A new method for suppressing excited-state contaminations on the nucleon form factors

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PRISMA

SFB 1044 THE LOW-ENERGY FRONTIER OF THE STANDARD MODEL

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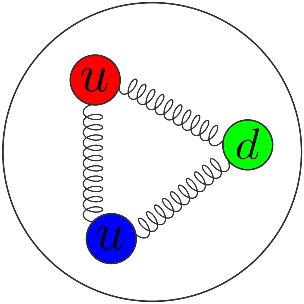
Introduction

- Study intrinsic structure of the nucleon
  $\Rightarrow$ electroweak form factors
- Charge distribution, mean squared radii, spin structure
- Axial charge g_A precisely known from neutron β-decay: $n \rightarrow p + e^- + \bar{\nu}_e$
- Experimental studies
  - Electron scattering
  - Neutrino scattering
  - Muon capture
- Lattice calculation: Analyze uncertainties from **excited-state contributions**



Introduction

- Consider the local vector and axial vector currents

$$V_\mu(x) = \bar{\psi}(x)\gamma_\mu\psi(x)$$
$$A_\mu(x) = \bar{\psi}(x)\gamma_\mu\gamma_5\psi(x)$$

- General decomposition of the nucleon matrix elements into form factors

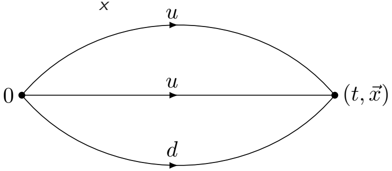
$$\langle N, \vec{p}', s' | V_\mu(0) | N, \vec{p}, s\rangle = \bar{u}(\vec{p}', s') \left[\gamma_\mu F_1(q^2) + i\frac{\sigma_{\mu\nu}q^\nu}{2m_N} F_2(q^2)\right] u(\vec{p}, s)$$
$$\langle N, \vec{p}', s' | A_\mu(0) | N, \vec{p}, s\rangle = \bar{u}(\vec{p}', s') \left[\gamma_\mu\gamma_5 G_A(q^2) + \gamma_5\frac{q_\mu}{2m_N} G_P(q^2)\right] u(\vec{p}, s)$$

  - Dirac and Pauli form factors: $F_1(q^2), F_2(q^2)$
  - **Axial** and **induced pseudoscalar** form factors: $G_A(q^2)$, $G_P(q^2)$

Nucleon Correlation Functions

- Euclidean nucleon 2-point correlation function ($t > 0$)

$$C_2(t,\vec{p}) = \sum_{\vec{x}} e^{-i\vec{p}\cdot\vec{x}} \Gamma_{\beta\alpha} \langle \mathcal{N}_\alpha(t,\vec{x}) \bar{\mathcal{N}}_\beta(0) \rangle$$



- Use a smeared version of the nucleon interpolating operator

$$\mathcal{N}_\alpha(x) = \epsilon_{abc}\, u^a_\alpha(x) \left(u^b_\beta(x)\, (C\gamma_5)_{\beta\gamma}\, d^c_\gamma(x) \right)$$

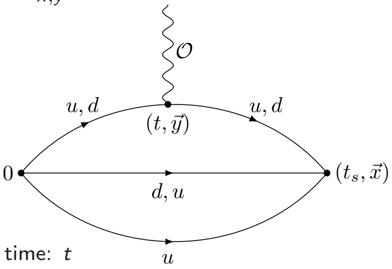
- Projector for positive parity ($J^P = 1/2^+$) and spin polarization in $\vec{s}$-direction

$$\Gamma = \frac{1}{2}(1+\gamma_0)(1 + i\gamma_5\, \vec{s}\cdot\vec{\gamma})$$

Nucleon Correlation Functions

- Euclidean nucleon 3-point correlation function

$$C_{3,\mathcal{O}}(t_s, t, \vec{p}, \vec{p}') = \sum_{\vec{x},\vec{y}} e^{i(\vec{p}' - \vec{p})\cdot\vec{y}} e^{-i\vec{p}'\cdot\vec{x}} \Gamma_{\beta\alpha} \langle \mathcal{N}_\alpha(t_s, \vec{x}) \mathcal{O}(t, \vec{y}) \bar{\mathcal{N}}_\beta(0) \rangle$$



  - Operator insertion time: t
  - Source-sink separation: t_s

- Local quark-bilinear operator: $\mathcal{O}(x) = \bar{\psi}(x)\mathcal{O}\psi(x)$

$$\langle N, \vec{p}', s' | \mathcal{O}(0) | N, \vec{p}, s \rangle = \bar{u}_{s'}(\vec{p}') \mathcal{O}(q) u_s(\vec{p}), \quad q = p' - p$$

Nucleon Correlation Functions

- **Spectral decomposition** of correlation functions (ground state): Insert complete set of one-nucleon states
- Overlap of interpolating operator with one-nucleon state

$$\langle 0|\mathcal{N}_\alpha(0,\vec{x})|N,\vec{p},s\rangle = e^{i\vec{p}\cdot\vec{x}} Z(\vec{p}) u^s_\alpha(\vec{p})$$
$$\langle N,\vec{p},s|\bar{\mathcal{N}}_\alpha(0,\vec{x})|0\rangle = e^{-i\vec{p}\cdot\vec{x}} Z^*(\vec{p}) \bar{u}^s_\alpha(\vec{p})$$

- Expressions for the nucleon 2-point and 3-point correlation function in the ground-state-dominating limit: $t, (t_s - t) \gg 0$

$$C_2(t,\vec{p}) = e^{-E_{\vec{p}}t}|Z(\vec{p})|^2 \left(1 + \frac{m}{E_{\vec{p}}}\right)$$
$$C_{3,\mathcal{O}}(t_s,t,\vec{p},\vec{p}') = \frac{e^{-E_{\vec{p}'}(t_s-t)}e^{-E_{\vec{p}}t}}{4E_{\vec{p}}E_{\vec{p}'}} Z(\vec{p}')Z^*(\vec{p})\mathrm{Tr}\left(\Gamma\left((\not{p}'+m)O(q)(\not{p}+m)\right)\right)$$

Nucleon Correlation Functions

- Spin projection: $\vec{s} = (0,0,1)^T$
- Set the nucleon at the sink at rest: $\vec{p}' = 0 \Rightarrow \vec{q} = -\vec{p}$
- Standard method: ratio of correlation functions [1]

$$R_{\mathcal{O}}(t_s, t, \vec{q}) = \frac{C_{3,\mathcal{O}}(t_s, t, \vec{q})}{C_2(t_s, 0)} \sqrt{\frac{C_2(t_s - t, \vec{q})C_2(t, 0)C_2(t_s, 0)}{C_2(t_s - t, 0)C_2(t, \vec{q})C_2(t_s, \vec{q})}}$$

- Asymptotic behavior of the ratio for A_μ

$$R_{A_0}(t_s, t, \vec{q}) = \frac{q_3}{\sqrt{2E_{\vec{q}}(m + E_{\vec{q}})}} \left(G_A(q^2)\delta_{j3} - \frac{m - E_{\vec{q}}}{2m_N} G_P(q^2) \right)$$

$$R_{A_j}(t_s, t, \vec{q}) = \frac{i}{\sqrt{2E_{\vec{q}}(m + E_{\vec{q}})}} \left((m + E_{\vec{q}})G_A(q^2)\delta_{j3} - \frac{G_P(q^2)}{2m_N} q_3 q_j \right)$$

[1] C. Alexandrou et al, arXiv:0811.0724 [hep-lat]

Setup

- D200 ensemble, generated by CLS (Coordinated Lattice Simulations)
  - $N_f = 2+1$ dynamical quark flavors
  - Non-perturbatively $\mathcal{O}(a)$-improved Wilson clover fermions
  - Tree-level Symanzik-improved gauge action
  - Open boundary conditions in time direction

β	a/fm	L/a	L/fm	T/a	m_π/MeV	N_c	$N_{\vec{p}}$
3.55	0.06440	64	4.1216	128	200	1021	179

- Four different source-sink separations t_s

$16a$	$18a$	$20a$	$22a$
1.03 fm	1.15 fm	1.28 fm	1.41 fm

- Standard truncated solver method [2] [3]

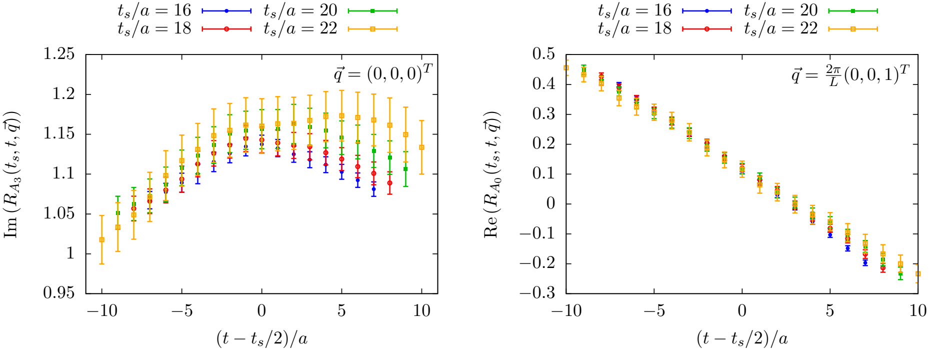
$$\langle \mathcal{O} \rangle = \left\langle \frac{1}{N_{\mathrm{LP}}} \sum_{k=1}^{N_{\mathrm{LP}}} \mathcal{O}_k^{\mathrm{LP}} \right\rangle + \langle \mathcal{O}_{\mathrm{bias}} \rangle, \quad \mathcal{O}_{\mathrm{bias}} = \frac{1}{N_{\mathrm{HP}}} \sum_{k=1}^{N_{\mathrm{HP}}} \left(\mathcal{O}_k^{\mathrm{HP}} - \mathcal{O}_k^{\mathrm{LP}} \right)$$

[2] G. Bali et al, Comp. Phys. Comm., Vol. 181 (2010)

[3] E. Shintani et al, Phys. Rev. D91, 114511 (2015)

Excited States

- Excited state contaminations on nucleon correlation functions



- See also talk by K. Ottnad (26/7 12:00 PM)

Excited States

- Perform a calculation in chiral effective theory

$$\mathcal{L}_{\text{eff}} = \frac{ig_A}{f}\bar{\psi}(x)\gamma_\mu\gamma_5\frac{\tau^a}{2}\psi(x)\partial_\mu\pi^a(x)+\ldots,$$
$$A^a_\mu(x) = -2if\partial_\mu\pi^a(x)+\ldots$$

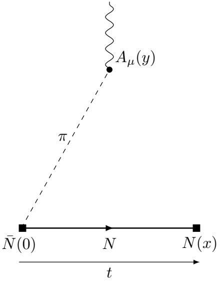
- Consider simplest case (1π-exchange)

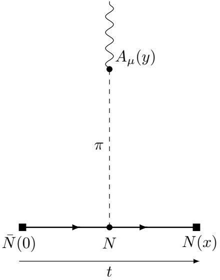
$$\int d^3x\,\Gamma_{\beta\alpha}\left\langle \mathcal{N}_{\alpha i}(x)A^3_0(y)\frac{1}{2}\left(1\pm\tau^3\right)_{ji}\bar{\mathcal{N}}_{\beta j}(0)\right\rangle_{1\pi}$$

- Position space representation of the above expression ($m_\pi \ll M_N$, $m_\pi^2 y_0/(2M_N) \ll 1$, $|\vec{y}|$ large)

$$\approx \pm 2Z_B g_A e^{-M_N x_0}(i\vec{s}\cdot\vec{\nabla}_{\vec{y}})\left[G_{m_\pi}(y)\text{-}G_{m_\pi}(x_0-y_0,\vec{y})+\frac{m_\pi^2}{2M_N}\frac{e^{-m_\pi|\vec{y}|}}{4\pi|\vec{y}|}\right]$$

- For $|\vec{y}| \gg x_0/2$: $N\pi$ or $N\pi\pi$ transition is almost unsuppressed (relative to ground state)
- Expect excited-state contamination to be suppressed for axial current **localized** around the spatial origin





Wave Packet Method

- Spatial localization of the axial current

$$C_{3,\vec{s}\cdot\vec{A}}[\psi](t_s,t)=\sum_{\vec{x},\vec{y}}\psi(\vec{y})\,\Gamma_{\beta\alpha}\langle\mathcal{N}_\alpha(t_s,\vec{x})\left(\vec{s}\cdot\vec{A}(t,\vec{y})\right)\bar{\mathcal{N}}_\beta(0)\rangle$$

- Use a **Gaussian shape** of the wave packet

$$\psi(\vec{y})=\frac{1}{L^3}\sum_{\vec{q}}e^{i\vec{q}\cdot\vec{y}}\tilde{\psi}(\vec{q}),\qquad \tilde{\psi}(\vec{q})=\exp\left(-\frac{\vec{q}^2}{2\Delta^2}\right)$$

- Consider $\vec{s}=(0,0,1)^T$ with $\vec{s}\cdot\vec{q}=0\rightarrow$ No pion pole contribution

$$\mathrm{Im}\left[C_{3,\vec{s}\cdot\vec{A}}(t_s,t,\vec{q})\right]=Z(0)Z^*(\vec{q})\left(1+\frac{m}{E_{\vec{q}}}\right)e^{-mt_s-(E_{\vec{q}}-m)t}G_A(q^2)$$

- Eliminate the t_s-dependence by dividing through the correlator $C_2(t_s,\vec{0})$

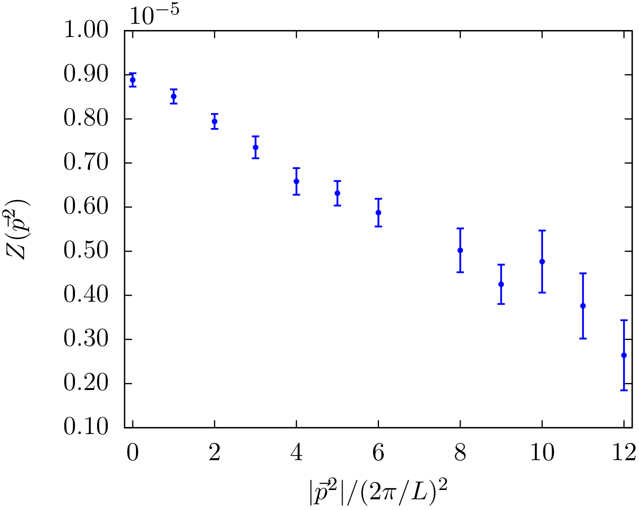
$$w(\Delta,t)=\sum_{\vec{q}}\tilde{\psi}(\vec{q})\frac{\mathrm{Im}\left[C_{3,A_3}(t_s,t,\vec{q})\right]}{C_2(t_s,\vec{0})}$$

- Use a **dipole form factor** parametrization for simplicity and perform a correlated fit of $w(\Delta,t)$ to the data

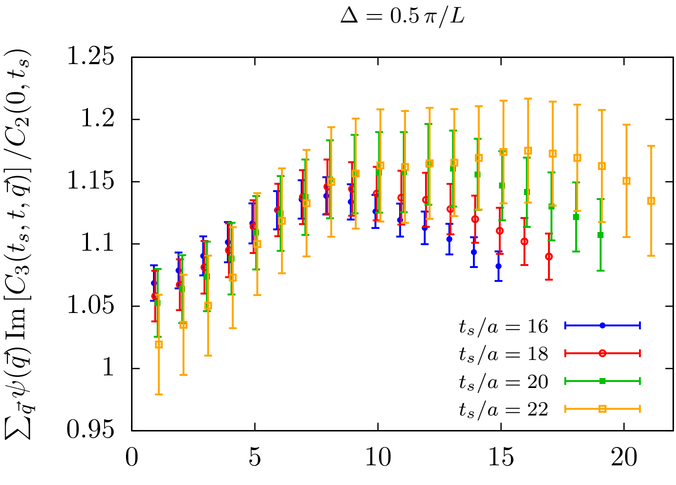
$$G_A(Q^2)=\frac{g_A}{\left(1+\frac{Q^2}{M_A^2}\right)^2}$$

Results

Perform correlated fit on 2-point correlation function: $C_2(t, \vec{p}) \propto e^{-E_{\vec{p}} t} |Z(\vec{p})|^2$

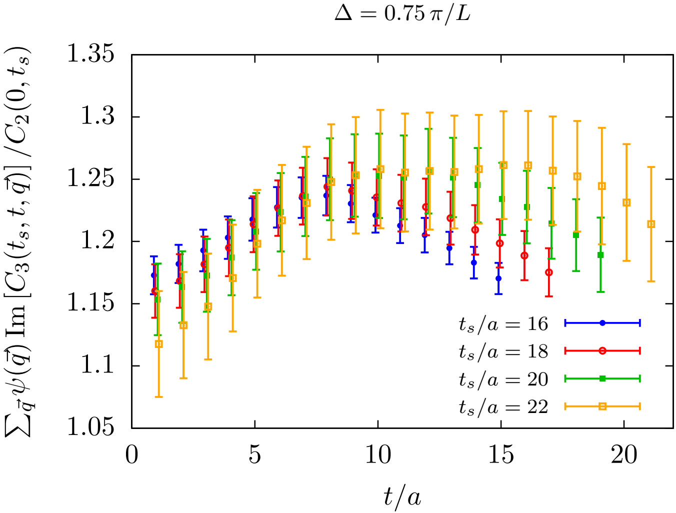


Results



Expected behavior: $w(\Delta, t) = \frac{1}{2} \sum_{\vec{q}} e^{-\vec{q}^2/(2\Delta^2)} \frac{Z(\vec{q})}{Z(\vec{0})} \left(1 + \frac{m}{E_{\vec{q}}}\right) e^{-(E_{\vec{q}}-m)t} G_A(q^2)$

Results



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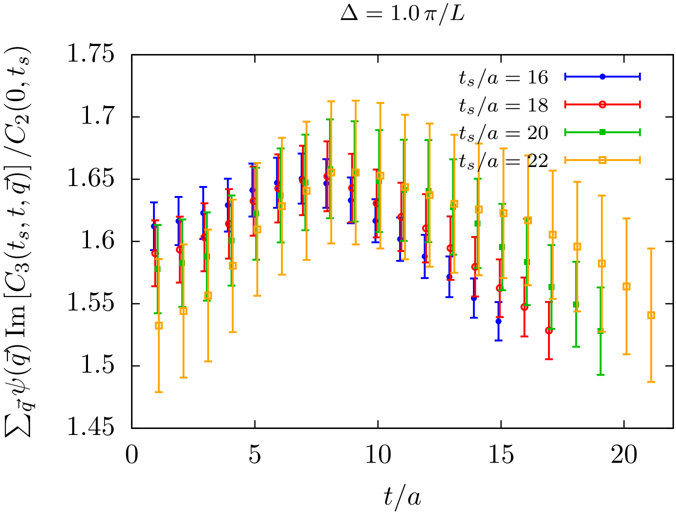

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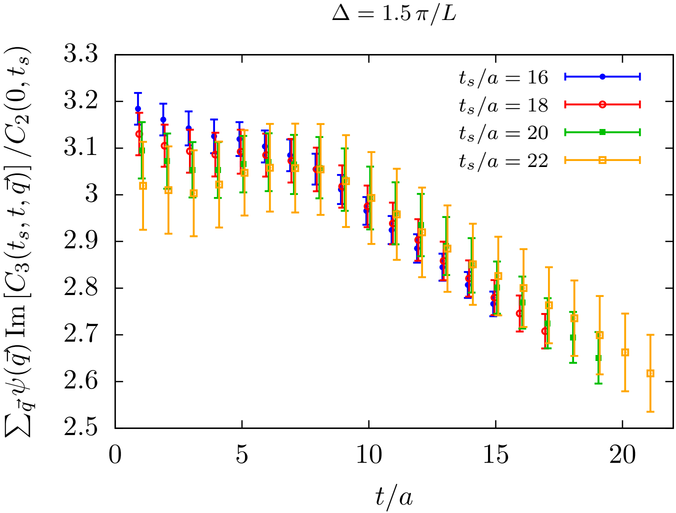

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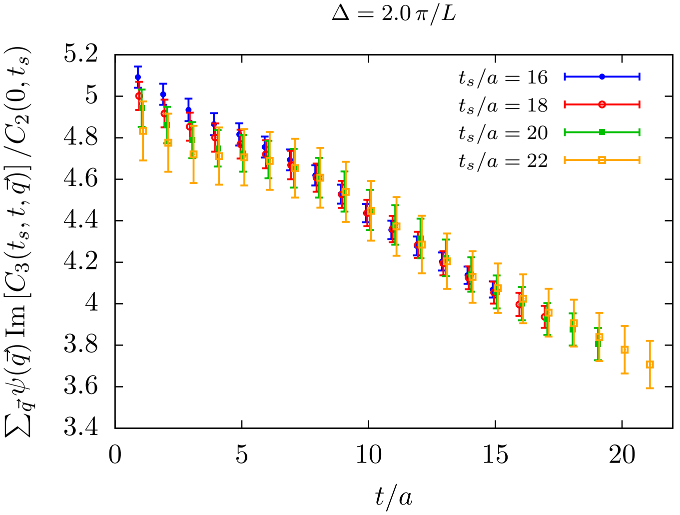

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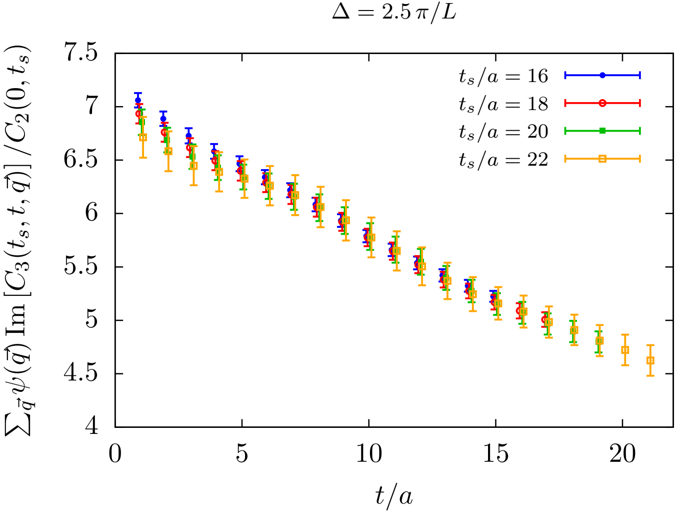

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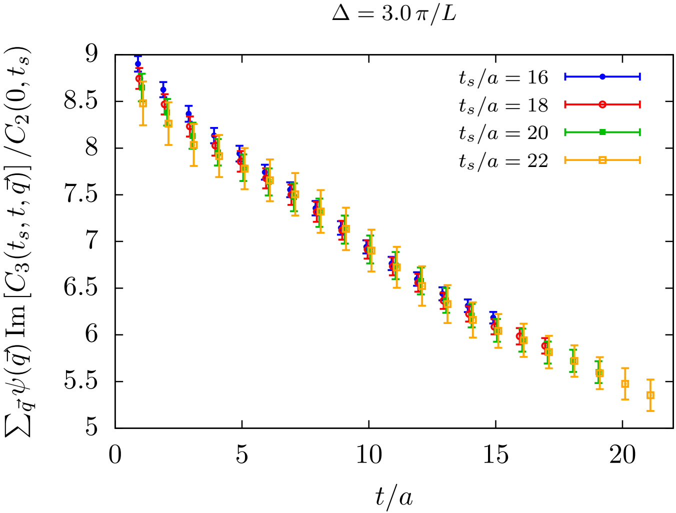

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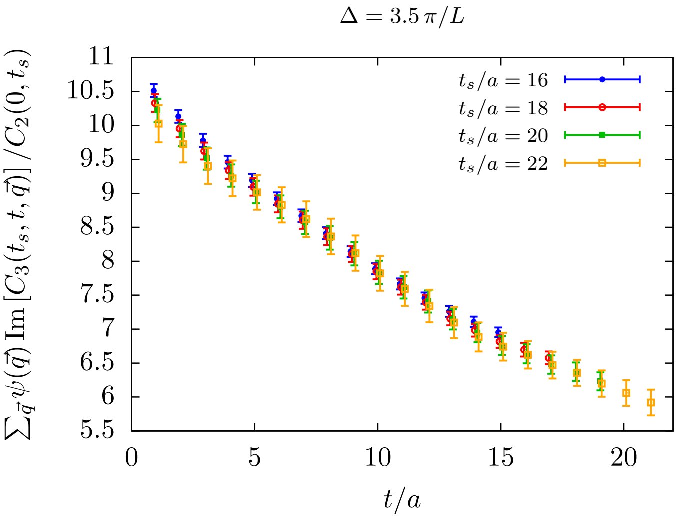

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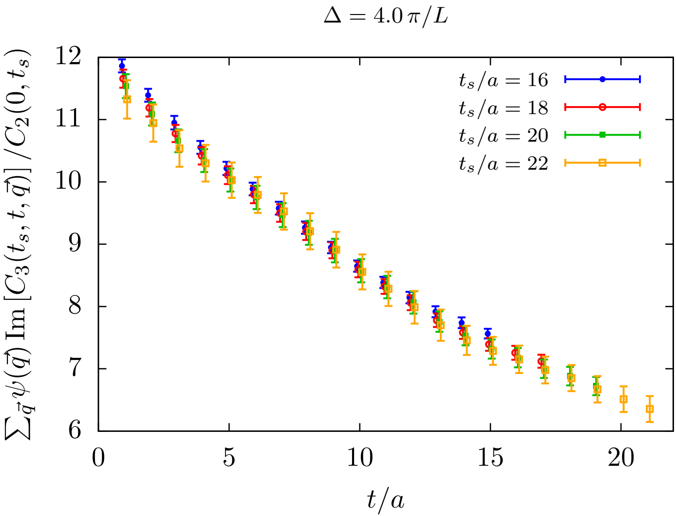

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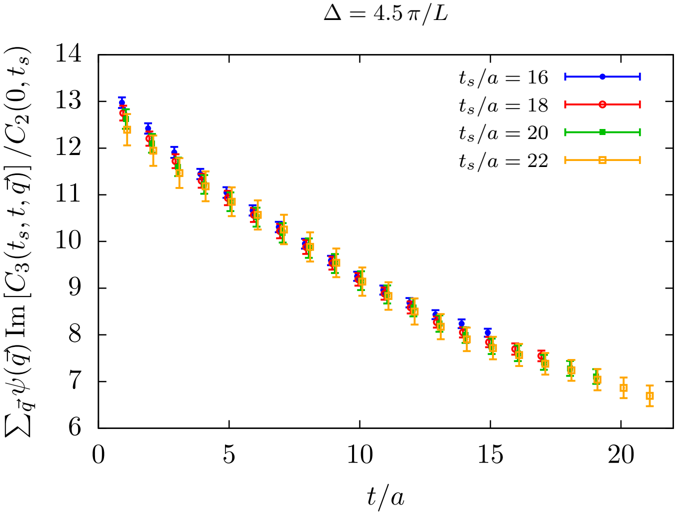

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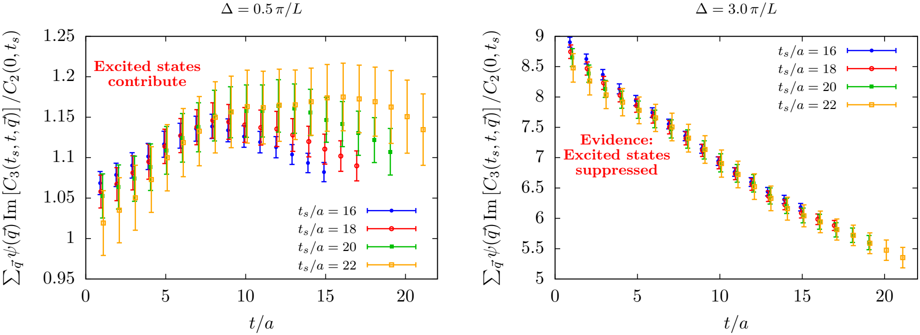
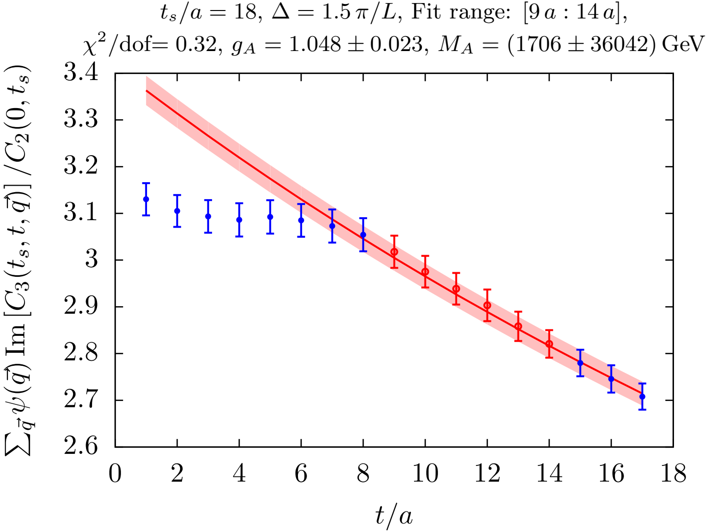
Results

- Results show t_s-independence for localized wave pakets
- **Evidence: Ground state dominates for increasing rate of localization**
- Perform a **correlated fit** on the data with the spectral representation as a model $w(\Delta, t)$

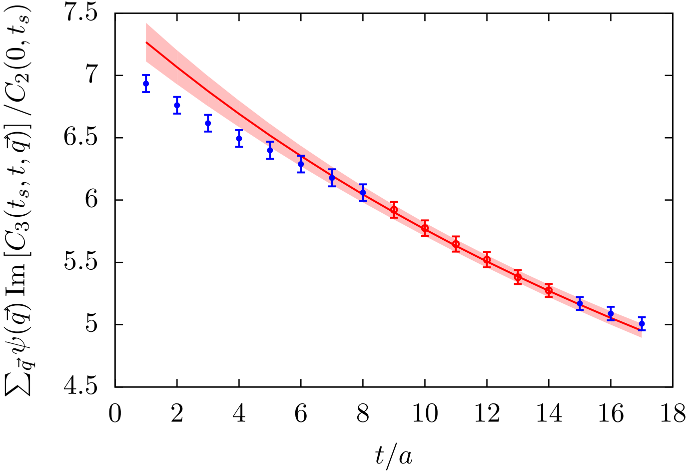


Results


$t_s/a = 18$, $\Delta = 1.5\,\pi/L$, Fit range: $[9\,a : 14\,a]$,
χ^2/dof= 0.32, $g_A = 1.048 \pm 0.023$, $M_A = (1706 \pm 36042)$ GeV
$\sum_{\vec{q}} \psi(\vec{q})\,\mathrm{Im}\,[C_3(t_s,t,\vec{q})]\,/C_2(0,t_s)$
3.4
3.3
3.2
3.1
3
2.9
2.8
2.7
2.6
0
2
4
6
8
10
12
14
16
18
t/a

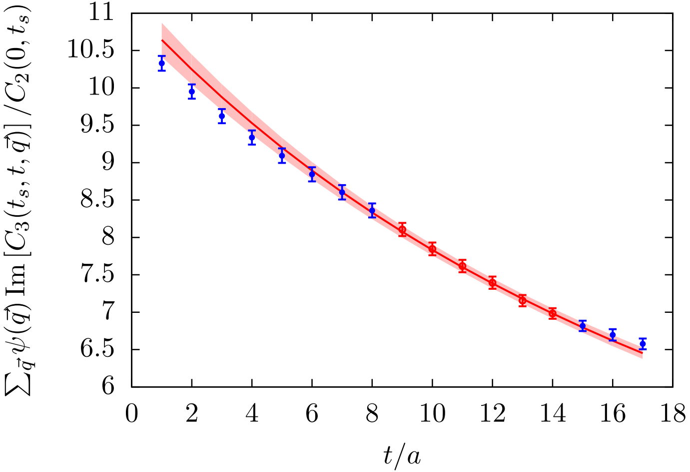
Results

$t_s/a = 18,\ \Delta = 2.5\,\pi/L$, Fit range: $[9\,a : 14\,a]$,
$\chi^2/\text{dof}= 0.73,\ g_A = 1.140 \pm 0.094,\ M_A = (1.443 \pm 0.53)$ GeV

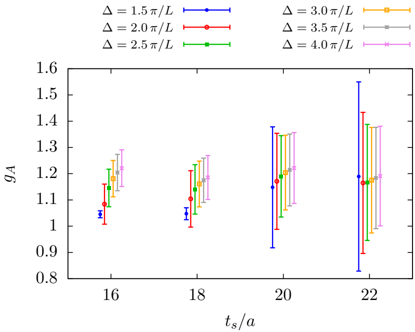


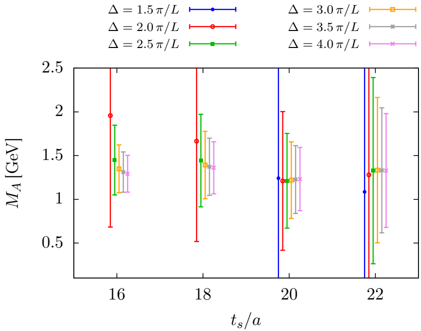
Results

$t_s/a = 18,\ \Delta = 3.5\,\pi/L$, Fit range: $[9\,a : 14\,a]$,
χ^2/dof$= 1.19,\ g_A = 1.175 \pm 0.084,\ M_A = (1.373 \pm 0.328)$ GeV



Results





g_A

$\Delta[\pi/L]/t_s[a]$	18	20
2.0	1.10 ± 0.11	1.17 ± 0.18
2.5	1.14 ± 0.09	1.19 ± 0.16
3.0	1.16 ± 0.09	1.20 ± 0.14
3.5	1.17 ± 0.08	1.21 ± 0.14
4.0	1.19 ± 0.08	1.22 ± 0.14

M_A [GeV]

$\Delta[\pi/L]/t_s[a]$	18	20
2.0	1.67 ± 1.15	1.21 ± 0.79
2.5	1.44 ± 0.53	1.21 ± 0.54
3.0	1.39 ± 0.39	1.21 ± 0.44
3.5	1.37 ± 0.33	1.23 ± 0.39
4.0	1.36 ± 0.30	1.23 ± 0.36

- Comparison: $g_A = 1.188 \pm 0.025$ (simultaneous fits, K. Ottnad, talk on 26/7 12:00 PM)

Conclusions

- Investigation of a new method to reduce excited-state contaminations on nucleon ground-state matrix elements
- There is evidence that for stronger localizations in position space excited states get suppressed (for fixed time separations)
- The ground state contribution provides a good description of the correlation function
- Reasonable results for g_A and M_A with rather large statistical errors
- Possible improvements
  - Perform simultaneous fits for different values of t_s and Δ ($\rightarrow$ decrease error)
  - Investigate wave packet profiles of "Gaussian$\times$Hermite polynomials"
  - Include excited-state contributions in the model function for the fit, such as $N\pi$, $N\pi\pi$ terms

Thank you very much for your attention!