Baryons and Interactions in Magnetic Fields
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Outline

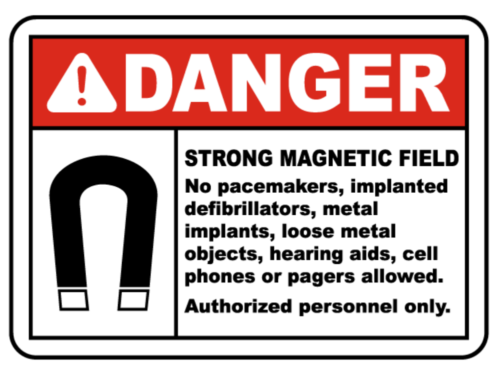
- Motivation
- PT in Large Magnetic Fields
- Octet Baryon Energies in Magnetic Fields
- Magnetic Polarizabilities
- Unitary Limit of Two-Nucleon Interactions
- Summary & Outlook

Magnetic Field scales

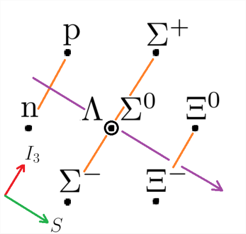
$$\frac{e}{2M_N} = 3{:}152 \quad 10^{\;14} \quad MeV\,T^{\;\;1}$$

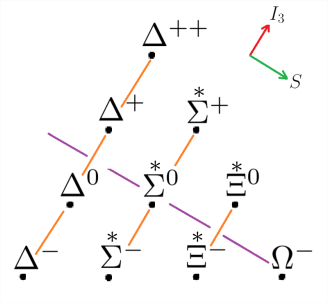
at $eB \quad m^2$:

$$eB \quad 3 \quad 10^{14}\mathrm{T} \quad 3 \quad 10^{18}\mathrm{G}$$

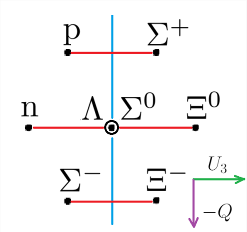


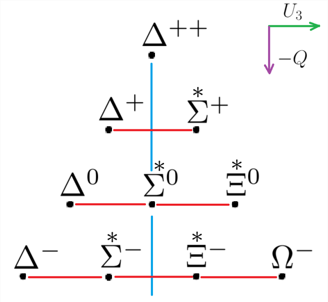
Low-Lying Baryons (Isospin multiplets)



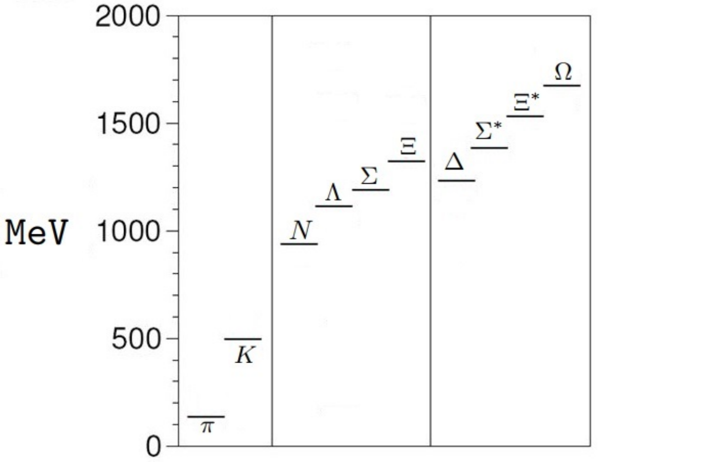


Low-Lying Baryons (U-spin multiplets)

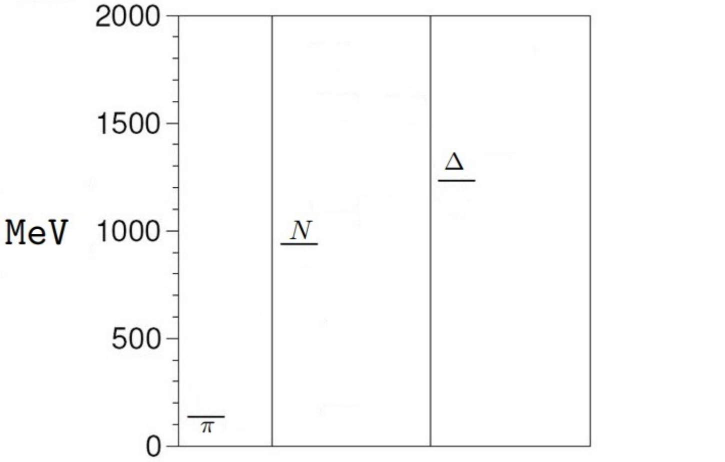




Mass Spectrum



Mass Spectrum (Special case)



Motivation

LQCD computations

- Background magnetic field calculations of electromagnetic properties
- Quantization due to prohibitive size of the lattices (t' Hooft, '79)

$$eB = \frac{6 \quad n}{L^2}$$

Example: $32^3 \quad 48$ lattice with $a = 0{:}11\mathtt{fm}$, produces $\quad 10^{18}\mathtt{G}$

Physical environments

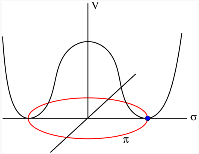
- **Magnetars**: Magnetic fields on the surface of (and within) highly magnetized neutron stars $\quad 10^{12} \quad 10^{15}\mathtt{G}$ (Duncan & Thomson, '92)
- **Relativistic Heavy-ion collision**:
  Non-central collisions $\quad 10^{19}\mathtt{G}$ (Kharzeev et al., '08)

Chiral Pertubation Theory

- PT is an effective field theory that makes use of the chiral symmetry of the underlying theory (QCD); $\frac{m_i}{QCD} = 0 \quad (i = u; d; s)$
- $SU(3)_L \quad SU(3)_R \; ! \; SU(3)_V$ due to the formation of the chiral condensate $h \quad i$
- The fluctuations about the condensate (i.e. vev) are parameterized by the coset space

$$SU(3)_L \quad SU(3)_R = SU(3)_V$$

and, these Goldstone bosons are identified with the low lying psuedo-scalar degrees of freedom (i.e. octet mesons)



Chiral Pertubation Theory

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- The fluctuations about the condensate (i.e. vev) are parameterized by the coset space

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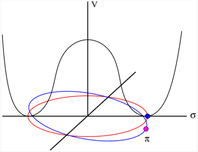
and, these pseudo Goldstone bosons are identified with the low lying psuedo-scalar degrees of freedom (i.e. octet mesons)



Meson Chiral Pertubation Theory

- Glodstone bosons are parameterized as

$$ = \exp\left(\frac{2i}{f}\right);$$

where,

$$^{i}_{j} = \begin{pmatrix} \frac{1}{\sqrt{2}}\,^{0} + \frac{1}{\sqrt{6}} & ^{+} & K^{+} \\ & \frac{1}{\sqrt{2}}\,^{0} + \frac{1}{\sqrt{6}} & K^{0} \\ K & \overline{K}^{0} & \frac{2}{\sqrt{6}} \end{pmatrix}^{i}_{j} :$$

where f is the three-flavor chiral-limit value of the pseudoscalar meson decay constant.

- We assume the power counting

$$\frac{k^2}{^2} \quad \frac{m^2}{^2} \quad ^2;$$

- The $O(\ ^2)$ terms in the (Euclidean) Lagrangian density are

$$L = \frac{f^2}{8}\mathrm{Tr}(\partial\ ^{\dagger}\partial\ ) \quad \mathrm{Tr}(m_q^{\dagger}\ + m_q\ ^{\dagger});$$

Meson Chiral Pertubation Theory in External Magnetic Fields

- Glodstone bosons are parameterized as

$$= \exp \frac{2i}{f} \quad ;$$

where,

$$^i_j = \begin{pmatrix} \frac{1}{\sqrt{2}}\,^0 + \frac{1}{\sqrt{6}} & ^+ & K^+ \\ & \frac{1}{\sqrt{2}}\,^0 + \frac{1}{\sqrt{6}} & K^0 \\ K & \overline{K}^0 & \frac{2}{\sqrt{6}} \end{pmatrix}^i_j ;$$

where f is the three-flavor chiral-limit value of the pseudoscalar meson decay constant.

- We assume the power counting

$$\frac{k^2}{^2} \quad \frac{m^2}{^2} \quad \frac{(eA\ )^2}{^2} \quad \frac{eF}{^2} \quad ^2;$$

- The $O(\ ^2)$ terms in the (Euclidean) Lagrangian density are

$$L = \frac{f^2}{8}\mathrm{Tr}(D \quad ^y D \quad) \qquad \mathrm{Tr}(m_q^y \quad + m_q \quad ^y);$$

Heavy Baryon Chiral Pertubation Theory

- Power counting issue (M_B)
- Modified power counting ($p = M_B v + k$) (Jenkins & Manohar, '93)

$$\frac{k}{M_B} \qquad \frac{\ }{M_B}$$

$O(\ )$ Octet baryon lagrangian

$$L = i\,\mathrm{Tr}\ \overline{B} v \ DB + 2D\,\mathrm{Tr}\ \overline{B} S \ \{A \ ; B\} + 2F\,\mathrm{Tr}\ \overline{B} S \ [A \ ; B]$$

$$B^i_j = \begin{pmatrix} \frac{1}{\sqrt{2}} \ {}^0 + \frac{1}{\sqrt{6}} & {}^+ & p \\ & \frac{1}{\sqrt{2}} \ {}^0 + \frac{1}{\sqrt{6}} & n \\ & {}^0 & \frac{2}{\sqrt{6}} \end{pmatrix}^i_j$$

$O(\ )$ Decuplet baryon lagrangian

$$L = \overline{T} \ (\ iv \ D + \ )T \ + 2H\,\overline{T} \ S \ A\,T \ + 2C \ \overline{T} \ A\,B + \overline{B} A \ T$$

where, T are embedded in the completely symmetric flavor tensor: T_{ijk}, for e.g. $T_{111} = {}^{++}$; $T_{112} = \frac{1}{\sqrt{3}}\ {}^{+}$

Heavy Baryon Chiral Pertubation Theory

- Chirally covariant derivative

$$(D\ B)^i_j = \partial\ B^i_j + [V\ ; B]^i_j$$

$$(D\ T)_{ijk} = \partial\ T_{ijk} + (V\ )^{i'}_i T_{i'jk} + (V\ )^{j'}_j T_{ij'k} + (V\ )^{k'}_k T_{ijk'}$$

- Vector and axial-vector fields of mesons

$$V\ = \frac{1}{2f^2}[\ ; \partial\ \ ] +$$

$$A\ = \frac{1}{f}\partial\ \ +$$

Heavy Baryon Chiral Pertubation Theory in External Magnetic Field

- Chirally covariant derivative

$$(D\ \ B)^i_j = @\ \ B^i_j + [V\ \ ; B]^i_j$$

$$(D\ \ T)_{ijk} = @\ \ T_{ijk} + (V\ \ )^{i^0}_i T_{i^0jk} + (V\ \ )^{j^0}_j T_{ij^0k} + (V\ \ )^{k^0}_k T_{ijk^0}$$

- Vector and axial-vector fields of mesons

$$V\ \ = ieA\ \ Q + \frac{1}{2f^2}[\ \ ; D\ \ \ \ ] +$$

$$A\ \ = \ \ \frac{1}{f} D\ \ \ \ +$$

Propagators

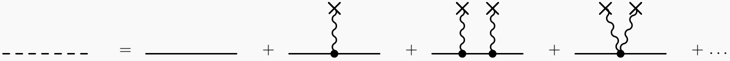
- Mesons

$$G_\phi(x;y) = \int_0^{\infty} \frac{ds}{(4\pi s)^2} e^{-m^2 s} \exp\left(-\frac{(x-y)^2}{4s}\right)$$

- Octet baryons

$$D_B(x;y) = \delta^{(3)}(\vec{x}-\vec{y})\,\delta(x_4-y_4)$$

Propagators in External Magnetic Field



- Mesons (Schwinger, '51)

$$G\ (x;y;B) = e^{ieQ\ \ B\ \ x_1\bar{x}_2} \int_0^{1} \frac{ds}{(4\ \ s)^2} e^{\ \ m^2 s} \frac{eQ\ \ Bs}{\sinh(eQ\ \ Bs)}$$

$$\exp\ \ \frac{eQ\ \ B\ \ \ x^2_?}{4\tanh(eQ\ \ Bs)}\ \ \frac{x_3^2 +\ \ x_4^2}{4s}\ \ ;$$

- This propagator satisfies

$$(\ \ D\ \ D\ \ + m^2)G\ (x;y;B) = \ \ (x\ \ \ y);$$

- Octet baryons propagators will enable us to calculate energies which involve tree and loop contributions.

Octet Baryon Energies (Tree-Level Contributions)

Local operators that contribute to $O(\ ^2)$

Octet baryon magnetic moment operators (Coleman & Glashow '61)

$$L = \frac{e}{2M_N} \quad {}_D\mathrm{Tr} \;\; \overline{B}\, S \quad fQ;Bg \;\; + \;\; {}_F\mathrm{Tr} \;\; \overline{B}\, S \quad [Q;B] \quad F \quad ;$$

where, $S \quad = \qquad v \ S$.

Kinetic-energy term (Landau Levels)

$$L = \quad \mathrm{Tr} \quad \overline{B}\, \frac{D^2_?}{2M_B} B \quad ;$$

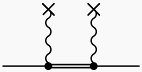
where $(D_?) \quad = D \qquad v \ (v \quad D)$.

Octet Baryon Energies (Tree-Level Contributions)

Local operator that contribute to $O(\ ^3)$

Magnetic dipole transition operator

$$L = \ _U \sqrt{\frac{3}{2}} \frac{ie}{M_N} \ \overline{B} S \ Q T \ + \overline{T} \ Q S \ B \ F \ :$$



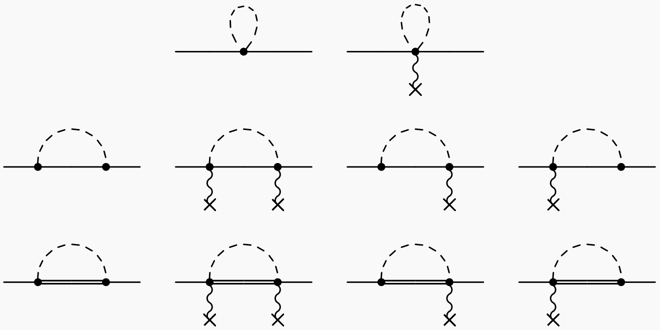
$_U$ can be determined using the measured values for the electromagnetic decay widths of the decuplet baryons. (Keller et al. (CLAS), '11,'12)

$$(T \ ! \ B \ ) = \ _T \frac{!^3}{2} \frac{M_B}{M_T} \ \frac{e \ \ _U}{2M_N} \ ^2$$

where,

$$! \ = \frac{M_T^2 \quad M_B^2}{2M_T}$$

Octet Baryon Energies (Meson-Loop Contributions)



Octet Baryon Energies (Meson-Loop Contributions, Continued...)

- Spin-dependent/Spin-independent contributions

$$E = \quad eB \quad {}_3 \; E_1 + \quad E_2$$

Explicit expressions

$$E_1 = \sum_{B} A_B \frac{Q \;\; m}{(4 \;\; f \;)^2} F_1 \left(\frac{|eB|}{m^2}, \frac{\;\;_B}{m} \right)$$

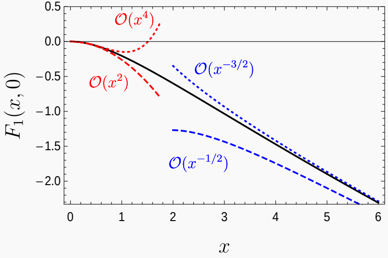
$$E_2 = \sum_{B} A_B \frac{m^3}{(4 \;\; f \;)^2} F_2 \left(\frac{|eB|}{m^2}, \frac{\;\;_B}{m} \right)$$

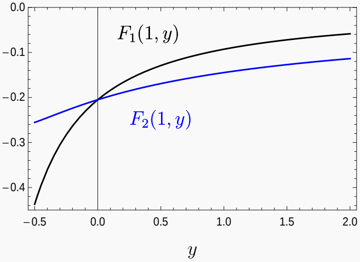
$$F_1(x; y) = \int_0^{\;1} ds \; f(x; s) g(y; s)$$

where

$$f(x; s) = \frac{^{1/2}}{s^{3/2}} \left(\frac{xs}{\sinh(xs)} \quad 1 \right) \qquad g(y; s) = e^{\;\; s(1 \;\; y^2)} \mathrm{Erfc}(y\sqrt{s}).$$

Behavior of loop functions





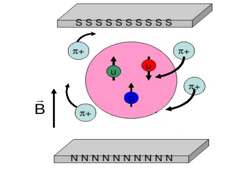
where, $x = \frac{|eB|}{m^2}$ and $y = \frac{B}{m}$.

Complete Third-order Calculation

- $(I_3 \neq 0)$ Octet baryon energies valid to $O(\ ^{3})$

$$E = M_B + \frac{|QeB|}{2M_B} \quad eB \quad {}_3 \quad \frac{{}_B}{2M_N} + E_1 \quad \frac{T \quad U^{2}}{T} \quad \frac{eB}{2M_N}^{2} + E_2$$

Magnetic Polarizabilities (Detour)



Empirical values (C. Patrignani et al. (PDG), '17)

$$\textit{Proton}: \ 2{:}5(4) \quad 10 \ ^4\mathrm{fm}^3$$
$$\textit{Neutron}: 3{:}7(12) \quad 10 \ ^4\mathrm{fm}^3$$

- The spin-averaged energy:

$$\overline{E} = M + \frac{jQeBj}{2M} \quad \frac{1}{2}4 \ (\ ^{lp} + \ ^{tr})B^2 +$$

$$^{lp} = \frac{}{6} \sum_B \frac{A_B S_{2B}}{m \ (4 \ f \ )^2} G \quad \frac{B}{m}$$

$$^{tr} = \frac{T}{2 \quad T} \quad \frac{e \quad U}{2M_N} \ ^2$$

Magnetic Polarizabilities (Contributions)

B	β^{lp}	β^{tr}	β^{ct}	$\beta^{\mathrm{lp}}+\beta^{\mathrm{tr}}+\beta^{\mathrm{ct}}$	$\beta_M^{\mathrm{expt.}}$
p	1.37	6.89	−5.76	∗ 2.50	2.5(4)
n	1.32	6.89	−4.51	∗ 3.70	3.7(12)
Σ^+	0.96	14.05	−5.76	9.26	–

Counterterm Promotion

- Higher order (short distance) operators as counterterms

$$L = \frac{1}{2}4 \quad B^2 \sum_{i=1}^{4} \quad {}^{ct}_{i} O_i;$$

where,

$$O_1 = \mathrm{Tr} \quad \overline{B}B \quad \mathrm{Tr} \quad Q^2 \quad ;$$
$$O_2 = \mathrm{Tr} \quad \overline{B} \, f Q; f Q; B g g \quad ;$$
$$O_3 = \mathrm{Tr} \quad \overline{B} \, f Q; [Q; B] g \quad ;$$
$$O_4 = \mathrm{Tr} \quad \overline{B} \, [Q; [Q; B]]$$

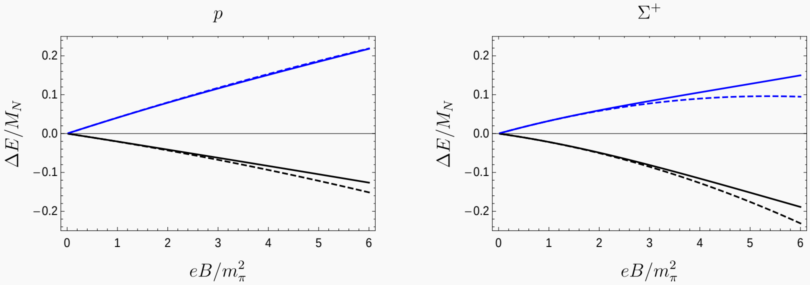
- Coleman-Glashow like relations for counterterm coefficients:

$${}^{ct}_{p} = {}^{ct}_{+}; \qquad {}^{ct}_{n} = {}^{ct}_{0}; \qquad {}^{ct} = {}^{ct}$$

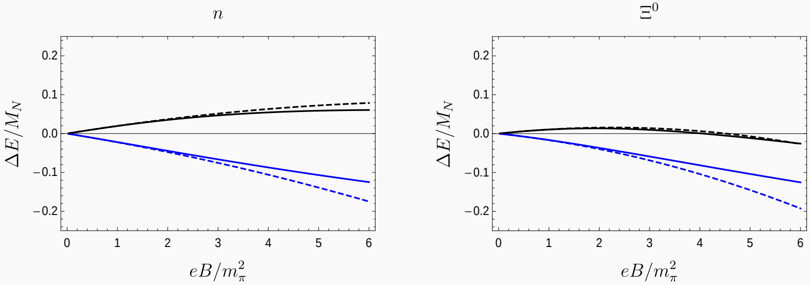
and

$$\frac{1}{\sqrt{3}} \; {}^{ct}_{0} = {}^{ct} \qquad {}^{ct}_{n} = \tfrac{1}{2}(\; {}^{ct}_{0} \qquad {}^{ct})$$

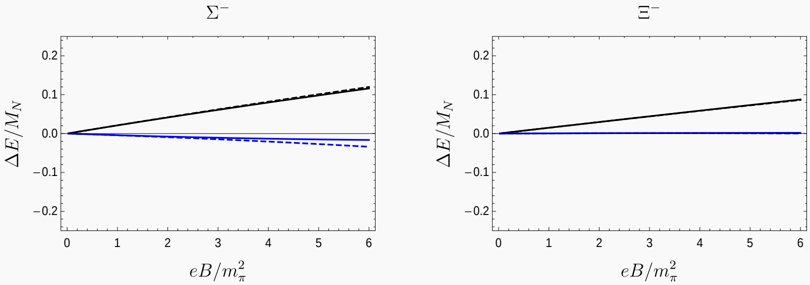
Baryon Energies



Baryon Energies

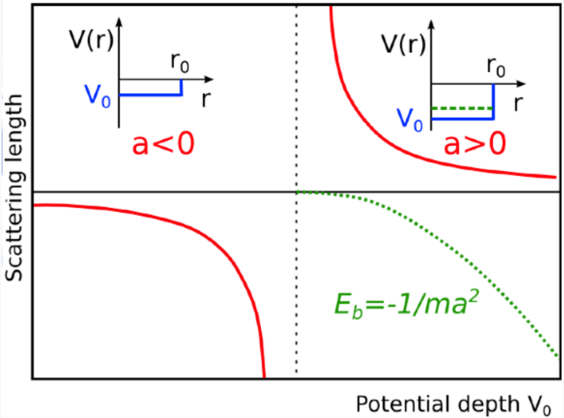


Baryon Energies



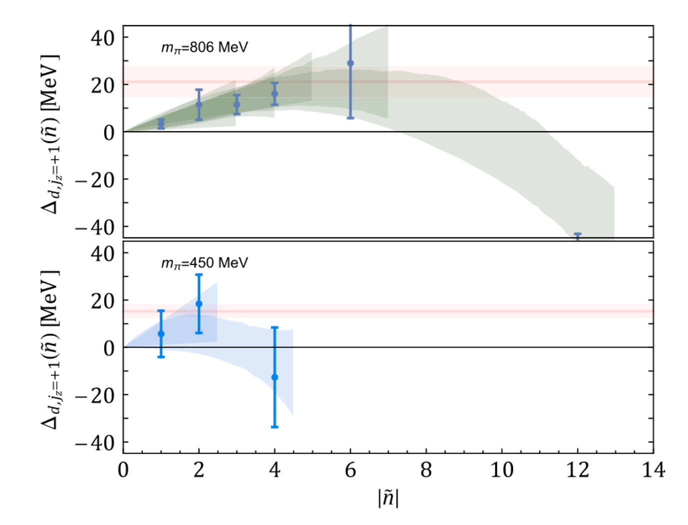
Two-Nucleon Interactions in Strong Magnetic Fields (Work in progress!)

- Unitary limit of two-nucleon interactions
- Scattering length *a ! 1*



Two-Nucleon Interactions in Strong Magnetic Fields

- LQCD results (NPLQCD Collaboration) (Detmold et al., '16)



Two-Nucleon Interactions in Strong Magnetic Fields

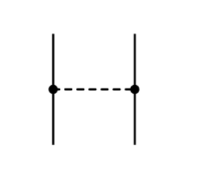
Yukawa potential ($B = 0$ case)

$$V(|\vec{x} - \vec{y}|) = \int_{-\infty}^{\infty} d(x_4 - y_4) G_4(x; y) = \int_0^{\infty} \frac{ds}{(4\pi s)^{3/2}} e^{-m^2 s} \exp\left(-\frac{(\vec{x} - \vec{y})^2}{4s}\right)$$

$$= \frac{e^{-m|\vec{x} - \vec{y}|}}{4\pi |\vec{x} - \vec{y}|}$$

Yukawa potential in cylindrical coordinates

$$V(\vec{\rho}; \vec{\rho}\,') = \int_0^{\infty} \frac{ds}{(4\pi s)^{\frac{3}{2}}} e^{-m^2 s} \exp\left(-\frac{x_3^2}{4s} - \frac{\rho^2 + \rho'^2}{4s}\right) \sum_{m=-\infty}^{\infty} e^{im\phi} I_{|m|}\left(\frac{\rho\rho'}{2s}\right)$$

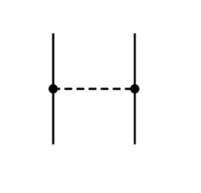


Two-Nucleon Interactions in Strong Magnetic Fields

Yukawa potential ($B \neq 0$ case)

$$V(\vec{r};\vec{r}\,';B) = -\frac{1}{\sqrt{4\pi}} \int_0^{\infty} \frac{ds}{(\pi s)^{\frac{1}{2}}} e^{-m^2 s} \exp\left(-\frac{x_3^2}{4s}\right) \frac{eQB\sqrt{z}}{2\pi(1-z)} e^{-\frac{\rho^2+\rho'^2}{2}\frac{1+z}{1-z}} \sum_{m=-\infty}^{\infty} \frac{e^{im\phi}}{z^{m/2}} I_{|m|}\left(\frac{2\sqrt{\rho\rho' z}}{1-z}\right)$$

where, $z = exp(-2|eQB|s)$ and $\xi = \frac{|eQB|\,\rho^{2}}{2}$.

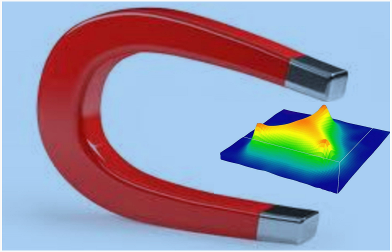


Summary and Outlook

- Energies of the octet baryons in large, uniform magnetic fields using heavy baryon PT.
- Issues with the large values of the magnetic polarizabilities
- U-spin predictions
- Interesting behavior of two-nucleon interaction

Acknowledgement / Thanks!

- NP@CCNY: Brian C. Tiburzi, Johannes Kirscher.
- NPLQCD Collaboration.



Thank you for your attention!

Decuplet Contribution

- Decuplet baryon propagator

$$[D_T(x;y)] = P \quad ^{(3)}(\vec{x} \quad \vec{y})\ (x_4 \quad y_4) e^{\quad (x_4 \quad y_4)};$$

where, $P = \quad v \; v \quad \frac{4}{3} S \; S$

- Decuplet contribution

$$D_B(x^0;x) = [D_T(x^0;x)]_{ij} D^0_i D_j G \; (x^0;x)$$

The three state mixing