# Lattice Calculation of Hadronic Tensor

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$\chi QCD$ collaboration

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# Hadronic tensor on the lattice

## ✦ Minkowski

$$W_{\mu\nu} = \frac{1}{4\pi}\int d^4 z e^{iq\cdot z}\left\langle P,S\left|\left[J_\mu^\dagger(z)J_\nu(0)\right]\right|P,S\right\rangle$$

**hadronic tensor is scale independent!**

$$W_{\mu\nu} = \left(-g_{\mu\nu} + \frac{q_\mu q_\nu}{q^2}\right)F_1(x,Q^2) + \frac{\hat{P}_\mu \hat{P}_\nu}{P\cdot q}F_2(x,Q^2)$$

**structure functions are frame independent!**

## ✦ Euclidean

$$C_4 = \sum_{\boldsymbol{x_f}} e^{-i\boldsymbol{p}\cdot\boldsymbol{x_f}}\sum_{\boldsymbol{x_2 x_1}} e^{-i\boldsymbol{q}\cdot(\boldsymbol{x_2}-\boldsymbol{x_1})}\left\langle \chi_N(\boldsymbol{x_f},t_f)J_\mu(\boldsymbol{x_2},t_2)J_\nu(\boldsymbol{x_1},t_1)\bar{\chi}_N(\boldsymbol{0},t_0)\right\rangle$$

$$C_2 = \sum_{\boldsymbol{x_f}} e^{-i\boldsymbol{p}\cdot\boldsymbol{x_f}}\left\langle \chi_N(\boldsymbol{x_f},t_f)\bar{\chi}_N(\boldsymbol{0},t_0)\right\rangle$$

$$\tilde{W}_{\mu\nu}(\boldsymbol{p},\boldsymbol{q},\tau) = \frac{E_p}{m_N}\frac{\mathrm{Tr}[\Gamma_e C_4]}{\mathrm{Tr}[\Gamma_e C_2]} \to \sum_{\boldsymbol{x_2 x_1}} e^{-i\boldsymbol{q}\cdot(\boldsymbol{x_2}-\boldsymbol{x_1})}\langle P,S\,|\,J_\mu(\boldsymbol{x_2},t_2)J_\nu(\boldsymbol{x_1},t_1)\,|\,P,S\rangle = \sum_n A_n e^{-(E_n-E_p)\tau}, \tau \equiv t_1 - t_2$$

K.F. Liu and S. J. Dong, PRL 72, 1790 (1994)

K.F. Liu, PRD 62, 074501 (2000)

$$W_{\mu\nu}(\boldsymbol{p},\boldsymbol{q},\nu) = \frac{1}{i}\int_{c-i\infty}^{c+i\infty} d\tau e^{\nu\tau}\tilde{W}_{\mu\nu}(\boldsymbol{p},\boldsymbol{q},\tau)$$

**formally, back to Minkowski space by inverse Laplace transform**

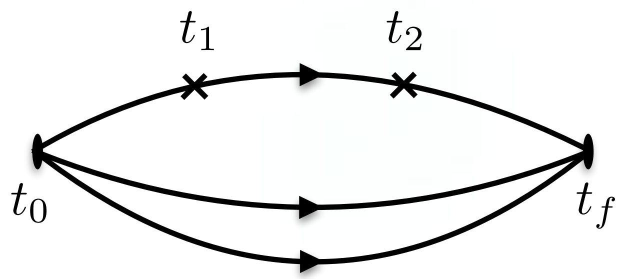
$$\tilde{W}_{\mu\nu}(\boldsymbol{p},\boldsymbol{q},\tau) = \int d\nu W_{\mu\nu}(\boldsymbol{p},\boldsymbol{q},\nu)e^{-\nu\tau}, \nu = E_n - E_p$$

**numerically, need to solve the inverse problem of the Laplace transform**

# Contractions

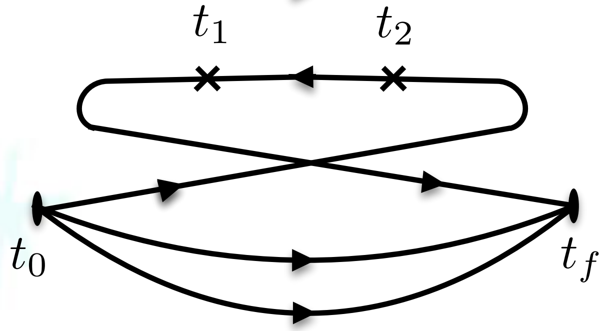
$$C_4 = \sum_{\boldsymbol{x_f}} e^{-ip\cdot \boldsymbol{x_f}} \sum_{\boldsymbol{x_2 x_1}} e^{-iq\cdot(\boldsymbol{x_2}-\boldsymbol{x_1})} \left\langle \chi_N(\boldsymbol{x_f}, t_f) J_\mu(\boldsymbol{x_2}, t_2) J_\nu(\boldsymbol{x_1}, t_1) \bar{\chi}_N(\boldsymbol{0}, t_0) \right\rangle$$

1. **valence and connected-sea parton**



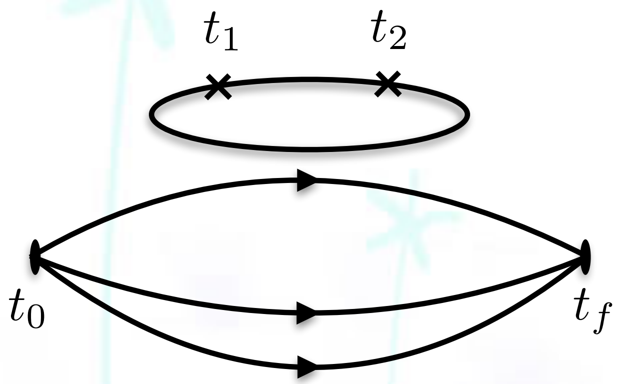
2. **connected-sea anti-parton**

**responsible for the Gottfried sum rule violation!**

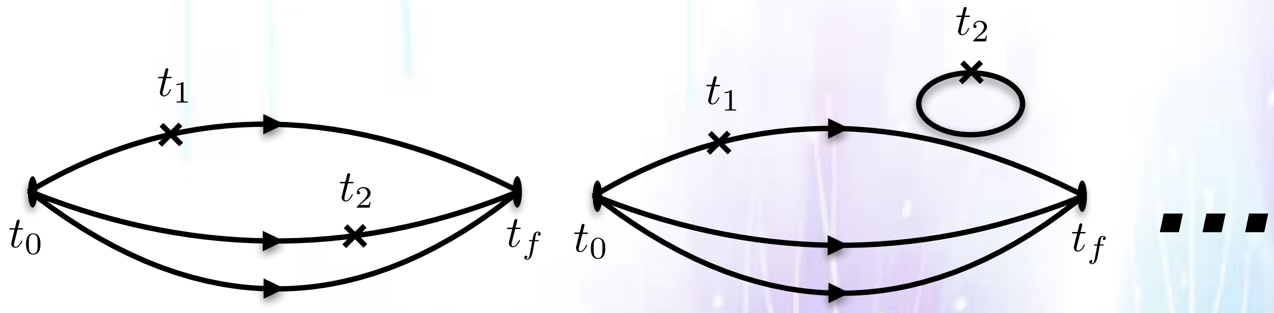


*K.F. Liu and S. J. Dong, PRL 72, 1790 (1994)*

3. **disconnected-sea parton and anti-parton**

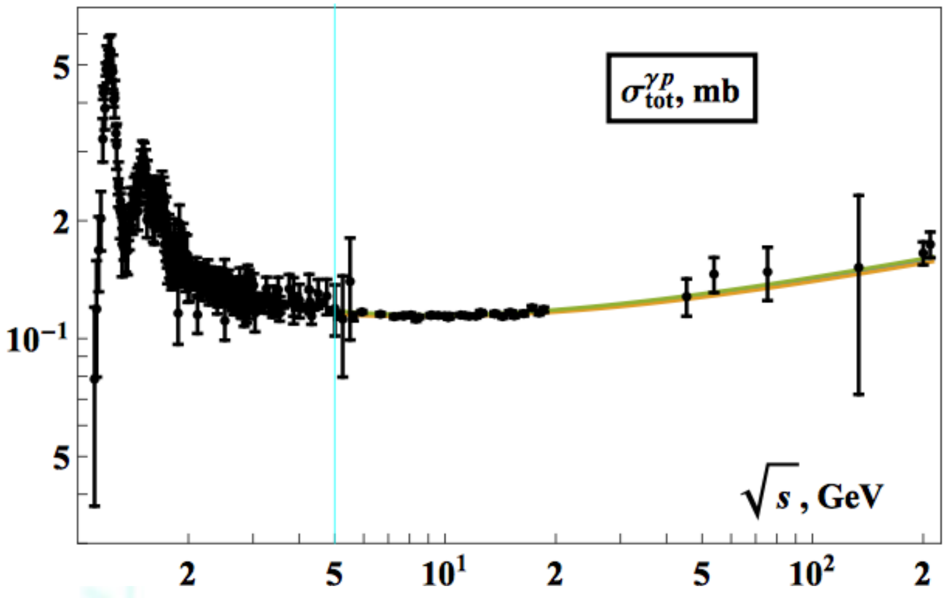


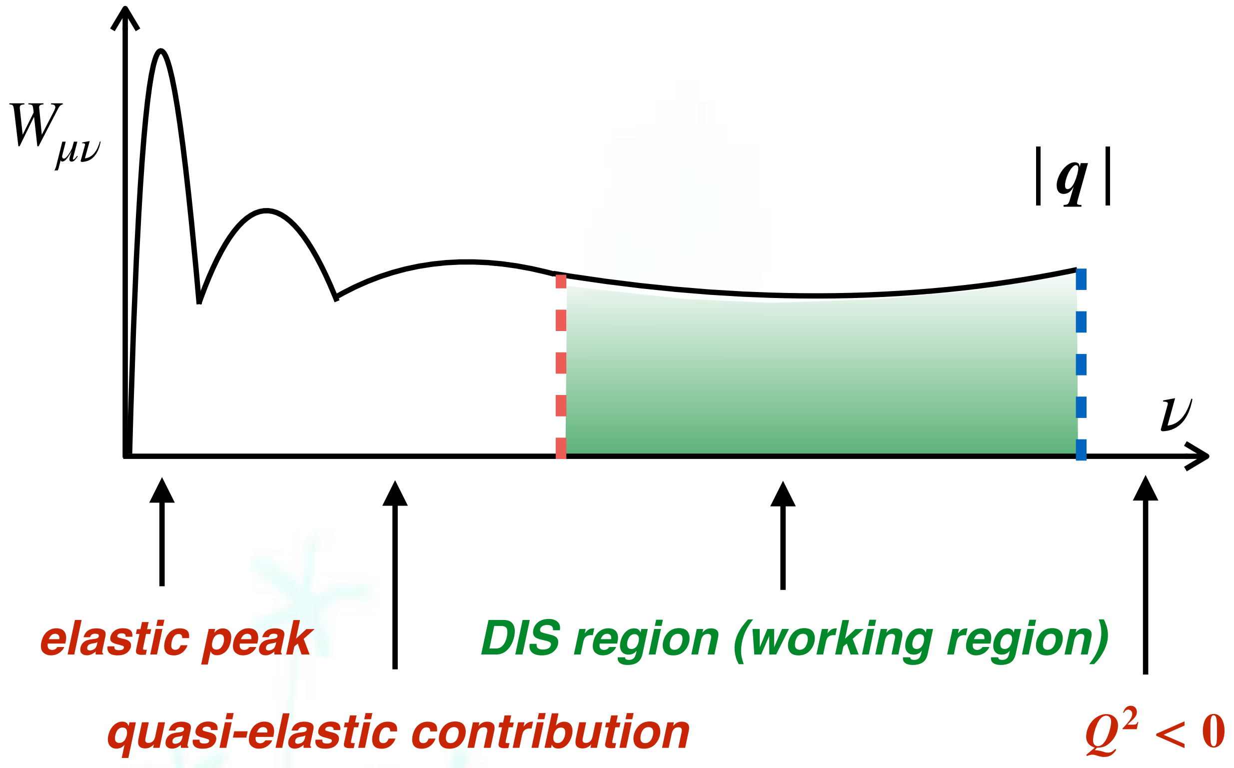
4. **pure higher-twist ones**



# Computation setups

## ✦ Sketch the hadronic tensor





***elastic peak***

***DIS region (working region)***

***quasi-elastic contribution***

$Q^2 < 0$

$\nu > (E_{n=0} - E_p) + \Delta E$ (away from the elastic peak)  $\nu < |\boldsymbol{q}|$ (physical $x$ and $Q^2$)

## ✦ Lattice setups

clover anisotropic lattice, $24^3 \times 128,\ a_t \sim 0.035\ \text{fm},\ m_\pi \sim 380\ \text{MeV},\ \dfrac{2\pi}{L} \sim 0.42\ \text{GeV}$

*H.-W. Lin et al., PRD 79, 034502 (2009)*

$\mu = \nu = 1$ and $p_1 = q_1 = 0$  $W_{11}(\nu) = F_1(x, Q^2)$

two sequential-sources for each 4-point function, 554 confs, 16 source positions

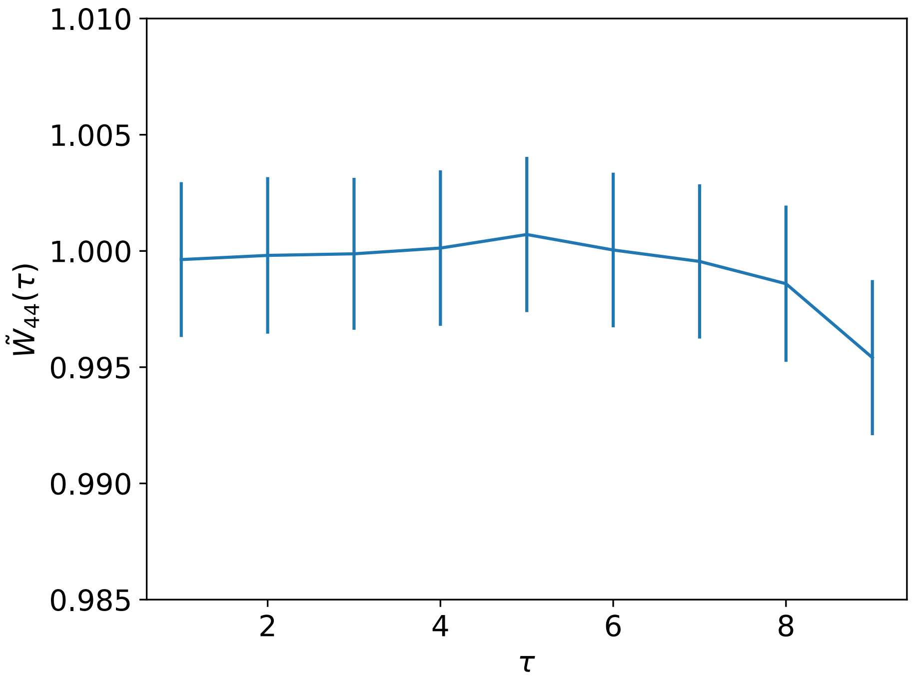
the *x*-range we can reach on this lattice is roughy [0.05, 0.3] by different kinematic setups

| $\boldsymbol{p}$ | $\boldsymbol{q}$ | $E_p$ | $E_{n=0}$ | $\lvert\boldsymbol{q}\rvert$ | $\nu$ | $Q^2$ | $x$ |
|---|---|---|---|---|---|---|---|
| (0,3,3) | (0,-6,-6) | 2.15 | 2.15 | 3.57 | [2.96, 3.68] | [4, 2] | [0.16, 0.07] |

# Check of the elastic case

normalized vector current $J_4 = \bar{\psi}\gamma_4\psi$

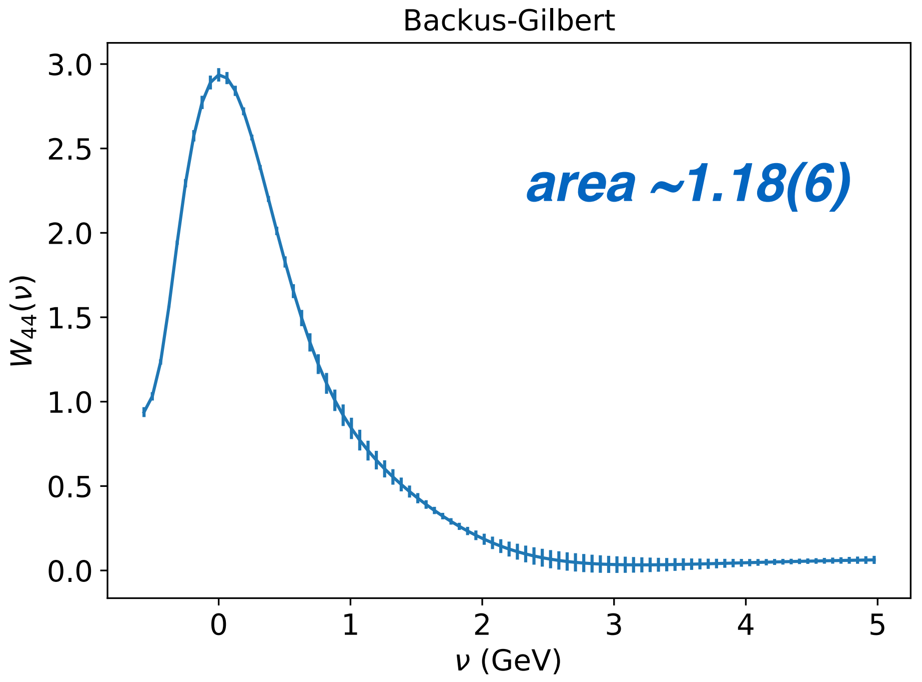
$$\tilde{W}_{44}(\boldsymbol{p}=0,\,\boldsymbol{q}=0,\,\tau) \overset{\tau\to\infty}{=} \langle N|J_4|N\rangle\langle N|J_4|N\rangle$$
$$= F_1^2(q^2=0) = g_V^2 = 1$$



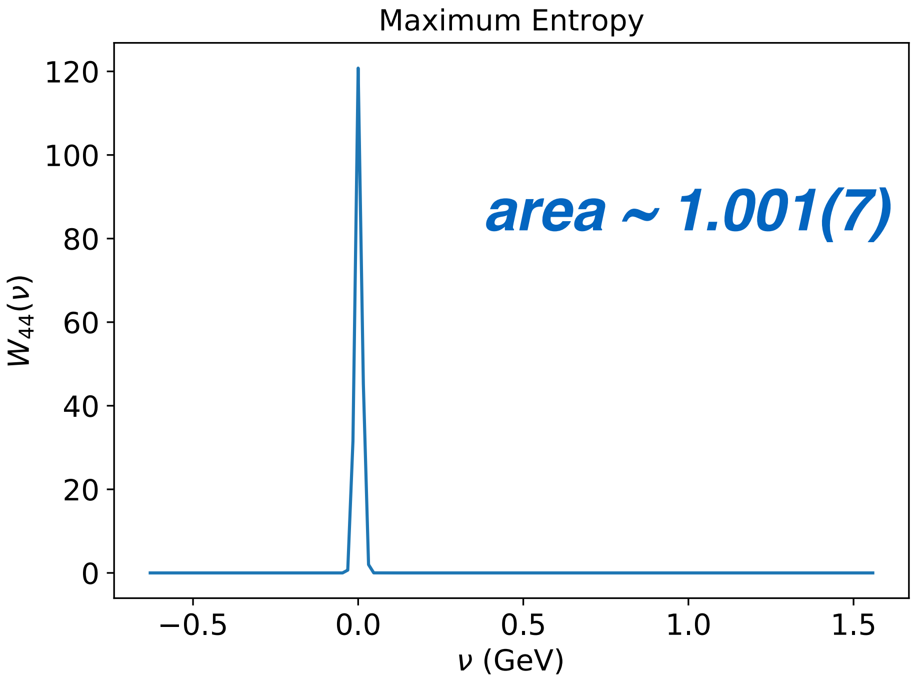
$$\tilde{W}_{\mu\nu}(\boldsymbol{p},\boldsymbol{q},\tau) = \int d\nu W_{\mu\nu}(\boldsymbol{p},\boldsymbol{q},\nu)e^{-\nu\tau}$$

**Backus-Gilbert or Maximum Entropy Method**

$$W_{44}(q^2,\,\nu) = \delta(q^2+2m_N\nu)\frac{2m_N}{1-q^2/4m_N{}^2}\left(G_E^2(q^2) - \frac{q^2}{4M_N^2}G_M^2(q^2)\right)$$
$$\overset{q^2=0}{=} \delta\nu G_E^2(q^2=0) = \delta\nu g_V^2 = \delta\nu$$



*area ~1.18(6)*



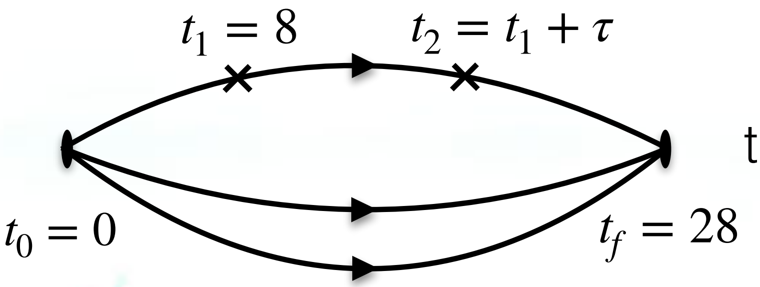
*area ~ 1.001(7)*

# Euclidean hadronic tensor

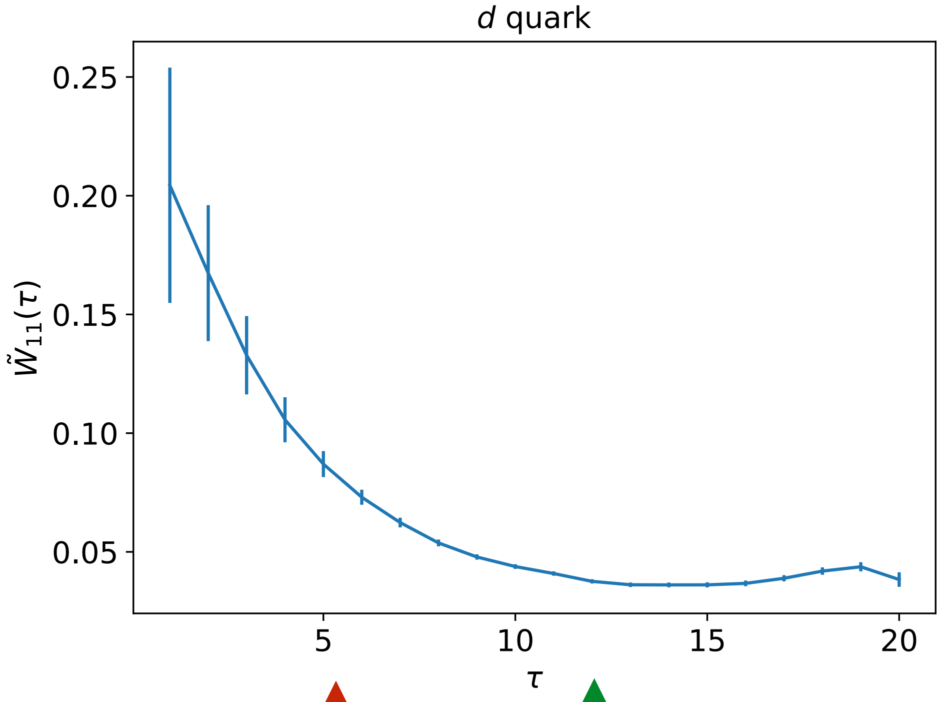
$\tilde{W}_{\mu\nu}(\boldsymbol{p},\boldsymbol{q},\tau)=\sum_n A_n e^{-(E_n-E_p)\tau}$ $\boldsymbol{p}$=(033), $\boldsymbol{q}$=(0-6-6) $\boldsymbol{p}+\boldsymbol{q}=-\boldsymbol{p}$ $E_0=(m_N^2+|\boldsymbol{p}+\boldsymbol{q}|^2)=E_p$

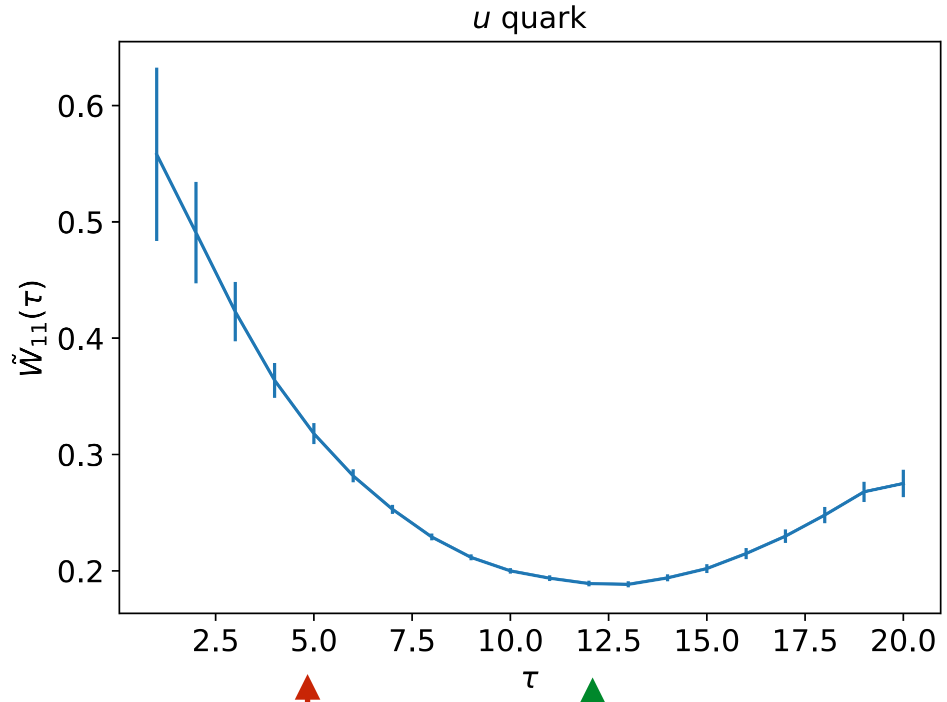
for small $\tau$, higher intermediate states contribute, exponentially decay

for large $\tau$, lowest intermediate state (elastic contribution) dominates, keep constant since $E_0=E_p$



to avoid the contact point and sink excited stats, choose $\tau\in[1,\ 12]$





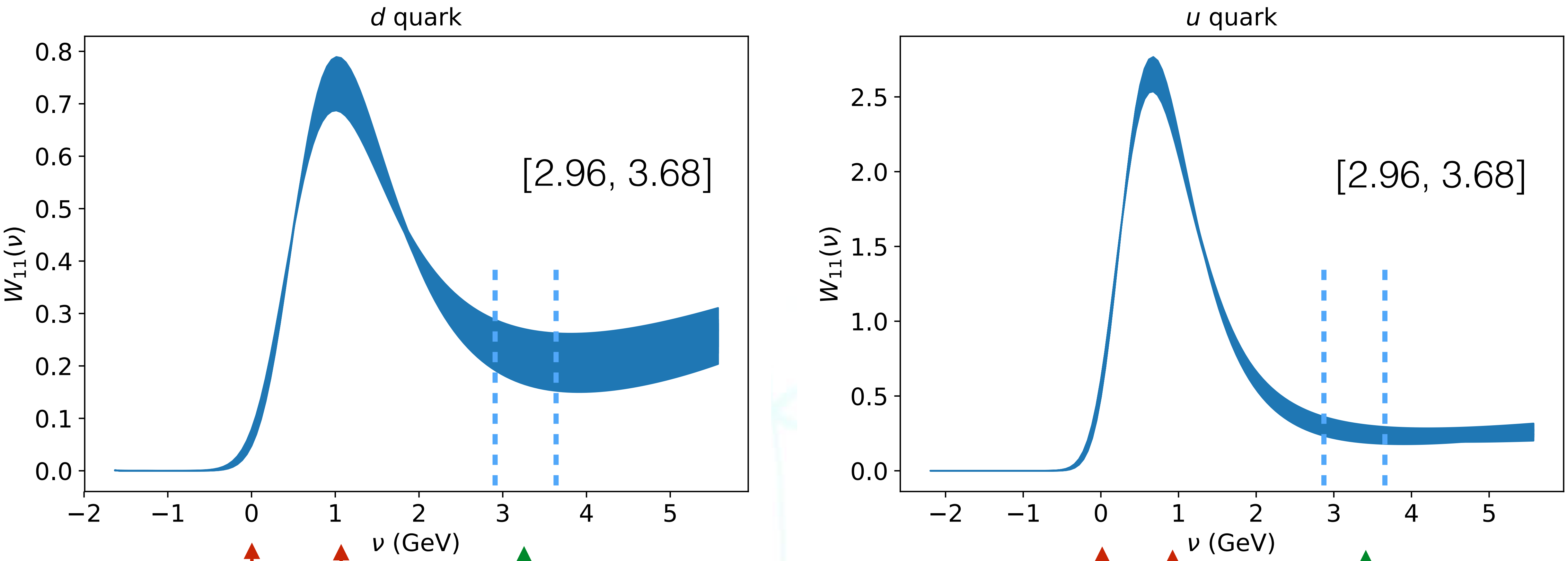
***elastic contribution (flat)***

***higher intermediate-states contribution (exponentially decay)***

***elastic contribution (flat)***

***higher intermediate-states contribution (exponentially decay)***

# Minkowski hadronic tensor (after MEM)



**elastic contribution**

**quasi-elastic contribution**

**DIS region (working region)**

**elastic contribution**

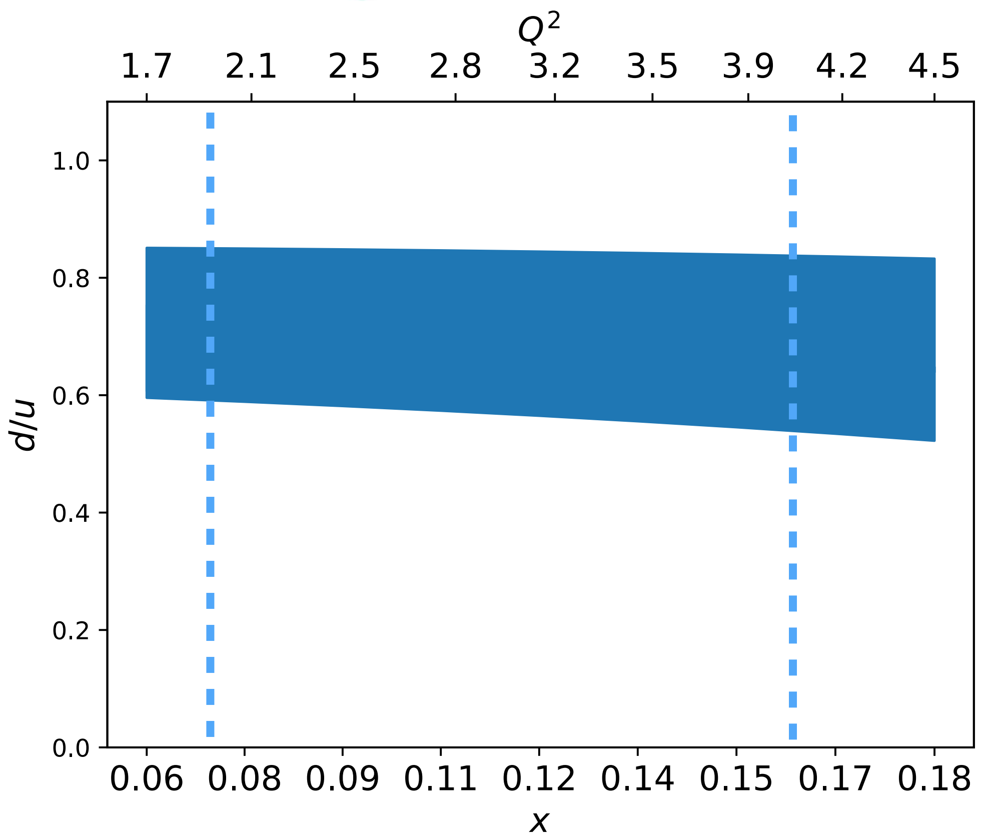
**quasi-elastic contribution**

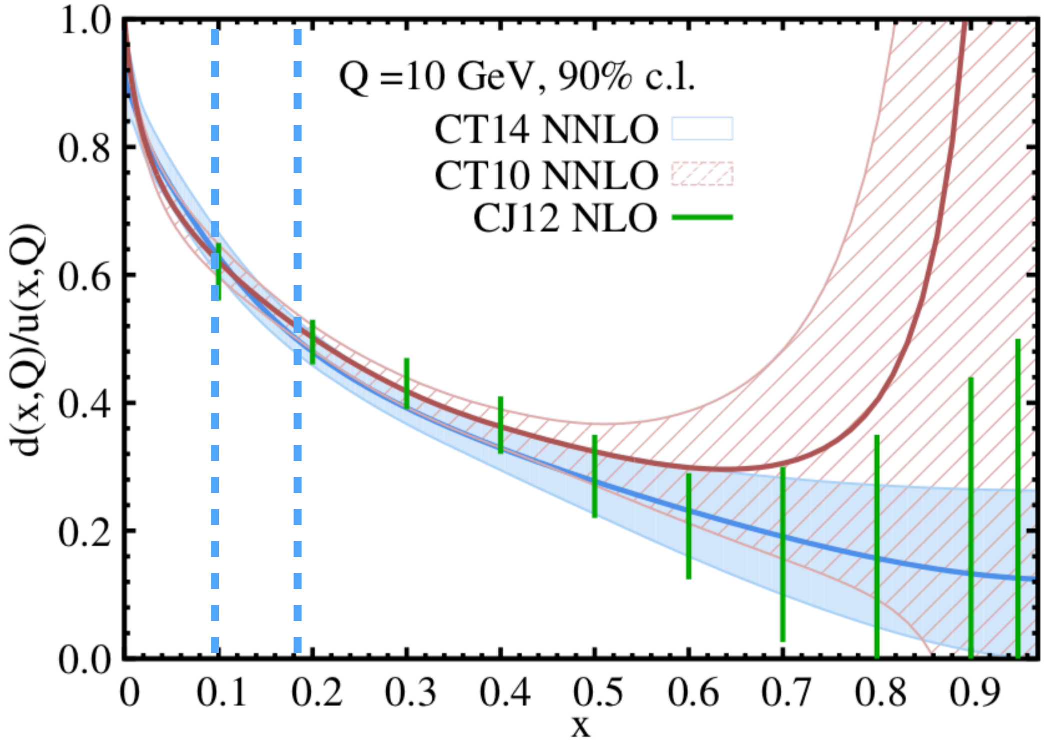
**DIS region (working region)**

- **Elastic contribution is suppressed by the large momentum transfer.** $G^2(0) \propto \dfrac{1}{\left(1+\dfrac{Q_{\rm el}^2}{\Lambda^2}\right)^4}$

  $Q^2 \sim 13\ \mathrm{GeV}^2,\ G^2(0) \sim 10^{-5}$
- **Quasi-elastic contribution is still large.**
- **MEM results are flat and relatively stable in the working region.**

# Ratio of *d* and *u*





$x \in [0.07,0.16]$ at $Q^2 = 2 \sim 4$ GeV

*S. Dulat et al., PRD 93, 033006 (2016)*

- ✦ **The systematic uncertainty of MEM is included in the blue band.**
- ✦ **Assuming a ratio of *d* and *u* will cancel partially the lattice artifacts, we determine *d/u* to be ~0.65 which is roughly consistent with the CT14 values.**

# Push to the physical limit

✦ **finite volume limit**

- **Higher Fock spaces (multi-hadron states in exclusive scatterings) are significant in the DIS region, the finite volume effect can be large.**

✦ **physical pion mass**

- **Higher pion mass will make it harder to generate multi-hadron intermediate states.**

✦ **smaller lattice spacing**

- **larger *p* and *q*, and therefore large $Q^2$ and *x* range**
- **better MEM resolution**

✦ **excited-state effects, etc.**

**They are all important and entangled with each other**
**we need to push everything to the physical limit.**

# Summary and outlook

- We are beginning to have some preliminary results from this hadronic tensor approach.
- More careful studies are needed to handle the lattice artifacts.
- Other inverse methods will be considered.
- We are working to extract the connected-sea anti-parton contribution.
- We can calculate the pure higher-twist contribution in the next stage.

***Thank you for your attention!***