Lattice quantum gravity with scalar fields

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Outline

- Motivation for lattice gravity
- Asymptotic safety conjecture
- Lattice discretization
- EDT coupled to scalar fields
- Future directions

Motivation

Can we understand gravity as a quantum field theory in four dimensions assuming that gravity is asymptotically safe using lattice in the non-perturbative regime.

Weinberg's conjecture

It is well-known that gravity is perturbatively non-renormalizable (infinite number of parameters have to be fixed), however Weinberg in 1979 conjectured that it might be asymptotically safe. This means,

- Gravity may be non-perturbatively renormalizable
- Non-trivial, strongly interacting fixed point with finite number of unstable directions (dimensionality of UV critical surface)

Continuum to lattice

The Einstein-Hilbert action for a metric $g_{\mu\nu}$ has the form,

$$S_{EH} = \frac{1}{16\pi G} \int d^4x \sqrt{-g}\,(2\Lambda - R) \tag{1}$$

where R is the Ricci scalar and Λ and G are cosmological constant and Newton's constant respectively. On a triangulation, we discretize according to

$$V_4 = \int d^4x \sqrt{-g} \rightarrow N_4[T] \tag{2}$$

The simple form of Euclidean Einstein-Regge discrete action is given by :

$$S_E = -\kappa_2 N_2 + \kappa_4 N_4 \tag{3}$$

where, N_i is the number of simplices of dimension i. And κ_2 and κ_4 are related to the Newton's constant G_N and cosmological constant Λ respectively. κ_4 must be tuned to a critical value such that an infinite volume can be taken. This leaves two parameters in the theory κ_2 and β. The discrete Euclidean-Regge action is,

$$S_E = -\kappa \sum V_2 \left(2\pi - \sum \theta\right) + \lambda \sum V_4 \tag{4}$$

where $\kappa = \frac{1}{8\pi G}$, $\theta = \cos^{-1}(1/4)$ and $\lambda = \kappa\Lambda$. Also, the volume of a d-simplex is given by :

$$V_d = l^d \frac{\sqrt{d+1}}{\sqrt{2^d} d!} \tag{5}$$

Simplifying, we get

$$S_E = -\frac{\sqrt{3}}{2}\pi\kappa N_2 + N_4\left(\kappa\frac{5\sqrt{3}}{2}\cos^{-1}\left(\frac{1}{4}\right) + \frac{\sqrt{5}}{96}\lambda\right) \tag{6}$$

We define new variables $\kappa_2 = \frac{\sqrt{3}}{2}\pi\kappa$ and $\kappa_4 = \kappa\ \frac{5\sqrt{3}}{2}\cos^{-1}(\frac{1}{4}) + \frac{\sqrt{5}}{96}\ \lambda$ and recover the form written above in eq. (3).

The dynamical triangulation approach is a modified version of calculus due to Regge. The space in supposed to be flat inside the $d = 4$ simplexes, the curvature being concentrated in $d - 2 = 2$ dimensional hinges, i.e. triangles. The angle between two tetrahedra faces, sharing a triangle is $\cos^{-1}(1/d)$. Each four-dimensional simplex has 5 nodes(vertices), 5 tetrahedral faces, 10 links and 10 triangles. The manifold triangulated is usually S^d, which has Euler characteristic given by

$$\chi = \sum_{0}^{d} (-1)^p \ N_p = 2 - 2h = 2$$

$\chi = N_0 - N_1 + N_2 - N_3 + N_4$ and we have,

$$N_3 = \frac{(d+1)}{2} N_4$$

$$N_2 = 2N_0 + 2N_4 - 4$$

$$N_1 = 3N_0 + \frac{N_4}{2} - 6$$

Problem with conformal scaling and behavior at high energies (case against asymptotic safety)

In hep-th/9812237 [Banks & Aharony] and later in 0709.3555 [Shomer] claimed the following :

The very-high energy spectrum of any d-dimensional quantum field theory is that of a d-dimensional conformal field theory. This is not true for gravity.

But why ?

The entropy of a renormalizable theory must scale as $S \sim E^{\frac{d-1}{d}}$. For gravity, the high-energy spectrum must be dominated by black holes. The Bekenstein-Hawking entropy area law tells us that $S \sim E^{\frac{d-2}{d-3}}$. Also note that, $(d-1)/d = (d-2)/(d-3)$ for $d = 3/2$.
So, one of the following can be possible :

- Entropy scaling is incorrect, gravity *can be* AS [WAIT]
- Entropy scaling and asymptotic safety (AS) scenario is correct, and d=3/2 at short distances around the UV fixed point [VERY GOOD]
- Entropy scaling is correct, space-time is never fractal (non-integer) and gravity can't be AS [SETBACK]

Minimal coupling to scalar (no backreaction, aka quenched [Smit, deBakker 1994. Presented at Lattice 1995 Melbourne]

Let's start with the usual gravity action and add additional term as,

$$S = S_E[g] + S[g, \Phi] \tag{7}$$

where,

$$S[g, \Phi] = \int d^4x \sqrt{-g} \left(g^{\mu\nu} \partial_\mu \Phi \partial_\nu \Phi + m_0^2 \Phi^2 \right) \tag{8}$$

here, Φ is a test particle and the back reaction of the metric is ignored. The propagator for the scalar field in a *fixed background* decays as,

$$G(r) = A(r) e^{-Mr} \tag{9}$$

where, r is the geodesic distance between two points. Here, M, which is the renormalized mass of the particle.

The multiplicative renormalization follows from the shift symmetry of the discrete lattice action [Agishtein and Migdal, NPB 385 (1992) 395-412] and can be seen as follows,

$$\begin{aligned} S_{\text{lat}} &= \sum_{\langle xy \rangle} \left((\Phi_x - \Phi_y)^2 + \sum_x m_0^2 \Phi_x^2 \right) \\ &= \sum_{\langle xy \rangle} \left((\mathrm{D} + 1 + m_0^2)\delta_{xy} - C_{xy} \right) \Phi\Phi \end{aligned} \tag{10}$$

where, C is the simplex neighbor (or connectivity) matrix which will be discussed later, m_0 is the bare mass and D is the space-time dimensions. For zero bare mass, there is a shift symmetry,

$$\Phi \to \Phi + c$$

Discrete laplacian

On each degenerate dynamical triangulation, we calculate the propagator as follows,

$$G = (-\Box + m_0^2)^{-1} \tag{11}$$

The definition of discrete Laplacian is :

$$\Box = \begin{cases} \text{D+1} & \text{if x = y} \\ -1 & \text{if x \& y are nearest neighbors} \\ 0 & \text{otherwise} \end{cases} \tag{12}$$

Here, $D+1$ is the coordination number of a D-simplex. Unlike the combinatorial case studied previously, we study degenerate triangulations in $D = 4$ dimension. A given four-simplex can have up to four same neighbors. This enables us to construct the laplacian for configurations such that sum of any particular row is just m_0^2. In the limit of vanishing bare mass ($m_0 \to 0$), we have an exact zero mode of the operator corresponding to the zero eigenvalue of the Laplacian.

$$L_b = \frac{1}{2}\partial_\mu \varphi_0 \partial^\mu \varphi_0 - \frac{1}{2} m_0^2 \varphi^2 \tag{13}$$

$$L_r = \frac{1}{2} Z_\phi \partial_\mu \varphi \partial^\mu \varphi - \frac{1}{2} Z_m m^2 \varphi^2 \tag{14}$$

Comparing them we get,

$$\phi_0(x) = Z_\phi^{1/2} \phi(x) \tag{15}$$

, and

$$m = \sqrt{\frac{Z_\phi}{Z_m}}\, m_0 \tag{16}$$

Taking log of (16) followed by derivative w.r.t $\ln \mu$ and replacing $\mu = 1/a$, we get:

$$\frac{d \ln(m)}{d \ln a} = \frac{1}{2} \Big(\frac{d \ln F}{d \ln a} \Big) \tag{17}$$

where, $F = Z_\phi / Z_m$

The mass anomalous dimension is defined as, $\gamma_m = \frac{d \ln(m)}{d \ln \mu}$, hence we have,

$$\gamma_m = -\frac{1}{2} \Big(\frac{d \ln F}{d \ln a} \Big) \tag{18}$$

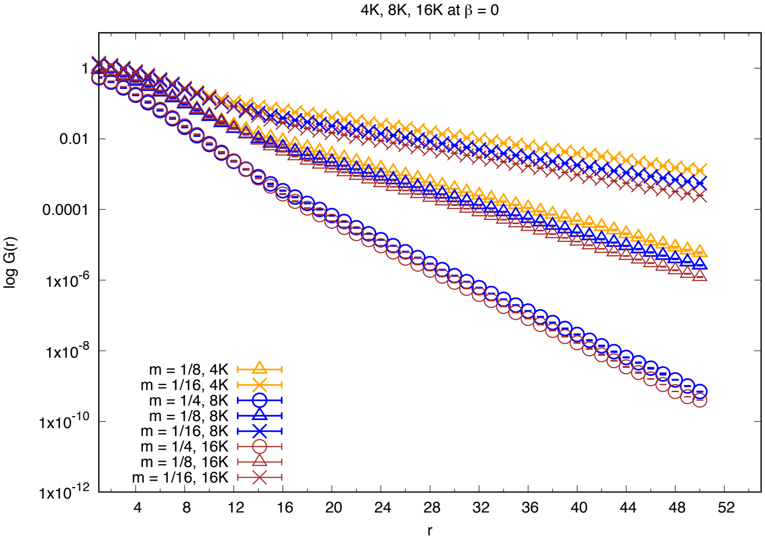
Fitting range for the scalar propagator

Semi-classical regimes :

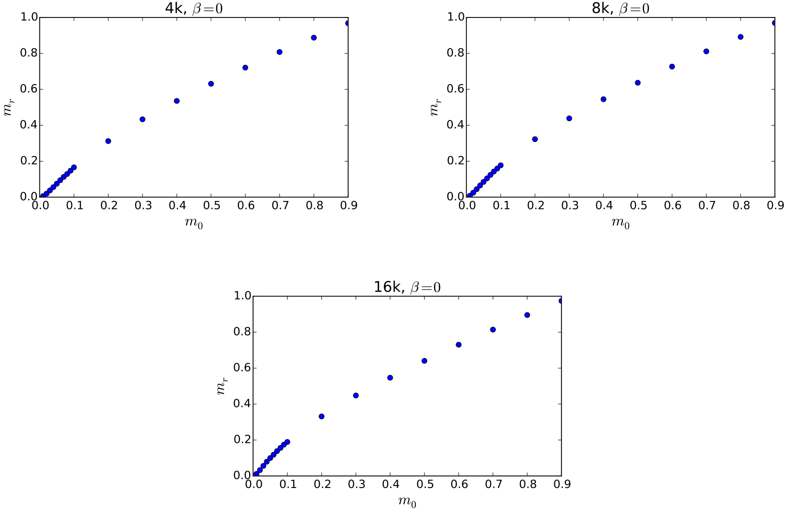
- 4k: $8 \leq r \leq 20$
- 8k: $10 \leq r \leq 23$
- 16k: $12 \leq r \leq 28$

Space-time on average is S^4 in this regime. We fit the propagators inside this range.

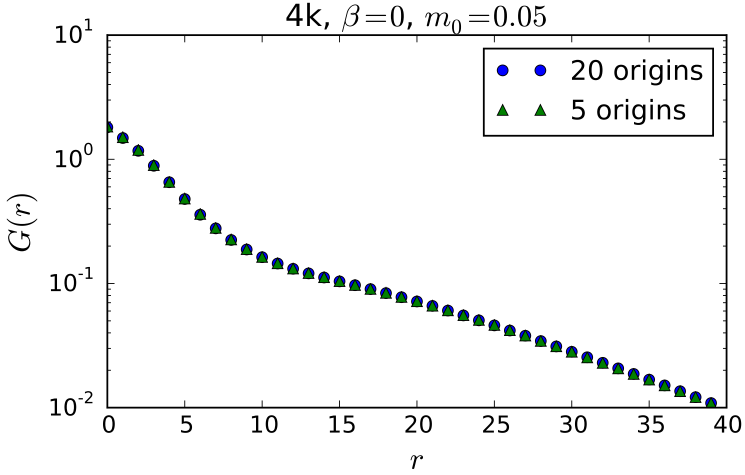
Scalar propagator results



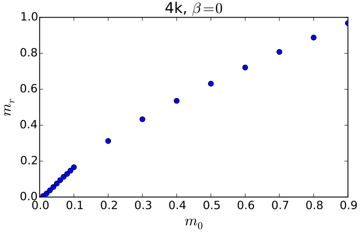
Multiplicative mass renormalization - for different volumes

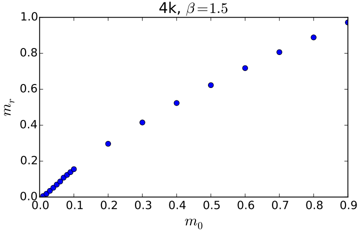


Dependence on number of origins ?



Multiplicative mass renormalization - for different β values





Thank you !