Three Neutrons from Lattice QCD

Jan-Lukas Wynen[1]

E. Berkowitz[1] J. Bulava[2] T. Luu[1,3] A. Shindler[4]

[1]Institute for Advanced Simulation — Forschungszentrum Jülich

[2]CP^3 Origins — University of Southern Denmark

[3]Helmholtz-Institut für Strahlen- und Kernphysik — Bonn University

[4]FRIB — Michigan State University

Motivation

- 3N forces relevant for neutron rich isotopes
  - Location of neutron dripline
  - Neutron stars
- ^{3}H and ^{3}He measured in experiment and on lattice

  [HAL QCD Prog.Theor.Phys. 127 (2012)]
- 3n (currently) not accessible by experiment
- Hard problem on lattice:
  - Weak compared to 2N forces
  - Signal-to-noise problem
- **Goal**: Develop methodology and proof of concept

Interpolating Operators

- 3 Neutrons at the sinks (primed arguments):

$$\mathcal{O}_{3n}^{S\,S_3} = \left(n^{\alpha'}(x_1')\,\Gamma^{\alpha'\beta'}_{s\,s_3}\; n^{\beta'}(x_2') \right) \Gamma^{s\,s_3\gamma'}_{S\,S_3}\; n^{\gamma'}(x_3')$$

- Particular choice of spin decomposition
- Naive number of Wick contractions: $N_u!\,N_d! = 3!\,6! = 4320$
- Three different sites (Pauli Principle)

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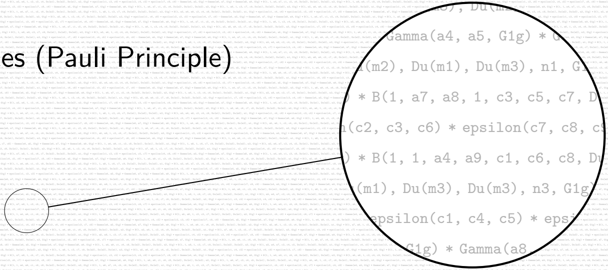

Baryon Blocks

- Provide fine control over
  - source displacements
  - sink momenta
- Combine quarks at sinks into Baryon Blocks:

$$\mathcal{B}^{\alpha'\,\alpha\beta\gamma}_{\;\;\;abc}(x'|f_1, x_1; f_2, x_2; f_3, x_3) = \varepsilon_{a'b'c'} S^{\alpha'\alpha}_{a'a}(f_1, x' \leftarrow x_1) \times \left[S^{\beta'\beta}_{b'b}(f_2, x' \leftarrow x_2) \Gamma^{\beta'\gamma'} S^{\gamma'\gamma}_{c'c}(f_3, x' \leftarrow x_3) \right]$$

- Abbreviate

$$\mathcal{B}^{\alpha'\,\alpha\beta\gamma}_{\;\;\;abc}(x'|f_1, x_1; f_2, x_2; f_3, x_3) \equiv \mathcal{B}_I(x'|X)$$

[Doi, Endres Comput. Phys. Commun 184 (2013)] [Detmold, Orginos PRD 87 (2013)]

Contractions

Correlator:

$$\mathcal{C}_{SM}(n^2) = \mathcal{F}_{n^2}\left(\sum_{\substack{IJK \\ X_1 X_2 X_3}} T^{SM}_{IJK}(X_1, X_2, X_3)\, \mathcal{B}_I(x_1'|X_1)\mathcal{B}_J(x_2'|X_2)\mathcal{B}_K(x_3'|X_3) \right)$$

- 3 blocks w/ distinct parameters

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- Contract to desired isospin, spin, colour

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Our choice for $\mathcal{F}$:

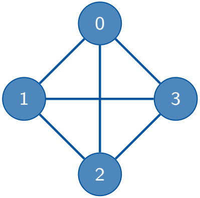
$$x_1' \mapsto +p \qquad x_2' \mapsto -p \qquad x_3' \mapsto 0$$

$$\mathcal{O}^{S\,S_3}_{3n} = \left(n^{\alpha'}(x_1')\,\Gamma^{\alpha'\beta'}_{s\,s_3}\; n^{\beta'}(x_2')\right)\Gamma^{s\,s_3\gamma'}_{S\,S_3}\; n^{\gamma'}(x_3')$$

Implementation / Code Generation

$$n^{\alpha_1}(n_1)\,\Gamma^{\alpha_1\alpha_2}_{T_1^1}\,n^{\alpha_2}(n_2)\,n^2(n_3) + \dots$$

FORM: Expand, sort, gather



Python: Graph optimisation,
reuse of intermediate results, ...

```
block0 = makeBaryonBlock(propd2,propu2,propd2,Gamma_G1g);
block2 = makeBaryonBlock(propd1,propu1,propd1,Gamma_G1g);
contract(corr_nnn_s12_m12, block0, block1, block2, coeffs[1], blockIdxs[0], blockIdxs[3], blockIdxs[4]);
contract(corr_nnn_s32_m12, block0, block1, block2, coeffs[537], blockIdxs[0], blockIdxs[2], blockIdxs[4]);
```

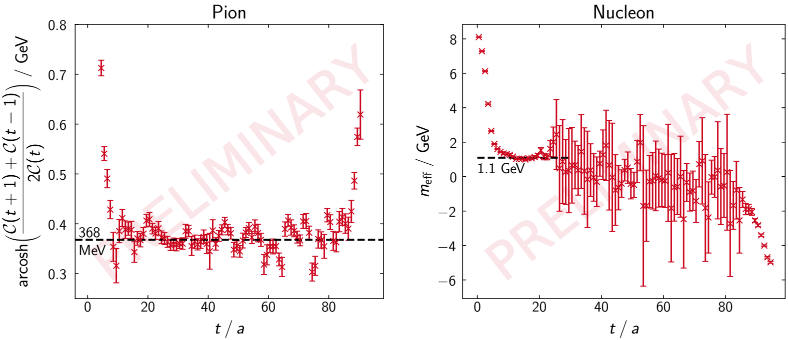
Ensemble

Clover-Wilson, generated in JARA-HPC

- $N_{\text{cfg}} = 175$ (ongoing)
- $N_t \times N_s^3 = 96 \times 48^3$
- Physical strange quark mass
- $a \approx 0.085\,\text{fm}$
- $m_\pi \approx 368\,\text{MeV}$
- $m_\pi L \approx 7.7$

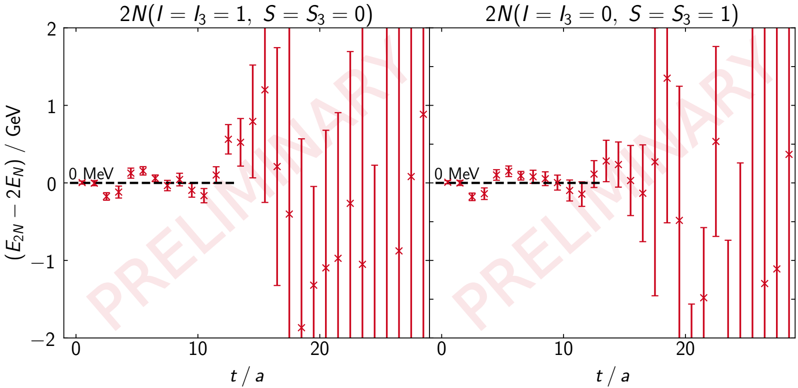
Parameters as in CLS H107 (96×32^3) [RQCD PRD 94 (2016)]

Pion and Nucleon



- *s*-wave
- No fits yet, need more statistics
- m_π consistent with measurement on CLS ensemble

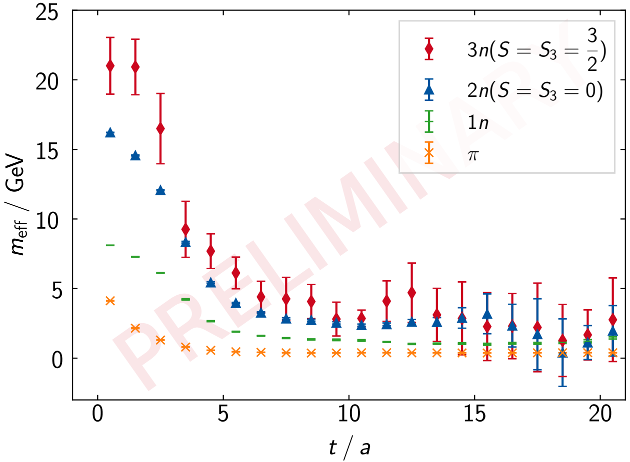
Two Nucleons



- s-wave
- Within statistics consistent with zero

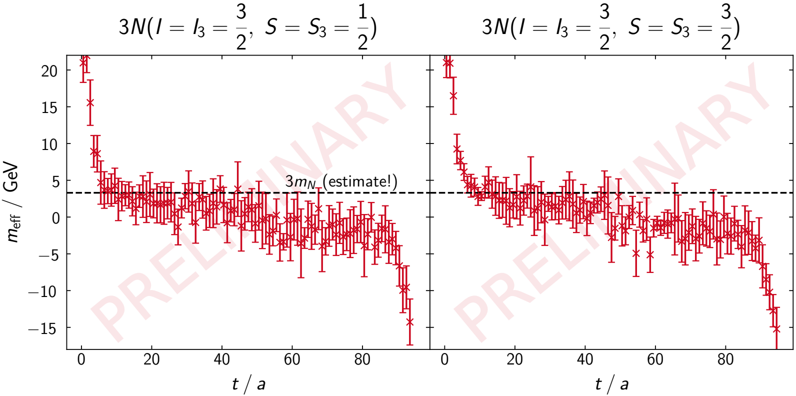
[NPLQCD PRD 92 (2015)] [Yamazaki et al. PRD 92 (2015)] [CalLat PLB 765 (2017)]
and many more

From π to $3n$



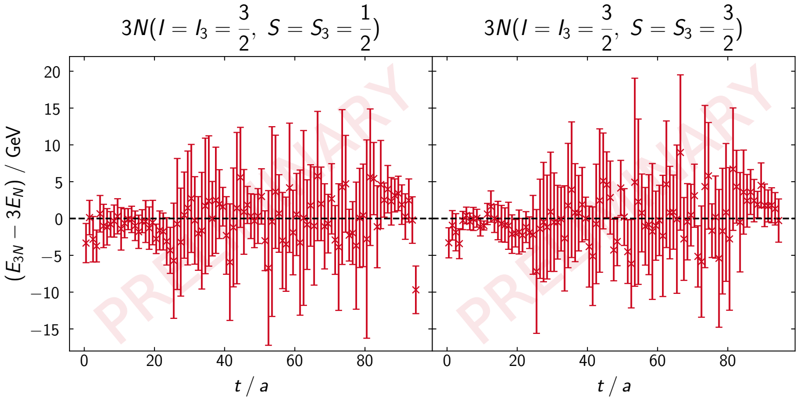
- (Semblance of) plateaus in the same range
- Large noise for $3n$

Three Neutrons — Effective Mass



- *p*-wave
- Clear plateau given low statistics
- However: Uncertainties large across whole t range

Three Neutrons — Energy Shift



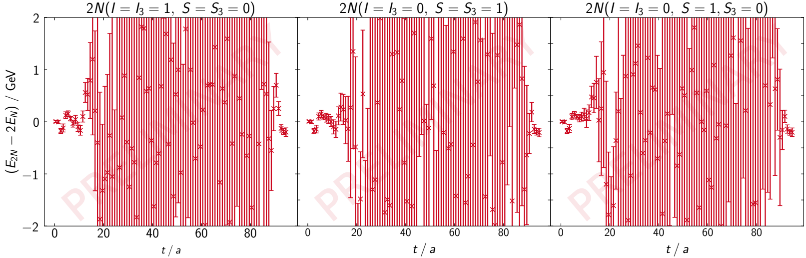
- Within statistics, consistent with zero
- Expected to be small

Conclusion and Outlook

- Generating new gauge ensemble
- Developed formalism for full $3N$ calculation
- Software suite for automatic code generation
  - General, can be used for other channels as well
  - Optimizes generated program
- Promising results, can see indications of plateaus
- Need (and will collect) more statistics for proper signal

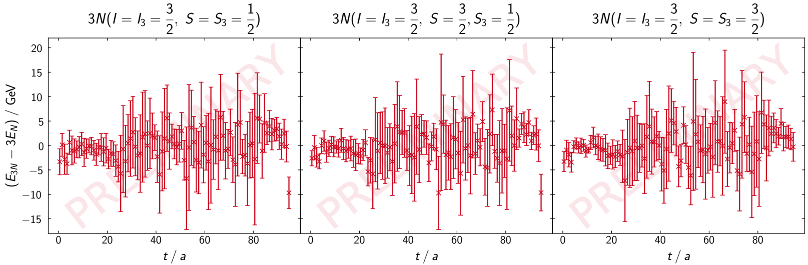
Backup Slides

All 2N Energy Shifts



- *s*-wave
- Other channels are zero due to required symmetries

All 3N Energy Shifts



Optimizations

Two Strategies

- Reduce number of contractions
  - Use symmetries
  - (Semi-) manual at the moment
- Reuse Baryon Blocks
  - Constructing blocks is expensive
  - Order contractions to use blocks multiple times
  - Automated, but *Traveling Salesperson*
  - Simple graph structure $\Rightarrow$ Can solve exactly