

Indications for infrared conformal behaviour of SU(2) gauge theory with $N_f = 3/2$ flavours of adjoint fermions

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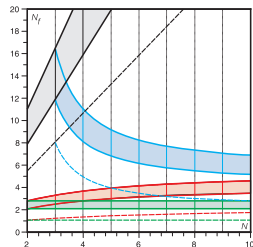
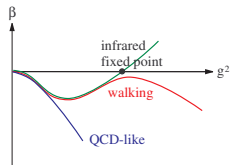
Lattice 2018



Motivation

- ▶ Exploration of the conformal window for adjoint fermions
- ▶ $N_f = 1/2$: Super YM, "QCD-like"
- ▶ $N_f = 1$: conformal? walking?
- ▶ $N_f = 2$: conformal fixed point
- ▶ Here: $N_f = 3/2$

[Bergner, Guidice, Münster, Montway,
Piemonte, PS: 1712.04692]



[Dietrich, Sannino
hep-ph/0611341]

Setup

- ▶ tree-level Symanzik-improved gauge action
- ▶ Wilson-fermions

$$D_w(x, y) = \delta_{xy} - \kappa \sum_{\mu=\pm 1}^{\pm 4} (1 - \gamma_\mu) V_\mu(x) \delta_{x+\mu, y}$$

- ▶ adjoint links

$$V_\mu(x)^{ab} = 2\text{Tr}[U_\mu^\dagger(x) T^a U_\mu(x) T^b]$$

- ▶ 3 flavours of Majorana fermions with 3 levels of Stout smearing

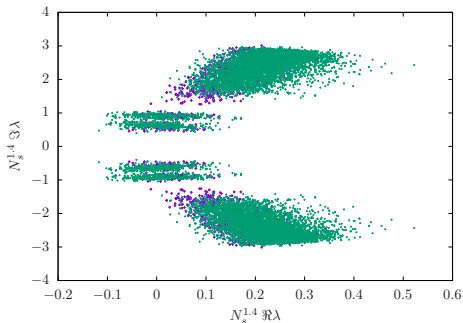
$$\bar{\psi} = \psi^T C$$

- ▶ possible sign problem

$$\int [d\psi] e^{-\bar{\psi} D_w \psi} = \text{Pf}(C D_w)^3 = \pm (\text{Det} D_w)^{3/2}$$

Simulation

- ▶ two-step Polynomial HMC algorithm
- ▶ ensembles for $\beta = 1.5$ and 1.7 with several values for κ/am_{PCAC}
- ▶ sign problem can be cured by reweighting, however not necessary



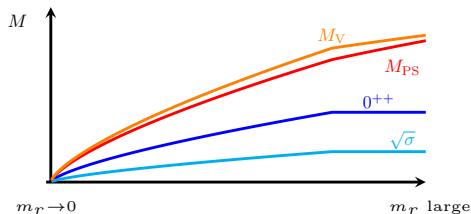
Particle Spectrum and Scaling

- ▶ mesons: m_{PS}, m_S, m_V, m_{PV}
- ▶ glueballs, e.g. 0^{++}
- ▶ spin-1/2 mixed fermion-gluon state

$$\sum_{\mu,\nu} \sigma_{\mu\nu} \text{Tr}[F_{\mu\nu}\psi]$$

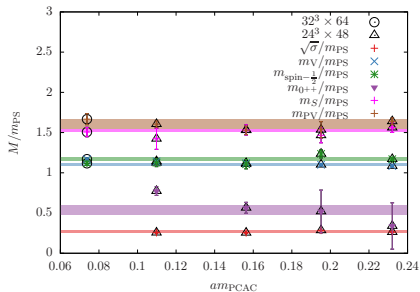
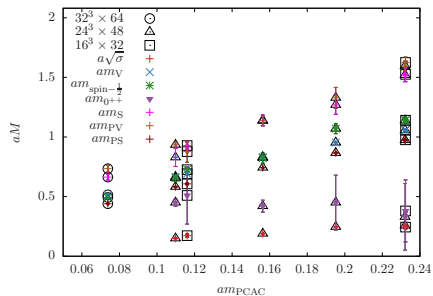
Expected behaviour of a (near) conformal theory:

- ▶ no light Goldstone boson
- ▶ scaling law: $M \propto m_r^{1/1+\gamma^*}$
- ▶ constant mass ratios



Particle Spectrum and Scaling

$$\beta = 1.5$$

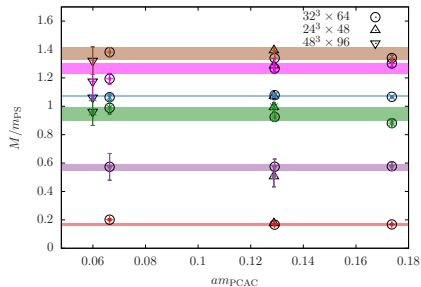
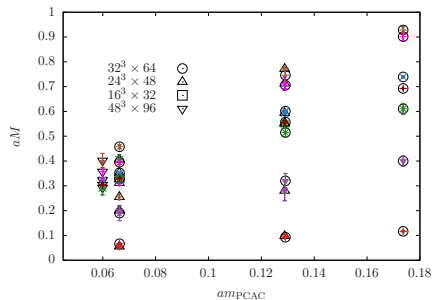


[1712.04692]

mass anomalous dimension: $\gamma^* = 0.499(12)$

Particle Spectrum and Scaling

$$\beta = 1.7$$



[1712.04692]

mass anomalous dimension: $\gamma^* = 0.33(13)$

Mode Number

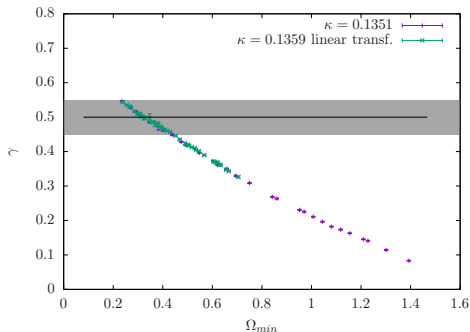
alternative method to determine the mass anomalous dimension:

$$\nu(\Omega) = \nu_0 + a_1(\Omega^2 - a_2^2)^{2/(1+\gamma^*)}$$

- ▶ determine $\gamma(\Omega)$ by a fit of ν in some interval $[\Omega_{\min}, \Omega_{\max}]$
- ▶ $\Omega \rightarrow \infty$: $\gamma(\Omega) \rightarrow 0$, gaussian fixed point value
- ▶ $\Omega \rightarrow 0$: scaling violations from finite size and finite m_{PCAC}
- ▶ carefully choose some intermediate $[\Omega_{\min}, \Omega_{\max}]$ for fitting
- ▶ conformal: slow change in $\gamma(\Omega) \rightarrow$ plateau at γ^*

Mode Number

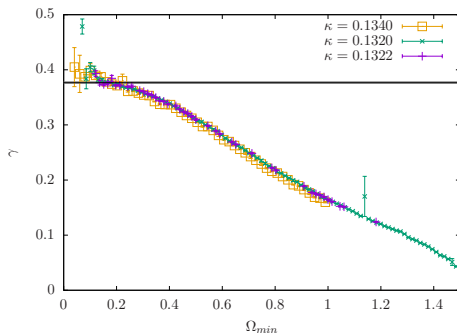
$\beta = 1.5$ with $\kappa = 0.1351$ and $\kappa = 0.1359$



- ▶ no plateau visible
- ▶ grey band: best χ^2 for both ensembles
- ▶ $\gamma^* \approx 0.50(5)$

Mode Number

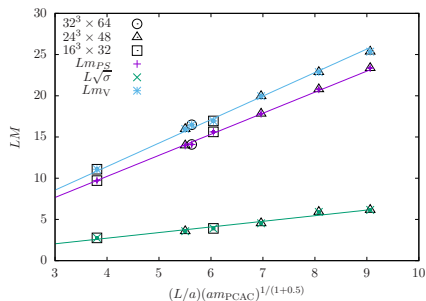
$\beta = 1.7$ with $\kappa = 0.13540$, $\kappa = 0.1320$ and $\kappa = 0.1322$



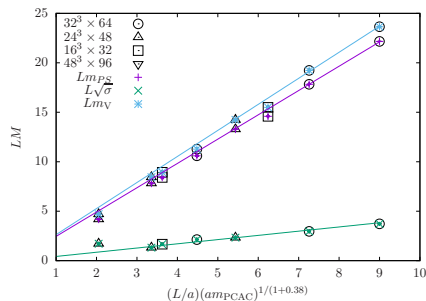
- ▶ plateau forming in the infrared
- ▶ $\gamma^* \approx 0.38(2)$

Crosscheck

$$\beta = 1.5$$



$$\beta = 1.7$$



[1712.04692]

- ▶ γ^* from the mode number together with the hyperscaling of the mass spectrum

Running Coupling

[Bergner, Piemonte: 1709.074510]

- ▶ Landau gauge:

$$\alpha(p^2) = \alpha(\mu)Z(p^2)J^2(p^2)$$

MiniMOM scheme:

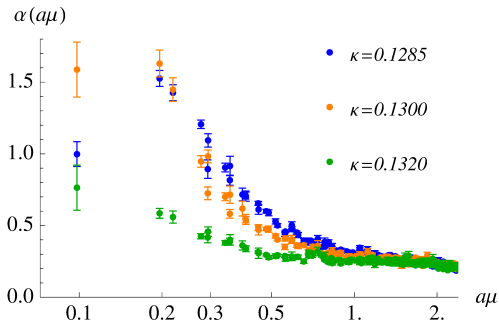
[von Smekal, Maltman, Sternbeck: 0903.1696]

- ▶ gluon dressing: $Z(\mu) = 1$
- ▶ ghost dressing: $J(\mu) = 1$

$$\alpha(p^2) = \frac{g_{\text{lat}}^2}{4\pi} Z(p^2)J^2(p^2)$$

Running Coupling

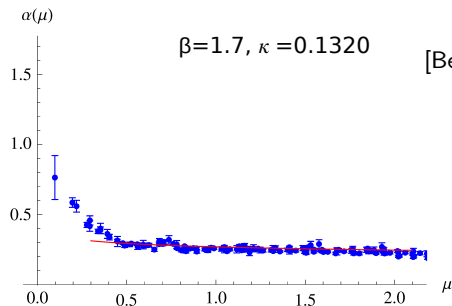
- ▶ determine $Z(p^2)$, $J(p^2)$ from gluon-, ghost-propagators in Landau gauge



$$\beta = 1.7, N_s = 32, N_t = 64$$

- ▶ κ is relevant parameter \rightarrow significant running for low p^2
- ▶ largest κ : α running slowly over a large momentum range

Running Coupling



$$a\Lambda(\beta = 1.5) = 0.0051(8),$$

$$a\Lambda(\beta = 1.6) = 0.0055(4),$$

$$a\Lambda(\beta = 1.7) = 0.0054(5).$$

- ▶ determine Λ^{MiniMOM} parameter by fitting to perturbation theory
- ▶ unlike QCD: $a\Lambda \ll aM \approx 0.3$
- ▶ Λ almost independent of β_{lat} \rightarrow slowly running β -function

Summary & Conclusion

- ▶ bound state spectrum indicates infrared conformality
- ▶ mass anomalous dimension obtained from mode number of the Dirac operator

$$\gamma^*(\beta = 1.5) \approx 0.5, \quad \gamma^*(\beta = 1.7) \approx 0.38$$

- ▶ cross check of γ^* with hyperscaling of the mass spectrum also points towards conformal behaviour
- ▶ running coupling in MiniMOM scheme: scaling violations from finite quark mass.
- ▶ determined Λ parameter
- ▶ slowly running coupling also indicates theory is close to or inside conformal window

Ensembles

	β	N_s	κ	am_{pcac}
A	1.5	16	0.137	0.02270(18)
B	1.5	16	0.135	0.11604(44)
C	1.5	16	0.132	0.23236(83)
D	1.5	24	0.1351	0.10986(12)
E	1.5	24	0.134	0.15632(15)
F	1.5	24	0.133	0.19515(20)
G	1.5	24	0.132	0.23207(22)
H	1.5	32	0.1359	0.07380(07)
J	1.7	16	0.130	0.12890(77)
K	1.7	24	0.133	0.03360(30)
L	1.7	24	0.132	0.06628(08)
M	1.7	24	0.130	0.12882(15)
N	1.7	32	0.132	0.06635(12)
O	1.7	32	0.130	0.12910(04)
P	1.7	32	0.1285	0.17366(04)
Q	1.7	48	0.1322	0.05990(05)
R1	1.7	24	0.134	-0.00097(22)
R3	1.7	32	0.134	-0.00052(11)