

Baryons and Interactions in Magnetic Fields (PhysRevD.97.014006)

Amol Deshmukh

The Graduate Center, CUNY
The City College of New York, CUNY

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Outline

- ▶ Motivation
- ▶ χ PT in Large Magnetic Fields
- ▶ Octet Baryon Energies in Magnetic Fields
- ▶ Magnetic Polarizabilities
- ▶ Unitary Limit of Two-Nucleon Interactions
- ▶ Summary & Outlook

Magnetic Field scales

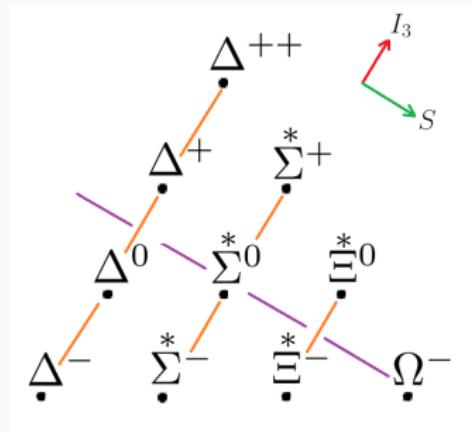
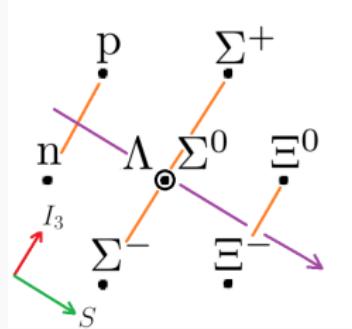
$$\frac{e}{2M_N} = 3.152 \times 10^{-14} \text{ MeV } T^{-1}$$

at $eB \approx m_\pi^2$:

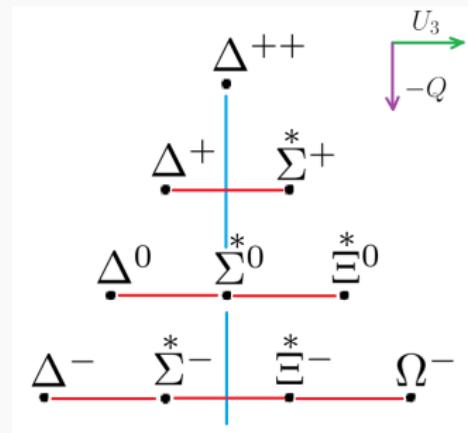
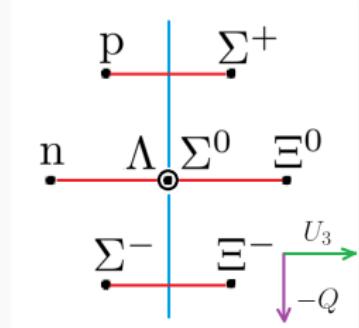
$$eB \approx 3 \times 10^{14} \text{ T} \approx 3 \times 10^{18} \text{ G}$$



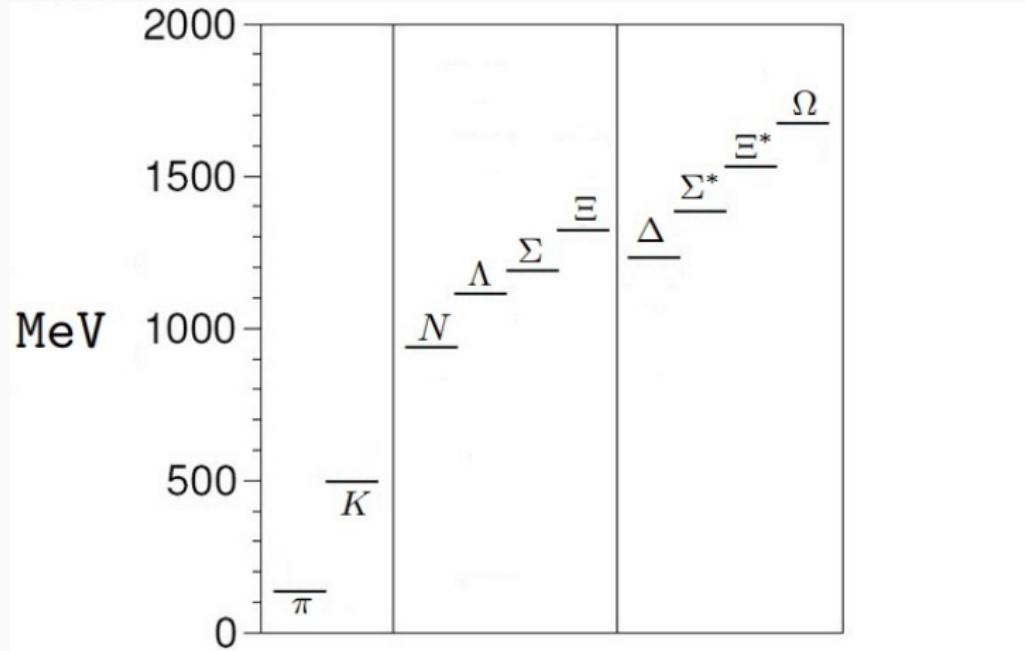
Low-Lying Baryons (Isospin multiplets)



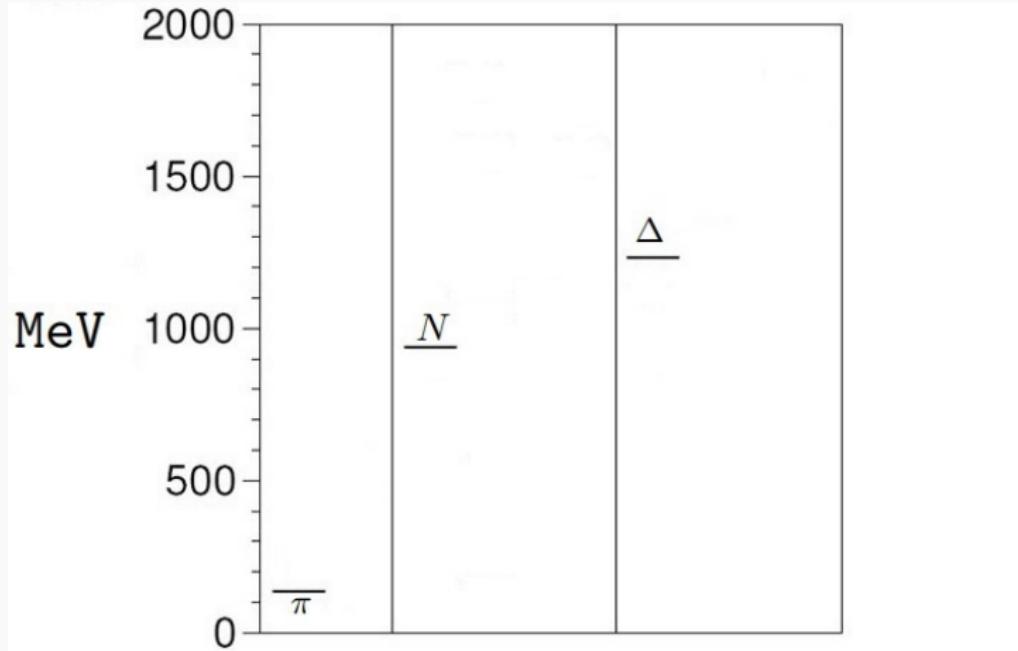
Low-Lying Baryons (U -spin multiplets)



Mass Spectrum



Mass Spectrum (Special case)



Motivation

LQCD computations

- ▶ Background magnetic field calculations of electromagnetic properties
- ▶ Quantization due to prohibitive size of the lattices (t' Hooft, '79)

$$eB = \frac{6\pi n}{L^2}$$

Example: $32^3 \times 48$ lattice with $a = 0.11\text{fm}$, produces $\approx 10^{18}\text{G}$

Physical environments

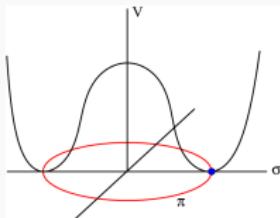
- ▶ **Magnetars:** Magnetic fields on the surface of (and within) highly magnetized neutron stars $\approx 10^{12} - 10^{15}\text{G}$ (Duncan & Thomson, '92)
- ▶ **Relativistic Heavy-ion collision:**
Non-central collisions $\approx 10^{19}\text{G}$ (Kharzeev et al., '08)

Chiral Perturbation Theory

- ▶ χPT is an effective field theory that makes use of the chiral symmetry of the underlying theory (QCD); $\frac{m_i}{\Lambda_{QCD}} = 0$ ($i = u, d, s$)
- ▶ $SU(3)_L \times SU(3)_R \rightarrow SU(3)_V$ due to the formation of the chiral condensate $\langle \bar{\psi} \psi \rangle$
- ▶ The fluctuations about the condensate (i.e. vev) are parameterized by the coset space

$$SU(3)_L \times SU(3)_R / SU(3)_V$$

and, these Goldstone bosons are identified with the low lying pseudo-scalar degrees of freedom (i.e. octet mesons)

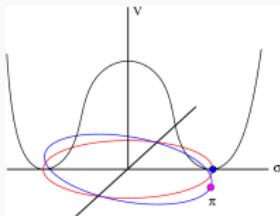


Chiral Perturbation Theory

- ▶ χPT is an effective field theory that makes use of the chiral symmetry of the underlying theory (QCD); $\frac{m_i}{\Lambda_{QCD}} \approx 0$ ($i = u, d, s$)
- ▶ $SU(3)_L \times SU(3)_R \rightarrow SU(3)_V$ due to the formation of the chiral condensate $\langle \bar{\psi} \psi \rangle$
- ▶ The fluctuations about the condensate (i.e. vev) are parameterized by the coset space

$$SU(3)_L \times SU(3)_R / SU(3)_V$$

and, these **pseudo Goldstone bosons** are identified with the low lying pseudo-scalar degrees of freedom (i.e. octet mesons)



Meson Chiral Perturbation Theory

- ▶ Goldstone bosons are parameterized as

$$\Sigma = \exp\left(\frac{2i\phi}{f_\phi}\right),$$

where,

$$\phi_j^i = \begin{pmatrix} \frac{1}{\sqrt{2}}\pi^0 + \frac{1}{\sqrt{6}}\eta & \pi^+ & K^+ \\ \pi^- & -\frac{1}{\sqrt{2}}\pi^0 + \frac{1}{\sqrt{6}}\eta & K^0 \\ K^- & \bar{K}^0 & -\frac{2}{\sqrt{6}}\eta \end{pmatrix}_j^i.$$

where f_ϕ is the three-flavor chiral-limit value of the pseudoscalar meson decay constant.

- ▶ We assume the power counting

$$\frac{k^2}{\Lambda_\chi^2} \sim \frac{m_\phi^2}{\Lambda_\chi^2} \sim \epsilon^2,$$

- ▶ The $\mathcal{O}(\epsilon^2)$ terms in the (Euclidean) Lagrangian density are

$$\mathcal{L} = \frac{f^2}{8} \text{Tr}(\partial_\mu \Sigma^\dagger \partial_\mu \Sigma) - \lambda \text{Tr}(m_q^\dagger \Sigma + m_q \Sigma^\dagger),$$

Meson Chiral Perturbation Theory in External Magnetic Fields

- ▶ Goldstone bosons are parameterized as

$$\Sigma = \exp\left(\frac{2i\phi}{f_\phi}\right),$$

where,

$$\phi_j^i = \begin{pmatrix} \frac{1}{\sqrt{2}}\pi^0 + \frac{1}{\sqrt{6}}\eta & \pi^+ & K^+ \\ \pi^- & -\frac{1}{\sqrt{2}}\pi^0 + \frac{1}{\sqrt{6}}\eta & K^0 \\ K^- & \bar{K}^0 & -\frac{2}{\sqrt{6}}\eta \end{pmatrix}_j^i.$$

where f_ϕ is the three-flavor chiral-limit value of the pseudoscalar meson decay constant.

- ▶ We assume the power counting

$$\frac{k^2}{\Lambda_\chi^2} \sim \frac{m_\phi^2}{\Lambda_\chi^2} \sim \frac{(eA_\mu)^2}{\Lambda_\chi^2} \sim \frac{eF_{\mu\nu}}{\Lambda_\chi^2} \sim \epsilon^2,$$

- ▶ The $\mathcal{O}(\epsilon^2)$ terms in the (Euclidean) Lagrangian density are

$$\mathcal{L} = \frac{f^2}{8} \text{Tr}(\textcolor{red}{D}_\mu \Sigma^\dagger \textcolor{red}{D}_\mu \Sigma) - \lambda \text{Tr}(m_q^\dagger \Sigma + m_q \Sigma^\dagger),$$

Heavy Baryon Chiral Perturbation Theory

- ▶ Power counting issue ($M_B \sim \Lambda_\chi$)
- ▶ Modified power counting ($p_\mu = M_B v_\mu + k_\mu$) (Jenkins & Manohar, '93)

$$\frac{k}{M_B} \sim \frac{\Delta}{M_B} \sim \epsilon$$

$\mathcal{O}(\epsilon)$ Octet baryon lagrangian

$$\mathcal{L} = -i \operatorname{Tr} (\bar{B} v \cdot \mathcal{D} B) + 2D \operatorname{Tr} (\bar{B} S_\mu \{A_\mu, B\}) + 2F \operatorname{Tr} (\bar{B} S_\mu [A_\mu, B])$$

$$B_j^i = \begin{pmatrix} \frac{1}{\sqrt{2}} \Sigma^0 + \frac{1}{\sqrt{6}} \Lambda & \Sigma^+ & p \\ \Sigma^- & -\frac{1}{\sqrt{2}} \Sigma^0 + \frac{1}{\sqrt{6}} \Lambda & n \\ \Xi^- & \Xi^0 & -\frac{2}{\sqrt{6}} \Lambda \end{pmatrix}_j^i$$

$\mathcal{O}(\epsilon)$ Decuplet baryon lagrangian

$$\mathcal{L} = \bar{T}_\mu (-iv \cdot \mathcal{D} + \Delta) T_\mu + 2\mathcal{H} \bar{T}_\mu S \cdot A T_\mu + 2\mathcal{C} (\bar{T} \cdot A B + \bar{B} A \cdot T)$$

where, T are embedded in the completely symmetric flavor tensor: T_{ijk} , for e.g. $T_{111} = \Delta^{++}$, $T_{112} = \frac{1}{\sqrt{3}} \Delta^+$

Heavy Baryon Chiral Perturbation Theory

- ▶ Chirally covariant derivative

$$(\mathcal{D}_\mu B)_j^i = \partial_\mu B_j^i + [\mathcal{V}_\mu, B]_j^i$$

$$(\mathcal{D}_\mu T)_{ijk} = \partial_\mu T_{ijk} + (\mathcal{V}_\mu)_i^{i'} T_{i'jk} + (\mathcal{V}_\mu)_j^{j'} T_{ij'k} + (\mathcal{V}_\mu)_k^{k'} T_{ijk'}$$

- ▶ Vector and axial-vector fields of mesons

$$\mathcal{V}_\mu = \frac{1}{2f^2} [\phi, \partial_\mu \phi] + \dots$$

$$\mathcal{A}_\mu = -\frac{1}{f} \partial_\mu \phi + \dots$$

Heavy Baryon Chiral Perturbation Theory in External Magnetic Field

- ▶ Chirally covariant derivative

$$(\mathcal{D}_\mu B)_j^i = \partial_\mu B_j^i + [\mathcal{V}_\mu, B]_j^i$$

$$(\mathcal{D}_\mu T)_{ijk} = \partial_\mu T_{ijk} + (\mathcal{V}_\mu)_i^{i'} T_{i'jk} + (\mathcal{V}_\mu)_j^{j'} T_{ij'k} + (\mathcal{V}_\mu)_k^{k'} T_{ijk'}$$

- ▶ Vector and axial-vector fields of mesons

$$\mathcal{V}_\mu = ieA_\mu Q + \frac{1}{2f^2} [\phi, D_\mu \phi] + \dots$$

$$A_\mu = -\frac{1}{f} D_\mu \phi + \dots$$

Propagators

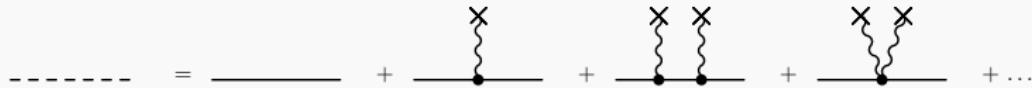
- ▶ Mesons

$$G_\phi(x, y) = \int_0^\infty \frac{ds}{(4\pi s)^2} e^{-m_\phi^2 s} \exp \left[-\frac{(x-y)^2}{4s} \right]$$

- ▶ Octet baryons

$$D_B(x, y) = \delta^{(3)}(\vec{x} - \vec{y}) \theta(x_4 - y_4)$$

Propagators in External Magnetic Field



- ▶ Mesons (Schwinger, '51)

$$G_\phi(x, y, B) = e^{ieQ_\phi B \Delta x_1 \bar{x}_2} \int_0^\infty \frac{ds}{(4\pi s)^2} e^{-m_\phi^2 s} \frac{eQ_\phi Bs}{\sinh(eQ_\phi Bs)} \\ \times \exp \left[-\frac{eQ_\phi B \Delta \vec{x}_\perp^2}{4 \tanh(eQ_\phi Bs)} - \frac{\Delta x_3^2 + \Delta x_4^2}{4s} \right],$$

- ▶ This propagator satisfies

$$(-D_\mu D_\mu + m^2) G_\phi(x, y, B) = \delta(x - y),$$

- ▶ Octet baryons propagators will enable us to calculate energies which involve tree and loop contributions.

Octet Baryon Energies (Tree-Level Contributions)

Local operators that contribute to $\mathcal{O}(\epsilon^2)$

Octet baryon magnetic moment operators

(Coleman & Glashow '61)

$$\mathcal{L} = \frac{e}{2M_N} [\mu_D \text{Tr} (\bar{B} S_{\mu\nu} \{Q, B\}) + \mu_F \text{Tr} (\bar{B} S_{\mu\nu} [Q, B])] F_{\mu\nu},$$

where, $S_{\mu\nu} = \epsilon_{\mu\nu\alpha\beta} v_\alpha S_\beta$.

Kinetic-energy term (Landau Levels)

$$\mathcal{L} = -\text{Tr} \left(\bar{B} \frac{\mathcal{D}_\perp^2}{2M_B} B \right),$$

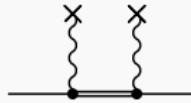
where $(\mathcal{D}_\perp)_\mu = \mathcal{D}_\mu - v_\mu(v \cdot \mathcal{D})$.

Octet Baryon Energies (Tree-Level Contributions)

Local operator that contribute to $\mathcal{O}(\epsilon^3)$

Magnetic dipole transition operator

$$\mathcal{L} = \mu_U \sqrt{\frac{3}{2}} \frac{ie}{M_N} (\bar{B} S_\mu Q T_\nu + \bar{T}_\mu Q S_\nu B) F_{\mu\nu}.$$



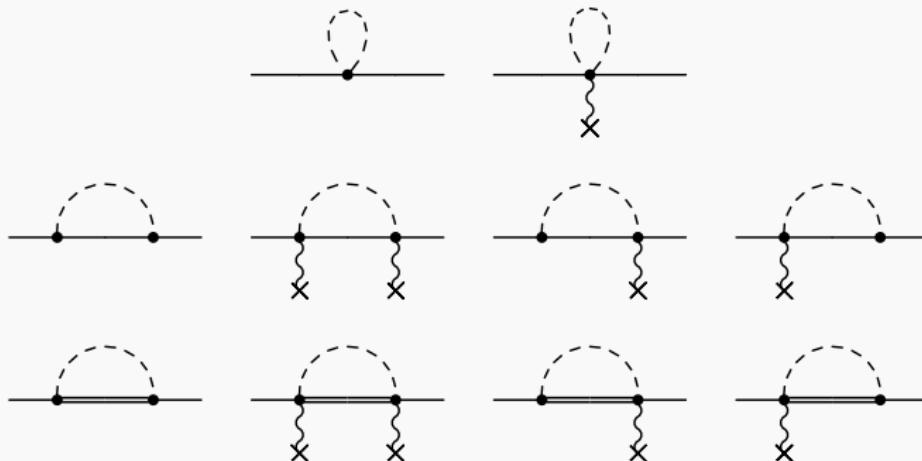
μ_U can be determined using the measured values for the electromagnetic decay widths of the decuplet baryons. (Keller et al. (CLAS), '11, '12)

$$\Gamma(T \rightarrow B\gamma) = \alpha_T \frac{\omega^3}{2\pi} \frac{M_B}{M_T} \left(\frac{e \mu_U}{2M_N} \right)^2$$

where,

$$\omega = \frac{M_T^2 - M_B^2}{2M_T}$$

Octet Baryon Energies (Meson-Loop Contributions)



Octet Baryon Energies (Meson-Loop Contributions, Continued...)

- ▶ Spin-dependent/Spin-independent contributions

$$\delta E = -eB\sigma_3 \delta E_1 + \delta E_2$$

Explicit expressions

$$\delta E_1 = \sum_{\mathcal{B}} \mathcal{A}_{\mathcal{B}} \frac{Q_{\phi} m_{\phi}}{(4\pi f_{\phi})^2} F_1 \left(\frac{|eB|}{m_{\phi}^2}, \frac{\Delta_{\mathcal{B}}}{m_{\phi}} \right)$$

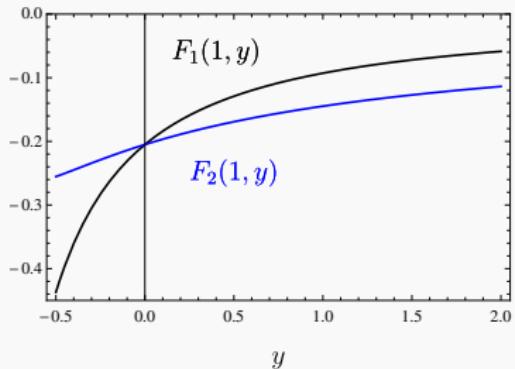
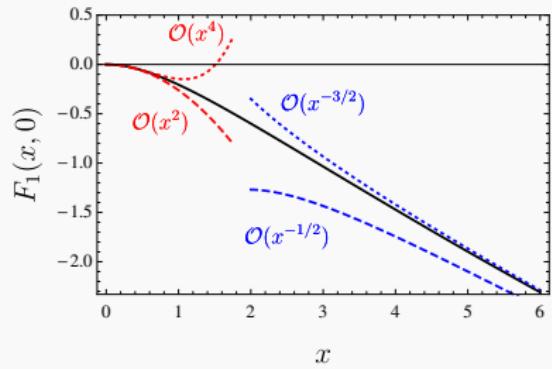
$$\delta E_2 = \sum_{\mathcal{B}} \mathcal{A}_{\mathcal{B}} \frac{m_{\phi}^3}{(4\pi f_{\phi})^2} F_2 \left(\frac{|eB|}{m_{\phi}^2}, \frac{\Delta_{\mathcal{B}}}{m_{\phi}} \right)$$

$$F_1(x, y) = \int_0^\infty ds \ f(x, s) g(y, s)$$

where

$$f(x, s) = \frac{\pi^{1/2}}{s^{3/2}} \left(\frac{xs}{\sinh(xs)} - 1 \right) \quad g(y, s) = e^{-s(1-y^2)} \text{Erfc}(y\sqrt{s}).$$

Behavior of loop functions



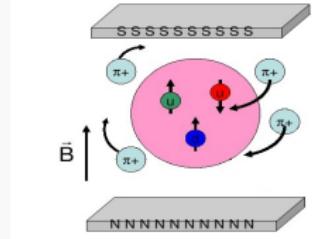
where, $x = \frac{|eB|}{m_\phi^2}$ and $y = \frac{\Delta_B}{m_\phi}$.

Complete Third-order Calculation

- ▶ ($I_3 \neq 0$) Octet baryon energies valid to $\mathcal{O}(\epsilon^3)$

$$E = M_B + \frac{|QeB|}{2M_B} - eB\sigma_3 \left(\frac{\mu_B}{2M_N} + \delta E_1 \right) - \frac{\alpha_T \mu_U^2}{\Delta_T} \left(\frac{eB}{2M_N} \right)^2 + \delta E_2$$

Magnetic Polarizabilities (Detour)



Empirical values

(C. Patrignani et al. (PDG), '17)

$$\text{Proton : } 2.5(4) \times 10^{-4} \text{ fm}^3$$

$$\text{Neutron : } 3.7(12) \times 10^{-4} \text{ fm}^3$$

- ▶ The spin-averaged energy:

$$\bar{E} = M + \frac{|QeB|}{2M} - \frac{1}{2} 4\pi (\beta^{\text{lp}} + \beta^{\text{tr}}) B^2 + \dots$$

$$\beta^{\text{lp}} = \frac{\alpha}{6} \sum_{\mathcal{B}} \frac{\mathcal{A}_{\mathcal{B}} \mathcal{S}_{2\mathcal{B}}}{m_\phi (4\pi f_\phi)^2} \mathcal{G} \left(\frac{\Delta_{\mathcal{B}}}{m_\phi} \right)$$

$$\beta^{\text{tr}} = \frac{\alpha \tau}{2\pi \Delta \tau} \left(\frac{e \mu_U}{2M_N} \right)^2$$

Magnetic Polarizabilities (Contributions)

B	β^{lp}	β^{tr}	β^{ct}	$\beta^{\text{lp}} + \beta^{\text{tr}} + \beta^{\text{ct}}$	$\beta_M^{\text{expt.}}$
p	1.37	6.89	-5.76	* 2.50	2.5(4)
n	1.32	6.89	-4.51	* 3.70	3.7(12)
Σ^+	0.96	14.05	-5.76	9.26	—

Counterterm Promotion

- ▶ Higher order (short distance) operators as counterterms

$$\Delta \mathcal{L} = -\frac{1}{2} 4\pi B^2 \sum_{i=1}^4 \beta_i^{\text{ct}} \mathcal{O}_i,$$

where,

$$\begin{aligned}\mathcal{O}_1 &= \text{Tr}(\bar{B}B) \text{ Tr}(Q^2), \\ \mathcal{O}_2 &= \text{Tr}(\bar{B}\{Q, \{Q, B\}\}), \\ \mathcal{O}_3 &= \text{Tr}(\bar{B}\{Q, [Q, B]\}), \\ \mathcal{O}_4 &= \text{Tr}(\bar{B}[Q, [Q, B]])\end{aligned}$$

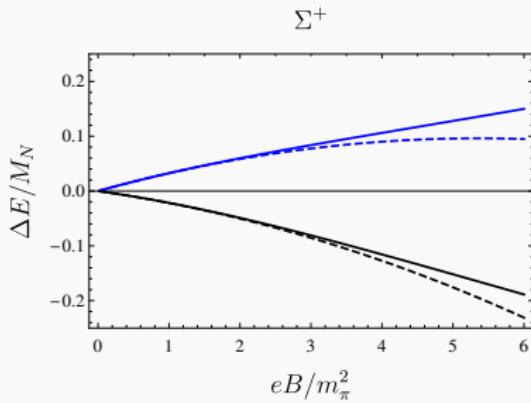
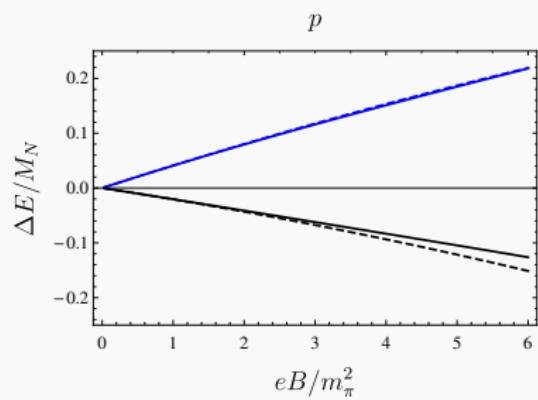
- ▶ Coleman-Glashow like relations for counterterm coefficients:

$$\beta_p^{\text{ct}} = \beta_{\Sigma^+}^{\text{ct}}, \quad \beta_n^{\text{ct}} = \beta_{\Xi^0}^{\text{ct}}, \quad \beta_{\Sigma^-}^{\text{ct}} = \beta_{\Xi^-}^{\text{ct}}$$

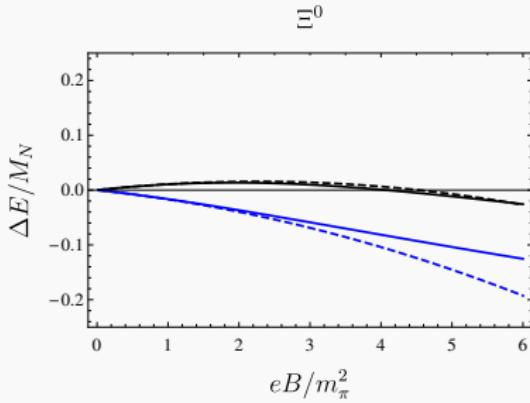
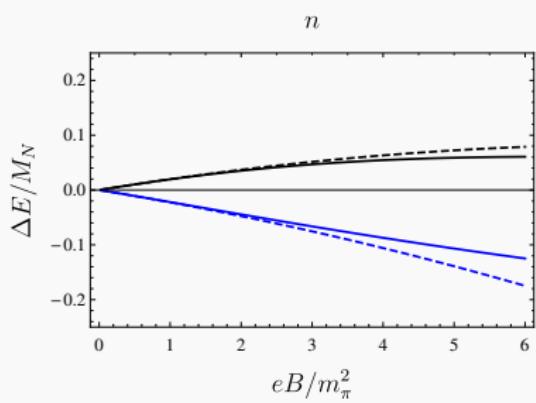
and

$$\frac{1}{\sqrt{3}} \beta_{\Sigma^0 \Lambda}^{\text{ct}} = \beta_{\Lambda}^{\text{ct}} - \beta_n^{\text{ct}} = \frac{1}{2} (\beta_{\Sigma^0}^{\text{ct}} - \beta_{\Lambda}^{\text{ct}})$$

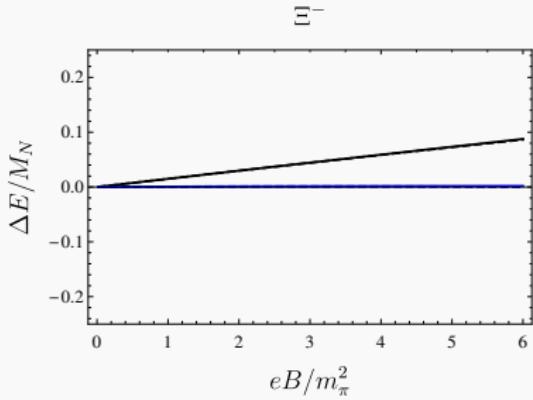
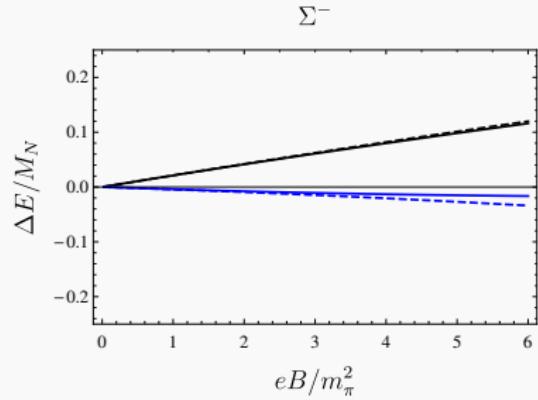
Baryon Energies



Baryon Energies

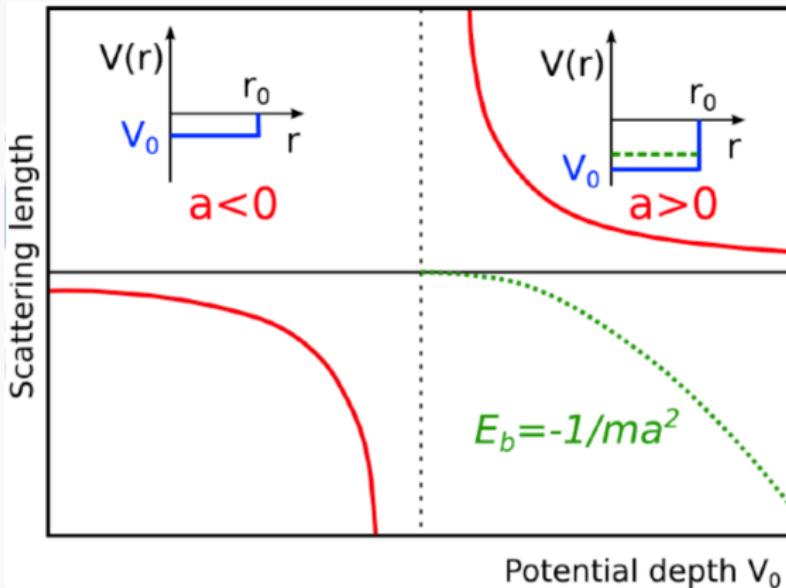


Baryon Energies



Two-Nucleon Interactions in Strong Magnetic Fields (Work in progress!)

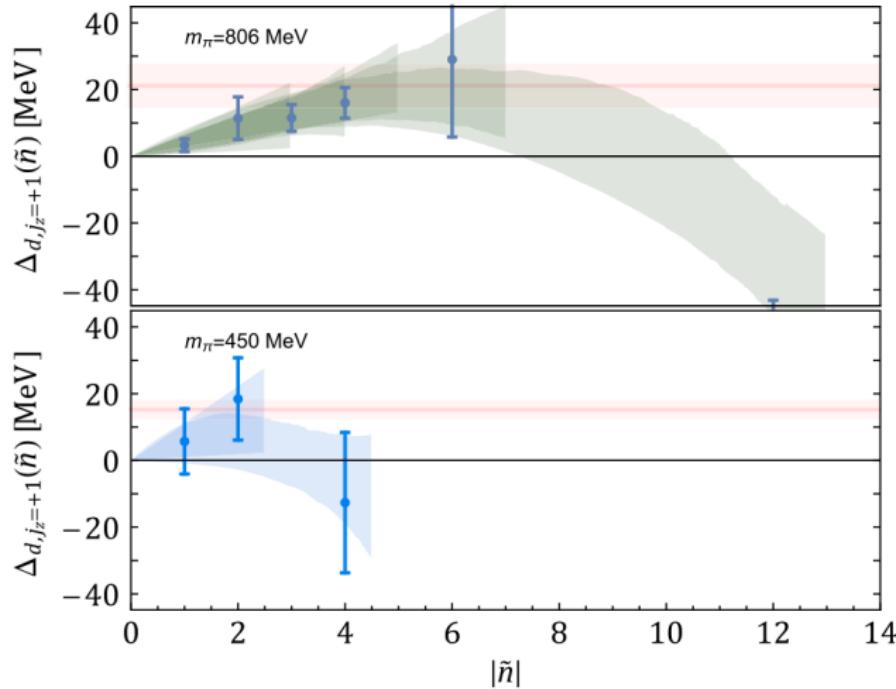
- ▶ Unitary limit of two-nucleon interactions
- ▶ Scattering length $a \rightarrow \infty$



Two-Nucleon Interactions in Strong Magnetic Fields

► LQCD results (NPLQCD Collaboration)

(Detmold et al., '16)



Two-Nucleon Interactions in Strong Magnetic Fields

- Yukawa potential ($B = 0$ case)

$$\begin{aligned} -V(|\vec{x} - \vec{y}|) &= \int_{-\infty}^{\infty} d(x_4 - y_4) G_4(x, y) = \int_0^{\infty} \frac{ds}{(4\pi s)^{3/2}} e^{-m^2 s} \exp \left[-\frac{(\vec{x} - \vec{y})^2}{4s} \right] \\ &= \frac{e^{-m|\vec{x} - \vec{y}|}}{4\pi|\vec{x} - \vec{y}|} \end{aligned}$$

- Yukawa potential in cylindrical coordinates

$$-V(\vec{\rho}, \vec{\rho}') = \int_0^{\infty} \frac{ds}{(4\pi s)^{\frac{3}{2}}} e^{-m^2 s} \exp \left[-\frac{\Delta x_3^2}{4s} - \frac{\rho^2 + \rho'^2}{4s} \right] \sum_{m=-\infty}^{m=\infty} e^{im\Delta\phi} I_{|m|} \left(\frac{\rho\rho'}{2s} \right)$$



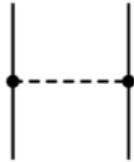
Two-Nucleon Interactions in Strong Magnetic Fields

- Yukawa potential ($B \neq 0$ case)

$$-V(\vec{\rho}, \vec{\rho}', B) = \frac{1}{\sqrt{4\pi}} \int_0^\infty \frac{ds}{(s)^{\frac{1}{2}}} e^{-m^2 s} \exp\left[-\frac{\Delta x_3^2}{4s}\right] \frac{eQB\sqrt{z}}{2\pi(1-z)} e^{\frac{\sigma+\sigma'}{2} \frac{1+z}{1-z}}$$

$$\times \sum_{m=-\infty}^{m=\infty} \frac{e^{im\Delta\phi}}{z^{m/2}} I_{|m|} \left(\frac{2\sqrt{\sigma'\sigma z}}{1-z} \right)$$

where, $z = \exp(-2|eQB|s)$ and $\sigma = \frac{|eQB|\rho^2}{2}$.

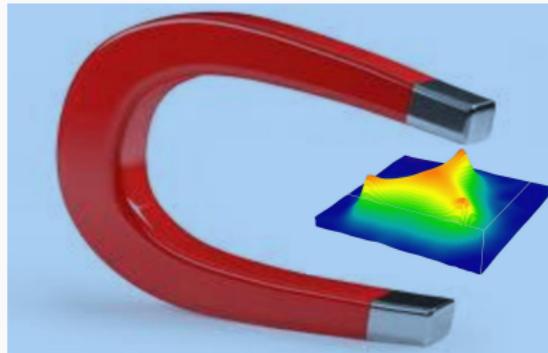


Summary and Outlook

- ▶ Energies of the octet baryons in large, uniform magnetic fields using heavy baryon χPT .
- ▶ Issues with the large values of the magnetic polarizabilities
- ▶ U -spin predictions
- ▶ Interesting behavior of two-nucleon interaction

Acknowledgement/Thanks!

- ▶ NP@CCNY: Brian C. Tiburzi, Johannes Kirscher.
- ▶ NPLQCD Collaboration.



Thank you for your attention!

Decuplet Contribution

- ▶ Decuplet baryon propagator

$$[D_T(x, y)]_{\mu\nu} = \mathcal{P}_{\mu\nu} \delta^{(3)}(\vec{x} - \vec{y}) \theta(x_4 - y_4) e^{-\Delta(x_4 - y_4)},$$

where, $\mathcal{P}_{\mu\nu} = \delta_{\mu\nu} - v_\mu v_\nu - \frac{4}{3} S_\mu S_\nu$

- ▶ Decuplet contribution

$$\delta D_B(x', x) = [D_T(x', x)]_{ij} D'_i D_j G_\phi(x', x)$$

The three state mixing

