QCD at non-zero density and phenomenology

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LATTICE 18

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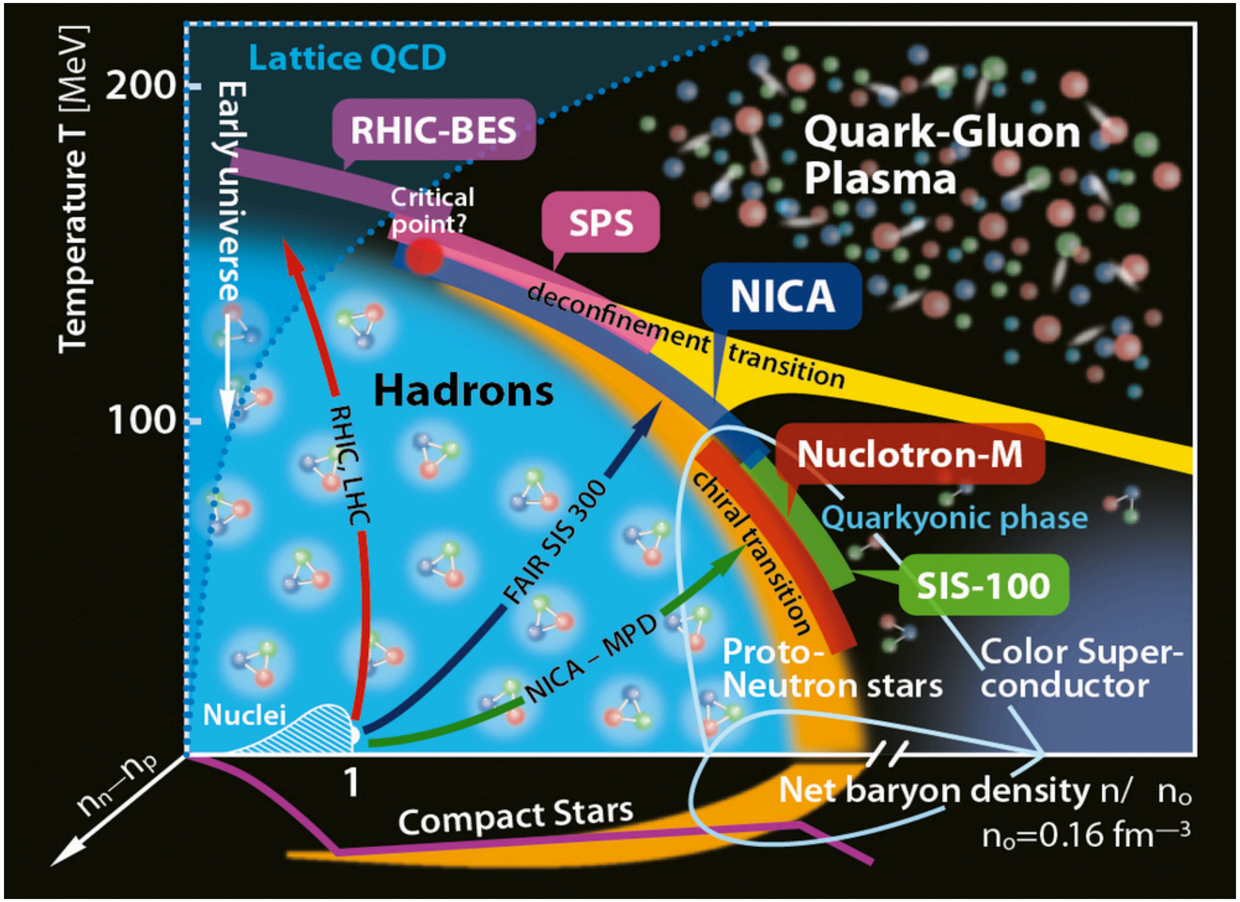
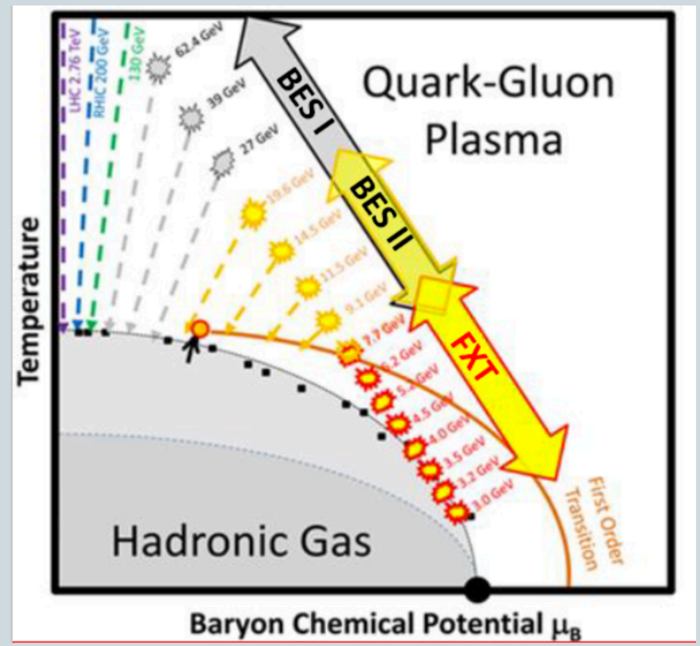
- Is there a critical point in the QCD phase diagram?
- What are the degrees of freedom in the vicinity of the phase transition?
- Where is the transition line at high density?
- What are the phases of QCD at high density?
- Are we creating a thermal medium in experiments?

Open Questions



Second Beam Energy Scan (BESII) at RHIC

- Planned for 2019-2020
- 24 weeks of runs each year
- Beam Energies have been chosen to keep the μ_B step ~50 MeV
- Chemical potentials of interest: μ_B/T~1.5…4



$\sqrt{s}$ (GeV)	19.6	14.5	11.5	9.1	7.7	6.2	5.2	4.5
μ_B (MeV)	205	260	315	370	420	487	541	589
# Events	400M	300M	230M	160M	100M	100M	100M	100M

Collider　　Fixed Target

Comparison of the facilities

Compilation by D. Cebra

Facilty	RHIC BESII	SPS	NICA	SIS-100 SIS-300	J-PARC HI
Exp.:	STAR +FXT	NA61	MPD + BM@N	CBM	JHITS
Start:	2019-20 2018	2009	2020 2017	2022	2025
Energy: $\sqrt{s_{NN}}$ (GeV)	7.7– 19.6 2.5-7.7	4.9-17.3	2.7 - 11 2.0-3.5	2.7-8.2	2.0-6.2
Rate: At 8 GeV	100 HZ 2000 Hz	100 HZ	<10 kHz	<10 MHZ	100 MHZ
Physics:	CP&OD	CP&OD	OD&DHM	OD&DHM	OD&DHM
	Collider Fixed target	Fixed target Lighter ion collisions	Collider Fixed target	Fixed target	Fixed target

CP=Critical Point   OD= Onset of Deconfinement   DHM=Dense Hadronic Matter

How can lattice QCD support the experiments?

- Equation of state
  - Needed for hydrodynamic description of the QGP
- QCD phase diagram
  - Transition line at finite density
  - Constraints on the location of the critical point
- Fluctuations of conserved charges
  - Can be simulated on the lattice and measured in experiments
  - Can give information on the evolution of heavy-ion collisions
  - Can give information on the critical point

Hadron Resonance Gas model

Dashen, Ma, Bernstein; Prakash, Venugopalan; Karsch, Tawfik, Redlich

- Interacting hadronic matter in the ground state can be well approximated by a non-interacting resonance gas
- The pressure can be written as:

$$p^{HRG}/T^4 = \frac{1}{VT^3} \sum_{i \in \, mesons} \ln \mathcal{Z}^M_{m_i}(T,V,\mu_{X^a}) + \frac{1}{VT^3} \sum_{i \in \, baryons} \ln \mathcal{Z}^B_{m_i}(T,V,\mu_{X^a})$$

where

$$\ln \mathcal{Z}^{M/B}_{m_i} = \mp \frac{V d_i}{2\pi^2} \int_0^\infty dk k^2 \ln(1 \mp z_i e^{-\varepsilon_i/T}) \quad ,$$

with energies $\varepsilon_i = \sqrt{k^2 + m_i^2}$, degeneracy factors d_i and fugacities

$$z_i = \exp\left((\sum_a X^a_i \mu_{X^a})/T \right) \; .$$

X^a: all possible conserved charges, including the baryon number B, electric charge Q, strangeness S.

- Fugacity expansion for μ_S=μ_Q=0: $\frac{p_B}{T^4} = \sum_{i \in B} \frac{d_i}{\pi^2} \left(\frac{m_i}{T}\right)^2 \sum_{N=1}^{\infty} (-1)^{N+1} N^{-2} K_2(N \frac{m_i}{T}) \cosh\left[N \frac{\mu_B}{T}\right]$

Boltzmann approximation: N=1

QCD Equation of State at finite density

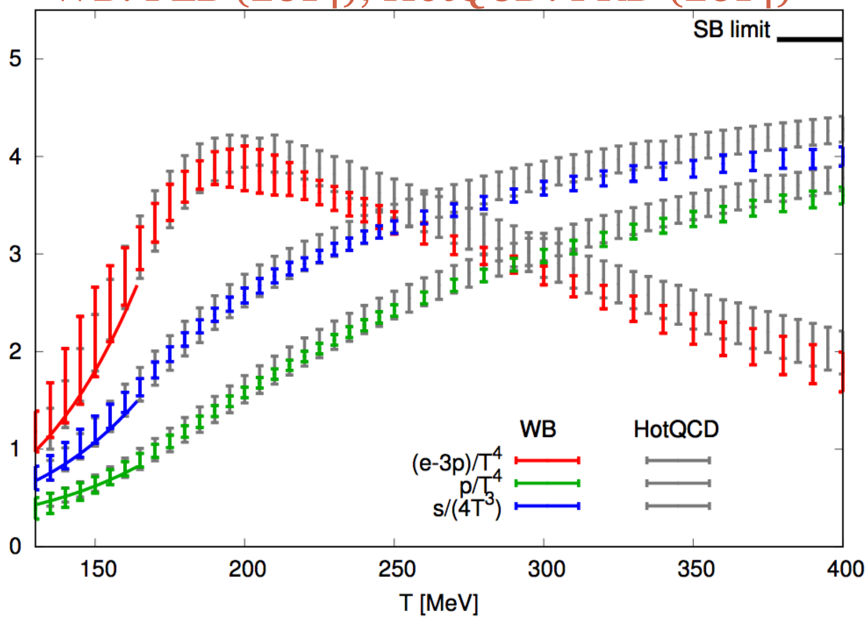
TAYLOR EXPANSION

ANALYTICAL CONTINUATION FROM IMAGINARY CHEMICAL POTENTIAL

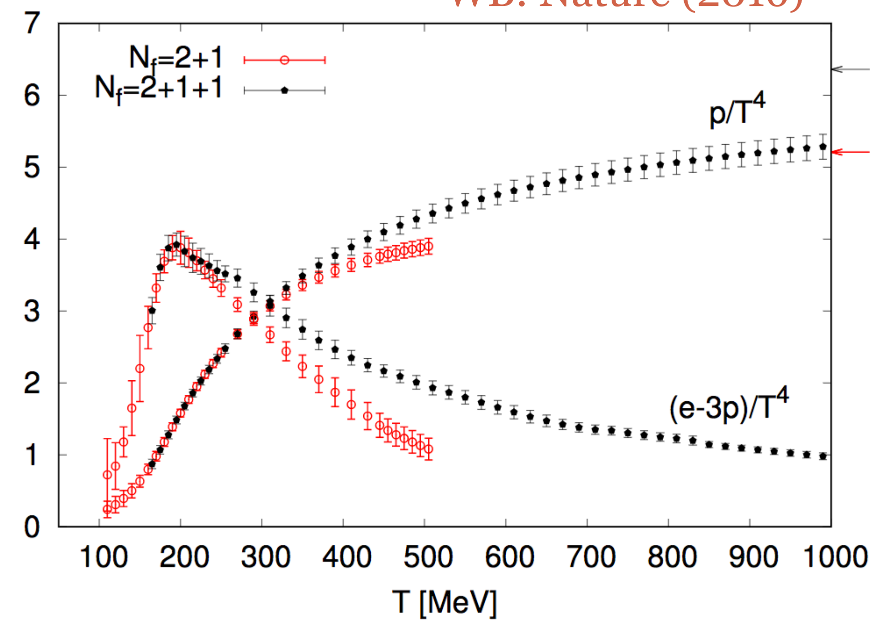
ALTERNATIVE EQUATIONS OF STATE AT LARGE DENSITIES

QCD EoS at μ_B=0

WB: PLB (2014); HotQCD: PRD (2014)



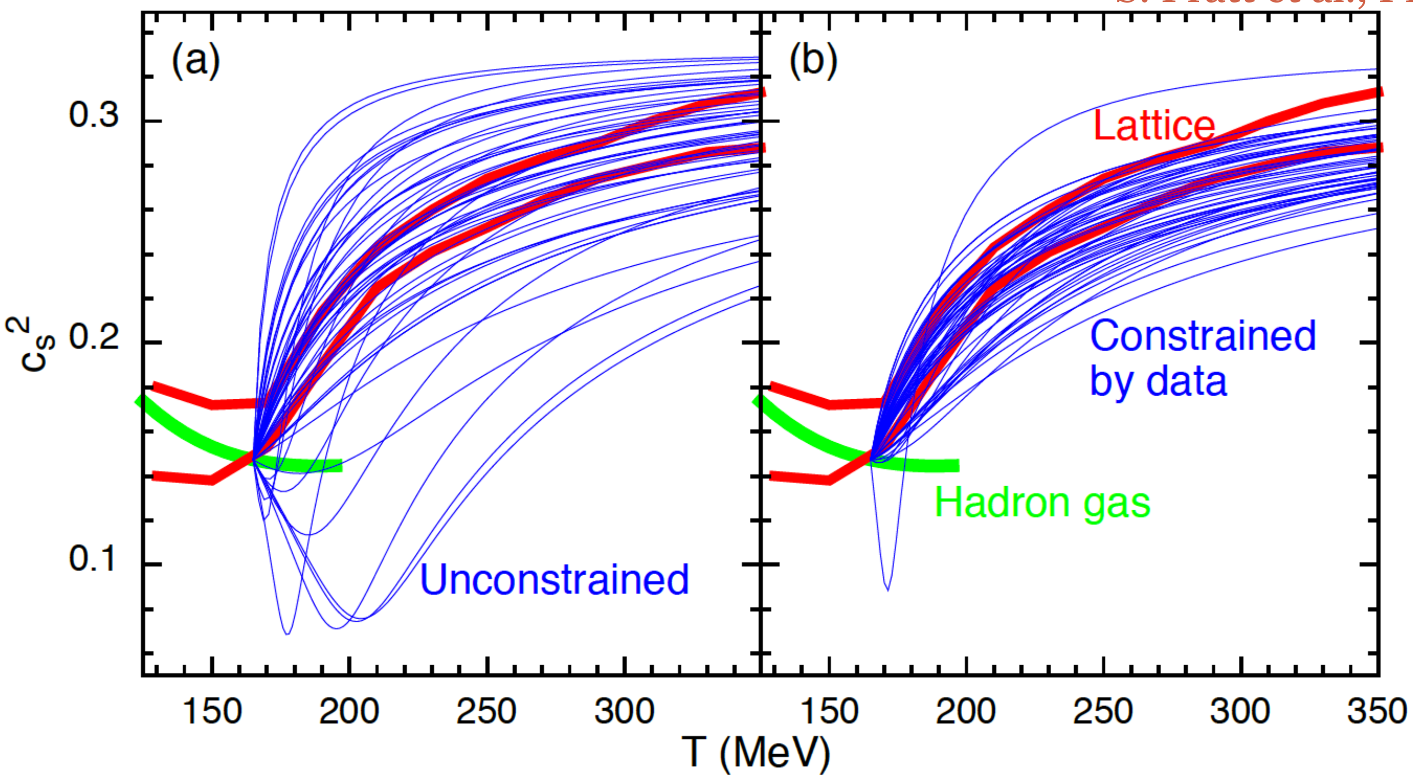
WB: Nature (2016)



- EoS for N_f=2+1 known in the continuum limit since 2013
- Good agreement with the HRG model at low temperature
- Charm quark relevant degree of freedom already at T~250 MeV

Constraints on the EoS from the experiments

S. Pratt et al., PRL (2015)



- Comparison of data from RHIC and LHC to theoretical models through Bayesian analysis
- The posterior distribution of EoS is consistent with the lattice QCD one

Taylor expansion of EoS

- Taylor expansion of the pressure:

$$\frac{p(T,\mu_B)}{T^4} = \frac{p(T,0)}{T^4} + \sum_{n=1}^{\infty} \frac{1}{(2n)!} \frac{\mathrm{d}^{2\mathrm{n}}(p/T^4)}{d(\frac{\mu_B}{T})^{2n}}\Bigg|_{\mu_B=0} \left(\frac{\mu_B}{T}\right)^{2n} = \sum_{n=0}^{\infty} c_{2n}(T) \left(\frac{\mu_B}{T}\right)^{2n}$$

- Two ways of extracting the Taylor expansion coefficients:
  - Direct simulation
  - Simulations at imaginary μ_B

- Two physics choices:
  - $\mu_B \neq 0$, $\mu_S = \mu_Q = 0$
  - μ_S and μ_Q are functions of T and μ_B to match the experimental constraints:

$\langle n_S \rangle = 0$ $\qquad\qquad$ $\langle n_Q \rangle = 0.4 \langle n_B \rangle$

Pressure coefficients: direct simulation

<u>Direct simulation</u>:

- Calculate derivatives of *lnZ*, where *Z* in the staggered formulation is given by:

$$Z = \int \mathcal{D}U \; e^{-S_g} (\det M_1)^{1/4} (\det M_2)^{1/4} (\det M_3)^{1/4} = \int \mathcal{D}U \; e^{-S_{\rm eff}}$$

where M_i is the fermionic determinant of flavor *i* and *Sg* the gauge action

- The derivatives with respect to the chemical potential of flavor *i* are

$$\begin{aligned}
A_j &= \tfrac{d}{d\mu_j}(\det M_j)^{1/4} = \tilde{\rm tr} M_j^{-1} M_j' \,, \\
B_j &= \tfrac{d^2}{(d\mu_j)^2}(\det M_j)^{1/4} = \tilde{\rm tr}\left(M_j'' M_j^{-1} - M_j' M_j^{-1} M_j' M_j {-} 1\right) \,, \\
C_j &= \tfrac{d^3}{(d\mu_j)^3}(\det M_j)^{1/4} = \tilde{\rm tr}\left(M_j' M_j^{-1} - 3 M_j'' M_j^{-1} M_j' M_j^{-1} \right.\\
&\qquad \left. + 2 M_j' M_j^{-1} M_j' M_j^{-1} M_j' M_j^{-1}\right) \,, \\
D_j &= \tfrac{d^4}{(d\mu_j)^4} \log(\det M_j)^{1/4} = \tilde{\rm tr}\left(M_j'' M_j^{-1} - 4 M_j' M_j^{-1} M_j' M_j^{-1} - 3 M_j'' M_j^{-1} M_j'' M_j^{-1} \right.\\
&\qquad + 12 M_j'' M_j^{-1} M_j' M_j^{-1} M_j' M_j^{-1} \\
&\qquad \left. - 6 M_j' M_j^{-1} M_j' M_j^{-1} M_j' M_j^{-1} M_j' M_j^{-1}\right) \,,
\end{aligned}$$

From which:

$$\begin{aligned}
\partial_i^4 \log Z &= \left\langle A_i^4 \right\rangle - 3\left\langle A_i^2 \right\rangle^2 + 3\left(\left\langle B_i^2\right\rangle - \langle B_i\rangle^2\right) \\
&\quad + 6\left(\left\langle A_i^2 B_i\right\rangle - \left\langle A_i^2\right\rangle \langle B_i\rangle\right) + 4\langle A_i C_i\rangle + \langle D_i\rangle
\end{aligned}$$

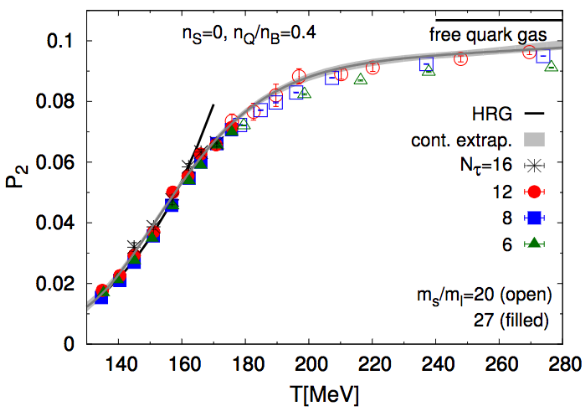
and so on…

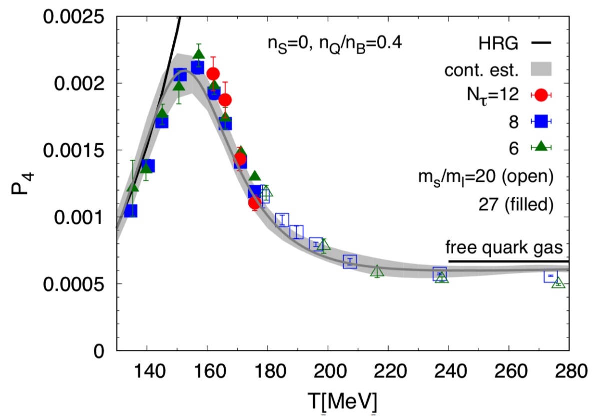
Pressure coefficients

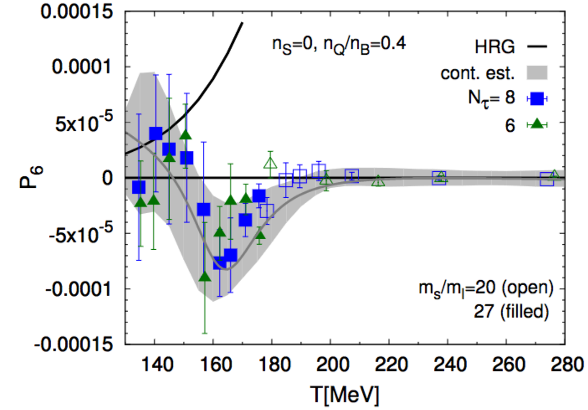
Direct simulation:

O(10^5) configurations (hotQCD: PRD (2017) and update 06/2018)

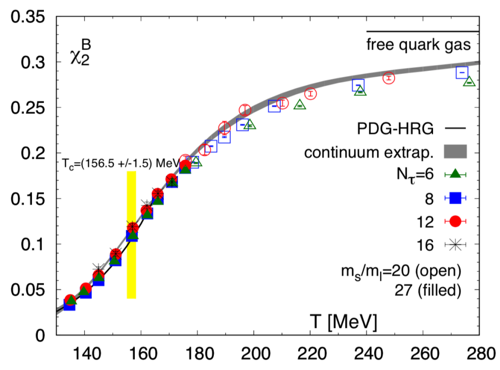
Strangeness neutrality

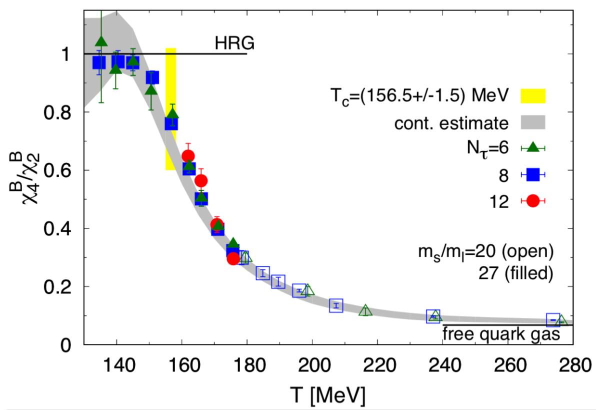


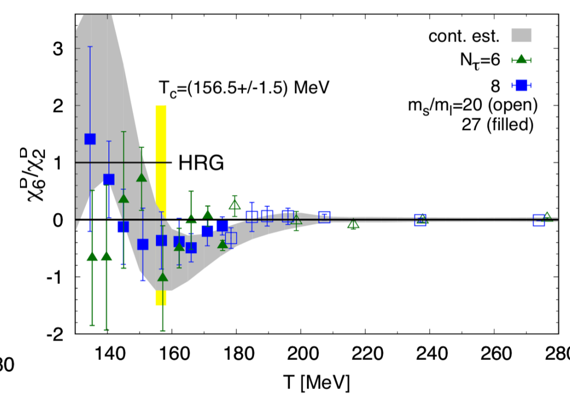




$\mu_S=\mu_Q=0$

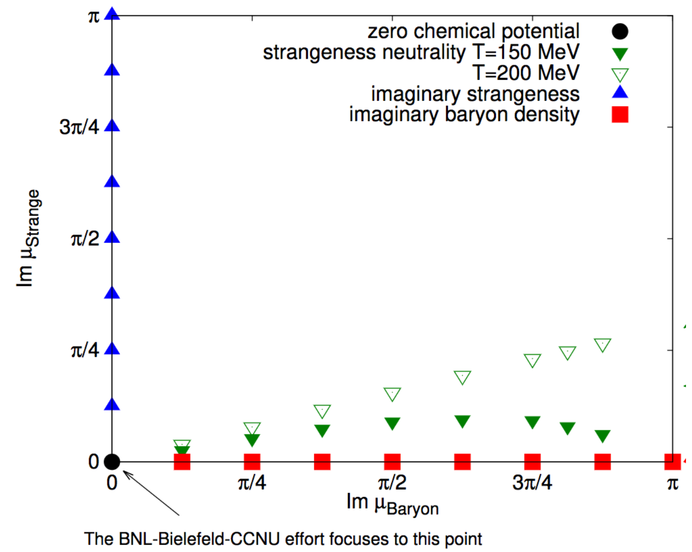






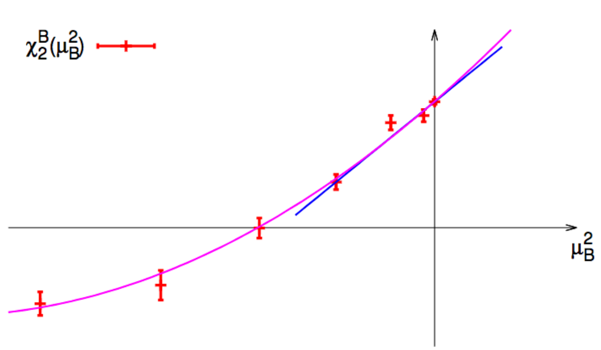
Pressure coefficients: simulations at imaginary μ_B

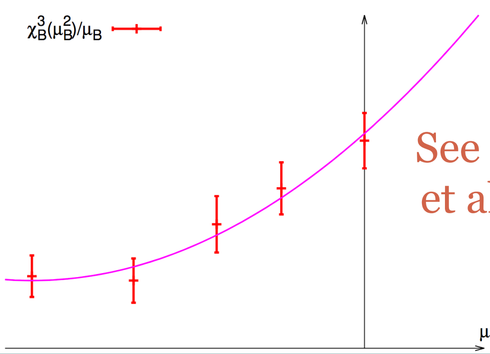
<u>Simulations at imaginary μ_B</u>:



Common technique: [de Forcrand, Philipsen (2002)], [D'Elia and Lombardo, (2002)], [Bonati et al., (2015), (2018)], [Cea et al., (2015)]

Strategy: simulate lower-order fluctuations and use them in a combined, correlated fit





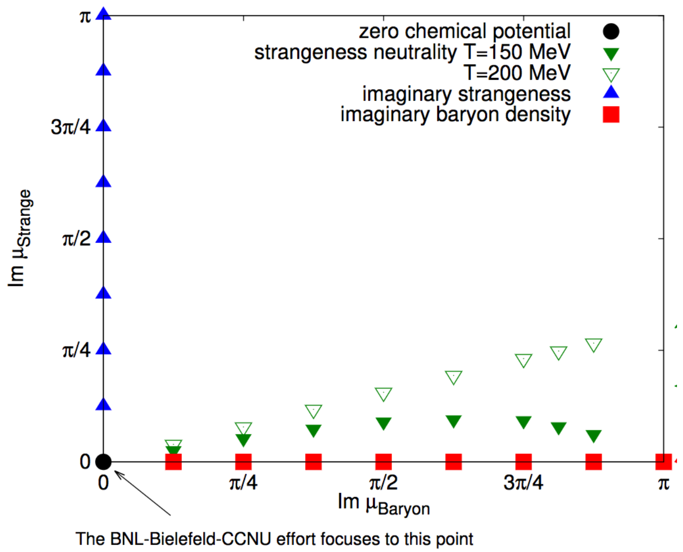
See also M. D'Elia et al., PRD (2017)

$$\chi_2^B(\mu_B^2) \approx \chi_2^B(0) + \frac{1}{2}\mu_B^2\chi_4^B(0) + \frac{1}{24}\mu_B^4\chi_6^B(0) + \dots$$

$$\frac{\chi_3^B(\mu_B^2)}{\mu_B} \approx \chi_4^B(0) + \frac{1}{6}\mu_B^2\chi_6^B(0) + \frac{1}{120}\mu_B^4\chi_8^B(0)$$

Pressure coefficients: simulations at imaginary μ_B

Simulations at imaginary μ_B:



Common technique: [de Forcrand, Philipsen (2002)], [D'Elia and Lombardo, (2002)], [Bonati et al., (2015), (2018)], [Cea et al., (2015)]

Strategy: simulate lower-order fluctuations and use them in a combined, correlated fit

$$\chi_1^B(\hat{\mu}_B) = 2c_2\hat{\mu}_B + 4c_4\hat{\mu}_B^3 + 6c_6\hat{\mu}_B^5 + \frac{4!}{7!}c_4\epsilon_1\hat{\mu}_B^7 + \frac{4!}{9!}c_4\epsilon_2\hat{\mu}_B^9$$

$$\chi_2^B(\hat{\mu}_B) = 2c_2 + 12c_4\hat{\mu}_B^2 + 30c_6\hat{\mu}_B^4 + \frac{4!}{6!}c_4\epsilon_1\hat{\mu}_B^6 + \frac{4!}{8!}c_4\epsilon_2\hat{\mu}_B^8$$

$$\chi_3^B(\hat{\mu}_B) = 24c_4\hat{\mu}_B + 120c_6\hat{\mu}_B^3 + \frac{4!}{5!}c_4\epsilon_1\hat{\mu}_B^5 + \frac{4!}{7!}c_4\epsilon_2\hat{\mu}_B^7$$

$$\chi_4^B(\hat{\mu}_B) = 24c_4 + 360c_6\hat{\mu}_B^2 + c_4\epsilon_1\hat{\mu}_B^4 + \frac{4!}{6!}c_4\epsilon_2\hat{\mu}_B^6.$$

See also M. D'Elia et al., PRD (2017)

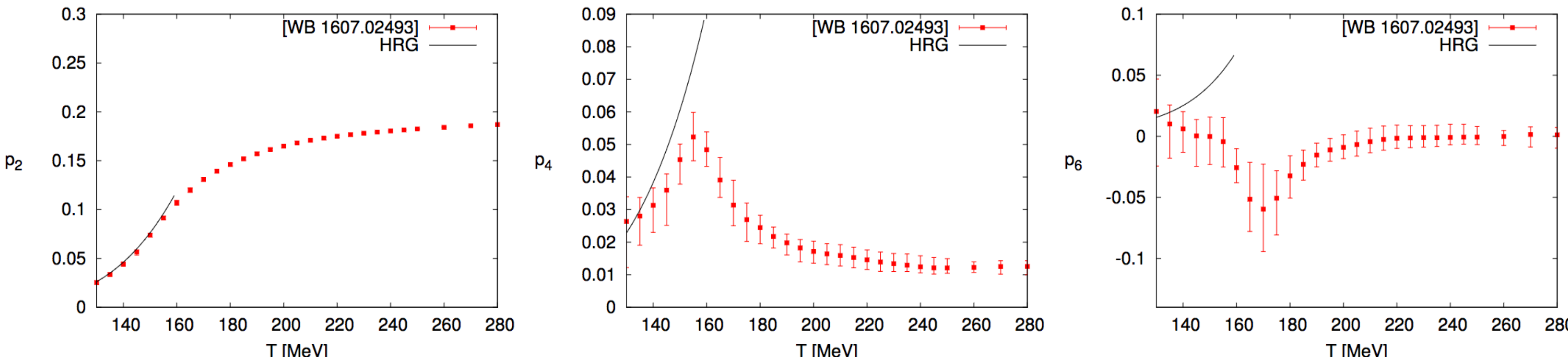
Pressure coefficients

See talk by Jana Guenther on Wednesday

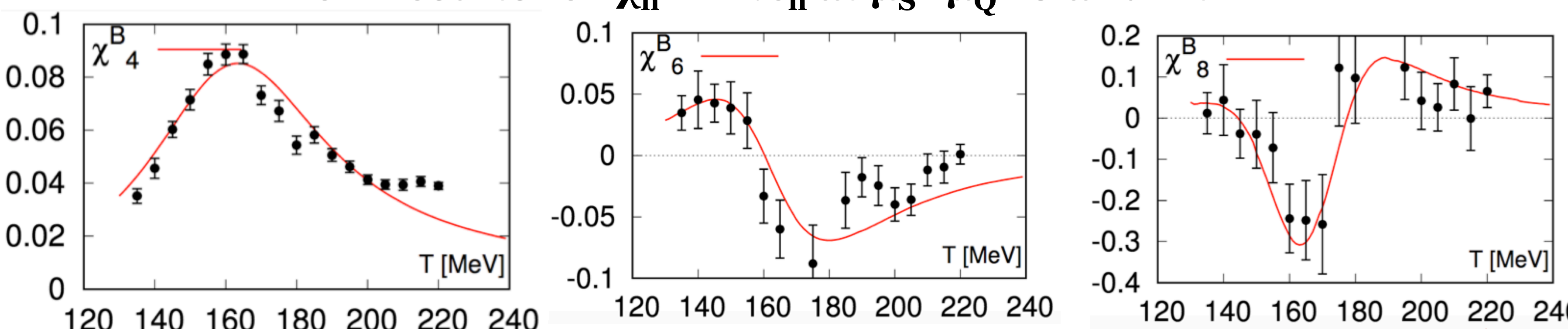
Simulations at imaginary μ_B:

Continuum, O(10[4]) configurations, errors include systematics (WB: NPA (2017))

Strangeness neutrality



New results for $\chi_n^B = n!c_n$ at $\mu_S=\mu_Q=0$ and Nt=12



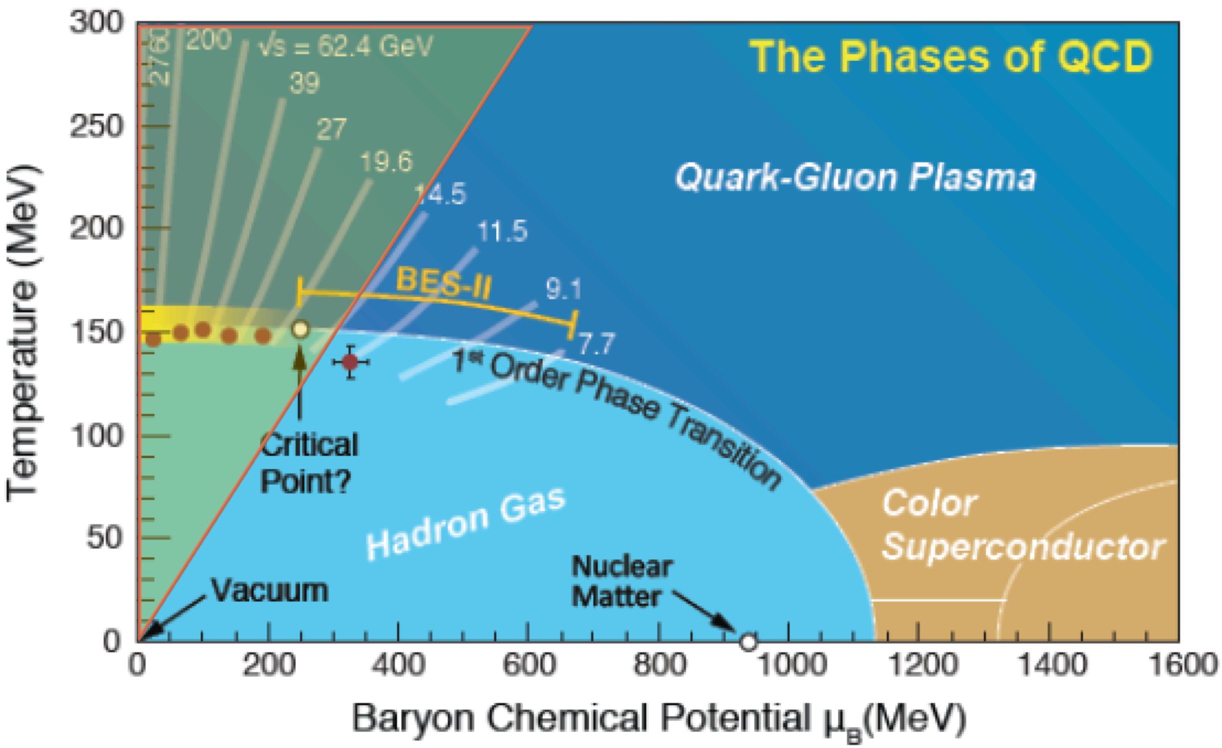
WB, 1805.04445 (2018)

Red curves are obtained by shifting χ_1^B/μ_B to finite μ_B: consistent with no-critical point

Range of validity of equation of state

- We now have the equation of state for $\mu_B/T \leq 2$ or in terms of the RHIC energy scan:

$$\sqrt{s} = 200,\ 62.4,\ 39,\ 27,\ 19.6,\ 14.5\mathrm{GeV}$$



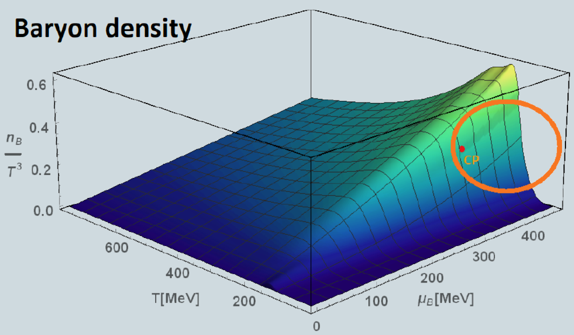
Alternative EoS at large densities

EoS for QCD with a 3D-Ising critical point

$T^4 c_n^{LAT}(T) = T^4 c_n^{Non\text{-}Ising}(T) + T_c^4 c_n^{Ising}(T)$

P. Parotto et al., 1805.05249 (2018)

- Implement scaling behavior of 3D-Ising model EoS
- Define map from 3D-Ising model to QCD
- Estimate contribution to Taylor coefficients from 3D-Ising model critical point
- Reconstruct full pressure



- Density discontinuous at $\mu_B > \mu_{Bc}$

Cluster expansion model

Vovchenko, Steinheimer, Philipsen, Stoecker, 1711.01261

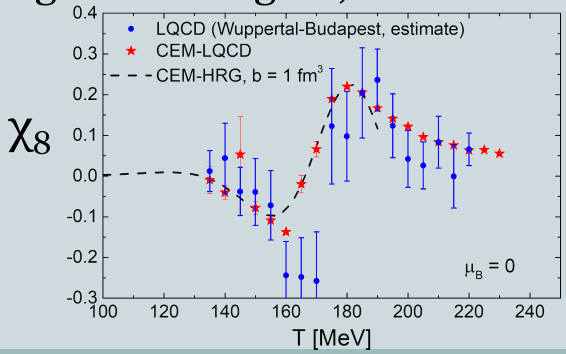
- HRG-motivated fugacity expansion for ρ_B

$$\frac{\rho_B(T,\mu_B)}{T^3} = \chi_1^B(T,\mu_B) = \sum_{k=1}^{\infty} b_k(T)\,\sinh(k\mu_B/T)$$

- b1(T) and b2(T) are model inputs
- All higher order coefficients predicted:

$$b_k(T) = \alpha_k^{SB} \frac{[b_2(T)]^{k-1}}{[b_1(T)]^{k-2}}$$

- Physical picture: HRG with repulsion at moderate T, "weakly" interacting quarks and gluons at high T, no CP



- Plan: integrate ρ_B and get $p(T,\mu_B)$

QCD phase diagram

TRANSITION TEMPERATURE

CURVATURE

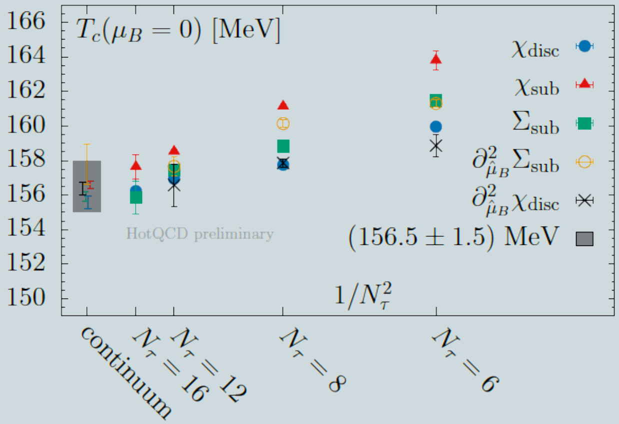
RADIUS OF CONVERGENCE OF TAYLOR SERIES

QCD transition temperature and curvature

Plenary talk by Sayantan Sharma on Tuesday

$$\frac{T_c(\mu_B)}{T_0} = 1 - \kappa_2\left(\frac{\mu_B}{T_0}\right)^2 - \kappa_4\left(\frac{\mu_B}{T_0}\right)^4 + O(\mu_B^6)$$

- QCD transition at μ_B=0 is a crossover
  Aoki et al., Nature (2006)
- Latest results on T_0 from HotQCD based on subtracted chiral condensate and chiral susceptibility

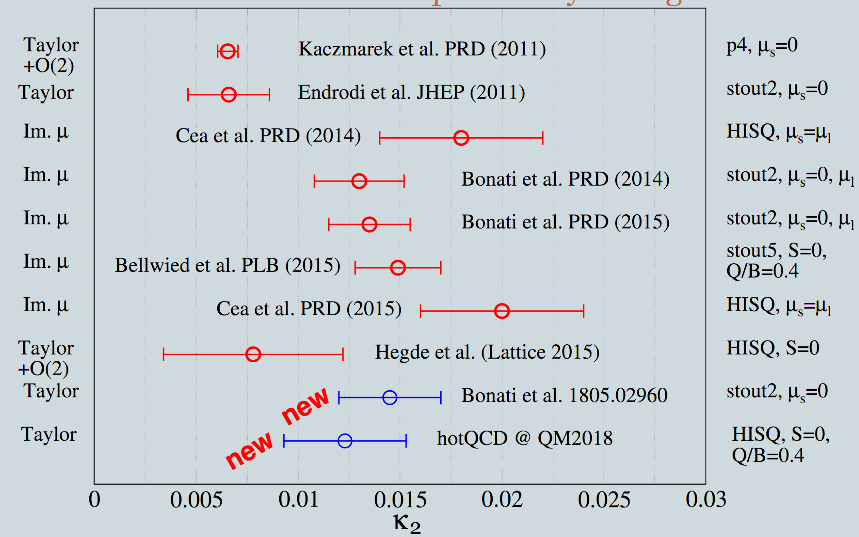


P. Steinbrecher for HotQCD, 1807.05607

T_0=156.5±1.5 MeV

See talk by Patrick Steinbrecher on Wednesday

Compilation by F. Negro



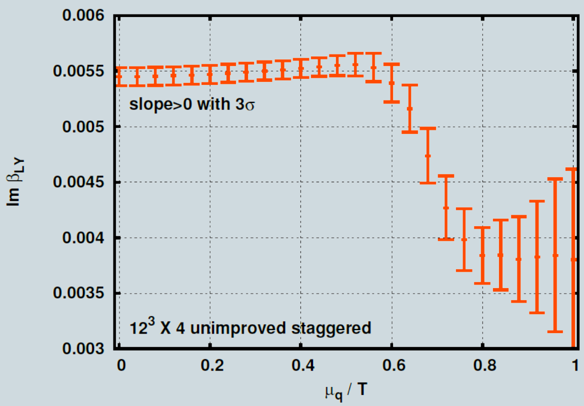
- Curvature very small at μ_B=0
- New results from HotQCD and from Bonati et al. agree with previous findings

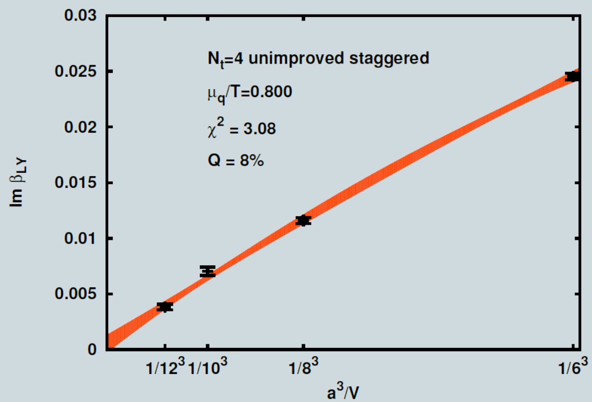
Radius of convergence of Taylor series

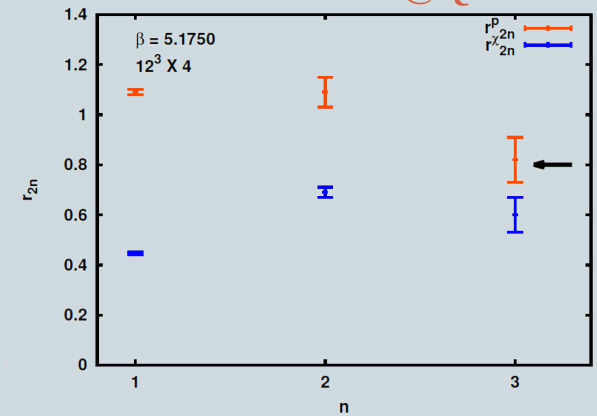
Plenary talk by Sayantan Sharma on Tuesday

- For a genuine phase transition, we expect the ∞-volume limit of the Lee-Yang zero to be real

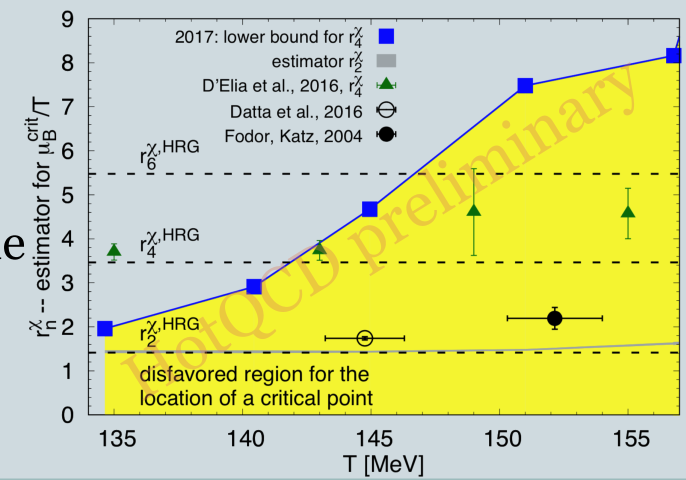
A. Pasztor for WB @QM2018



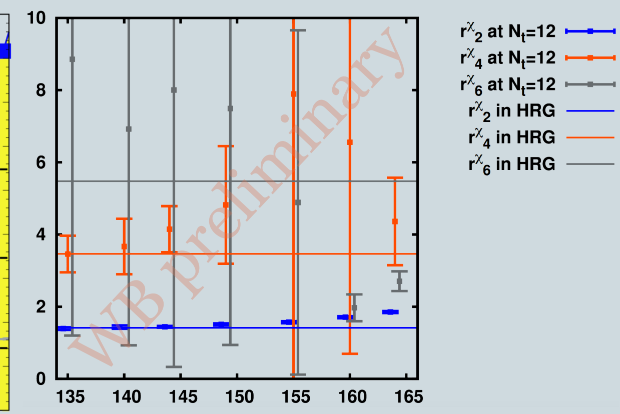




$$r^{\chi}_{2n} = \left|\frac{2n(2n-1)\chi^B_{2n}}{\chi^B_{2n+2}}\right|^{1/2}$$



- It grows as ~n in the HRG model

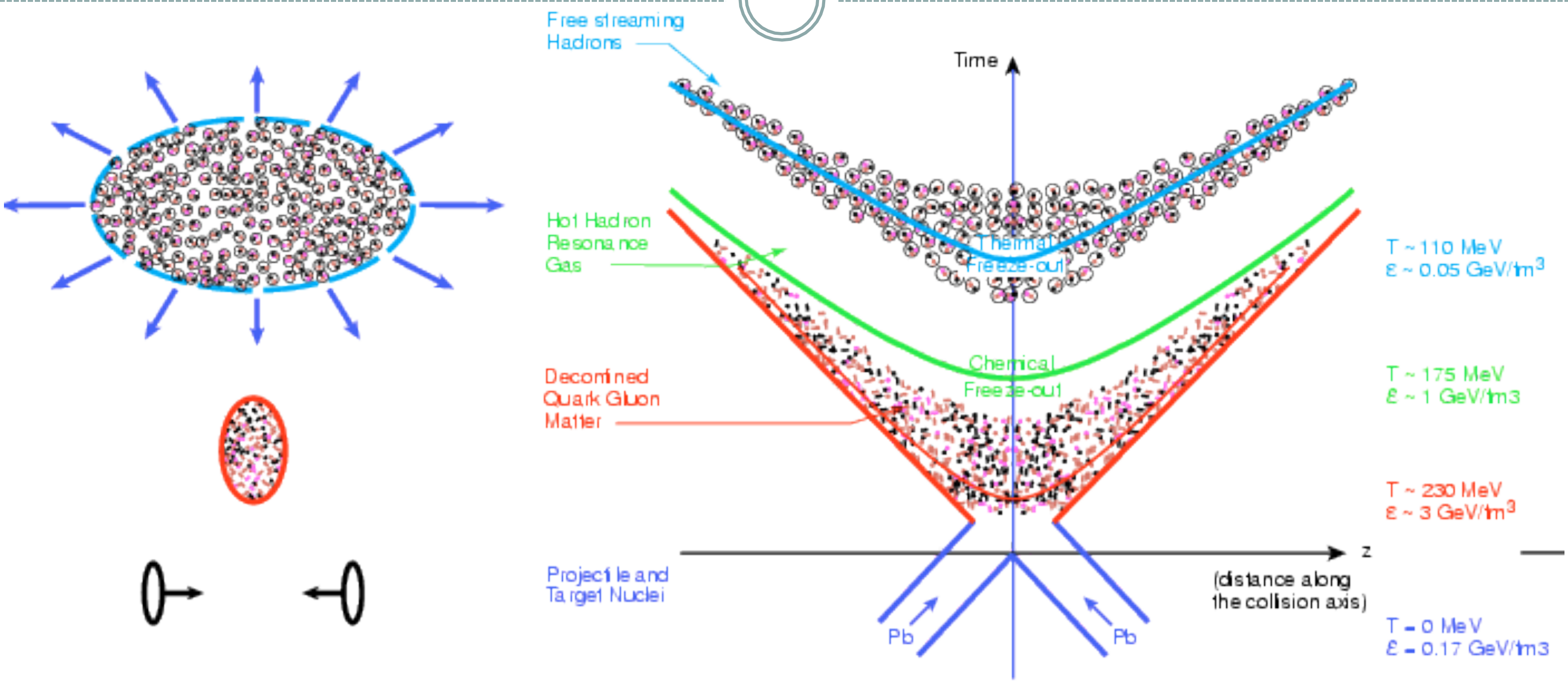


Fluctuations of conserved charges

**COMPARISON TO EXPERIMENT:
CHEMICAL FREEZE-OUT PARAMETERS**

**COMPARISON TO HRG MODEL:
SEARCH FOR THE CRITICAL POINT**

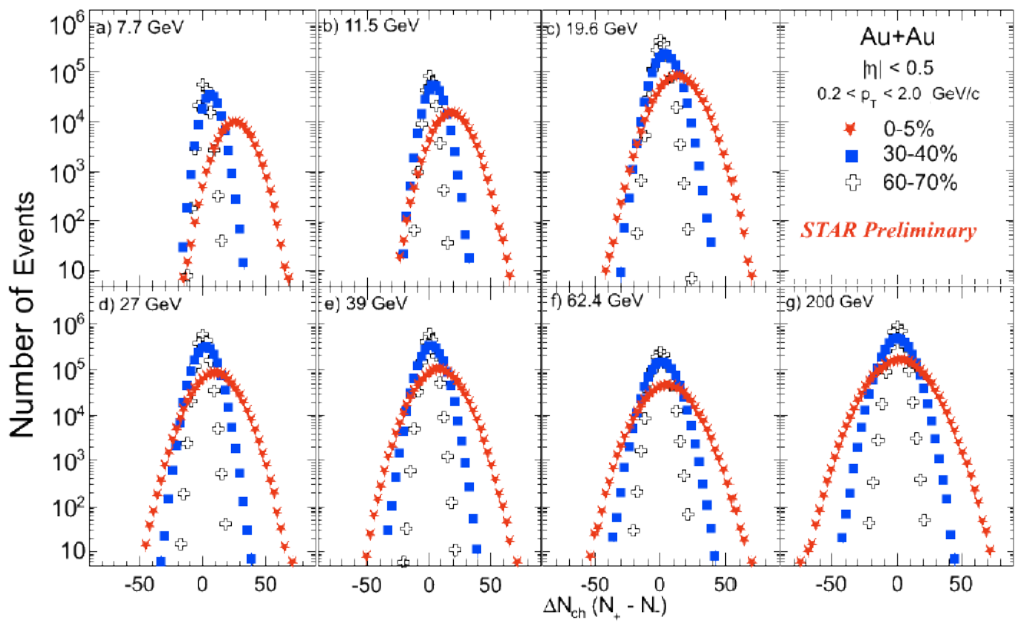
Evolution of a heavy-ion collision



- Chemical freeze-out: inelastic reactions cease: the chemical composition of the system is fixed (particle yields and fluctuations)
- Kinetic freeze-out: elastic reactions cease: spectra and correlations are frozen (free streaming of hadrons)
- Hadrons reach the detector

Distribution of conserved charges

- Consider the number of electrically charged particles N_Q
- Its average value over the whole ensemble of events is $\langle N_Q \rangle$
- In experiments it is possible to measure its event-by-event distribution



STAR Collab.: PRL (2014)

Fluctuations on the lattice

- Fluctuations of conserved charges are the cumulants of their event-by-event distribution

- Definition: $$\chi_{lmn}^{BSQ} = \frac{\partial^{l+m+n} p/T^4}{\partial(\mu_B/T)^l \partial(\mu_S/T)^m \partial(\mu_Q/T)^n}.$$

- They can be calculated on the lattice and compared to experiment

- variance: $\sigma^2=\chi_2$ Skewness: $S=\chi_3/(\chi_2)^{3/2}$ Kurtosis: $\kappa=\chi_4/(\chi_2)^2$

$$S\sigma = \chi_3/\chi_2 \qquad \kappa\sigma^2 = \chi_4/\chi_2$$

$$M/\sigma^2 = \chi_1/\chi_2 \qquad S\sigma^3/M = \chi_3/\chi_1$$

Things to keep in mind

- Effects due to volume variation because of finite centrality bin width
  - Experimentally corrected by centrality-bin-width correction method

V. Skokov et al., PRC (2013), P. Braun-Munzinger et al., NPA (2017), V. Begun and M. Mackowiak-Pawlowska (2017)

- Finite reconstruction efficiency
  - Experimentally corrected based on binomial distribution

A.Bzdak,V.Koch, PRC (2012)

- Spallation protons
  - Experimentally removed with proper cuts in p_T
- Canonical vs Gran Canonical ensemble
  - Experimental cuts in the kinematics and acceptance

V. Koch, S. Jeon, PRL (2000)

- Baryon number conservation
  - Experimental data need to be corrected for this effect

P. Braun-Munzinger et al., NPA (2017)

- Proton multiplicity distributions vs baryon number fluctuations
  - Recipes for treating proton fluctuations

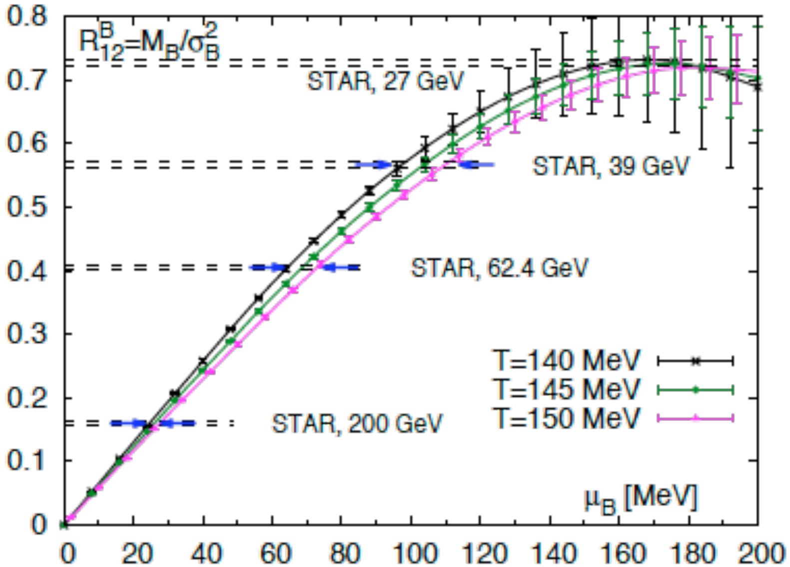
M. Asakawa and M. Kitazawa, PRC(2012), M. Nahrgang et al., 1402.1238

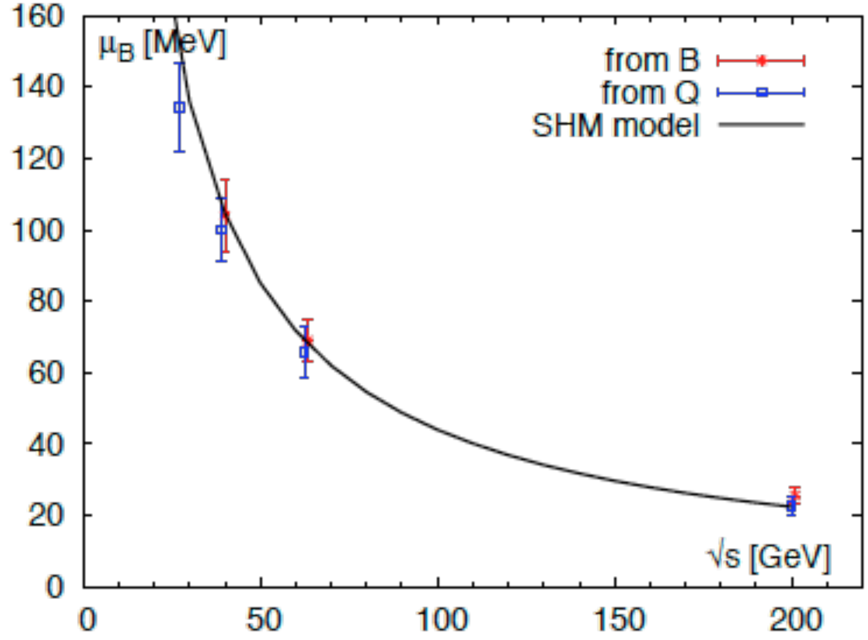
- Final-state interactions in the hadronic phase
  - Consistency between different charges = fundamental test

J.Steinheimer et al., PRL (2013)

Consistency between freeze-out of B and Q

- Independent fit of of R_{12}^Q and R_{12}^B: consistency between freeze-out chemical potentials





$\sqrt{s}[GeV]$	μ_B^f [MeV] (from B)	μ_B^f [MeV] (from Q)
200	25.8±2.7	22.8±2.6
62.4	69.7±6.4	66.6±7.9
39	105±11	101±10
27	-	136±13.8

WB: PRL (2014)
STAR collaboration, PRL (2014)

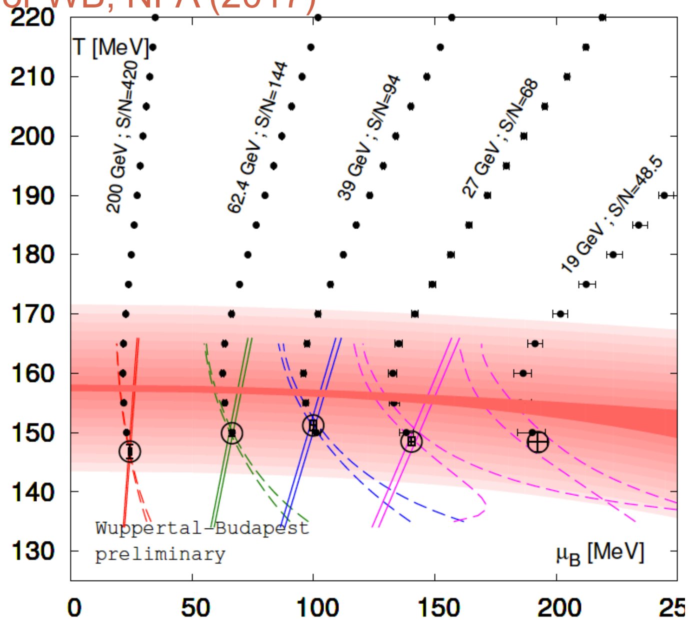
Freeze-out line from first principles

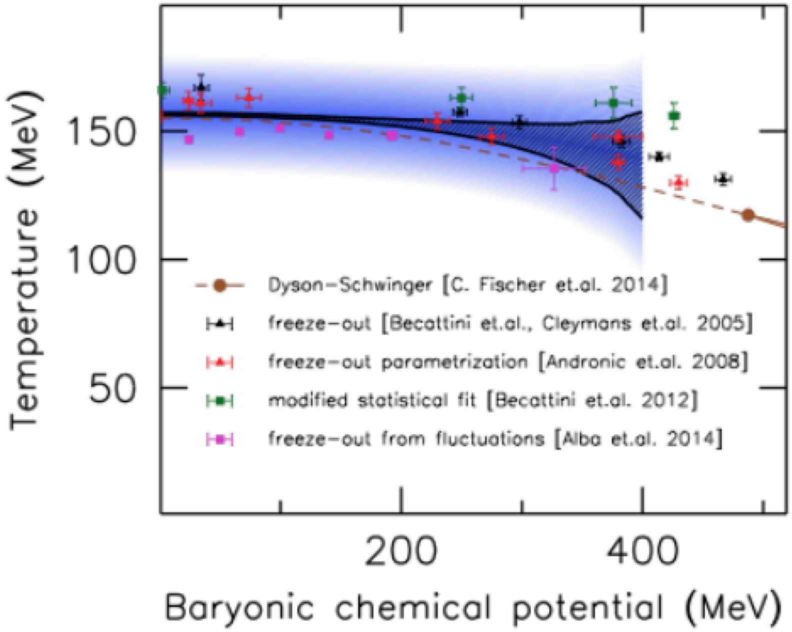
- Use T- and μ_B-dependence of R_{12}^{Q} and R_{12}^{B} for a combined fit:

$$R_{12}^{Q}(T,\mu_B)=\frac{\chi_1^Q(T,\mu_B)}{\chi_2^Q(T,\mu_B)}=\frac{\chi_{11}^{QB}(T,0)+\chi_2^Q(T,0)q_1(T)+\chi_{11}^{QS}(T,0)s_1(T)}{\chi_2^Q(T,0)}\frac{\mu_B}{T}+\mathcal{O}(\mu_B^3).$$

$$R_{12}^{B}(T,\mu_B)=\frac{\chi_1^B(T,\mu_B)}{\chi_2^B(T,\mu_B)}=\frac{\chi_2^B(T,0)+\chi_{11}^{BQ}(T,0)q_1(T)+\chi_{11}^{BS}(T,0)s_1(T)}{\chi_2^B(T,0)}\frac{\mu_B}{T}+\mathcal{O}(\mu_B^3)$$

C. Ratti for WB, NPA (2017)





Kaon fluctuations on the lattice

J. Noronha-Hostler, C.R. et al., 1607.02527

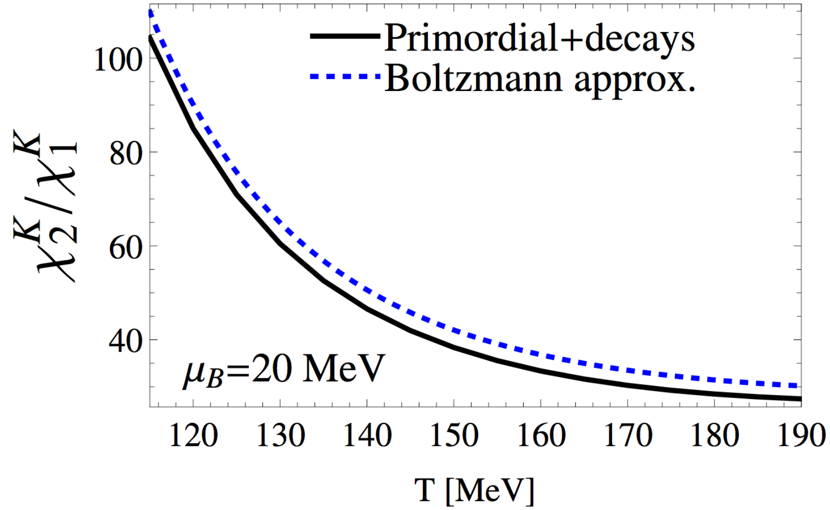
- Lattice QCD works in terms of conserved charges
- Challenge: isolate the fluctuations of a given particle species
- Assuming an HRG model in the Boltzmann approximation, it is possible to write the pressure as:

$$P(\hat{\mu}_B, \hat{\mu}_S) = P^{BS}_{00} + P^{BS}_{10}\cosh(\hat{\mu}_B) + P^{BS}_{01}\cosh(\hat{\mu}_S)$$
$$+ P^{BS}_{11}\cosh(\hat{\mu}_B - \hat{\mu}_S) + P^{BS}_{12}\cosh(\hat{\mu}_B - 2\hat{\mu}_S) + P^{BS}_{13}\cosh(\hat{\mu}_B - 3\hat{\mu}_S)$$

- Kaons in heavy ion collisions: primordial + decays
- Idea: calculate χ_2^K/χ_1^K in the HRG model for the two cases: only primordial kaons in the Boltzmann approximation vs primordial + resonance decay kaons

Kaon fluctuations on the lattice

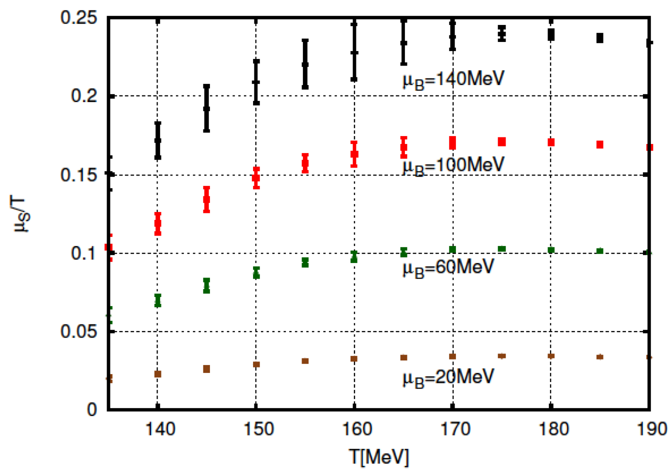
J. Noronha-Hostler, C.R. et al., forthcoming

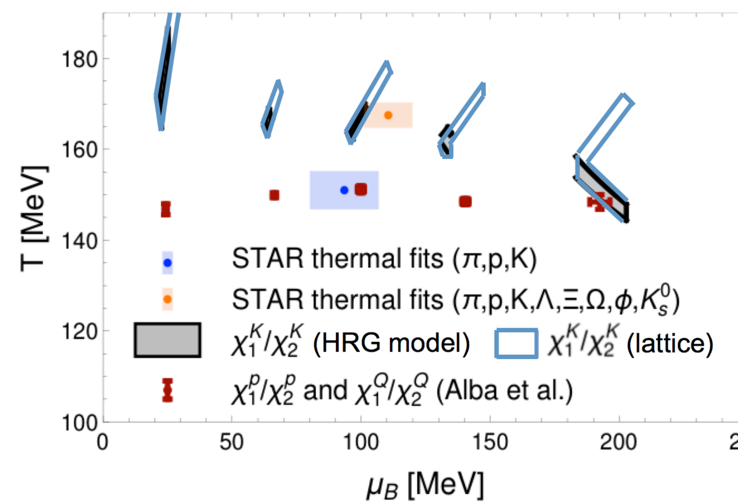


- Boltzmann approximation works well for lower order kaon fluctuations

$$\frac{\chi_2^K}{\chi_1^K} = \frac{\cosh(\hat{\mu}_S + \hat{\mu}_Q)}{\sinh(\hat{\mu}_S + \hat{\mu}_Q)}$$

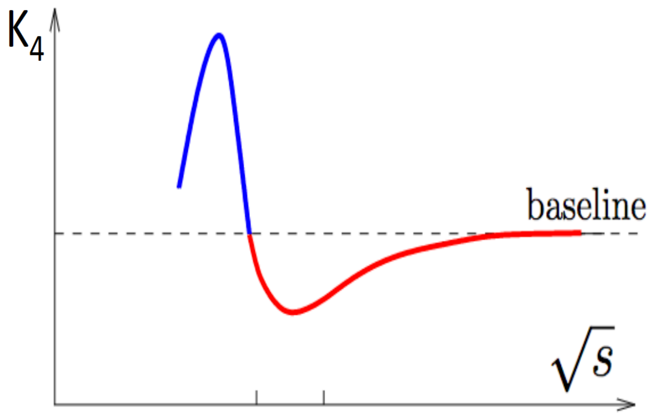
- χ_2^K/χ_1^K from primordial kaons + decays is very close to the Boltzmann approximation
- μ_S and μ_Q are functions of T and μ_B to match the experimental constraints:

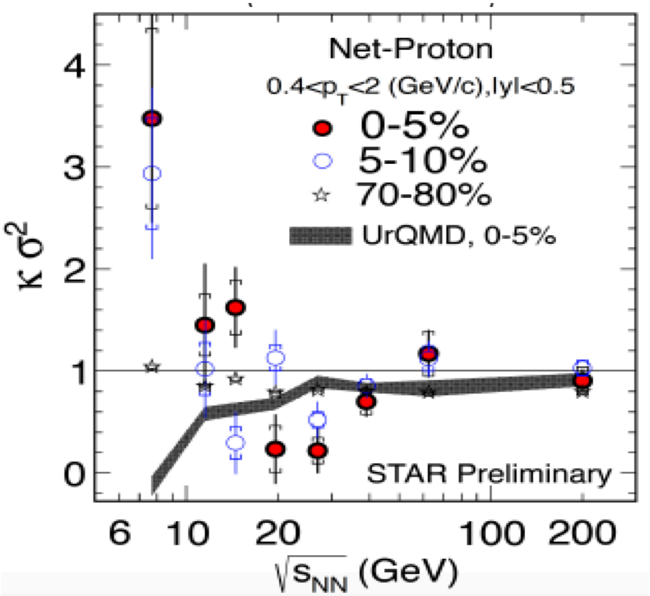




Fluctuations at the critical point

M. Stephanov, PRL (2009).





- Correlation length near the critical point

$\xi \sim |T - T_c|^{-\nu}$ where $\nu > 0$

$$\chi_2 = VT\xi^2$$
$$\chi_3 = 2VT^{3/2}\hat{\lambda}_3\xi^{9/2}$$
$$\chi_4 = 6VT^2[2\hat{\lambda}_3^2 - \hat{\lambda}_4]\xi^7$$

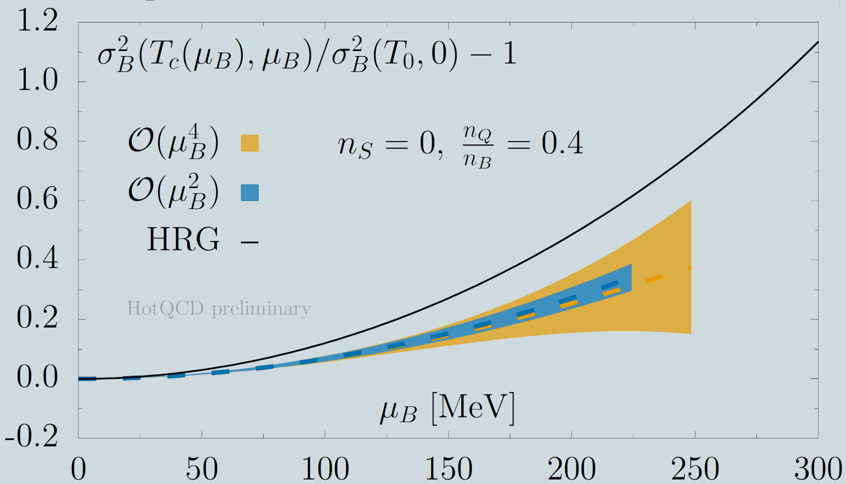
- Fluctuations are expected to diverge at the critical point
- Fourth-order fluctuations should have a non-monotonic behavior
- Preliminary STAR data seem to confirm this
- Can we describe this trend with lattice QCD?

Fluctuations along the QCD crossover

P. Steinbrecher for HotQCD, 1807.05607

Net-baryon variance

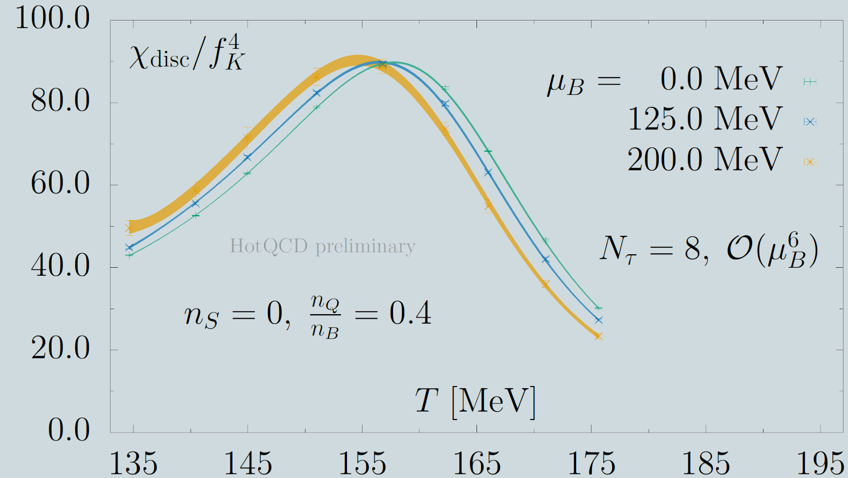
$$\frac{\sigma_B^2(T_c(\mu_B),\mu_B)-\sigma_B^2(T_0,0)}{\sigma_B^2(T_0,0)}=\lambda_2\left(\frac{\mu_B}{T_0}\right)^2+\lambda_4\left(\frac{\mu_B}{T_0}\right)^4+\mathcal{O}(\mu_B^6)$$



- Expected to be larger than HRG model result near the CP
- No sign of criticality

Disconnected chiral susceptibility

$$\chi_{\rm sub}\equiv\frac{T}{V}m_s\left(\frac{\partial}{\partial m_u}+\frac{\partial}{\partial m_d}\right)\Big[m_s(\Sigma_u+\Sigma_d)-(m_u+m_d)\Sigma_s\Big]$$

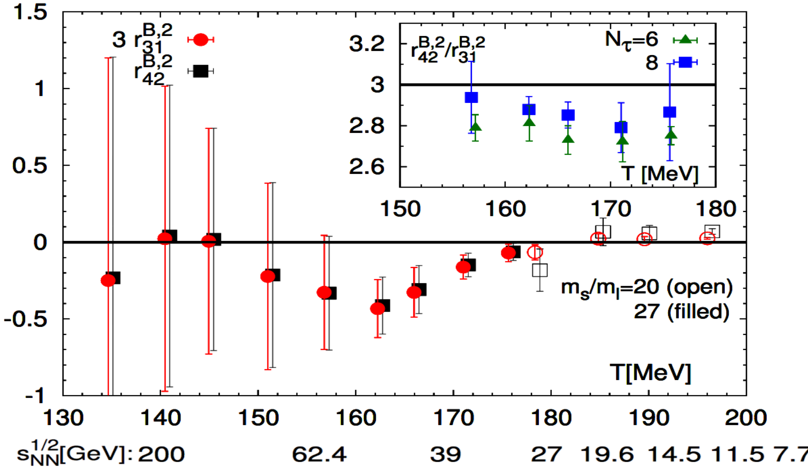


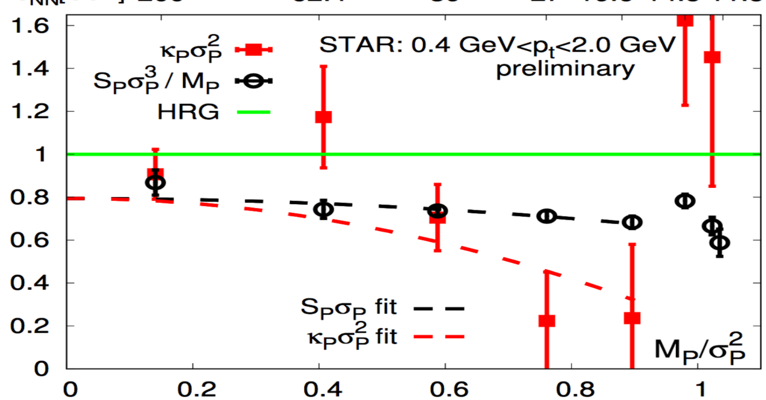
- Peak height expected to increase near the CP
- No sign of criticality

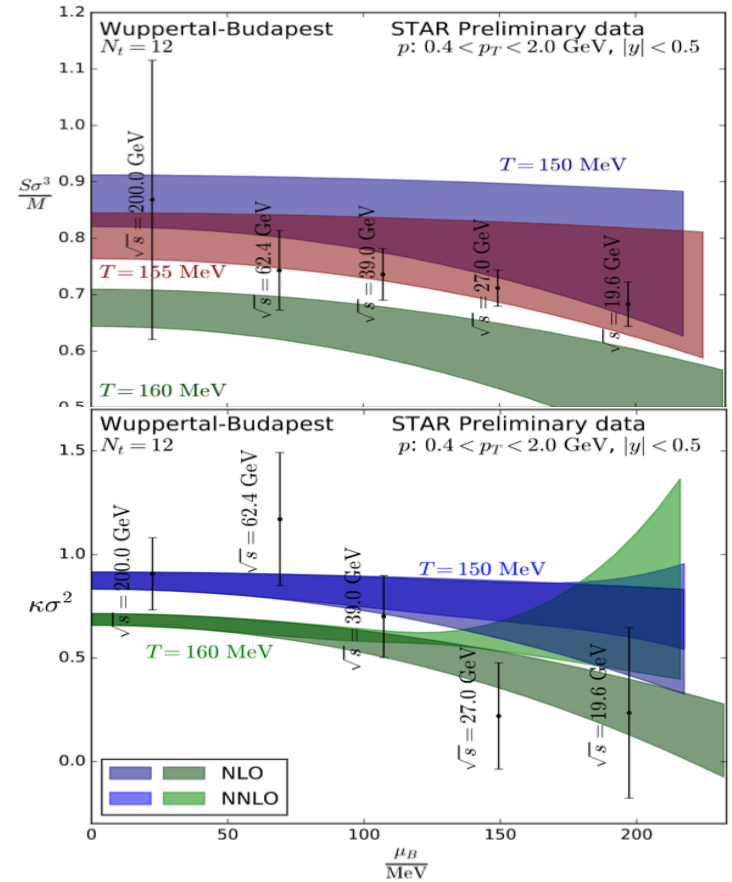
Higher order fluctuations

HotQCD, PRD (2017)

WB, 1805.04445 (2018)







$$\frac{S_B\sigma_B^3}{M_B} = \frac{\chi_3^B(T,\mu_B)}{\chi_1^B(T,\mu_B)} = \frac{\chi_4^B + s_1\chi_{31}^{BS} + q_1\chi_{31}^{BQ}}{\chi_2^B + s_1\chi_{11}^{BS} + q_1\chi_{11}^{BQ}} + \mathcal{O}(\mu_B^2) \equiv r_{31}^{B,0} + r_{31}^{B,2}\hat{\mu}_B^2 + \mathcal{O}(\mu_B^4)$$

$$\kappa_B\sigma_B^2 = \frac{\chi_4^B(T,\mu_B)}{\chi_2^B(T,\mu_B)} = \frac{\chi_4^B}{\chi_2^B} + \mathcal{O}(\mu_B^2) \equiv r_{42}^{B,0} + r_{42}^{B,2}\hat{\mu}_B^2 + \mathcal{O}(\mu_B^4)\,,$$

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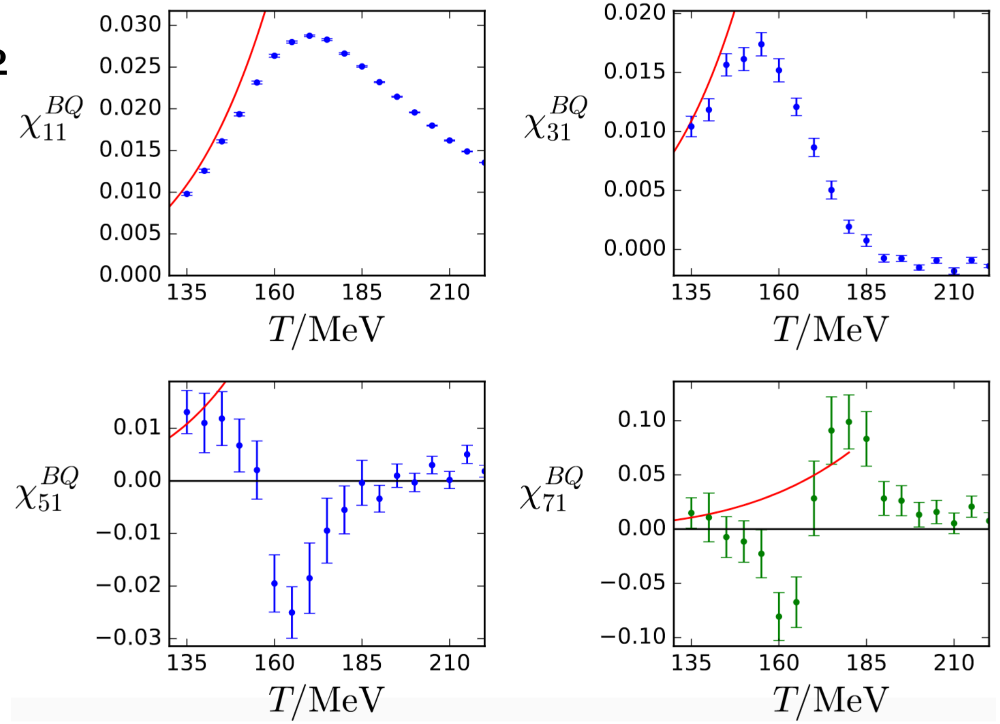
Alternative explanation: canonical suppression

WB, 1805.04445 (2018)

- Simulation of the lower order correlators at imaginary μ_B
- Fit to extract higher order terms
- Results exist also for BS, QS and BQS correlators

Nt=12



$$\chi_{11}^{BS}(\hat{\mu}_B) = \chi_{11}^{BS} + \frac{1}{2!}\chi_{31}^{BS}\hat{\mu}_B^2 + \frac{1}{4!}\chi_{51}^{BS}\hat{\mu}_B^4 + \frac{1}{6!}\chi_{71}^{BS}\hat{\mu}_B^6 + \frac{1}{8!}\chi_{91}^{BS}\hat{\mu}_B^8$$

$$\chi_{21}^{BS}(\hat{\mu}_B) = \chi_{31}^{BS}\hat{\mu}_B + \frac{1}{3!}\chi_{51}^{BS}\hat{\mu}_B^3 + \frac{1}{5!}\chi_{71}^{BS}\hat{\mu}_B^5 + \frac{1}{7!}\chi_{91}^{BS}\hat{\mu}_B^7$$

$$\chi_{31}^{BS}(\hat{\mu}_B) = \chi_{31}^{BS} + \frac{1}{2!}\chi_{51}^{BS}\hat{\mu}_B^2 + \frac{1}{4!}\chi_{71}^{BS}\hat{\mu}_B^4 + \frac{1}{6!}\chi_{91}^{BS}\hat{\mu}_B^6$$

Forthcoming experimental data at RHIC

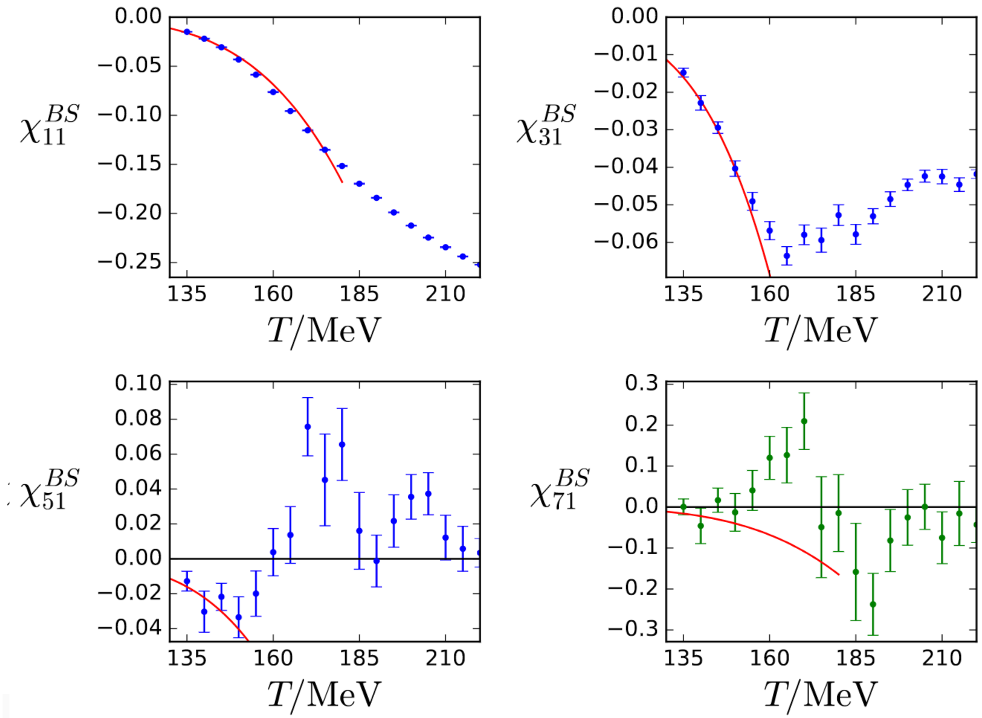
See talk by Jana Guenther on Wednesday

Off-diagonal correlators

- Simulation of the lower order correlators at imaginary μ_B
- Fit to extract higher order terms
- Results exist also for BS, QS and BQS correlators

WB, 1805.04445 (2018)

Nt=12



$$\chi_{11}^{BS}(\hat\mu_B) = \chi_{11}^{BS} + \frac{1}{2!}\chi_{31}^{BS}\hat\mu_B^2 + \frac{1}{4!}\chi_{51}^{BS}\hat\mu_B^4 + \frac{1}{6!}\chi_{71}^{BS}\hat\mu_B^6 + \frac{1}{8!}\chi_{91}^{BS}\hat\mu_B^8$$

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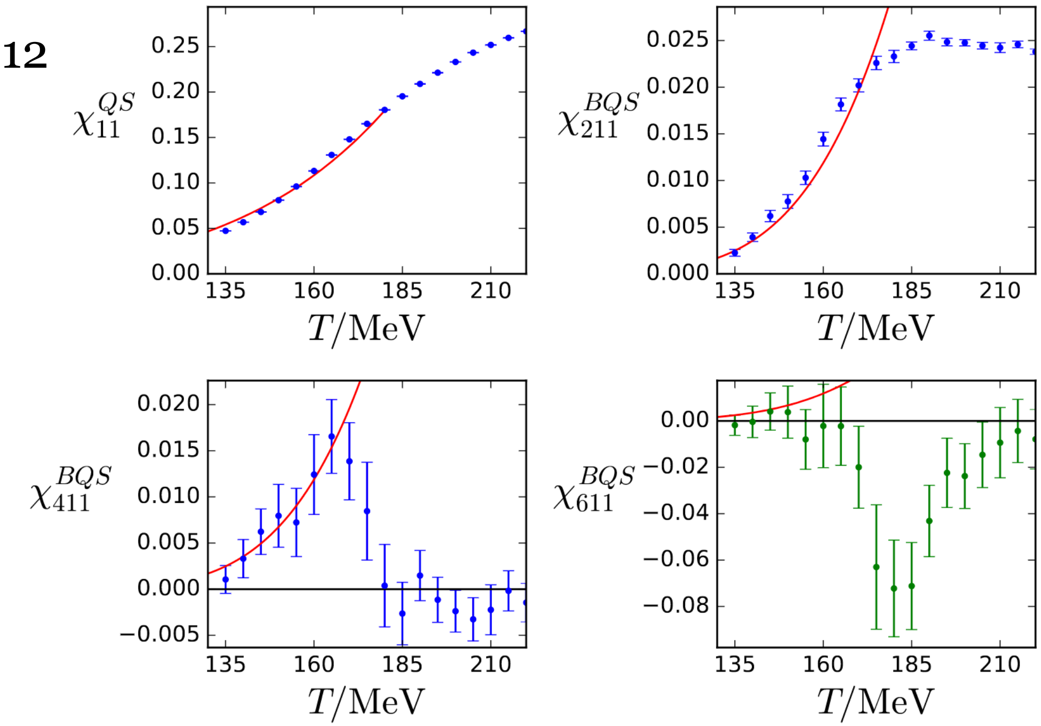


$$\chi_{11}^{BS}(\hat\mu_B) = \chi_{11}^{BS} + \frac{1}{2!}\chi_{31}^{BS}\hat\mu_B^2 + \frac{1}{4!}\chi_{51}^{BS}\hat\mu_B^4 + \frac{1}{6!}\chi_{71}^{BS}\hat\mu_B^6 + \frac{1}{8!}\chi_{91}^{BS}\hat\mu_B^8$$

$$\chi_{21}^{BS}(\hat\mu_B) = \chi_{31}^{BS}\hat\mu_B + \frac{1}{3!}\chi_{51}^{BS}\hat\mu_B^3 + \frac{1}{5!}\chi_{71}^{BS}\hat\mu_B^5 + \frac{1}{7!}\chi_{91}^{BS}\hat\mu_B^7$$

$$\chi_{31}^{BS}(\hat\mu_B) = \chi_{31}^{BS} + \frac{1}{2!}\chi_{51}^{BS}\hat\mu_B^2 + \frac{1}{4!}\chi_{71}^{BS}\hat\mu_B^4 + \frac{1}{6!}\chi_{91}^{BS}\hat\mu_B^6$$

Forthcoming experimental data at RHIC

See talk by Jana Guenther on Wednesday

Other approaches I did not have time to address

- Reweighting techniques (Fodor & Katz)
- Canonical ensemble (Alexandru et al., Kratochvila, de Forcrand, Ejiri, Bornyakov, Goy, Lombardo, Nakamura)
- Density of state methods (Fodor, Katz & Schmidt, Alexandru et al.)
- Two-color QCD (ITEP Moscow lattice group, Kogut et al., S. Hands et al., von Smekal et al.)
- Scalar field theories with complex actions (See talk by M. Ogilvie on Tuesday)
- Complex Langevin (see talks by D. Sinclair, S. Tsutsui, F. Attanasio, Y. Ito, A. Joseph on Monday)
- Lefshetz Thimble (see talks by K. Zambello, S. Lawrence, N. Warrington, H. Lamm on Monday)
- Phase unwrapping (see talks by G. Kanwar and M. Wagman on Friday)

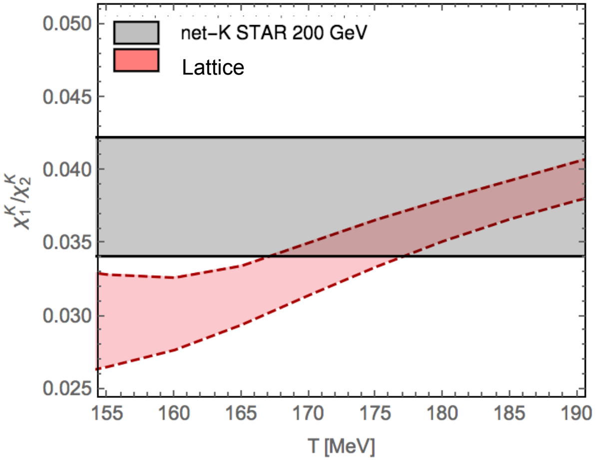
Conclusions

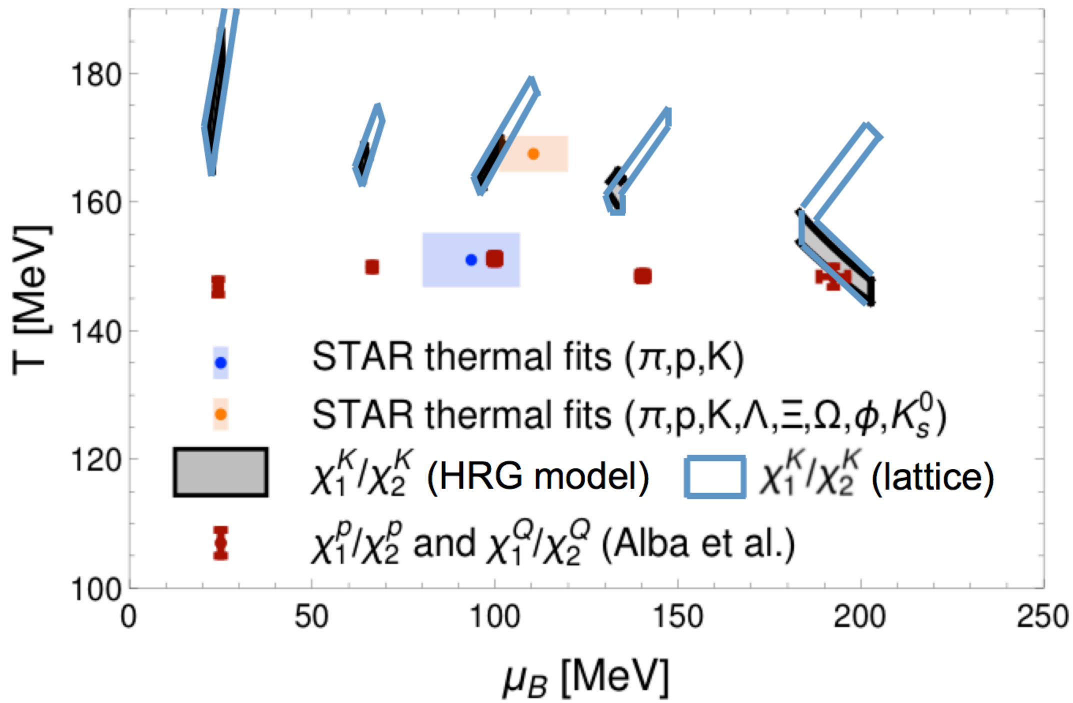
- Need for quantitative results at finite-density to support the experimental programs
  - Equation of state
  - Phase transition line
  - Fluctuations of conserved charges
- Current lattice results for thermodynamics up to $\mu_B/T \leq 2$
- Extensions to higher densities by means of lattice-based models
- No indication of Critical Point from lattice QCD in the explored μ_B range

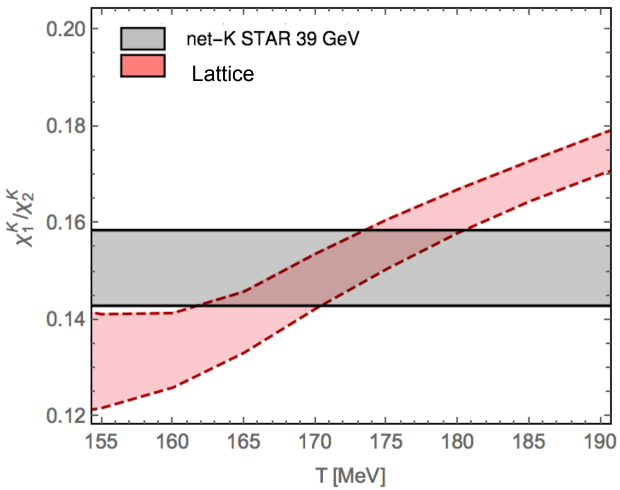
Backup slides

Kaon fluctuations on the lattice

J. Noronha-Hostler, C.R. et al. forthcoming



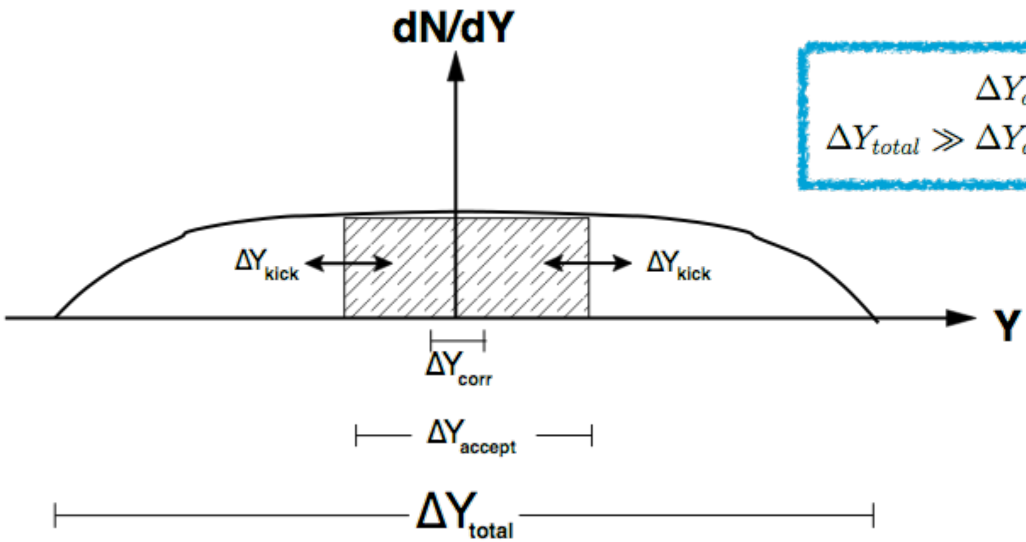




- Lattice QCD temperatures have a large uncertainty but they are above the light flavor ones

Fluctuations of conserved charges?

- If we look at the entire system, none of the conserved charges will fluctuate
- By studying a sufficiently small subsystem, the fluctuations of conserved quantities become meaningful



$$\Delta Y_{accept} \gg \Delta Y_{corr}$$
$$\Delta Y_{total} \gg \Delta Y_{accept} \gg \Delta Y_{kick}$$

- ❑ ΔYtotal: range for total charge multiplicity distribution
- ❑ ΔYaccept: interval for the accepted charged particles
- ❑ ΔYkick: rapidity shift that charges receive during and after hadronization

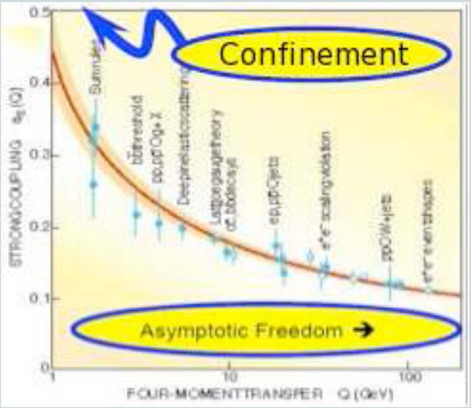
QCD matter under extreme conditions

To address these questions we need fundamental theory and experiment

Theory: Quantum Chromodynamics

- QCD is the fundamental theory of strong interactions
- It describes interactions among quarks and gluons

$$L_{QCD} = \sum_{i=1}^{n_f} \overline{\psi}_i \gamma_\mu \left(i\partial^\mu - gA_a^\mu \frac{\lambda_a}{2} \right) \psi_i - m_i \overline{\psi}_i \psi_i - \frac{1}{4} \sum_a F_a^{\mu\nu} F_a^{\mu\nu}$$



Experiment: heavy-ion collisions

- Quark-gluon plasma (QGP) discovery at RHIC and the LHC
- QGP is a strongly interacting (almost) perfect fluid

Cumulants of multiplicity distribution

- Deviation of N_Q from its mean in a single event: $\delta N_Q = N_Q - \langle N_Q \rangle$
- The cumulants of the event-by-event distribution of NQ are:

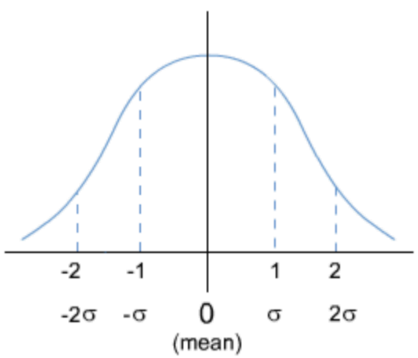
$$\chi_2 = \langle (\delta NQ)^2 \rangle \qquad \chi_3 = \langle (\delta NQ)^3 \rangle \qquad \chi_4 = \langle (\delta NQ)^4 \rangle - 3\langle (\delta NQ)^2 \rangle^2$$

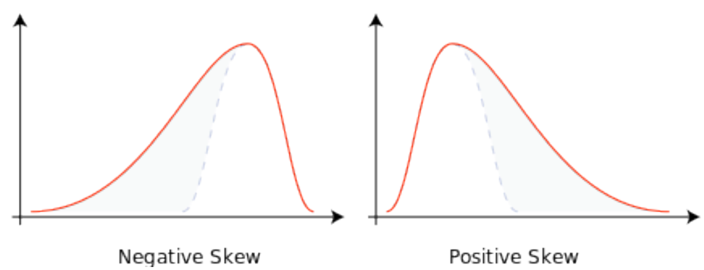
- The cumulants are related to the central moments of the distribution by:

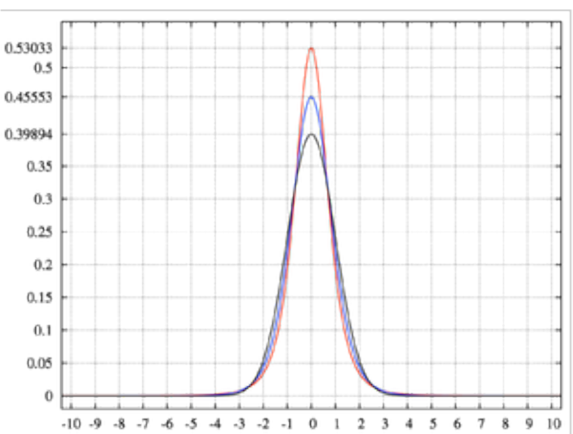
variance: $\sigma^2 = \chi_2$

Skewness: $S = \chi_3/(\chi_2)^{3/2}$

Kurtosis: $\kappa = \chi_4/(\chi_2)^2$





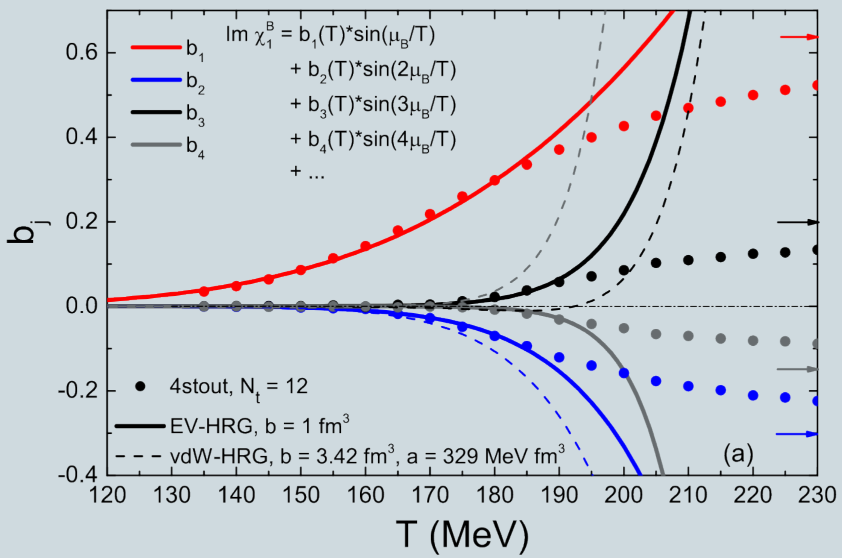


Fluctuations and hadrochemistry

$$\chi_1^B(T,\mu_B) = \frac{\rho_B(T,\mu_B)}{T^3} = \sum_{k=1}^{\infty} b_k(T)\sinh(k\,\mu_B/T)$$

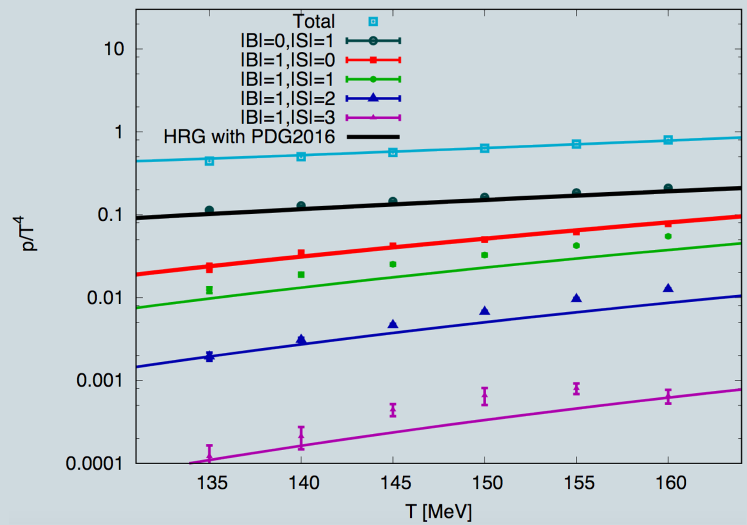
$$P(\hat{\mu}_B,\hat{\mu}_S) = P_{00}^{BS} + P_{10}^{BS}\cosh(\hat{\mu}_B) + P_{01}^{BS}\cosh(\hat{\mu}_S) + P_{11}^{BS}\cosh(\hat{\mu}_B-\hat{\mu}_S) + P_{12}^{BS}\cosh(\hat{\mu}_B-2\hat{\mu}_S) + P_{13}^{BS}\cosh(\hat{\mu}_B-3\hat{\mu}_S)$$

V. Vovchenko et al., PLB (2017)



- Consistent with HRG at low temperatures
- Consistent with approach to ideal gas limit
- b_2 departs from zero at T~160 MeV
- Deviation from ideal HRG

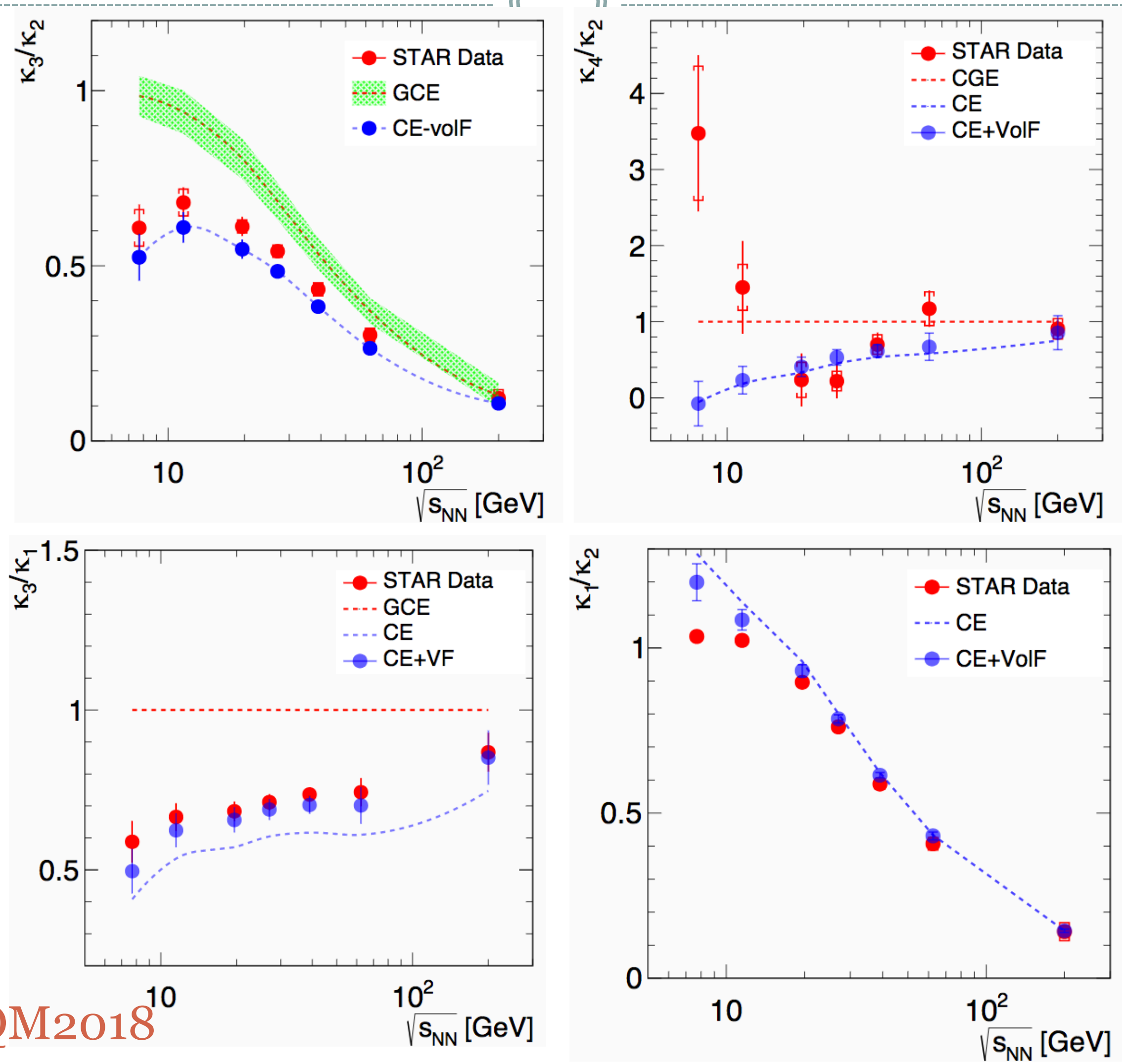
P. Alba et al., PRD (2017)



- Need of additional strange hadrons, predicted by the Quark Model but not yet detected
- First pointed out in Bazavov et al., PRL(2014)

(see talk by J. Glesaaen on Friday)

Canonical suppression



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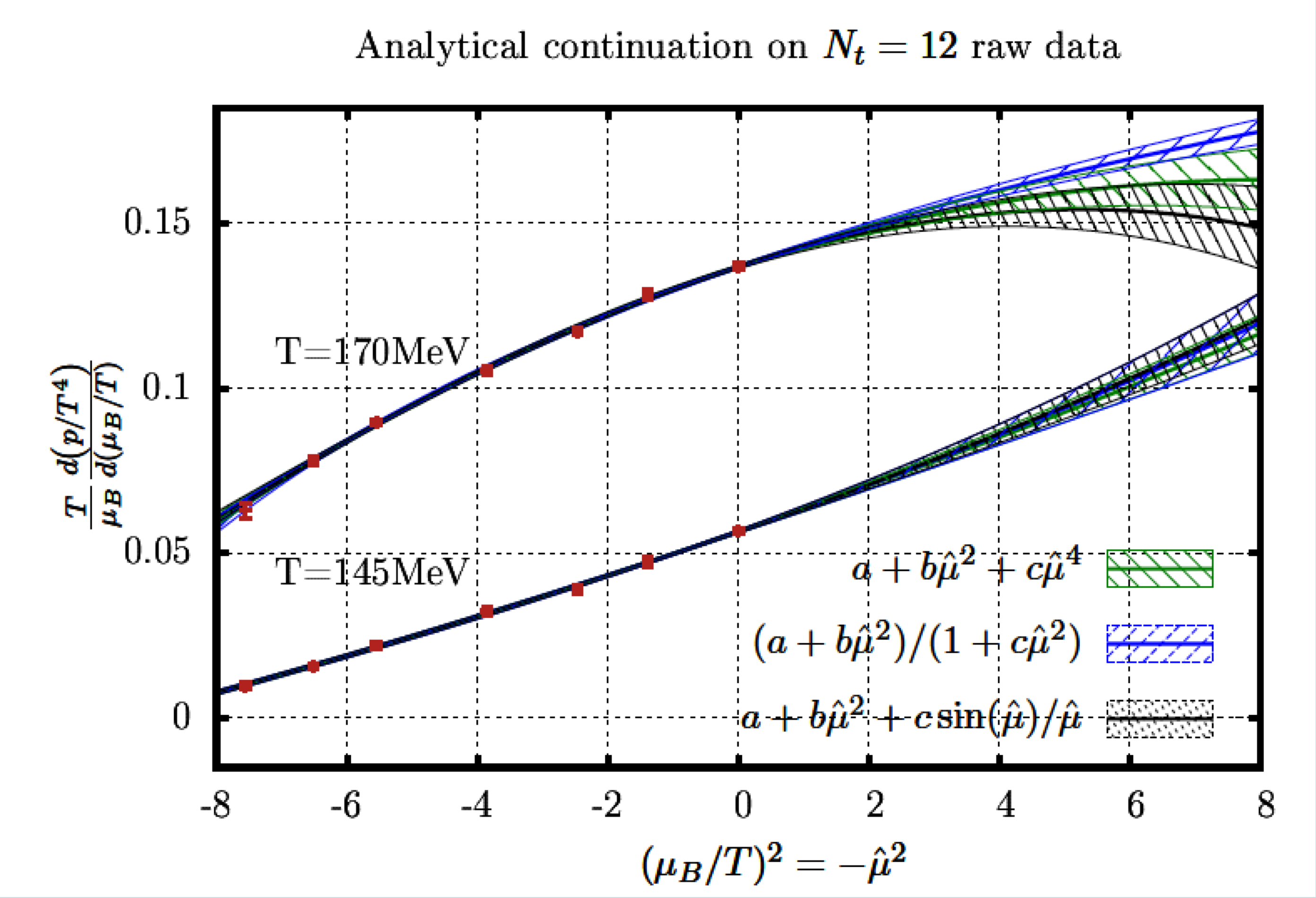
above 11.5 GeV CE suppression accounts for measured deviations from GCE

Analytical continuation – illustration of systematics



Analytical continuation – illustration of systematics

Condition: $\chi_8 \lesssim \chi_4 \longrightarrow f(\hat{\mu}_B) = a + b\hat{\mu}_B^2 + c\hat{\mu}_B^4 + \frac{b\epsilon}{840}\hat{\mu}_B^6$