}{1-\xi}\ln\frac{\xi}{\xi-1}+1+\frac{3}{2\xi}\right)_{+(1)}^{[1,\infty]}-\frac{3}{2\xi} & \xi>1\\ \left(\frac{1+\xi^2}{1-\xi}\left[-\ln\frac{\mu^2}{y^2P_z^2}+\ln\left(4\xi(1-\xi)\right)\right]-\frac{\xi(1+\xi)}{1-\xi}\right)_{+(1)}^{[0,1]} & 0<\xi<1\\ \left(-\frac{1+\xi^2}{1-\xi}\ln\frac{-\xi}{1-\xi}-1+\frac{3}{2(1-\xi)}\right)_{+(1)}^{[-\infty,0]}-\frac{3}{2(1-\xi)} & \xi<0\end{cases}$$
$$+\frac{\alpha_s C_F}{2\pi}\delta(1-\xi)\left(\frac{3}{2}\ln\frac{\mu^2}{4y^2P_z^2}+\frac{5}{2}\right).$$

Numerical results

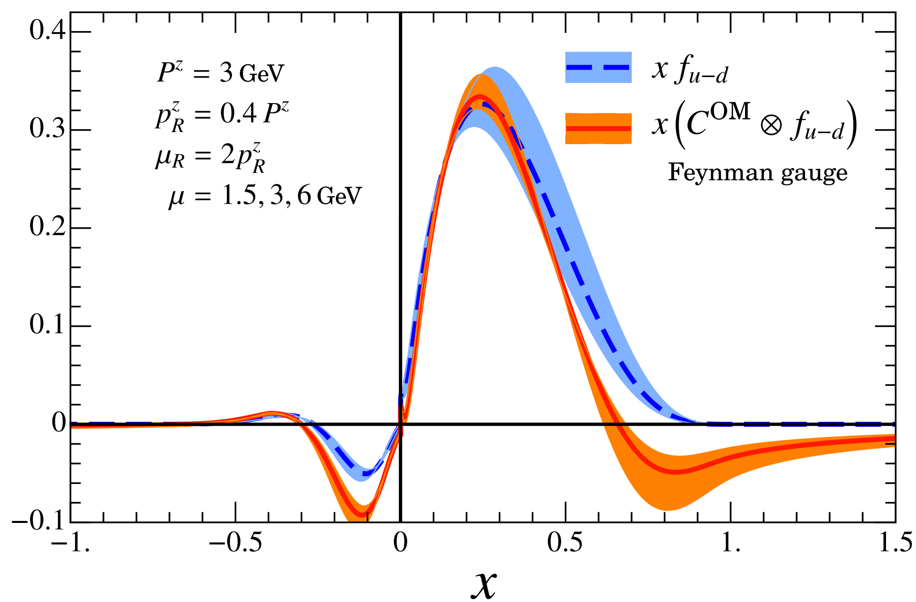
- Take the iso-vector parton distribution $f_{u\text{-}d}$ as example:

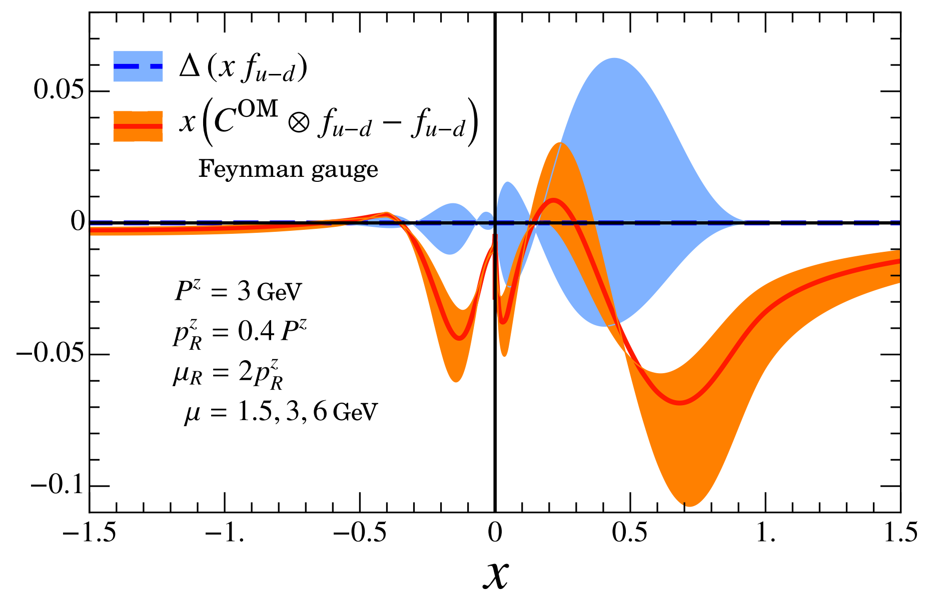
$$f_{u-d}(x,\mu) = f_u(x,\mu) - f_d(x,\mu) - f_{\bar{u}}(-x,\mu) + f_{\bar{d}}(-x,\mu) \ ,$$

$$f_{\bar{u}}(-x,\mu) = -f_{\bar{u}}(x,\mu) \ , \qquad f_{\bar{d}}(-x,\mu) = -f_{\bar{d}}(x,\mu) \ .$$

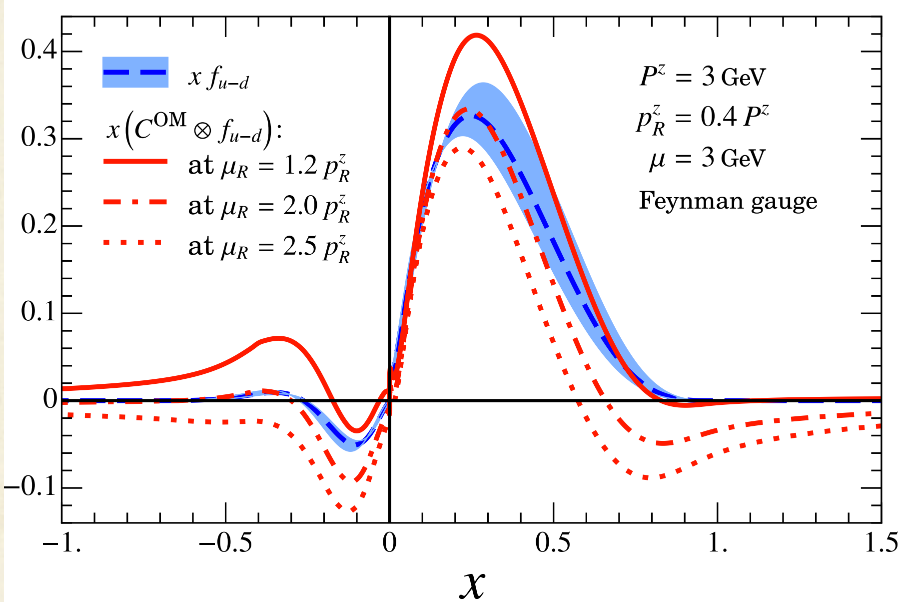
- Input:
  - "MSTW 2008" PDF
  - NLO $\alpha_s(\mu)$

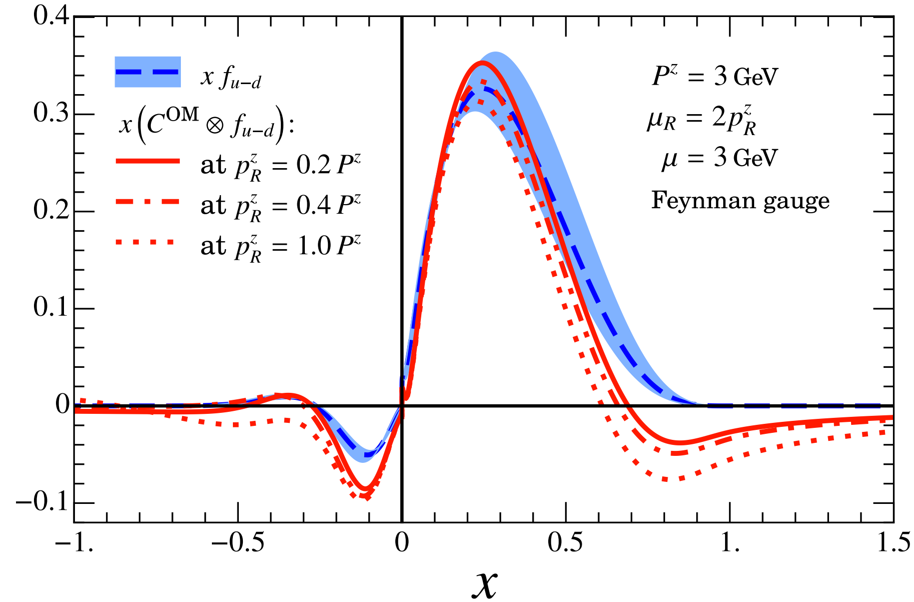
Variation of factorization scale μ



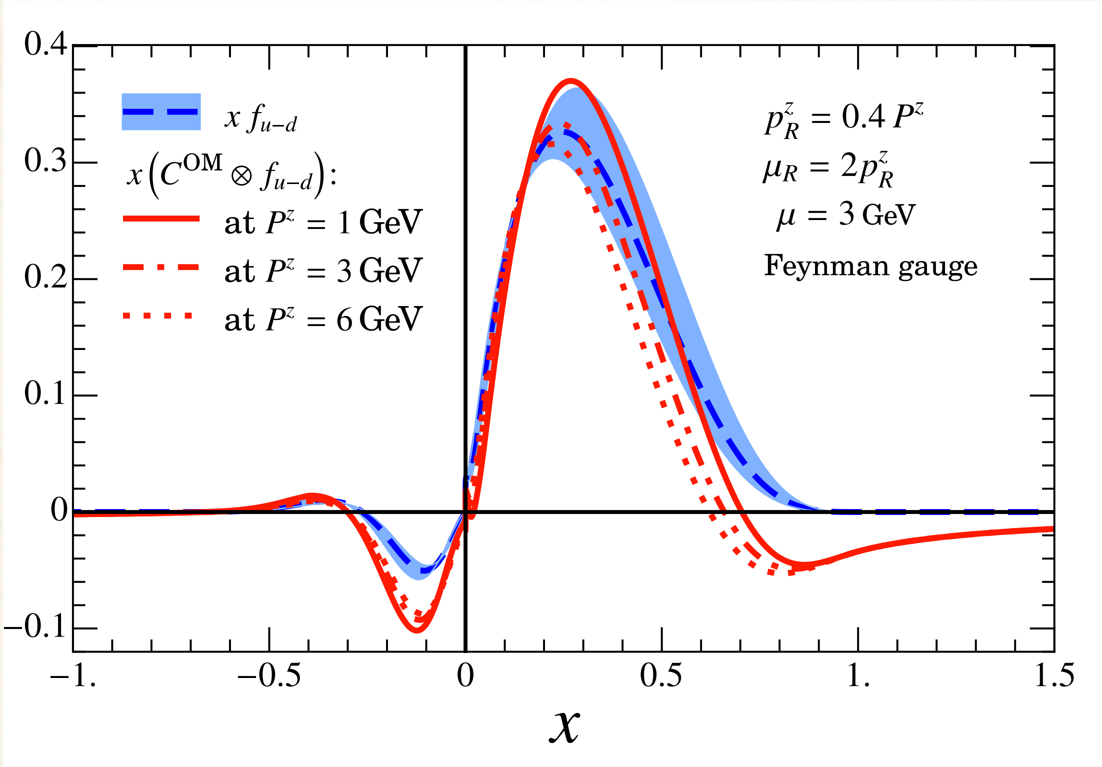


Variation of RI/MOM scales μ_R, $p_R{}^z$



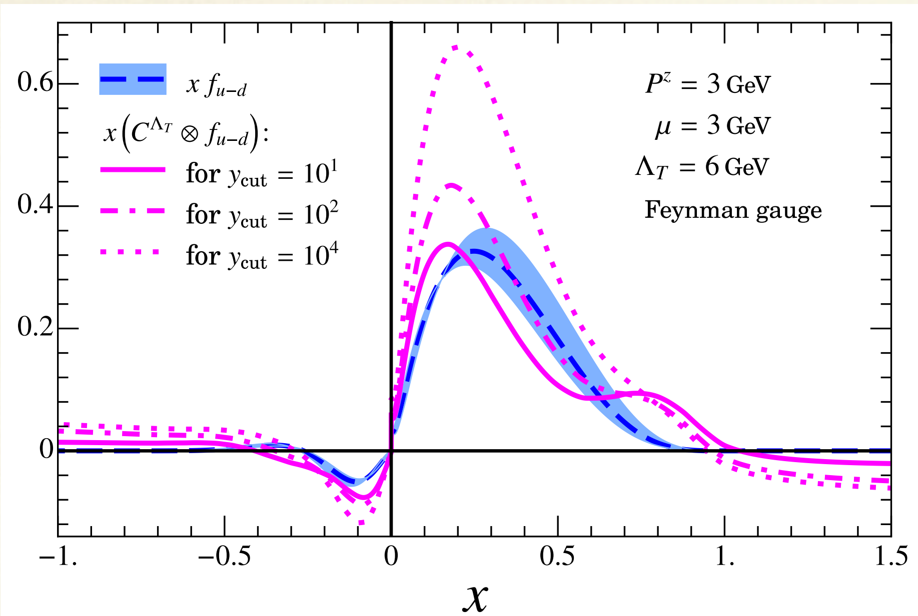


Variation of nucleon momentum P^z

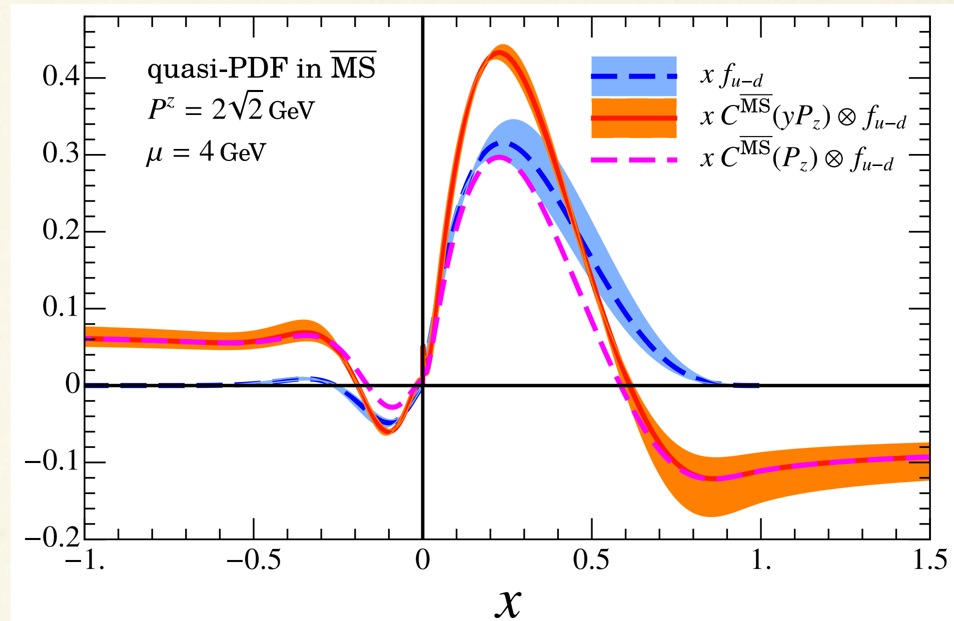


Other schemes

Transverse momentum cut-off scheme



MSbar scheme



Xiong, Ji, Zhang, Zhao, 2014;

Izubuchi, Ji, Jin, Stewart and Y.Z., 2018

Recall unregulated UV divergence when *x/y->∞*, and *y/x->∞*, use a hard cut-off $y_{cut}=10^{\pm n}$.