

# Non-perturbative generation of elementary fermion mass: a numerical study

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**36th International Symposium on Lattice Field Theory**  
**June 22 - 28, 2018, East Lansing, MI, USA**

- conjecture of a non-perturbative mechanism for the elementary particle mass generation R. Frezzotti, G. Rossi ('15)
- We test this conjecture in the "simplest" appropriate  $d = 4$  "toy model": may be the first non-perturbative simulation of gauge + scalars + fermions

$$\begin{aligned} \mathcal{L}_{\text{toy}}(Q, A, \Phi) = & \frac{1}{4} F_{\mu\nu}^a F_{\mu\nu}^a + \frac{1}{2} \text{Tr}[\partial_\mu \Phi^\dagger \partial_\mu \Phi] + \frac{m_\phi^2}{2} \text{Tr}[\Phi^\dagger \Phi] + \frac{\lambda_0}{4} (\text{Tr}[\Phi^\dagger \Phi])^2 \\ & + \bar{Q}_L \gamma_\mu \mathcal{D}_\mu Q_L + \bar{Q}_R \gamma_\mu \mathcal{D}_\mu Q_R + \eta (\bar{Q}_L \Phi Q_R + h.c.) \\ & + \frac{b^2}{2} \rho (\bar{Q}_L \overset{\leftarrow}{\mathcal{D}}_\mu \Phi \mathcal{D}_\mu Q_R + h.c.), \quad \Phi \equiv \varphi_0 \mathbb{1} + i \tau_i \varphi_i \end{aligned}$$

- "Wilson-like"  $\propto \rho$  (naively irrelevant)
- UV cutoff  $\sim b^{-1}$

$$\begin{aligned} \mathcal{L}_{\text{toy}}(Q, A, \Phi) = & \frac{1}{4} F_{\mu\nu}^a F_{\mu\nu}^a + \frac{1}{2} \text{Tr} [\partial_\mu \Phi^\dagger \partial_\mu \Phi] + \frac{m_\phi^2}{2} \text{Tr} [\Phi^\dagger \Phi] + \frac{\lambda_0}{4} (\text{Tr} [\Phi^\dagger \Phi])^2 \\ & + \overline{Q}_L \gamma_\mu \mathcal{D}_\mu Q_L + \overline{Q}_R \gamma_\mu \mathcal{D}_\mu Q_R + \frac{b^2}{2} \rho (\overline{Q}_L \overset{\leftarrow}{\mathcal{D}}_\mu \Phi \mathcal{D}_\mu Q_R + h.c.) \\ & + \eta (\overline{Q}_L \Phi Q_R + h.c.), \quad \Phi \equiv [\varphi| - i\tau^2 \varphi^*] \end{aligned}$$

- Symmetries & power counting  $\binom{\text{in suitable}}{\text{UV-regul.}}$   $\implies$  Renormalizability
- Invariant under  $\chi$  (global)  $SU(2)_L \times SU(2)_R$  transformations

$$\begin{aligned} \bullet \chi_{L,R} : \quad & (\Phi \rightarrow \Omega_L \Phi) \otimes \tilde{\chi}_L \quad \text{and/or} \quad (\Phi \rightarrow \Phi \Omega_R^\dagger) \otimes \tilde{\chi}_R \\ \tilde{\chi}_{L,R} : & \begin{cases} Q_{L,R} \rightarrow \Omega_{L,R} Q_{L,R} & \Omega_{L,R} \in SU(2)_{L,R} \\ \overline{Q}_{L,R} \rightarrow \overline{Q}_{L,R} \Omega_{L,R}^\dagger & \end{cases} \end{aligned}$$

- $\chi$  invariance forbids  $\frac{1}{b} \overline{Q} Q$  terms and softens power like U.V. divergences
- Fermionic chiral transformations  $\tilde{\chi}$  are not a symmetry if  $(\rho, \eta) \neq (0, 0)$

- A value of the bare parameter  $\eta$  exist where the effective Yukawa term vanishes
- The renormalized Schwinger-Dyson equation (SDE) of the Axial  $\tilde{\chi}_L \otimes \tilde{\chi}_R$  read

$$Z_{\tilde{A}} \partial_\mu \langle \tilde{J}_\mu^{A,i}(x) \hat{O}(0) \rangle =$$

$$-[\eta - \bar{\eta}(\eta, g_s, \rho, \lambda_0)] \langle \left[ \overline{Q}_L \left\{ \frac{\tau^i}{2}, \Phi \right\} Q_R - Q_R \left\{ \frac{\tau^i}{2}, \Phi \right\} Q_L \right] (x) \hat{O}(0) \rangle + O(b^2) + \dots$$

- $\bar{\eta}$  ( $\mu_0^2$  independent) controls the  $O(b^{-2})$  mixing of the Wilson-like term with the Yukawa
- $\tilde{J}_\mu^{A,i} = \overline{Q}_L \gamma_\mu \gamma_5 \frac{\tau^i}{2} Q_L$  and  $Z_{\tilde{A}}$  its renormalization constant
- At  $\eta = \eta_{cr}$  such that

$$\eta_{cr} - \bar{\eta}(\eta_{cr}, g_s, \rho, \lambda_0) = 0$$

the current  $\tilde{J}_\mu^{L,i}$  is conserved up to  $O(b^2)$  and possible NP mixing contributions (...)

- At  $\eta = \eta_{cr}$  the SDE equation

$$Z_{\tilde{A}} \partial_\mu \langle \tilde{J}_\mu^{L,i}(x) \hat{\mathcal{O}}(0) \rangle = O(b^2) + \underline{\dots}$$

Wigner phase ( $\langle \Phi \rangle = 0$ )

$$Z_{\tilde{A}} \partial_\mu \langle \tilde{J}_\mu^{A,i}(x) \hat{\mathcal{O}}(0) \rangle = O(b^2)$$

$$M_{PS} = 0$$

conjecture

Nambu-Goldstone phase ( $\langle \Phi \rangle = v$ )

$$Z_{\tilde{A}} \partial_\mu \langle \tilde{J}_\mu^{A,i}(x) \hat{\mathcal{O}}(0) \rangle = c_1 " \Lambda_s " + O(b^2)$$

$$\text{thus } M_{PS} \neq 0$$

$$" \Lambda_s " = \Lambda_s f(v) \text{ with } f(0) = 0, \text{ e.g. } \frac{\Lambda_s v^2}{\Lambda_s^2 + v^2}$$

- In the NG phase, the occurrence of  $\propto c_1 \Lambda_s$  in the SDEs is equivalent to a NP term in the Low Energy Effective Lagrangian

$$\mathcal{L}_{4,NG}^{LEEL} \supset c_1 \Lambda_s [\bar{Q}_L \mathcal{U} Q_R + \text{hc}], \quad \mathcal{U} = \frac{\Phi}{\sqrt{\Phi^\dagger \Phi}} = \frac{(v + \sigma + i \vec{\tau} \vec{\varphi})}{\sqrt{(v + \sigma)^2 + \vec{\varphi} \vec{\varphi}}} = \mathbb{1} + i \frac{\vec{\tau} \vec{\varphi}}{v} + \dots$$

- The term  $c_1 \Lambda_s [\bar{Q}_L \mathcal{U} Q_R + \text{hc}]$  contain a quark mass

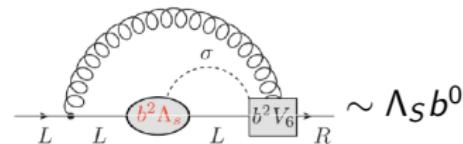
$$c_1 \Lambda_s [\bar{Q}_L \mathcal{U} Q_R + \text{hc}] \supset c_1 \Lambda_s \bar{Q} Q_R \left\{ \begin{array}{l} \text{Natural mass} \\ \neq \text{Yukawa term} \\ c_1 = O(\alpha_s^2) \rightarrow \text{Hierarchy} \end{array} \right.$$

see talk by R. Frezzotti

Simulations strategy to check the occurrence of the mechanism:

- We regularise the toy model on the lattice
  - In Wigner phase ( $\langle \phi \rangle = 0$ ): We look for the critical value  $\eta = \eta_{cr}$  of the Yukawa bare parameter
  - In Nambu-Goldstone phase ( $\langle \phi \rangle = v$ ): At  $\eta = \eta_{cr}$  we measure  $M_{PS}$  and the WI mass
  - Simulations at three values of the lattice spacing

- Quenched approximation sufficient R. Frezzotti, G. Rossi ('15). NP effective vertices  $\supset b^2 \Lambda_s \sigma \bar{Q} D Q$  combined with  $b^2 \bar{Q} D \Phi D Q \Rightarrow$  NP mass



- Lattice regularization of  $\int d^4x \mathcal{L}_{toy}$
- "Naive" fermions (convenient in the valence) with  $\tilde{chi}$  breaking terms

$$\mathcal{S}_{lat} = b^4 \sum_x \left\{ \mathcal{L}_{kin}^{YM}[U] + \mathcal{L}_{kin}^{sca}(\Phi) + \mathcal{V}(\Phi) + \sum_g \bar{\Psi}_g D_{lat}[U, \Phi] \Psi_g \right\}$$

$\mathcal{L}_{kin}^{YM}[U]$  : SU(3) plaquette action

$$\mathcal{L}_{kin}^{sca}(\Phi) + \mathcal{V}(\Phi) = \frac{1}{2} \text{tr} [\Phi^\dagger (-\partial_\mu^* \partial_\mu) \Phi] + \frac{m_\phi^2}{2} \text{tr} [\Phi^\dagger \Phi] + \frac{\lambda_0}{4} (\text{tr} [\Phi^\dagger \Phi])^2$$

where  $\Phi = \varphi_0 \mathbb{1} + i \varphi_j \tau^j$  and  $F(x) \equiv [\varphi_0 \mathbb{1} + i \gamma_5 \tau^j \varphi_j](x)$

$$(D_{lat}[U, \Phi] \Psi)(x) = \gamma_\mu \tilde{\nabla}_\mu \Psi(x) + \eta F(x) \Psi(x) - b^2 \rho \frac{1}{2} F(x) \tilde{\nabla}_\mu \tilde{\nabla}_\mu \Psi(x) + \\ - b^2 \rho \frac{1}{4} \left[ (\partial_\mu F)(x) U_\mu(x) \tilde{\nabla}_\mu \Psi(x + \hat{\mu}) + (\partial_\mu^* F)(x) U_\mu^\dagger(x - \hat{\mu}) \tilde{\nabla}_\mu \Psi(x - \hat{\mu}) \right]$$

## Properties of the lattice action in S. Capitani, et, al. ('18)

- Consider two generations of valence fermions  $\bar{\Psi}_\ell D_{latt} \Psi_\ell + \bar{\Psi}_h D_{latt} \Psi_h$  in order to have valence correlators involving no fermionic disconnected diagrams

e.g.  $\langle \bar{\Psi}_\ell(x) \Gamma \tau \Psi_h(x) \bar{\Psi}_h(y) \Gamma \tau \Psi_\ell(y) \rangle \quad \ell = (u, d) \quad h = (c, s)$

- Wilson-like ( $d = 6$ ) term  $\propto \rho$**  do not remove the doublers
- Flavour content: (16 doublers)  $\times$  (2 isospin)  $\times$  (2 generations)
- Spectrum Doubling Symmetry  $\implies$  at  $\eta = \eta_{cr}$   $\tilde{\chi}$  gets simultaneously restored for all tastes up to cutoff effects
- Quenched  $(U, \Phi) \implies$  exceptional configurations: at large  $|\eta|$  and  $|\rho|$  enhanced by  $\Phi$  fluctuations
- Add twisted mass term:  $S_{lat}^{toy+tm} = S_{lat} + i\mu b^4 \sum_x \bar{\Psi} \gamma_5 \tau_3 \Psi$   
control over exceptional confs. at the price of a soft breaking of  $\chi_{L,R}$  (and  $\tilde{\chi}_{L,R}$  when restored), safe  $\mu \rightarrow 0$  extrapolation

# Simulation details and Renormalization

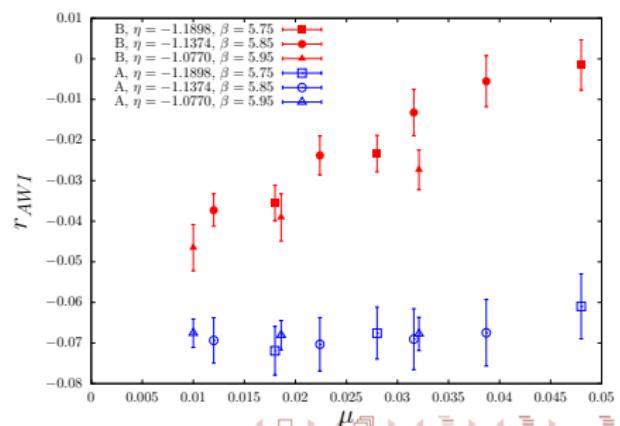
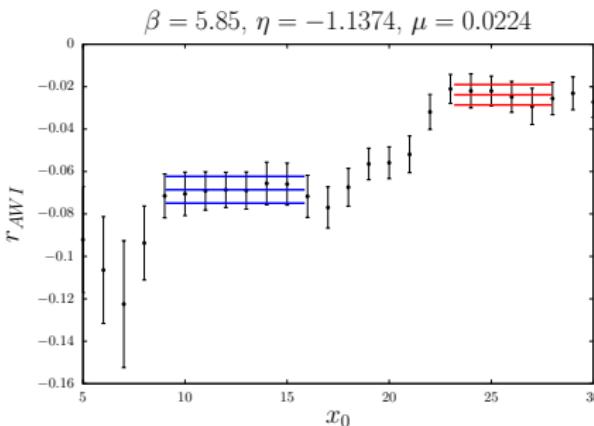
- Quenched lattice study: independent generations and renormalization of  $U$  and  $\Phi$
- In the fermionic sector effective Yukawa coupling vanishes at  $\eta = \eta_{cr}$
- Gauge coupling renormalized keeping  $r_0 \sim 0.5\text{fm}$  S. Necco, R. Sommer ('01)  
 $\beta = 5.75$  ( $b = 0.15$  fm),  $\beta = 5.85$  ( $b = 0.12$  fm) &  $\beta = 5.95$  ( $b = 0.10$  fm)
- Scalars parameters  $m_\phi^2, \lambda_0$  fixed by the renormalization condition  
 $m_\sigma^2 r_0^2 = 1.258$  and  $\lambda_R = \frac{m_\sigma^2}{2v_R^2} = 0.4408$
- Wilson-like coupling  $\rho \sim 1.96$ : free parameter since we are only interested to see whether the mechanism exists ( $\rho$  relevant for the magnitude of the NP mass)
- In Wigner phase we keep fixed  $(m_\phi^2 - m_{cr}^2)r_0^2 = 1.21$

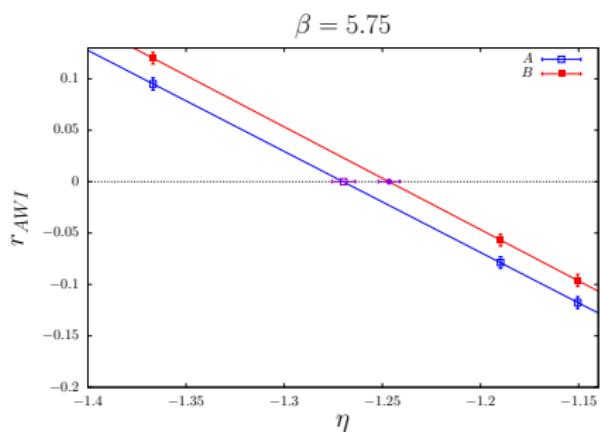
# Wigner phase $\langle \phi \rangle = 0$

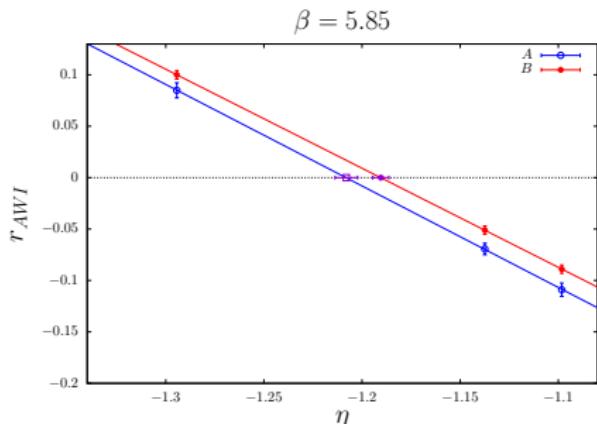
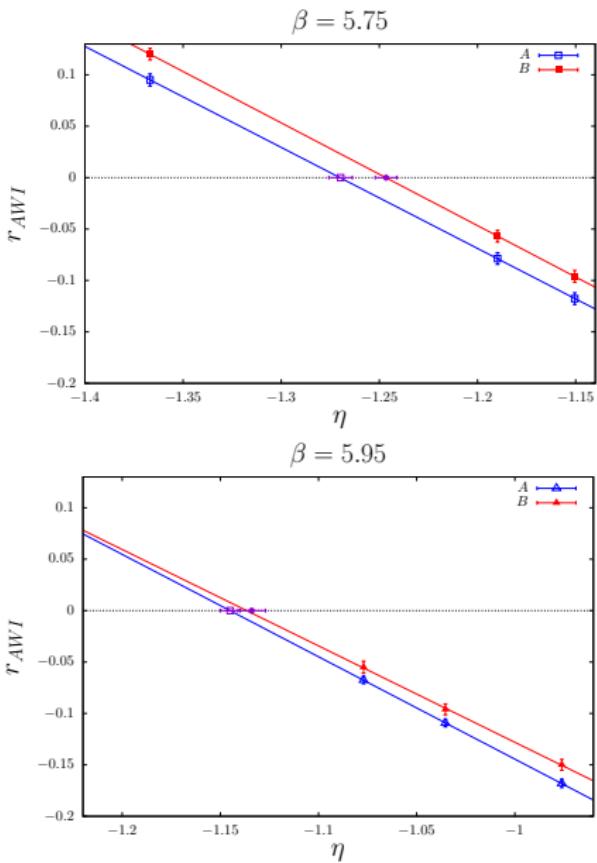
- find  $\eta_{cr}$  where  $\tilde{J}_0^{A,i} = \bar{\Psi} \gamma_\mu \gamma_5 \frac{\tau^i}{2} \Psi|_{1pt.\,split}$  is conserved

$$r_{AWI}(\eta; g_s^2, \lambda_0, \rho, \mu)|_{\eta_{cr}} = \frac{\sum_{\vec{x}} \sum_{\vec{y}} \langle P^1(0) [\partial_0 \tilde{J}_0^{A1}](x) \phi^0(y) \rangle}{\sum_{\vec{x}} \sum_{\vec{y}} \langle P^1(0) D^{P1}(x) \phi^0(y) \rangle} \Bigg|_{\eta_{cr}}^{\mu \rightarrow 0} = 0, \quad y_0 = x_0 + \tau$$

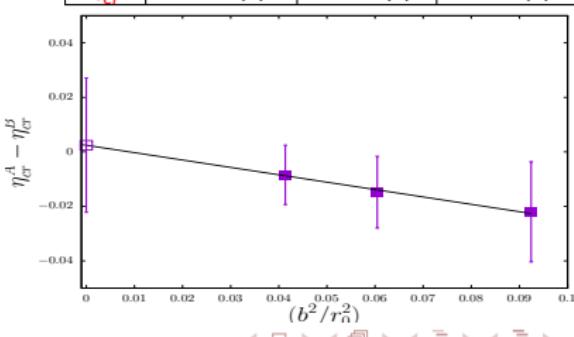
$$D^{P1} = \bar{\Psi}_L \left\{ \Phi, \frac{\tau \pm}{2} \right\} \Psi_R - h.c., \quad \frac{\tau}{fm} \sim 0.6 \quad \frac{x_0}{fm} \stackrel{A}{\sim} [0.9, 1.8] \quad \frac{x_0}{fm} \stackrel{B}{\sim} [2.7, 3.3]$$







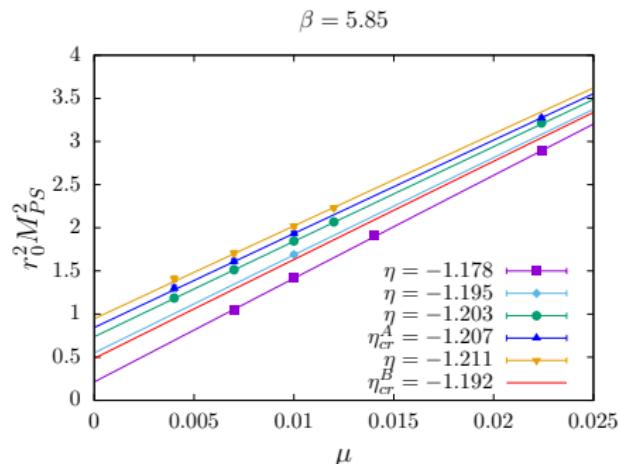
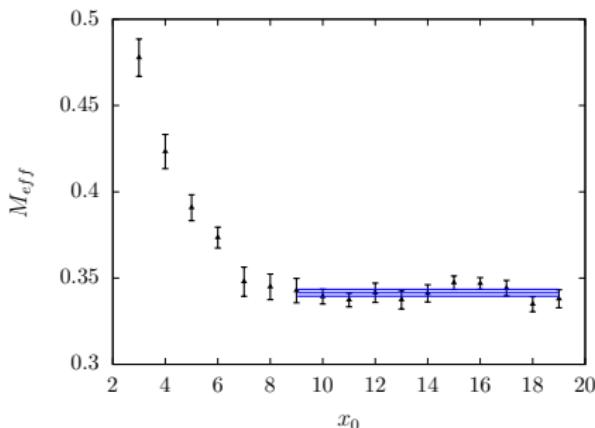
$\beta$	5.75	5.85	5.95
$\eta_{cr}^A$	-1.271(10)	-1.207(8)	-1.145(6)
$\eta_{cr}^B$	-1.249(8)	-1.192(6)	-1.136(6)



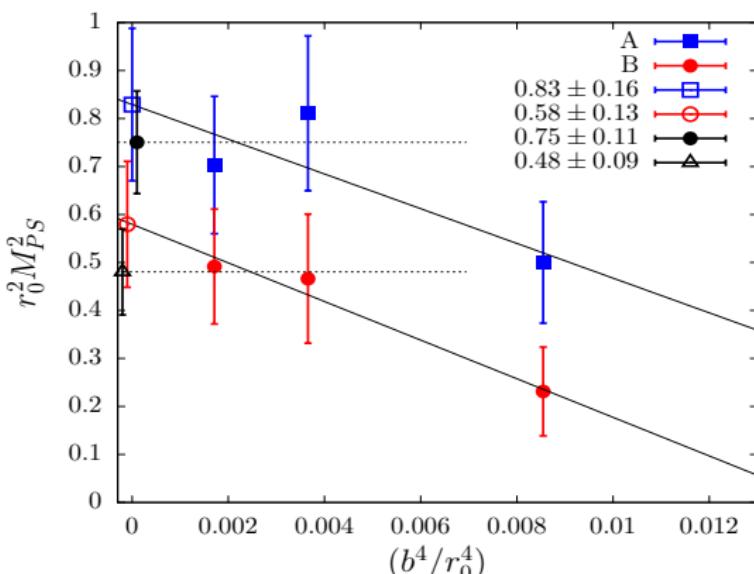
# Nambu-Goldston phase $\langle \phi \rangle = v$

- Calculate  $M_{PS}$  at  $\eta = \eta_{cr}$  and  $\mu \rightarrow 0$
- Global fit of  $M_{PS}^2 = a + b\mu + c\eta + d\mu^2 + e\eta^2 + f\eta\mu$

$$\beta = 5.85, \eta = -1.207, \mu = 0.0100$$

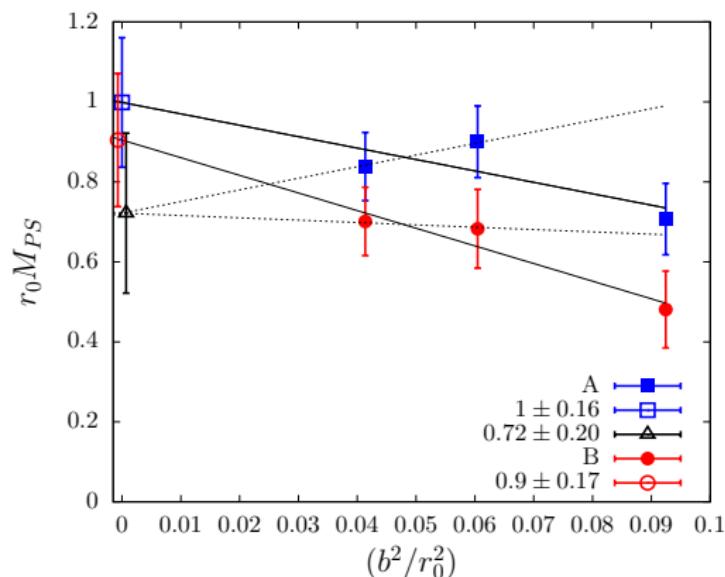


- If we assume the no mechanism hypothesis  $Z_j \partial_\mu \langle \tilde{J}_\mu^{A,i}(x) \hat{\mathcal{O}}(0) \rangle = O(b^2)$  then  $M_{PS}^2 = O(b^4) \Rightarrow \text{à la Symanzik: } \mathcal{L}_4^{EFF} + b^2 \mathcal{L}_6^{EFF} + \dots$
- We simulate at  $\eta = \eta_{cr}$  and  $\mu > 0$  and extrapolate to  $\mu \rightarrow 0^+$  thus twist angle  $\omega = \pi/2$ . All terms in  $b^2 \mathcal{L}_6^{EFF}$  are either parity odd or  $\tilde{\chi}$  invariant  
 $\lim_{\mu \rightarrow 0^+} \langle PS | \mathcal{L}_6^{EFF} | PS \rangle(\mu) = 0$

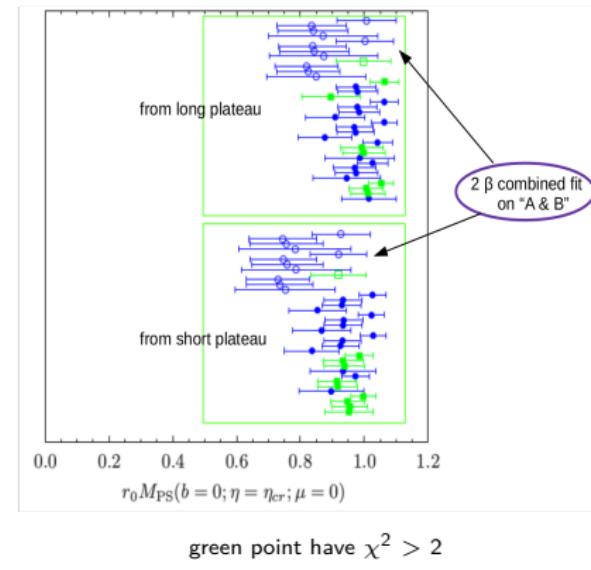
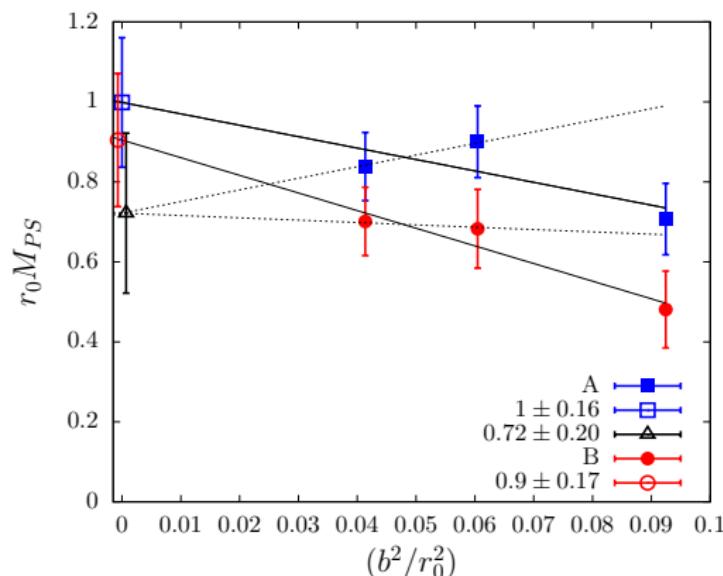


No mechanism hypothesis  
NOT supported by data

- Check occurrence of the mechanism:  $r_0 M_{PS}(\mu \rightarrow 0^+)$  vs  $b^2/r_0^2$ 
  - $M_{PS}$  at  $\eta_{cr}^A$  and  $\eta_{cr}^B$  give consistent continuum limit
  - Dotted lines, combined fit imposing common continuum limit & excluding the coarsest lattice

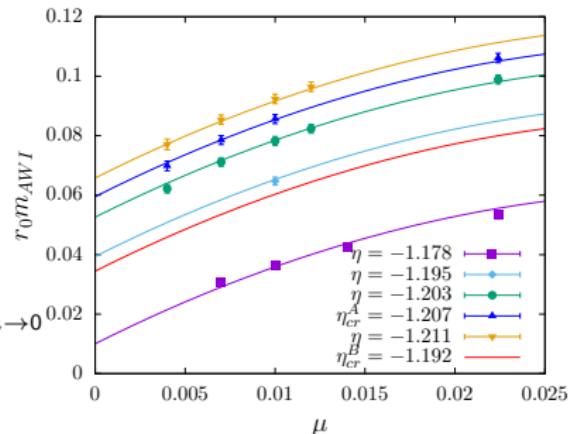
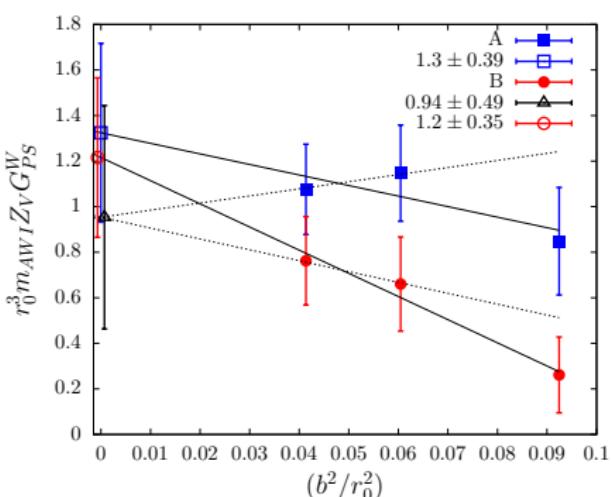


- Check occurrence of the mechanism:  $r_0 M_{PS}(\mu \rightarrow 0^+)$  vs  $b^2/r_0^2$ 
  - $M_{PS}$  at  $\eta_{cr}^A$  and  $\eta_{cr}^B$  give consistent continuum limit
  - Dotted lines, combined fit imposing common continuum limit & excluding the coarsest lattice



Median over 72 analyses (excluding  $\chi^2 > 2$ )  $r_0 M_{PS} = 0.927 \pm 0.094 \pm 0.095$

- WI mass  $m_{AWI}(\eta_{cr}) = \frac{\partial_0 \langle \tilde{J}_0^{A1}(x) P^1(y) \rangle}{\langle P^1(x) P^1(y) \rangle} \Big|_{\eta_{cr}}$
- Global fit  $m_{AWI} = a + b\mu + c\eta + d\mu^2$
- Renormalized quantity  $r_0^3 m_{AWI} Z_V G_{PS}^W$
- $Z_V$  from  $Z_V \langle \tilde{J}_0^{V2}(x) P^1(y) \rangle \Big|_{\mu \rightarrow 0} = 2\mu \langle P^1 P^1 \rangle \Big|_{\mu \rightarrow 0}$
- In the Wigner phase  $G_{PS}^W = \langle 0 | P^1 | PS \rangle$

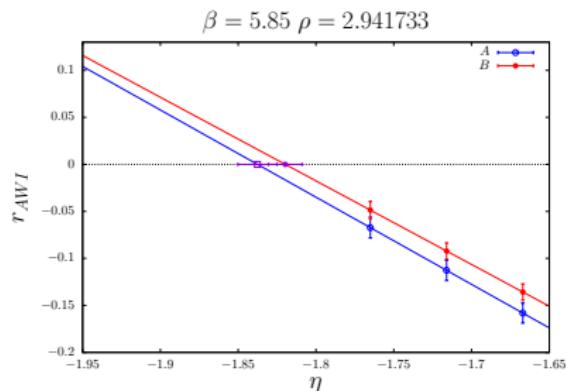


Median over 72 analyses ( as for  $M_{PS}$  excluding  $\chi^2 > 2$ )

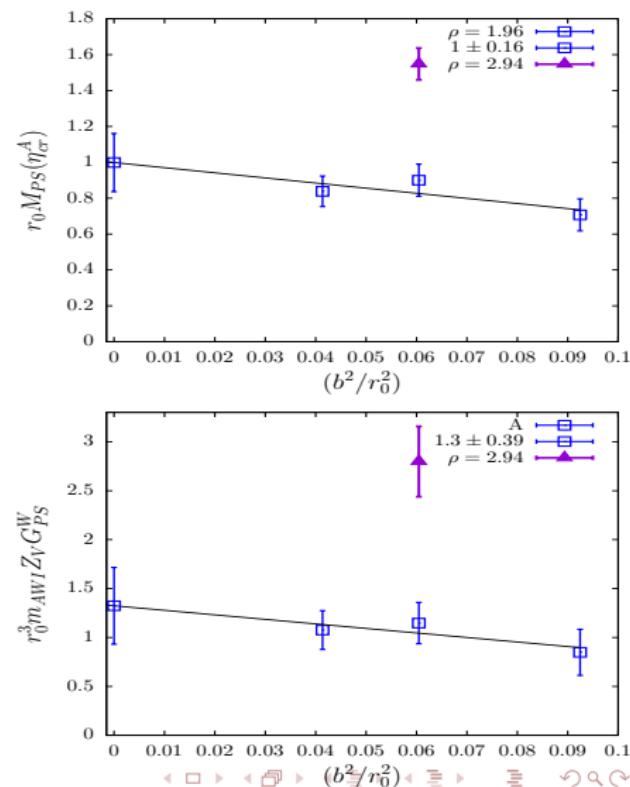
$$r_0^3 m_{AWI} Z_V G_{PS} = 1.204 \pm 0.394 \pm 0.193$$

# Increasing $\rho : 1.96 \rightarrow 2.94$

increase of  $m_{AWI}$  (and  $M_{PS}$ ) with  $\rho$  is expected according to  
 R. Frezzotti, G. Rossi ('15) mechanism



$\beta = 5.85$	$\rho = 1.86$	$\rho = 2.94$
$\eta_{cr}^A$	-1.207(8)	-1.838(13)
$\eta_{cr}^B$	-1.192(6)	-1.820(11)
$r_0 M_{PS}(\eta_{cr}^A)$	0.90(9)	1.54(9)
$r_0 m_{AWI}(\eta_{cr}^A)$	1.1(2)	2.8(4)



## Conclusions and Remarks

- We have evidence that the mechanism conjectured by R. Frezzotti, G. Rossi ('15) is supported by numerical simulation. The same data exclude the hypothesis of "no mechanism" with 5-6 sigmas of significance
- Other explanations of our data? Possible and searched for: till now none found
- NP mass term in  $\tilde{\chi}$  SDEs  $\iff$  NP mass term plus further  $\tilde{\chi}$ -breaking terms in the LE effective action valid at the scale  $\Lambda_S \ll p \ll \Lambda_{UV} \sim b^{-1}$
- $d > 4$   $\tilde{\chi}$ -breaking terms control the NP mass
- Fermion mass  $\sim \Lambda_S$  :
  - expected to be unrelated to  $v$  (in the limit  $v \gg \Lambda_S$ ) & vanishing as  $v \rightarrow 0$
  - natural a' la 't Hooft (at least in weak sense:  $\tilde{\chi}$  recovery)
- Extension of this mechanism to weak interactions and possible phenomenological implications are discussed in the talk  
R. Frezzotti "Towards models with an unified dynamical mechanism for elementary particle masses"

Thank you for your attention

# Backup slides

- Using staggered formalism to analyze naive valence fermions

- $\Psi(x)$  contain 4 replicas  $B = 1, \dots, 4$

$$\Psi(x) = \mathcal{A}_x \chi(x), \quad \mathcal{A}_x = \gamma_1^{x_1} \gamma_2^{x_2} \gamma_2^{x_2} \gamma_2^{x_2}, \quad \text{Spin diagonalization of } S_{latt} \text{ in } \chi^B \text{ basis}$$

- $\chi(x)$  contain 4 tastes  $a=1, \dots, 4$

$$q_{\alpha,a}^B(y) = \frac{1}{8} \sum_{\xi} \overline{U}(2y, 2y + \xi) [\Gamma_{\xi}]_{\alpha,a} (1 - b \sum_{\mu} \xi_{\mu} \tilde{\nabla}_{\mu}) \chi^B(2y + \xi),$$

$$q_{\alpha,a}^B(y) \text{ taste basis, } \quad x_{\mu} = 2y_{\mu} + \xi_{\mu}, \quad \xi_{\mu} = 0, 1$$

- Flavour content:  $\underbrace{(4 \text{ replicas: } B) \times (4 \text{ tastes: } a) \times}_{16 \text{ doublers}} (2 \text{ isospin}) \times \text{generations}$

- Following Kluberg-Stern et al. ('83), ..., Sharpe et al. ('93), Luo ('96) and adding scalars we get the small  $b$  expansion of  $S_{lat}^{fer}$  on smooth  $U,\Phi$  configuration

$$S_{lat}^{fer} = \sum_{y,B} \bar{q}^B(y) \left\{ \sum_{\mu} (\gamma_{\mu} \otimes \mathbb{1}) D_{\mu} + (\eta - \bar{\eta}) \mathcal{F}(y) \right\} q^B(y) + O(b^2)$$

$$\mathcal{F}(y) = \varphi_0(2y)(\mathbb{1} \otimes \mathbb{1}) + S^B i \tau^i \varphi_i(2y)(\gamma_5 \otimes t_5), \quad S^A = \pm 1, \quad \text{taste matrices } t_{\mu} = \gamma_{\mu}^*$$

- Quark bilinear in  $\Psi$  basis that have the classical continuum limit in  $q^B$  basis
- Point split vector current

$$\tilde{J}_\mu^{V^i}(x) = \bar{\Psi}(x - \hat{\mu})\gamma_\mu \frac{\tau^i}{2} U_\mu(x - \hat{\mu})\Psi(x) + \bar{\Psi}(x)\gamma_\mu \frac{\tau^i}{2} U_\mu^\dagger(x - \hat{\mu})\Psi(x - \hat{\mu})$$

$$\sum_{\xi} \tilde{J}_\mu^{V^i}(2y + \xi) = \sum_{B=1}^4 \bar{q}^B(y)(\gamma_\mu \otimes \mathbb{1}) \frac{\tau^i}{2} q^B(y) + O(b^2)$$

- Point split axial current

$$\tilde{J}_\mu^{A^i}(x) = \bar{\Psi}(x - \hat{\mu})\gamma_\mu \gamma_5 \frac{\tau^i}{2} U_\mu(x - \hat{\mu})\Psi(x) + \bar{\Psi}(x)\gamma_\mu \gamma_5 \frac{\tau^i}{2} U_\mu^\dagger(x - \hat{\mu})\Psi(x - \hat{\mu})$$

$$\sum_{\xi} \tilde{J}_\mu^{A^i}(2y + \xi) = \sum_{B=1}^4 \bar{q}^B(y)(\gamma_\mu \gamma_5 \otimes t_5) \frac{\tau^i}{2} q^B(y) + O(b^2)$$

- Correlators with generation off-diagonal operator  $\Rightarrow$  no disconnected diagrams

e.g.  $\langle \bar{\Psi}_\ell(x)\Gamma\tau\Psi_h(x)\bar{\Psi}_h(y)\Gamma\tau\Psi_\ell(y) \rangle \quad \ell = (u, d) \quad h = (c, s)$

- Loop effects do not generate  $d \leq 4$  operator besides  $F_{\mu\nu}F_{\mu\nu}$ ,  $\partial_\mu\Phi^\dagger\partial_\mu\Phi$ ,  $q^B(\gamma_\mu \otimes \mathbb{1})\tilde{\nabla}q^B$ ,  $\Phi^\dagger\Phi$ ,  $(\Phi^\dagger\Phi)^2$ ,  $\eta\bar{q}^B(y)\mathcal{F}^B(y)q^B(y)$  (all in  $S_{lat}$ )
- Argument: In  $S_{latt}$  there are only  $\tilde{\nabla}_\mu$  acting on fermions  $\implies$  Spectrum Doubling Symmetry :  
 $\Psi \rightarrow e^{-ix \cdot \pi_H} M_H \Psi$     $\bar{\Psi} \rightarrow \bar{\Psi} M_H^\dagger e^{+ix \cdot \pi_H}$ ,    $H = \{\mu_1, \dots, \mu_h\}$  ordered,  
16 vectors  $\pi_H (\pi_{H,\mu} = \pi$  if  $\mu \in H)$  with  $M_H = (i\gamma_5\gamma_1) \dots (i\gamma_5\gamma_{\mu_h})$
- It is a symmetry of  $S_{latt}$ , thus also of the  $\Gamma_{lat}[U, \Phi, \Psi]$ . So the latter can only have terms with symmetric covariant derivatives  $\tilde{\nabla}_\mu$  acting on  $\Psi$ .  
Close to the continuum limit among the local terms of  $\Gamma_{lat}$  only those with no or one  $\tilde{\nabla}_\mu$  fermionic derivative are relevant
- At  $\eta = \eta_{cr}$   $\tilde{\chi}$  gets simultaneously restored for all tastes up to cutoff effects

- Simulation parameters of the scalar sector
- Renormalization condition in the NG phase

$$m_\sigma^2 r_0^2 = 1.258, \quad \lambda_R = \frac{m_\sigma^2}{2v_R^2} = 0.4408, \quad Z_\phi = [m_\phi^2 G(0) - V_4 v^2]^{-1}$$

where  $G(0)$  is the two point function at zero momentum

$b(fm)$	$\beta$	$r_0^2 M_\sigma^2$	$\lambda_{NP}$	$b^2 m_\phi^2$	$\lambda_0$
0.152	5.75	1.278(4)	0.437(2)	-0.5941	0.5807
0.123	5.85	1.286(4)	0.441(2)	-0.5805	0.5917
0.102	5.95	1.290(5)	0.444(2)	-0.5756	0.6022

$\kappa$ : code hopping parameter, s.t.  $\kappa^{-1} - 2\kappa\lambda_0 - 8 = b^2 m_0^2$

- In Wigner phase we keep fixed  $(m_\phi^2 - m_{cr}^2)r_0^2 = 1.21$

$\beta$	$r_0/b$	$(m_\phi^2 - m_{cr}^2)b^2$	$b^2 m_{cr}^2$	$b^2 m_\phi^2$	$\lambda_0$	$\kappa$
5.75	3.29	0.1119(12)	-0.5269(12)	-0.4150	0.5807	0.129280
5.85	4.06	0.0742(11)	-0.5357(11)	-0.4615	0.5917	0.130000
5.95	4.91	0.0504(10)	-0.5460(10)	-0.4956	0.6022	0.130521

## Wigner phase simulations parameters

$\beta$	$a^{-4}(L^3 \times T)$	$\eta$	$a\mu$	$N_U \times N_\phi$
5.75 $(a \sim 0.152 \text{ fm})$	$16^3 \times 32$	-1.1505	0.0180 0.0280 0.0480	60x8
		-1.1898	0.0180 0.0280 0.0480	
		-1.3668	0.0180 0.0280 0.0480	
5.85 $(a \sim 0.123 \text{ fm})$	$16^3 \times 40$	-1.0983	0.0224 0.0316 0.0387	60x8
		-1.1375	0.0120 0.0172 0.0224 0.0387 0.0600	
		-1.2944	0.0224 0.0387	
5.95 $(a \sim 0.102 \text{ fm})$	$20^3 \times 48$	-0.9761	0.0186, 0.0321	60x8
		-1.0354	0.0186, 0.0321	
		-1.0771	0.0100 0.0186, 0.0321	

$\beta$	5.75	5.85	5.95
$\eta_{cr}^A$	-1.271(10)	-1.207(8)	-1.145(6)
$\eta_{cr}^B$	-1.249(8)	-1.192(6)	-1.136(6)

- $\bar{\eta}$  controls the mixing of the Wilson-like term  $b^2 \hat{O}_6 = \frac{b^2}{2} \rho (\overline{Q}_L \overleftarrow{\mathcal{D}}_\mu \Phi \mathcal{D}_\mu Q_R + h.c.)$  with the Yukawa  $\hat{O}_{Yuk} = \eta (\overline{Q}_L \Phi Q_R + h.c.).$

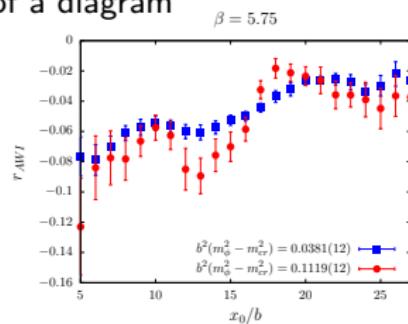
$$\hat{O}_6 = [O_6]_{sub} + \frac{Z_J - 1}{b^2} \tilde{J} + \frac{\bar{\eta}}{b^2} \hat{O}_{Yuk}$$

- The divergence  $1/b^2$  in the mixing pattern are independent on the scalar mass  $m_\phi^2$

$$\frac{1}{p^2 + m^2} = \frac{1}{p^2} - \frac{1}{p^2} m^2 \frac{1}{p^2} + \frac{1}{p^2} m^2 \frac{1}{p^2} m^2 \frac{1}{p^2} + \dots$$

mass insertion lowers the degree of divergence of a diagram

- We are also checking numerically that  $\eta_{cr}$  is independent on  $m_\phi^2$
- No change in  $r_{AWI}^A$ ,  $r_{AWI}^B$  within present errors both  $\beta = 5.75$  and  $5.95$



$\beta$	$\mu$	$\eta$	$r_{AWI}^A$	$r_{AWI}^B$	$b^2 m_\phi^2$	$b^2(m_\phi^2 - m_{cr}^2)$
5.75	0.028	-1.1898	-0.0675(64)	-0.0233(45)	-0.4150	0.1119(12)
5.75	0.028	-1.1842	-0.0596(34)	-0.0284(20)	-0.4887	0.0381(12)
5.95	0.0186	-1.0771	-0.0680(35)	-0.0390(58)	-0.4956	0.0504(10))
5.95	0.0186	-1.0748	-0.0613(21)	-0.0386(15)	-0.5289	0.0171(10)

## Nambu-Goldstone phase simulations parameters

$\beta$	$a^{-4}(L^3 \times T)$	$\eta$	$a\mu$	$N_U \times N_\phi$
5.75 ( $a \sim 0.152$ fm)	$16^3 \times 40$	-1.2714 -1.2656 -1.2539 -1.2404 -1.231	0.0050 0.0087, 0.0131, 0.0183, 0.0277 0.0131 0.0183 0.0131 0.0087, 0.0183	60x1
5.85 ( $a \sim 0.123$ fm)	$20^3 \times 40$	-1.2105 <b>-1.2068</b> -1.2028 -1.1949 -1.1776	0.0040, 0.0070, 0.0100, 0.0120 0.0040, 0.0070, 0.0100 ,0.0224 0.0040, 0.0070, 0.0100, 0.0120, 0.0224 0.0100 0.0070, 0.0100, 0.0140, 0.0224 , 0.0316	30x2
5.95 ( $a \sim 0.102$ fm)	$24^3 \times 48$	-1.1474 <b>-1.1449</b> -1.1215 -1.1134	0.0066, 0.0077, 0.0116, 0.0145, 0.0185 0.0060, 0.0077, 0.0116, 0.0145 0.0077 0.0077, 0.0108	30x1

blue point  $\sim \eta_{cr}^A$

# Low energy effective action in NG phase for $S_{lat}^{\text{toy-tm}}$

$$\Gamma^{NG} = \Gamma_4^{NG} + b^2 \Gamma_6^{NG} + b^4 \Gamma_8^{NG} + \dots \quad \text{where}$$

$$\begin{aligned} \Gamma_{4 \text{ loc}}^{NG} = & \frac{1}{4}(G \cdot G) + \bar{Q} \not{D} Q + \frac{1}{2} \text{tr} [\partial_\mu \Phi^\dagger \partial_\mu \Phi] + \frac{m_\Phi^2}{2} \text{tr} [\Phi^\dagger \Phi] + \frac{\lambda_\Phi}{4} (\text{tr} [\Phi^\dagger \Phi])^2 + \\ & + \eta_{\text{eff}} \bar{Q} F Q + \mu_{\text{eff}} \bar{Q} i \gamma_5 \tau^3 Q + c_1 \Lambda_s \bar{Q} U_F Q + (c_2 \Lambda_s^2 R + c_3 \Lambda_s^2) \frac{1}{2} \text{tr} [\partial_\mu U^\dagger \partial_\mu U] \end{aligned}$$

$$\Gamma_{6 \text{ loc}}^{NG} \supset O_{6,glu}^{\tilde{x}-\text{inv}} ; [ \sum_{\Gamma_A \Gamma_B} c_{AB} (\bar{Q} \Gamma_A Q) (\bar{Q} \Gamma_B Q) ]^{\tilde{x}-\text{inv}} ; [ D_\lambda \bar{Q}_L \Phi D_\lambda Q_R + h.c.] , \dots ;$$

here  $\eta_{\text{eff}} \propto \eta - \eta_{\text{cr}}$ ,  $\mu_{\text{eff}} \propto \mu$ ; vacuum choice (by “ $\Phi$ -projection”):  $\Phi \propto U \propto I$ ;

$$F \equiv (v + \zeta_0) U_F, \sigma_F \equiv \zeta_0 U_F, U_F = P_R U + P_L U^\dagger$$

In the No Mechanism hypothesis the twist angle is  $\omega = \frac{\pi}{2}$ , thus the physical basis is defined as  $\Psi = \exp(i \frac{\pi}{4} \gamma_5 \tau^3) Q$

The term  $[D_\lambda \bar{Q}_L \Phi D_\lambda Q_R + h.c.]$  and similar fermionic bilinears are parity odd