Isospin breaking corrections to the HVP at the physical point

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[RBC/UKQCD, Phys.Rev.Lett. 121 (2018) 022003]

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Muon $\mathbf{a}_\mu$ and the hadronic vacuum polarisation (HVP)

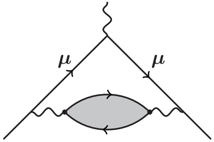
- experiment: polarized muons in a magnetic field [Bennet et al., Phys.Rev. **D73**, 072003 (2006)]

$$\mathbf{a}_\mu = \mathbf{11659208.9(5.4)(3.3)} \times \mathbf{10^{-10}}$$

- Standard Model [PDG]

$$\mathbf{a}_\mu = \mathbf{11659180.3(0.1)(4.2)(2.6)} \times \mathbf{10^{-10}}$$

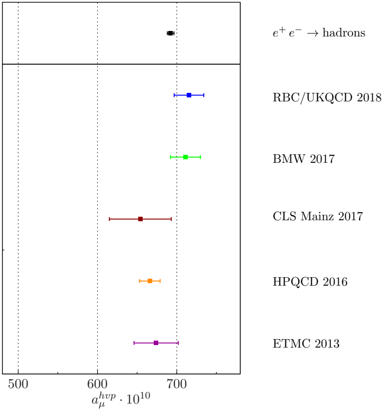
- Comparison of theory and experiment: $\mathbf{3.6\sigma}$ deviation
- largest error on SM estimate from HVP



- estimate from $\mathbf{e^+e^-} \longrightarrow$ hadrons [Davier et al., Eur.Phys.J. **C71**, 1515 (2011)]

$$\mathbf{(692.3 \pm 4.2 \pm 0.3)} \times \mathbf{10^{-10}}$$

HVP from the **R**-ratio $\leftrightarrow$ Lattice



- lattice result to be competitive with **R**-ratio requires precision of $\lesssim$ **1%**
  $\longrightarrow$ Isospin Breaking (IB) Corrections
- RBC/UKQCD 2018: HVP at physical point including IB corrections
  [C. Lehner, V.G. et al., Phys.Rev.Lett. 121 (2018) 022003]
  - $\longrightarrow$ Isospin symmetric HVP [C. Lehner, Friday 14:00]
  - $\longrightarrow$ this talk: Isospin Breaking Corrections

IB corrections to lattice calculations

- lattice calculations often done in isospin symmetric limit
- sources of IB corrections
  - different masses for up- and down quark (of $\mathcal{O}((\mathbf{m_d} - \mathbf{m_u})/\mathbf{\Lambda}_{\mathrm{QCD}}))$
  - Quarks have electrical charge (of $\mathcal{O}(\alpha))$

$\rightarrow$ need to be included in lattice calculations with precision $\lesssim \mathbf{1\%}$

Inclusion of IB Effects

- strong IB by using different input quark masses
- stochastic QED using $\mathbf{U(1)}$ gauge configurations
  [A. Duncan, E. Eichten, H. Thacker, Phys.Rev.Lett. **76**, 3894 (1996)]

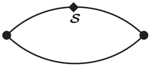
- Here: Expansion around isospin symmetric calculation
  [G.M. de Divitiis et al, JHEP 1204 (2012) 124],[RM123 Collaboration, Phys.Rev. D87, 114505 (2013)]

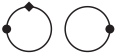
$$\mathcal{O} = \mathcal{O}^{\mathbf{0}} + \alpha\mathcal{O}^{\mathrm{QED}} + \sum_{\mathbf{f}} \mathbf{\Delta m_f}\mathcal{O}^{\mathbf{\Delta m_f}}$$

Expansion around IB symmetric

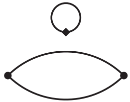
- perturbative expansion in $\mathbf{\Delta m_f = (m_f^0 - m_f)}$ [G.M. de Divitiis *et al*, JHEP 1204 (2012) 124]

$$\langle \mathbf{O} \rangle_{\mathbf{m_f}} = \langle \mathbf{O} \rangle_{\mathbf{m_f^0}} + \mathbf{\Delta m_f} \frac{\partial}{\partial \mathbf{m_f}} \langle \mathbf{O} \rangle \bigg|_{\mathbf{m_f^0}} + \mathcal{O}\left(\mathbf{\Delta m_f^2}\right)$$





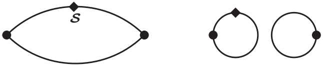
sea quark effects:
quark-disconnected diagrams



Expansion around IB symmetric

- perturbative expansion in $\mathbf{\Delta m_f = (m_f^0 - m_f)}$ [G.M. de Divitiis *et al*, JHEP 1204 (2012) 124]

$$\langle O \rangle_{m_f} = \langle O \rangle_{m_f^0} + \Delta m_f \, \frac{\partial}{\partial m_f} \langle O \rangle \Big|_{m_f^0} + \mathcal{O}\left(\Delta m_f^2\right)$$



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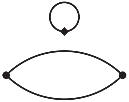
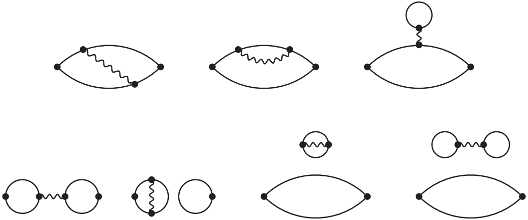


- expand the path integral in α [RM123 Collaboration, Phys.Rev. **D87**, 114505 (2013)]

$$\langle O \rangle = \langle O \rangle_{e=0} + \frac{1}{2} e^2 \, \frac{\partial^2}{\partial e^2} \langle O \rangle \Big|_{e=0} + \mathcal{O}(\alpha^2)$$



Expansion around IB symmetric

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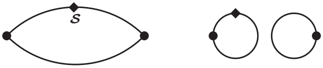

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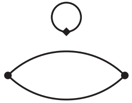

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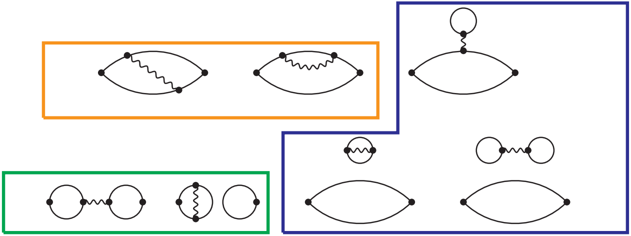


quark-connected

quark-disconnected

unquenched QED

IB corrections to HVP at physical point

- [C. Lehner, V.G. *et al.*, Phys.Rev.Lett. 121 (2018) 022003]

- $\mathbf{N_f = 2 + 1}$ Möbius DWF, $\mathbf{48^3} \times \mathbf{96}$ lattice, $\mathbf{a^{-1} = 1.730(4)}$ GeV
  [T. Blum *et al.* Phys.Rev. *D93* (2016) no.7, 074505]

- IB corrections from expansion around isospin symmetric calculation

$$\mathbf{C(t) = C^0(t) + \alpha C^{\mathrm{QED}}(t) + \sum_f \Delta m_f C^{\Delta m_f}(t)}$$

- photon propagator in Feynman gauge, $\mathrm{QED}_{\mathbf{L}}$

$$\mathbf{\Delta}_{\mu\nu}(\mathbf{x}-\mathbf{y}) = \delta_{\mu\nu}\,\frac{\mathbf{1}}{\mathbf{N}}\sum_{\mathbf{k},\,\vec{\mathbf{k}}\neq 0}\frac{\mathbf{e}^{\mathbf{ik}\cdot(\mathbf{x}-\mathbf{y})}}{\hat{\mathbf{k}}^2}$$

- photon propagator estimated from stochastic photon field
  [D. Giusti et al. Phys.Rev. D95 (2017) 114504]

$$\mathbf{\Delta}_{\mu\nu}(\mathbf{x}-\mathbf{y}) = \langle \mathbf{A}_\mu(\mathbf{x})\mathbf{A}_\nu(\mathbf{y})\rangle$$

Tuning the quark masses

- isospin symmetric calculation using quark masses determined without QED
  [T. Blum *et al.* Phys.Rev. *D93* (2016) no.7, 074505]
- physical quark masses including QED:

→ tune ($\mathbf{u}$,$\mathbf{d}$,$\mathbf{s}$) masses to reproduce experimental π^+, $\mathbf{K}^+$ and $\mathbf{K_0}$ mass (and check π^0 mass)

$$\mathbf{a\,m_{\pi^+}^{\text{exp}}} = \left[\mathbf{m_{\pi^+}^0} + \alpha \mathbf{m_{\pi^+}^{\text{QED}}} + \mathbf{\Delta m_d\; m_{\pi^+}^{\Delta m_d}} + \mathbf{\Delta m_u\; m_{\pi^+}^{\Delta m_u}}\right]$$

$$\mathbf{a\,m_{K^+}^{\text{exp}}} = \left[\mathbf{m_{K^+}^0} + \alpha \mathbf{m_{K^+}^{\text{QED}}} + \mathbf{\Delta m_u\; m_{K^+}^{\Delta m_u}} + \mathbf{\Delta m_s\; m_{K^+}^{\Delta m_s}}\right]$$

$$\mathbf{a\,m_{K^0}^{\text{exp}}} = \left[\mathbf{m_{K^0}^0} + \alpha \mathbf{m_{K^0}^{\text{QED}}} + \mathbf{\Delta m_d\; m_{K^0}^{\Delta m_d}} + \mathbf{\Delta m_s\; m_{K^0}^{\Delta m_s}}\right]$$

- lattice spacing: fix another mass including QED
  → here: Omega-Baryon

$$\mathbf{a} \rightarrow \mathbf{a(\Delta m_s)} = \left(\mathbf{m_\Omega^0} + \alpha \mathbf{m_\Omega^{\text{QED}}} + \mathbf{3\,\Delta m_s\; m_\Omega^{\Delta m_s}}\right) / \mathbf{m_\Omega^{\text{exp}}}$$

→ shift in **a** smaller then statistical error on lattice spacing

connected QED corrections to the HVP

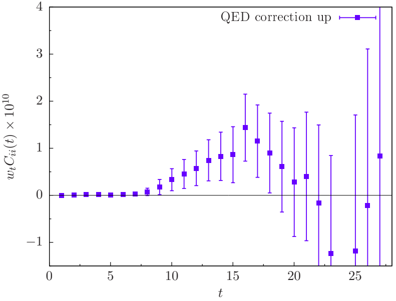
- vector two-point function

$$C_{\mu\nu}(t) = \sum_{\vec{x}} \langle J_\mu(t, \vec{x}) J_\nu(0) \rangle$$

- HVP contribution to a_μ [Bernecker and Meyer, Eur.Phys.J. A47, 148 (2011); Feng *et al.* Phys.Rev. **D88**, 034505 (2013)]

$$a_\mu = \sum_t w_t C_{ii}(t) \qquad\qquad i = 0, 1, 2$$

- connected QED correction

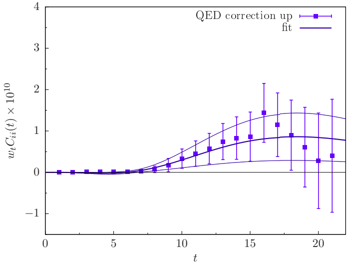


QED corrections to the HVP

- Ansatz for $\mathcal{O}(\alpha)$-correction to correlator

$$\delta \mathbf{C}(\mathbf{t}) = (\mathbf{c}_1 + \mathbf{c}_0 \mathbf{t})\mathbf{e}^{-\mathbf{E}\mathbf{t}}$$

- lowest lying state w/o QED $\pi\pi$
- lowest lying state with QED $\pi\gamma$
  $\rightarrow$ QED$_{\mathbf{L}}$: photon has one unit of momentum
- fit data to ansatz with $\mathbf{c}_0$ and $\mathbf{c}_1$ as parameters



$$\mathbf{a}_{\mu}^{\text{QED,con}} = \mathbf{5.9(5.7)} \times \mathbf{10}^{-10}$$

Systematic errors

- $\mathbf{a_\mu^{\mathrm{QED,con}} = 5.9(5.7)_S}$ $\times\ \mathbf{10^{-10}}$

Systematic errors

- $\mathbf{a}_{\boldsymbol{\mu}}^{\text{QED,con}} = \mathbf{5.9(5.7)_S(1.1)_E}$ $\times\, \mathbf{10^{-10}}$

- ansatz for extrapolation: vary energy between $\pi\pi$ and $\pi\gamma$ $\mathbf{(1.1)_E}$

Systematic errors

- $\mathbf{a_\mu^{\text{QED,con}} = 5.9(5.7)_S(1.1)_E(0.3)_C}$ $\times\ \mathbf{10^{-10}}$

- ansatz for extrapolation: vary energy between $\pi\pi$ and $\pi\gamma$ $\mathbf{(1.1)_E}$
- discretization error $\mathbf{(0.3)_C}$ estimated as $\mathbf{(a\Lambda)^2}$ with $\mathbf{\Lambda = 400}$ MeV

Systematic errors

- $a_\mu^{\mathrm{QED,con}} = 5.9(5.7)_S(1.1)_E(0.3)_C(1.2)_V \qquad \times 10^{-10}$

- ansatz for extrapolation: vary energy between $\pi\pi$ and $\pi\gamma$ $(1.1)_E$
- discretization error $(0.3)_C$ estimated as $(a\Lambda)^2$ with $\Lambda = 400$ MeV
- finite volume corrections
  - repeat calculation using infinite volume photon as estimate $\rightarrow (1.2)_V$

$$\Delta^{\mathrm{inf}}(x) = \int_{-\pi}^{\pi} \frac{d^4k}{(2\pi^4)} \frac{1}{\hat{k}^2} e^{ikx}$$

  - study of finite volume effects using scalar QED [J. Harrison, Wed 16:30]
  - analytical calculation [A. Portelli, J. Bijnens, N. Hermansson Truedsson, T. Janowski, ...]

$\longrightarrow$ most likely negligible

Systematic errors

- $a_\mu^{\mathrm{QED,con}} = 5.9(5.7)_S(1.1)_E(0.3)_C(1.2)_V(0.0)_A(0.0)_Z \times 10^{-10}$

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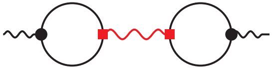
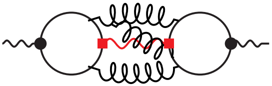
  $\rightarrow$ most likely negligible
- propagate uncertainties from lattice spacing $(0.0)_A$ and Z_V $(0.0)_Z$

leading disconnected QED correction

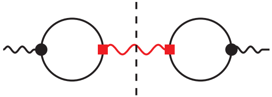


gluons between the quarks lines



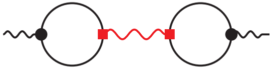
$\rightarrow$ QED correction to LO HVP

no gluons between the quarks lines



$\rightarrow$ included in NLO HVP

leading disconnected QED correction



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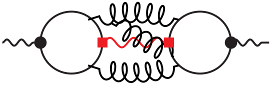

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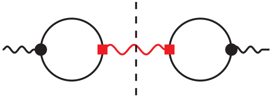
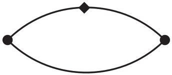


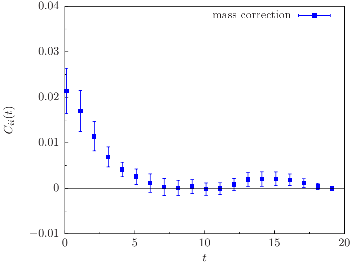
$\longrightarrow$ included in NLO HVP

- using data generated for light-by-light project
  [T. Blum *et al.* Phys. Rev. Lett. 118, 022005 (2017)]
- result

$$\mathbf{a}_{\mu}^{\text{QED, disc}} = -\mathbf{6.9(2.1)}_{\mathsf{S}}\mathbf{(1.3)}_{\mathsf{E}}\mathbf{(0.4)}_{\mathsf{C}}\mathbf{(0.4)}_{\mathsf{V}}\mathbf{(0.0)}_{\mathsf{A}}\mathbf{(0.0)}_{\mathsf{Z}} \times \mathbf{10^{-10}}$$

strong Isospin Breaking Corrections to the HVP





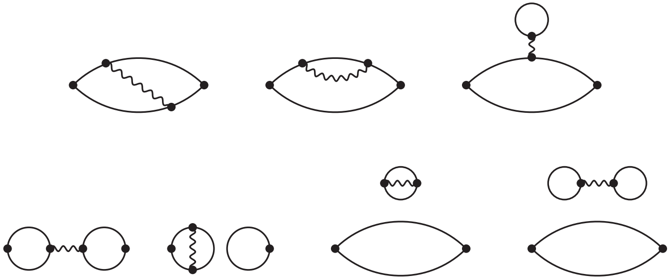
- Ansatz

$$\delta \mathbf{C}(\mathbf{t}) = (\mathbf{c}_1 + \mathbf{c}_0 \mathbf{t}) \mathbf{e}^{-\mathbf{E}\mathbf{t}}$$

- lowest lying state $\pi\pi$, vary between free and interacting
- result

$$\mathbf{a}_{\mu}^{\mathrm{sIB}} = \mathbf{10.6(4.3)_S(1.3)_E(0.6)_C(6.6)_V(0.1)_A(0.0)_Z \times 10^{-10}}$$

Summary IB corrections to the HVP



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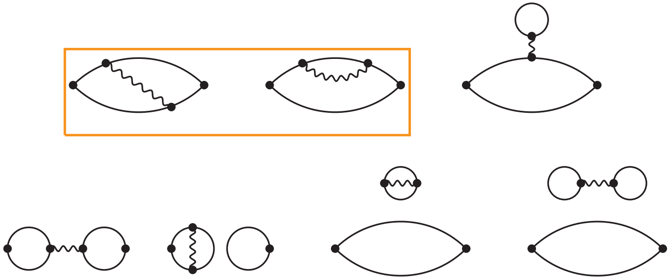

- connected $\mathbf{a}_{\mu}^{\text{QED, conn}} = \mathbf{5.9(5.7)(1.7)} \times \mathbf{10^{-10}}$

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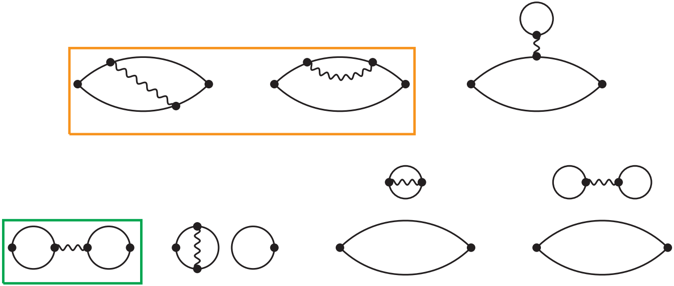

- connected $\mathbf{a}_{\mu}^{\text{QED, conn}} = \mathbf{5.9(5.7)(1.7)} \times \mathbf{10^{-10}}$
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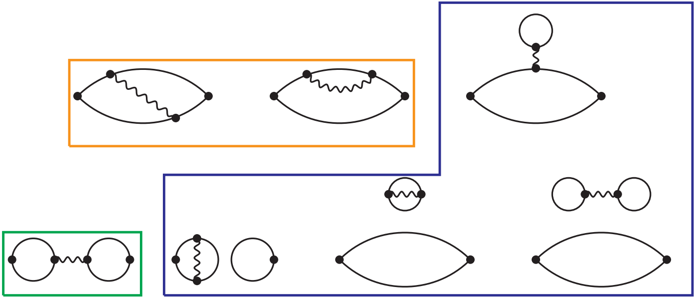

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- disconnected $\mathbf{a}_{\mu}^{\text{QED, disc}} = \mathbf{-6.9(2.1)(2.7)} \times \mathbf{10^{-10}}$
- at least $^1/_{N_c}$ suppressed $\rightarrow$ assign **30%** systematic error

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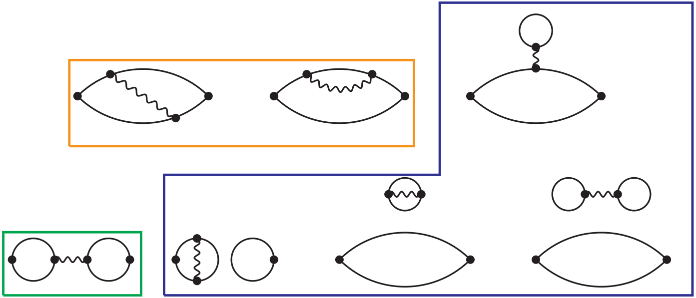

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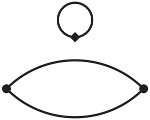

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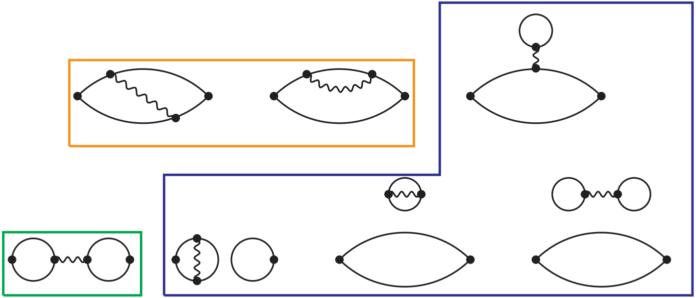

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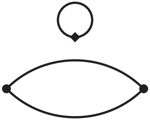
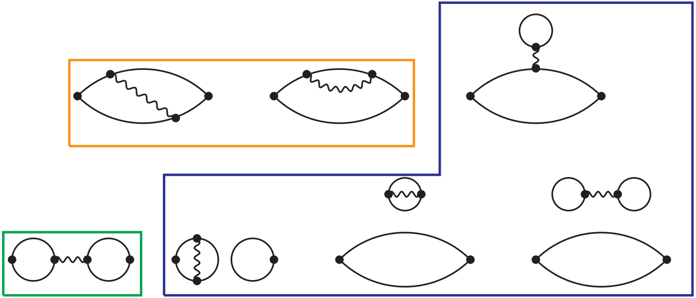
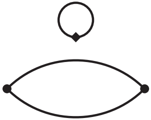


- strong IB correction $\mathbf{a}_\mu^{\text{sIB}} = \mathbf{10.6(4.3)(6.8) \times 10^{-10}}$

Summary IB corrections to the HVP

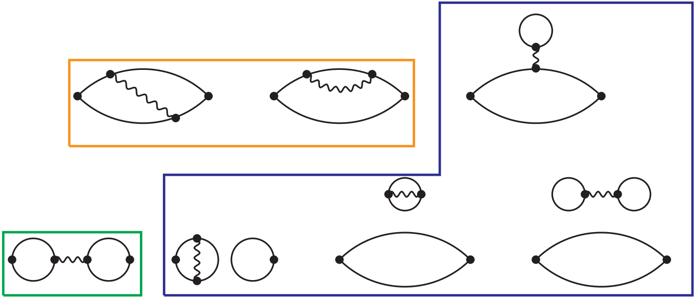


- connected $\mathbf{a}_\mu^{\text{QED, conn}} = \mathbf{5.9(5.7)(1.7) \times 10^{-10}}$
- disconnected $\mathbf{a}_\mu^{\text{QED, disc}} = \mathbf{-6.9(2.1)(2.7) \times 10^{-10}}$
- at least $^1/_{N_c}$ suppressed $\rightarrow$ assign **30%** systematic error



- strong IB correction $\mathbf{a}_\mu^{\text{sIB}} = \mathbf{10.6(4.3)(6.8) \times 10^{-10}}$
- disconnected sIB $\mathbf{SU(3)_f}$ and $\mathbf{1/N_c}$ suppressed $\rightarrow$ assign **10%** error

Summary IB corrections to the HVP



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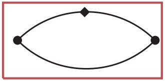
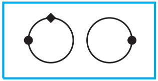
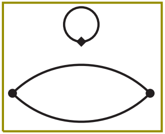
- at least $^1/_{\mathrm{N_c}}$ suppressed $\to$ assign **30%** systematic error







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- disconnected sIB $\mathbf{SU(3)_f}$ and $\mathbf{1/N_c}$ suppressed $\to$ assign **10%** error
- unquenched sIB $\mathbf{\Delta m_u \approx -\Delta m_d}$ suppressed

Conclusions

- Lattice HVP calculation at $\lesssim \mathbf{1\%}$ requires inclusion of isospin breaking
- we have calculated IB corrections directly at physical point
  $\rightarrow$ tuned ($\mathbf{u}$, $\mathbf{d}$, $\mathbf{s}$) masses including QED using $\boldsymbol{\pi}^{+}$, $\mathbf{K}^{+}$, $\mathbf{K}^{0}$ and $\boldsymbol{\Omega}$ for lattice spacing
- connected and one disconnected QED correction, connected strong IB correction

Conclusions

- Lattice HVP calculation at $\lesssim \mathbf{1\%}$ requires inclusion of isospin breaking
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  $\rightarrow$ tuned $(\mathbf{u}, \mathbf{d}, \mathbf{s})$ masses including QED using π^+, $\mathbf{K}^+$, $\mathbf{K}^0$ and $\mathbf{\Omega}$ for lattice spacing
- connected and one disconnected QED correction, connected strong IB correction

Outlook

- re-use light-by-light data to [+ M. Bruno]
  - increase statistics for connected QED diagrams
  - calculate the QED-unquenched diagrams
- second lattice spacing for QED corrections
- strong IB: effects from sea quark mass shift, second lattice spacing

Backup

Results HVP window method - total

see [C. Lehner, V.G. et al., Phys.Rev.Lett. 121 (2018) 022003]

$a_\mu^{\text{ud, conn, isospin}}$	$202.9(1.4)_\text{S}(0.2)_\text{C}(0.1)_\text{V}(0.2)_\text{A}(0.2)_\text{Z}$	$649.7(14.2)_\text{S}(2.8)_\text{C}(3.7)_\text{V}(1.5)_\text{A}(0.4)_\text{Z}(0.1)_\text{E48}(0.1)_\text{E64}$
$a_\mu^{\text{s, conn, isospin}}$	$27.0(0.2)_\text{S}(0.0)_\text{C}(0.1)_\text{A}(0.0)_\text{Z}$	$53.2(0.4)_\text{S}(0.0)_\text{C}(0.3)_\text{A}(0.0)_\text{Z}$
$a_\mu^{\text{c, conn, isospin}}$	$3.0(0.0)_\text{S}(0.1)_\text{C}(0.0)_\text{Z}(0.0)_\text{M}$	$14.3(0.0)_\text{S}(0.7)_\text{C}(0.1)_\text{Z}(0.0)_\text{M}$
$a_\mu^{\text{uds, disc, isospin}}$	$-1.0(0.1)_\text{S}(0.0)_\text{C}(0.0)_\text{V}(0.0)_\text{A}(0.0)_\text{Z}$	$-11.2(3.3)_\text{S}(0.4)_\text{V}(2.3)_\text{L}$
$a_\mu^{\text{QED, conn}}$	$0.2(0.2)_\text{S}(0.0)_\text{C}(0.0)_\text{V}(0.0)_\text{A}(0.0)_\text{Z}(0.0)_\text{E}$	$5.9(5.7)_\text{S}(0.3)_\text{C}(1.2)_\text{V}(0.0)_\text{A}(0.0)_\text{Z}(1.1)_\text{E}$
$a_\mu^{\text{QED, disc}}$	$-0.2(0.1)_\text{S}(0.0)_\text{C}(0.0)_\text{V}(0.0)_\text{A}(0.0)_\text{Z}(0.0)_\text{E}$	$-6.9(2.1)_\text{S}(0.4)_\text{C}(1.4)_\text{V}(0.0)_\text{A}(0.0)_\text{Z}(1.3)_\text{E}$
a_μ^{SIB}	$0.1(0.2)_\text{S}(0.0)_\text{C}(0.2)_\text{V}(0.0)_\text{A}(0.0)_\text{Z}(0.0)_\text{E48}$	$10.6(4.3)_\text{S}(0.6)_\text{C}(6.6)_\text{V}(0.1)_\text{A}(0.0)_\text{Z}(1.3)_\text{E48}$
$a_\mu^{\text{udsc, isospin}}$	$231.9(1.4)_\text{S}(0.2)_\text{C}(0.1)_\text{V}(0.3)_\text{A}(0.2)_\text{Z}(0.0)_\text{M}$	$705.9(14.6)_\text{S}(2.9)_\text{C}(3.7)_\text{V}(1.8)_\text{A}(0.4)_\text{Z}(2.3)_\text{L}(0.1)_\text{E48}$ $(0.1)_\text{E64}(0.0)_\text{M}$
$a_\mu^{\text{QED, SIB}}$	$0.1(0.3)_\text{S}(0.0)_\text{C}(0.2)_\text{V}(0.0)_\text{A}(0.0)_\text{Z}(0.0)_\text{E}(0.0)_\text{E48}$	$9.5(7.4)_\text{S}(0.7)_\text{C}(6.9)_\text{V}(0.1)_\text{A}(0.0)_\text{Z}(1.7)_\text{E}(1.3)_\text{E48}$
$a_\mu^{\text{R-ratio}}$	$460.4(0.7)_\text{RST}(2.1)_\text{RSY}$	
a_μ	$692.5(1.4)_\text{S}(0.2)_\text{C}(0.2)_\text{V}(0.3)_\text{A}(0.2)_\text{Z}(0.0)_\text{E}(0.0)_\text{E48}$ $(0.0)_\text{b}(0.1)_\text{c}(0.0)_{\overline{\text{S}}}(0.0)_{\overline{\text{Q}}}(0.0)_\text{M}(0.7)_\text{RST}(2.1)_\text{RSY}$	$715.4(16.3)_\text{S}(3.0)_\text{C}(7.8)_\text{V}(1.9)_\text{A}(0.4)_\text{Z}(1.7)_\text{E}(2.3)_\text{L}$ $(1.5)_\text{E48}(0.1)_\text{E64}(0.3)_\text{b}(0.2)_\text{c}(1.1)_{\overline{\text{S}}}(0.3)_{\overline{\text{Q}}}(0.0)_\text{M}$

TABLE I. Individual and summed contributions to a_μ multiplied by 10^{10}. The left column lists results for the window method with $t_0 = 0.4$ fm and $t_1 = 1$ fm. The right column shows results for the pure first-principles lattice calculation. The respective uncertainties are defined in the main text.

Window Method

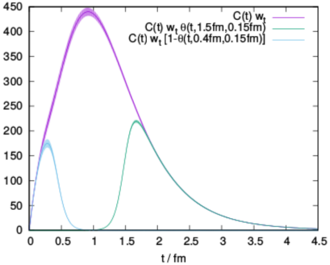
- combining lattice with **R**-ratio data [RBC/UKQCD, Phys.Rev.Lett. 121 (2018) 022003]
- very short and long distances from **R**-ratio, intermediate distances from lattice

$$\mathbf{a}_\mu = \mathbf{a}_\mu^{\mathrm{SD}} + \mathbf{a}_\mu^{\mathrm{W}} + \mathbf{a}_\mu^{\mathrm{LD}}$$

$$\mathbf{a}_\mu^{\mathrm{SD}} = \sum_t \mathbf{w}_t \mathbf{C}(t)[1 - \theta(t, t_0, \Delta)]$$

$$\mathbf{a}_\mu^{\mathrm{W}} = \sum_t \mathbf{w}_t \mathbf{C}(t)[\theta(t, t_0, \Delta) - \theta(t, t_1, \Delta)]$$

$$\mathbf{a}_\mu^{\mathrm{LD}} = \sum_t \mathbf{w}_t \mathbf{C}(t)\, \theta(t, t_1, \Delta)$$



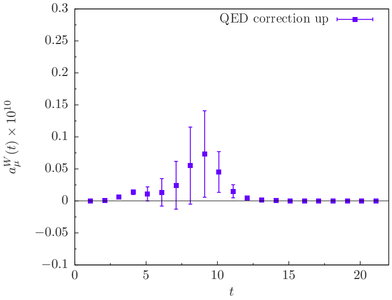
Window Contribution for QED corrections

- combining lattice with **R**-ratio data
  $\longrightarrow$ window method [RBC/UKQCD, Phys.Rev.Lett. 121 (2018) 022003],[C. Lehner, Friday 14:00]
- very short and long distances from **R**-ratio, intermediate distances from lattice

$$a_\mu = a_\mu^{SD} + a_\mu^{W} + a_\mu^{LD} \qquad a_\mu^{W} = \sum_t w_t C(t)[\theta(t,t_0,\Delta) - \theta(t,t_1,\Delta)]$$

$$\theta(t,t',\Delta) = (1 + \tanh[(t-t')/\Delta])/2$$

- window contribution from QED corrections



- $t_0 = 0.4$ fm
- $t_1 = 1.0$ fm
- $\Delta = 0.15$ fm

- $a_\mu^{W,QED} = 0.2(0.2) \times 10^{-10}$

Ansatz for IB correlator

- generic two-point function w/o QED

$$\mathbf{C_0(t) = A_0 e^{-m_0 t}}$$

- generic two-point function with QED (up to all orders)

$$\mathbf{C(t) = A e^{-mt}}$$

- expand

$$\mathbf{C(t) = A e^{-mt} = (A_0 + \alpha\delta A) e^{-(m_0 + \alpha\delta m)t} = (A_0 + \alpha\delta A) e^{-m_0 t}(1 - \delta m t)}$$

- perturbative method: $\mathcal{O}(\alpha)$ correction to correlator

$$\mathbf{\delta C(t) = A_0 e^{-m_0 t}\left(\frac{\delta A}{A_0} - \delta m t\right)}$$

Results quark masses tuning

- isospin symmetric calculation [T. Blum et al. Phys.Rev. D93 (2016) no.7, 074505]

$$\mathbf{am_\ell = 0.0006979(81)} \qquad \mathbf{am_s = 0.03580(16)}$$

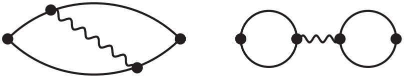
- tune (**u**,**d**,**s**) masses to reproduce experimental π^+, $\mathbf{K^+}$ and $\mathbf{K_0}$ mass (and check π^0 mass), fix lattice spacing using Ω^-

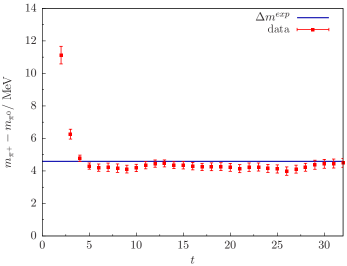
$$\mathbf{\Delta m_u = 0.00050(1)} \qquad \mathbf{\Delta m_d = -0.00050(1)} \qquad \mathbf{\Delta m_s = -0.0002(2)}$$

- ratio of quark masses

$$\mathbf{\frac{m_d}{m_u} = 0.449(22)}$$

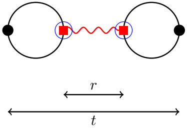
- pion mass difference





Disconnected QED correction to HVP - position space sampling

- position space sampling [L. Jin et al., PRD93, 014503 (2016)]
- Disconnected QED correction



- sampling of position of ◯
- point-to-all propagators
- short distances: sample all possible pairs
- long distances: sample points stochastically using some probability distribution
- exact photon propagator

stochastic method

- Feynman gauge

$$S_\gamma^{\mathrm{Feyn}}[A] = -\frac{a^4}{2}\sum_x\sum_\mu A_\mu(x)\partial^2 A_\mu(x) \qquad \text{with} \quad \partial^2 = \sum_\mu \partial_\mu^* \partial_\mu$$

- in momentum space

$$S_\gamma^{\mathrm{Feyn}}[A] = \frac{1}{2N}\sum_{k,\vec{k}\neq 0} \hat{k}^2 \sum_\mu \left|\tilde{A}_\mu(k)\right|^2 \qquad\qquad \hat{k}_\mu = \frac{2}{a}\sin\left(\frac{ak_\mu}{2}\right)$$

- remove all spatial zero modes $\rightarrow$ $\mathrm{QED_L}$
  [S. Uno and M. Hayakawa, Prog. Theor. Phys. **120**, 413 (2008)]
- draw $\tilde{A}_\mu(k)$ from Gaussian distribution with variance $2N/\hat{k}^2$
- electro quenched approximation
- multiply $SU(3)$ gauge links with $U(1)$ photon fields

$$U_\mu(x) \rightarrow e^{ieA_\mu(x)}U_\mu(x)$$

- remove $\mathcal{O}(e)$ noise by averaging over $+e$ and $-e$
- QED correction to all orders in α

a_μ: Experiment vs. Theory

- $\mathbf{a_\mu = (g_\mu - 2)/2}$
- measured and calculated very precisely $\longrightarrow$ test of the Standard Model
- experiment: polarized muons in a magnetic field [Bennet et al., Phys.Rev. **D73**, 072003 (2006)]

$$\mathbf{a_\mu = 11659208.9(5.4)(3.3) \times 10^{-10}}$$

- Standard Model

em	$\mathbf{(11658471.895 \pm 0.008) \times 10^{-10}}$	[Kinoshita et al., Phys.Rev.Lett. **109**, 111808 (2012)]
weak	$\mathbf{(15.36 \pm 0.10) \times 10^{-10}}$	[Gnendinger et al., Phys.Rev. **D88**, 053005 (2013)]
HVP	$\mathbf{(692.3 \pm 4.2 \pm 0.3) \times 10^{-10}}$	[Davier et al., Eur.Phys.J. **C71**, 1515 (2011)]
HVP(α^3)	$\mathbf{(-9.84 \pm 0.06) \times 10^{-10}}$	[Hagiwara et al., J.Phys. **G38**, 085003 (2011)]
LbL	$\mathbf{(10.5 \pm 2.6) \times 10^{-10}}$	[Prades et al.,Adv.Ser.Direct.High Energy Phys. 20, 303 (2009)]

- Comparison of theory and experiment: $\mathbf{3.6\sigma}$ deviation

$$\mathbf{\Delta a_\mu = a_\mu^{exp} - a_\mu^{SM} = 28.8(6.3)^{Exp}(4.9)^{SM} \times 10^{-10}}$$

- new physics?