Fate of a recent conformal fixed point and β-function in the $SU(3)$ BSM gauge theory with ten massless flavors

Daniel Nogradi

in collaboration with

Zoltan Fodor, Kieran Holland

Julius Kuti, Chik Him Wong

What, why and how?

$SU(3)$ gauge theory with $N_f = 10$ flavors
IR conformal or chirally broken?

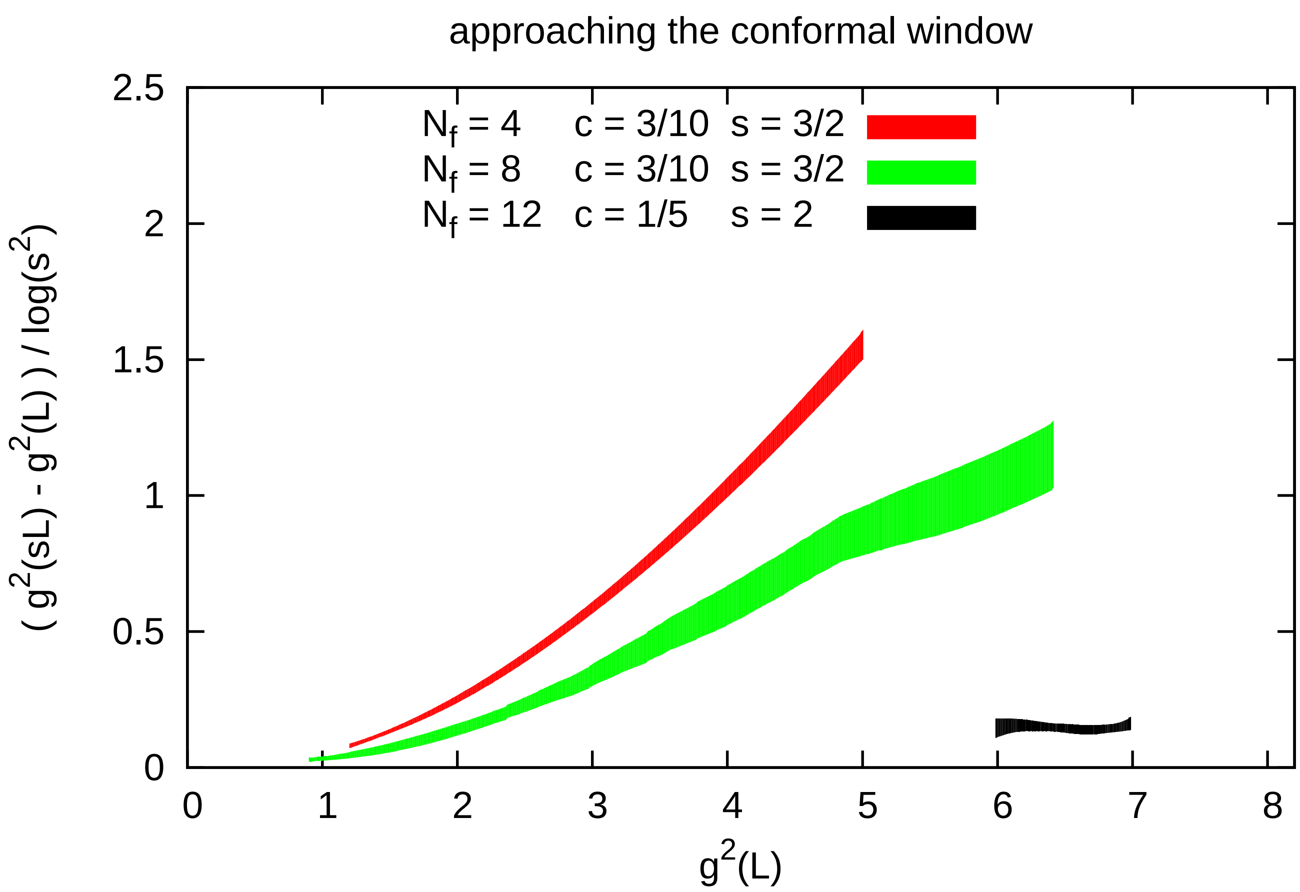
There was/is some controversy about $N_f = 12 \ldots\ldots$

$N_f = 10$ should be simpler

Relevant for model building: conformal walking: 4+6 model, tune masses $\rightarrow$ walking if 10 flavors conformal, if chirally broken: usual walking

What, why and how?

$N_f = 10$ interesting on its own



Last year: $N_f = 3$ sextet, $N_f = 14$ fund (both conformal, 1711.00130)

This conference: Kieran Holland: $N_f = 13$ fund (conformal)

What, why and how?

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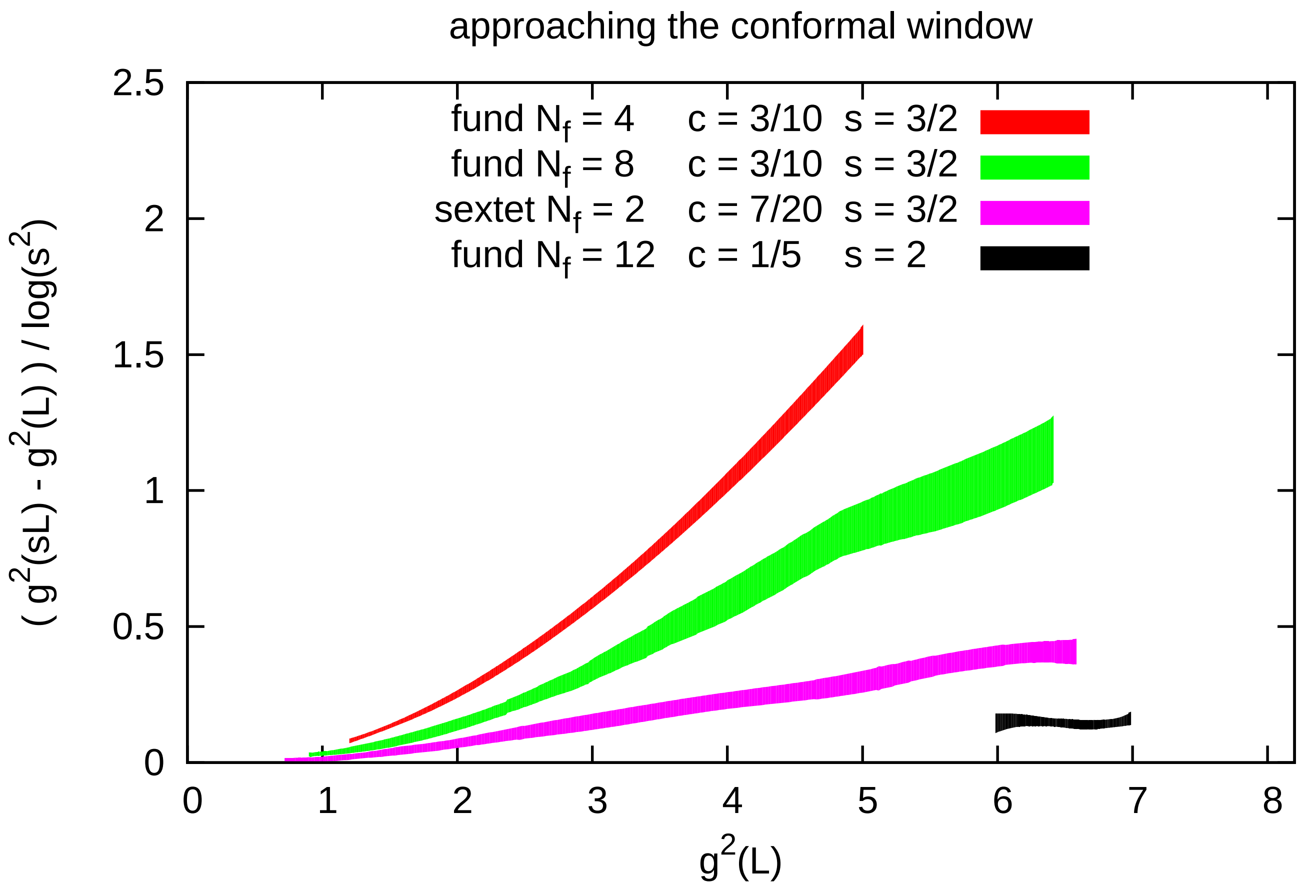

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What, why and how?

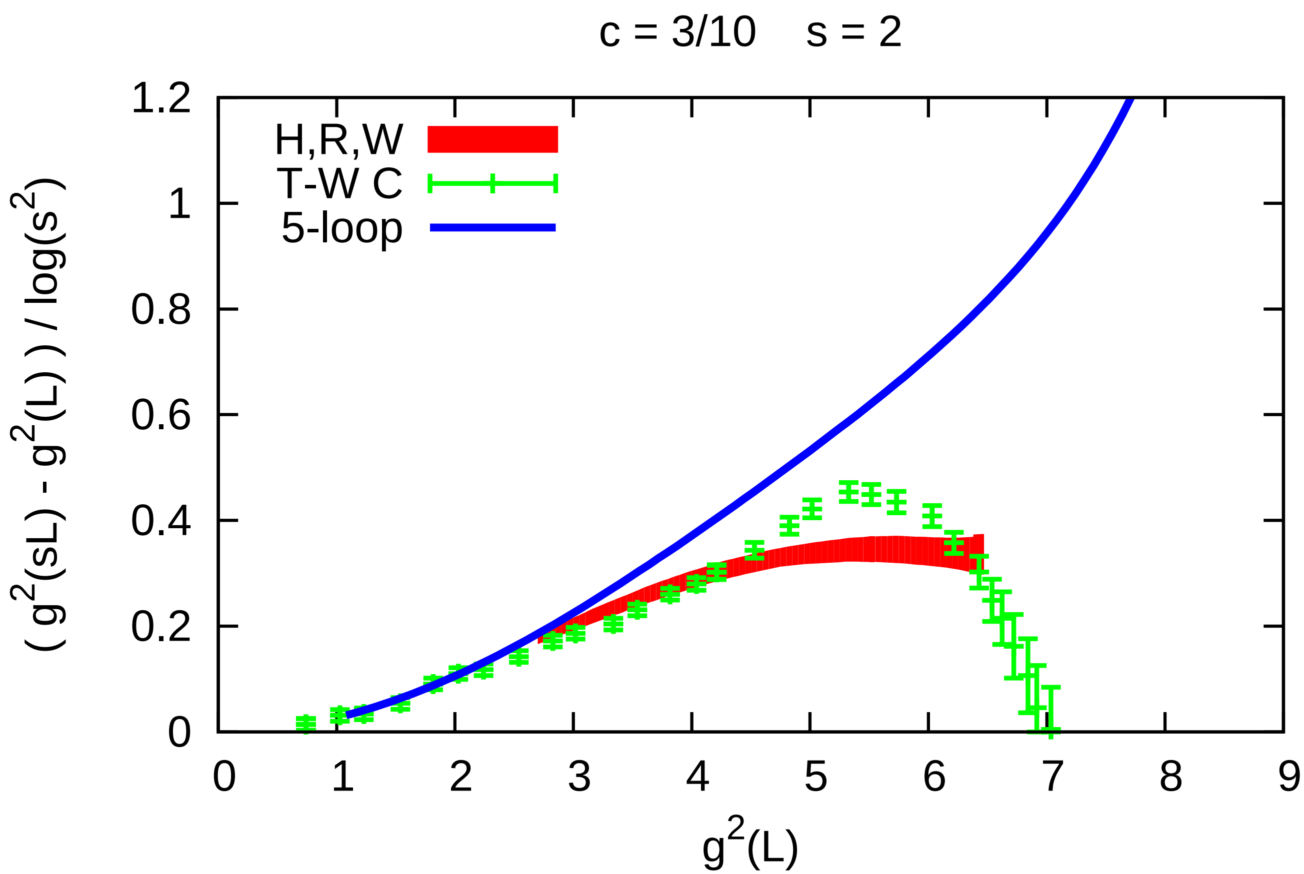
Calculate $N_f = 10$ running coupling, β-function in continuum

Periodic finite volume gradient flow scheme

Step scaling, $L \to 2L$, discrete β-function

What, why and how?

Results in literature - domain wall



Hasenfratz, Rebbi, Witzel: 1710.11578, $8 \to 16, 10 \to 20, 12 \to 24$

Chiu: PoS LATTICE2016 (2017) 228, $8 \to 16, 10 \to 20, 12 \to 24, 16 \to 32$

Discrepancy for $4.5 < g^2(L) < 6.0$

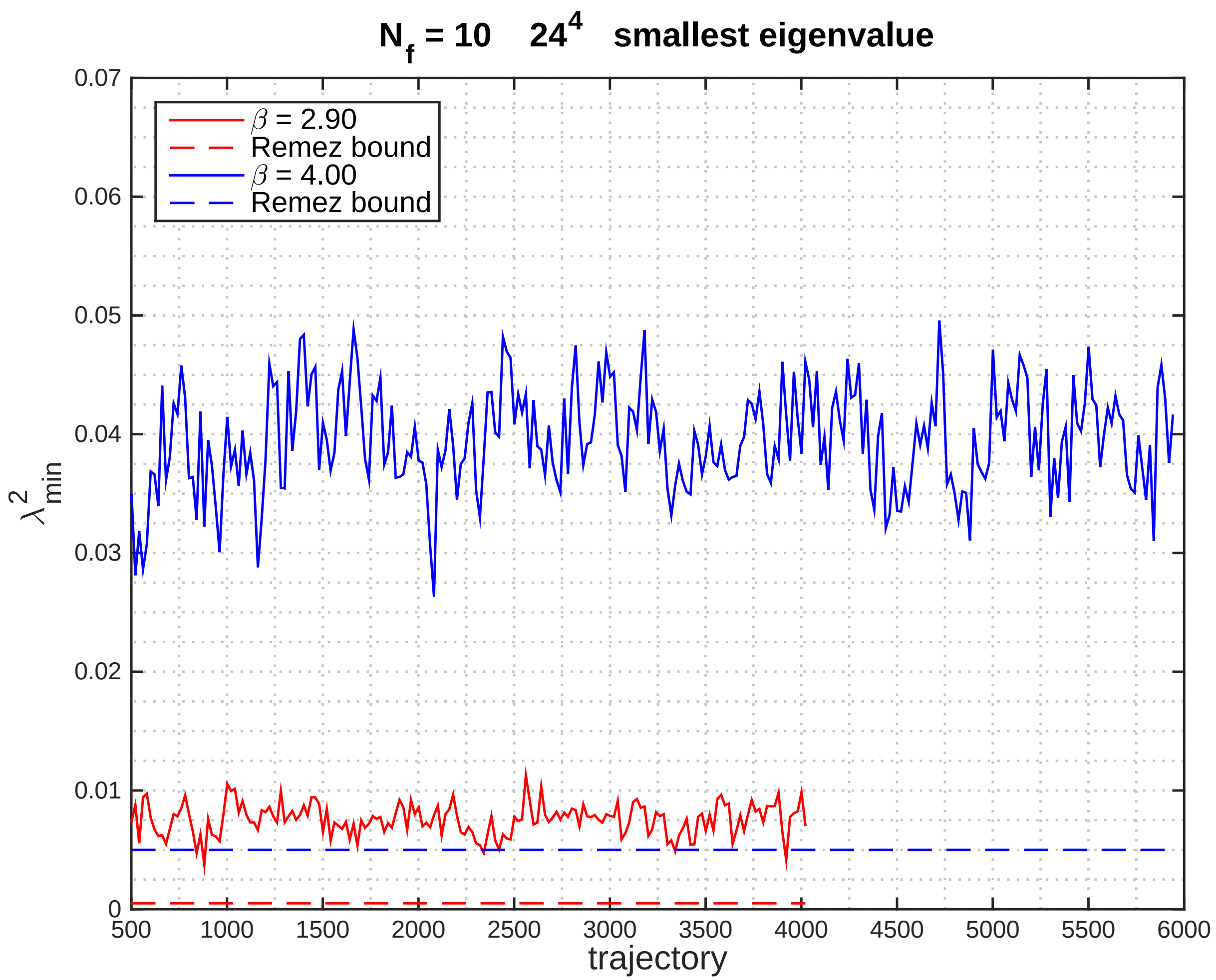
Outline

- Numerical setup
- Rooting, taste breaking, etc
- Continuum extrapolation
- Comparison with literature
- Conclusion and outlook

Numerical setup

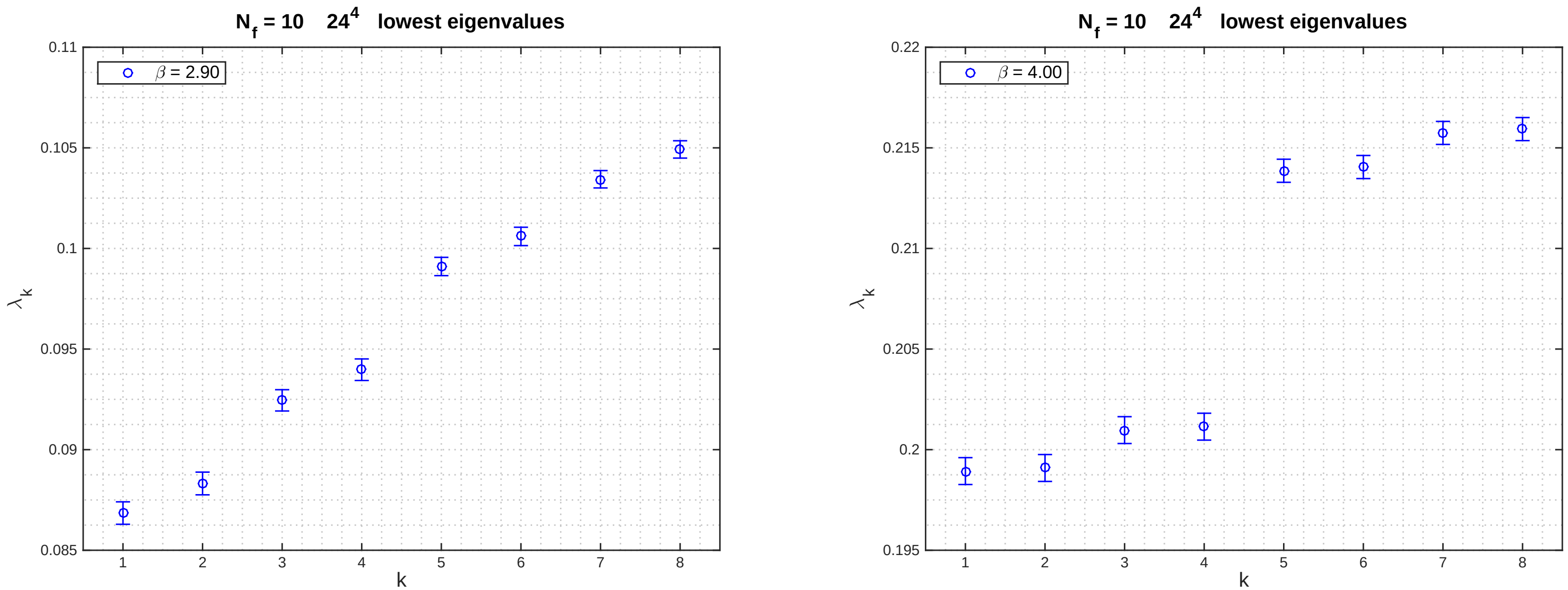
- Tree-level improved Symanzik gauge action
- Periodic gauge field
- 4-step stout-improved rooted staggered fermions ($\varrho = 0.12$)
- Anti-periodic in all directions
- $m = 0$
- $12 \to 24$, $16 \to 32$, $18 \to 36$, $20 \to 40$, $24 \to 48$

Rooting - eigenvalue gap - Remez algorithm



Infrared regulator $1/L$ acts similarly to m in large volumes
$\rightarrow$ stable algorithm

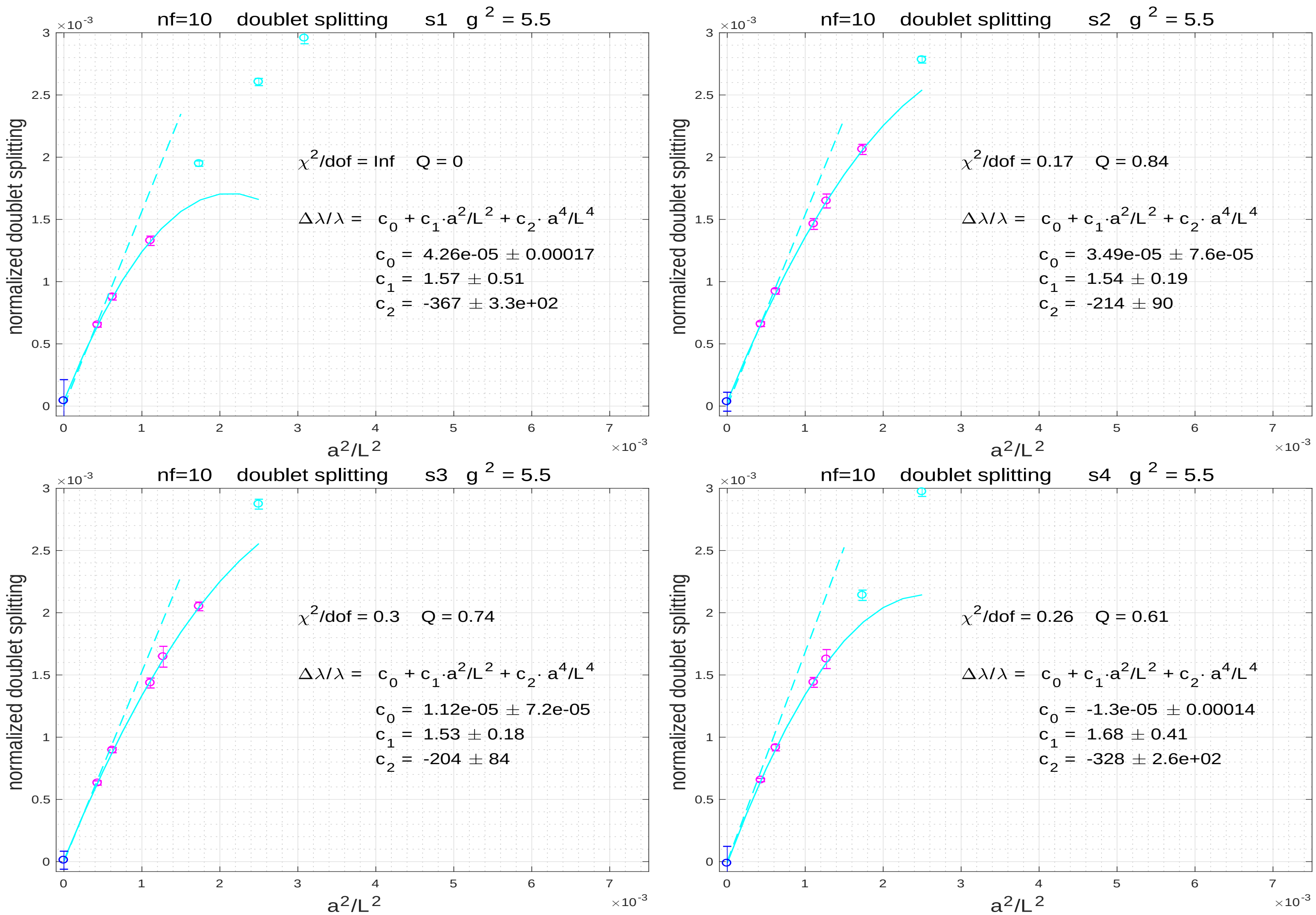
Rooting - taste breaking in Dirac eigenvalues



Lowest 8 eigenvalues

First (low β): doublets, then (high β): quartets

Rooting - taste breaking in Dirac eigenvalues

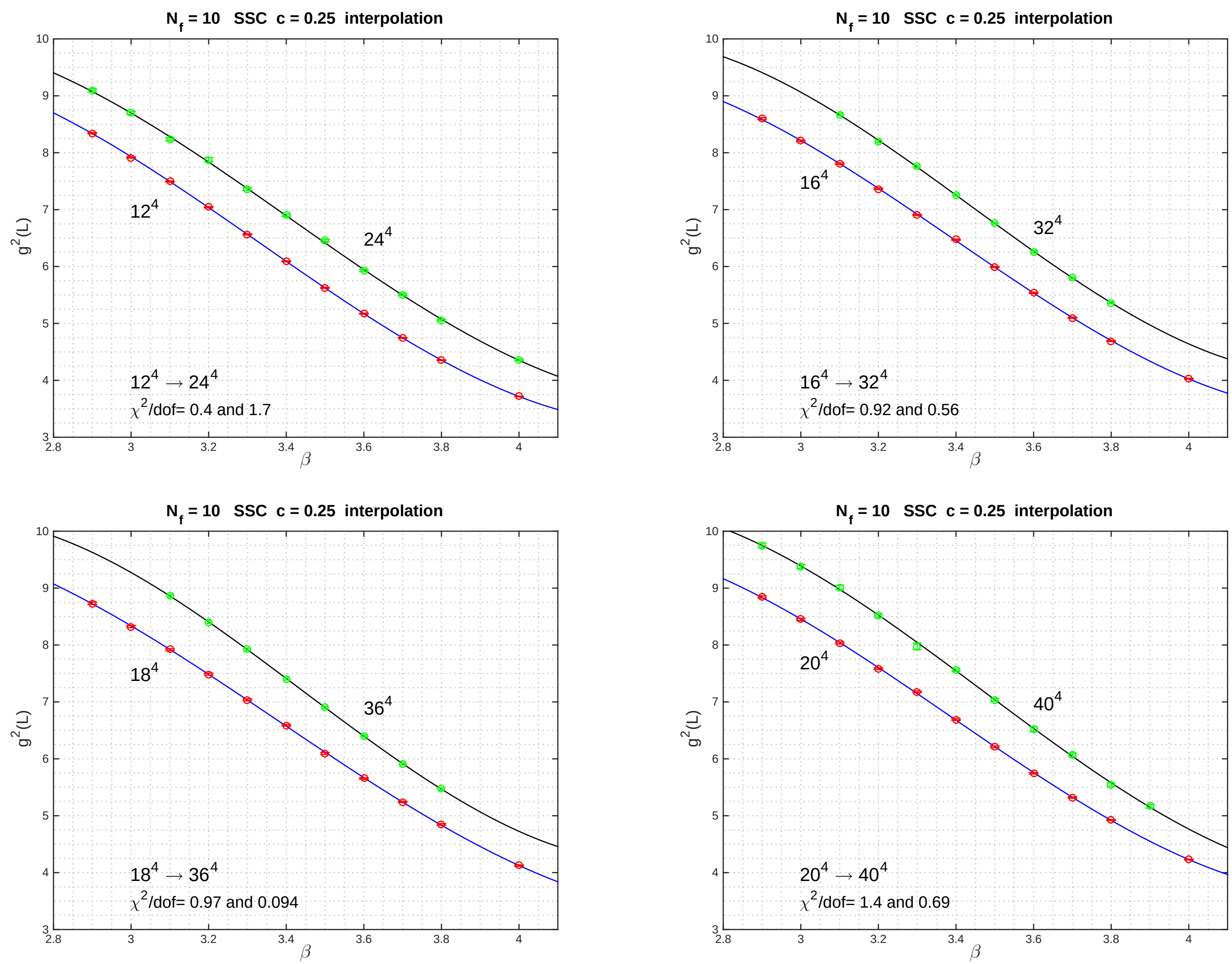


Fix $g^2(L) = 5.5$, taste breaking disappears in the continuum

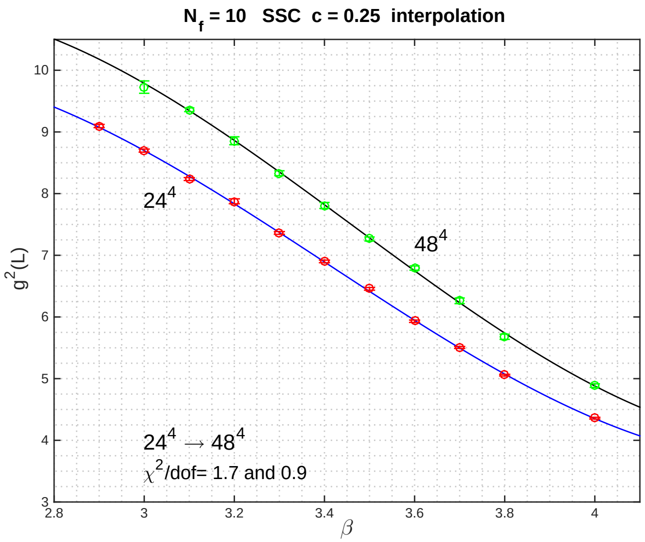
Continuum extrapolation

- Interpolate by polynomials (rather than tune)
- Larger c: smaller cut-off effects, larger stat errors (knew this already)
- Take $c = 1/4,\ 3/10$ and $s = 2$ (also $s = 3/2$)
- Check consistency of SSC and WSC discretizations

Continuum extrapolation



Continuum extrapolation



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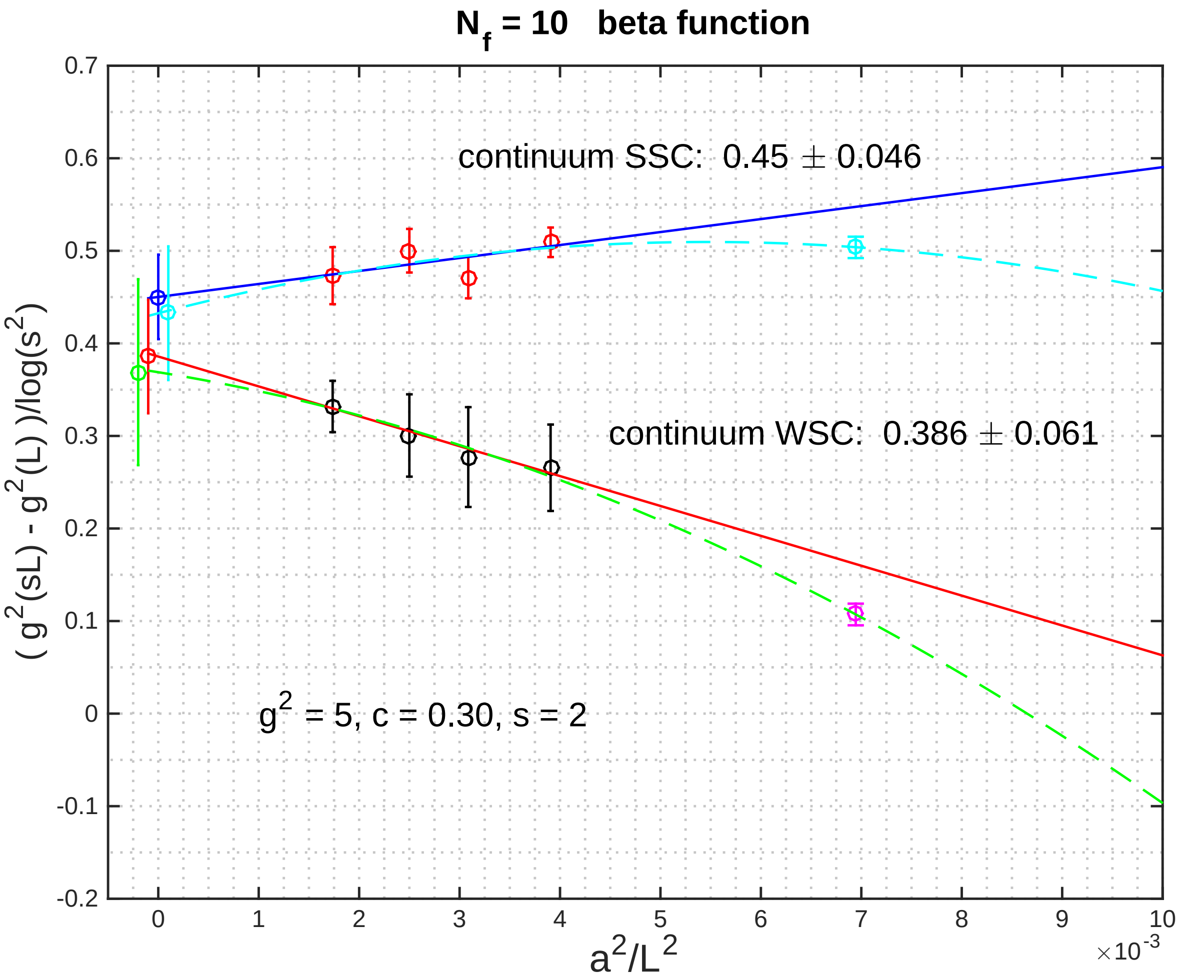
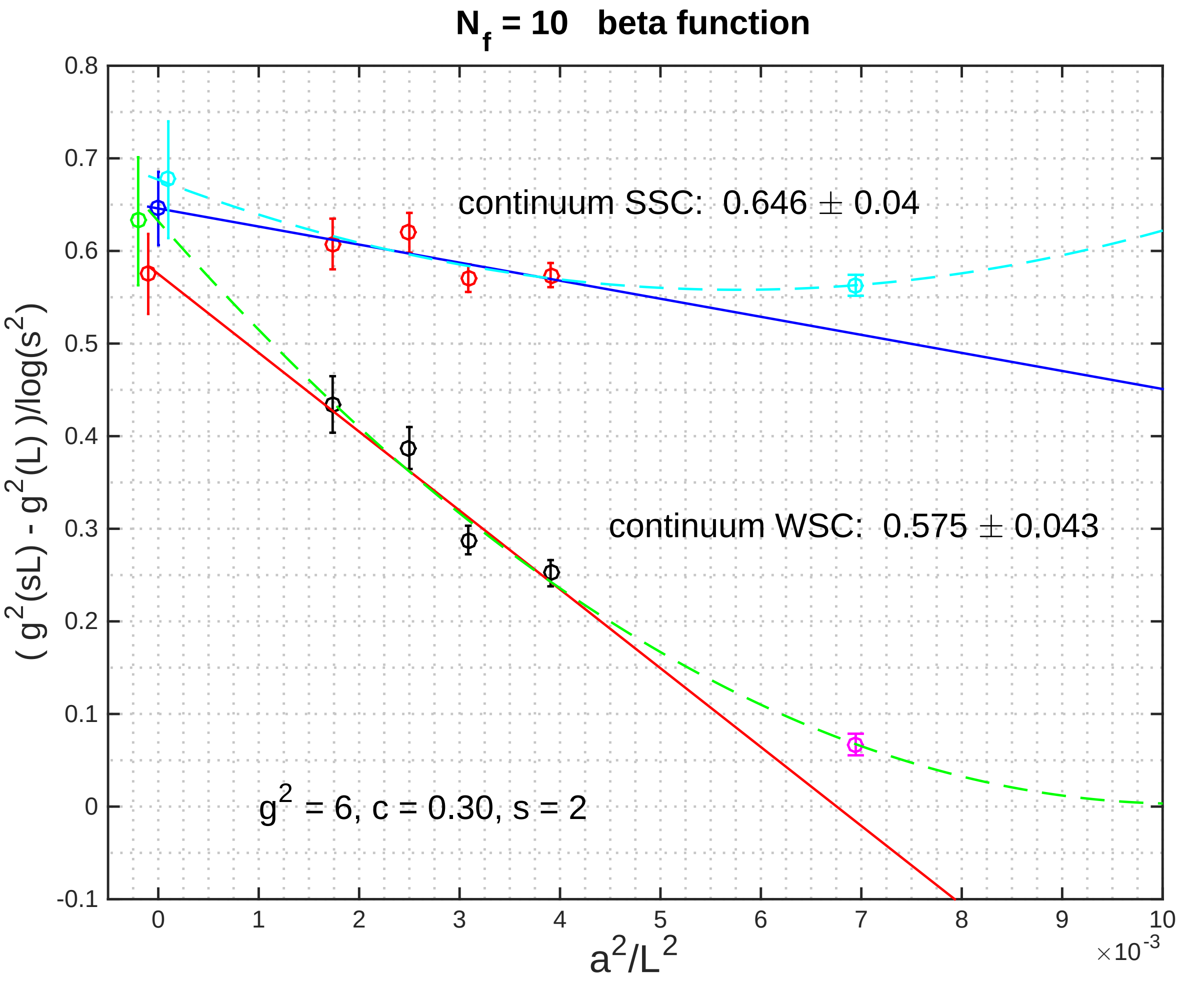
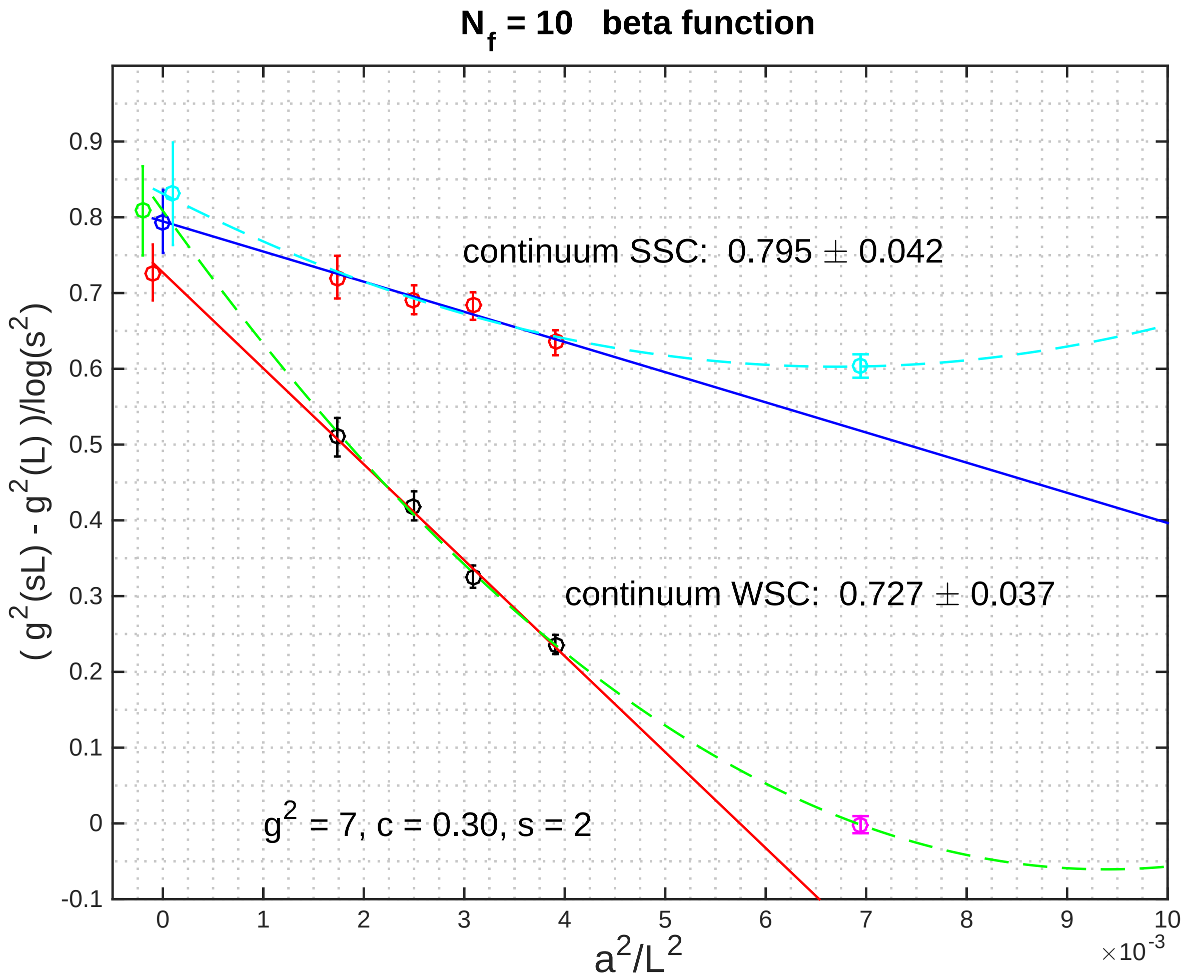


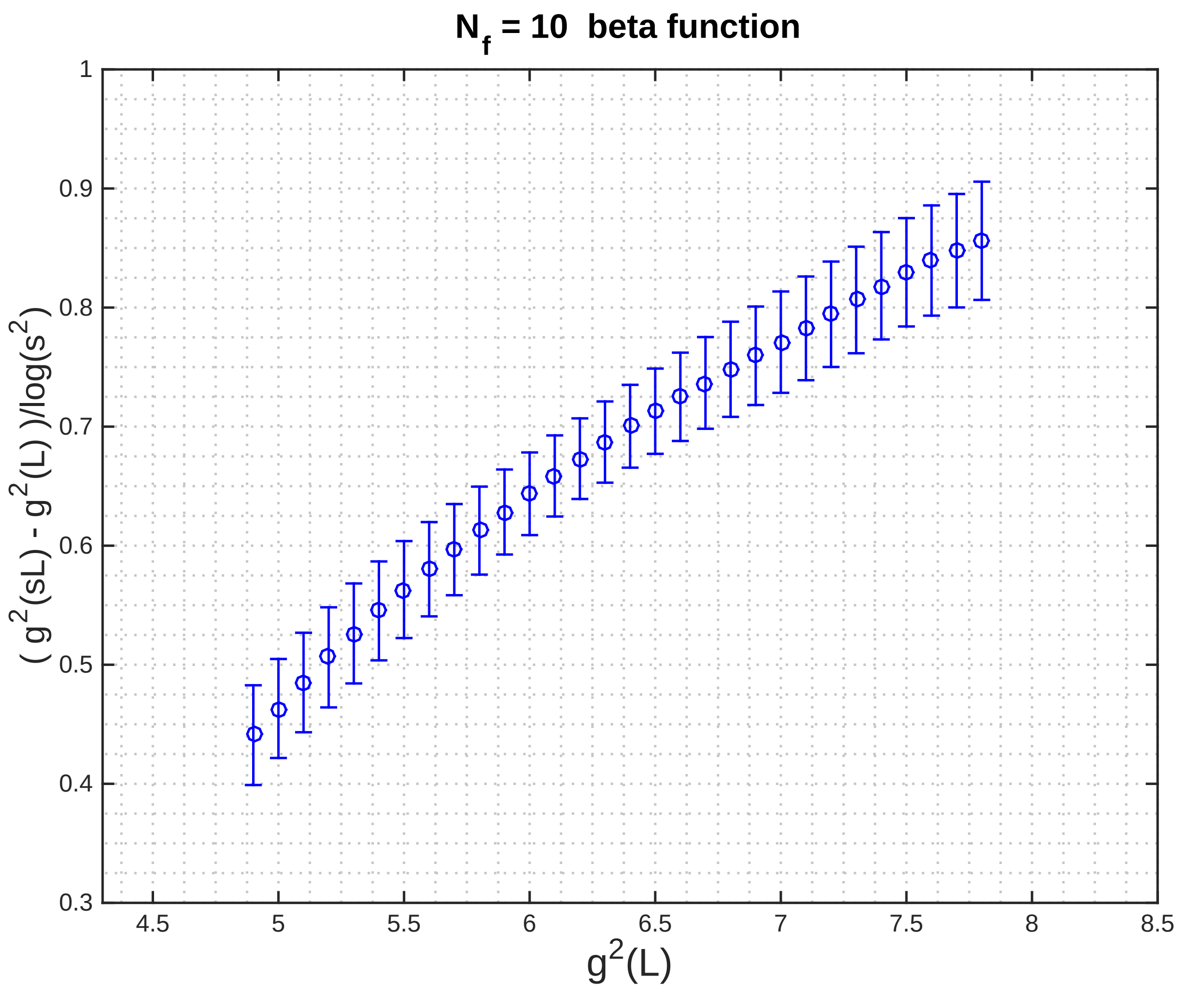
Continuum extrapolation



Continuum extrapolation



Final result from $12 \to 24, 16 \to 32, 18 \to 36, 20 \to 40, 24 \to 48$



$c = 1/4$ $s = 2$

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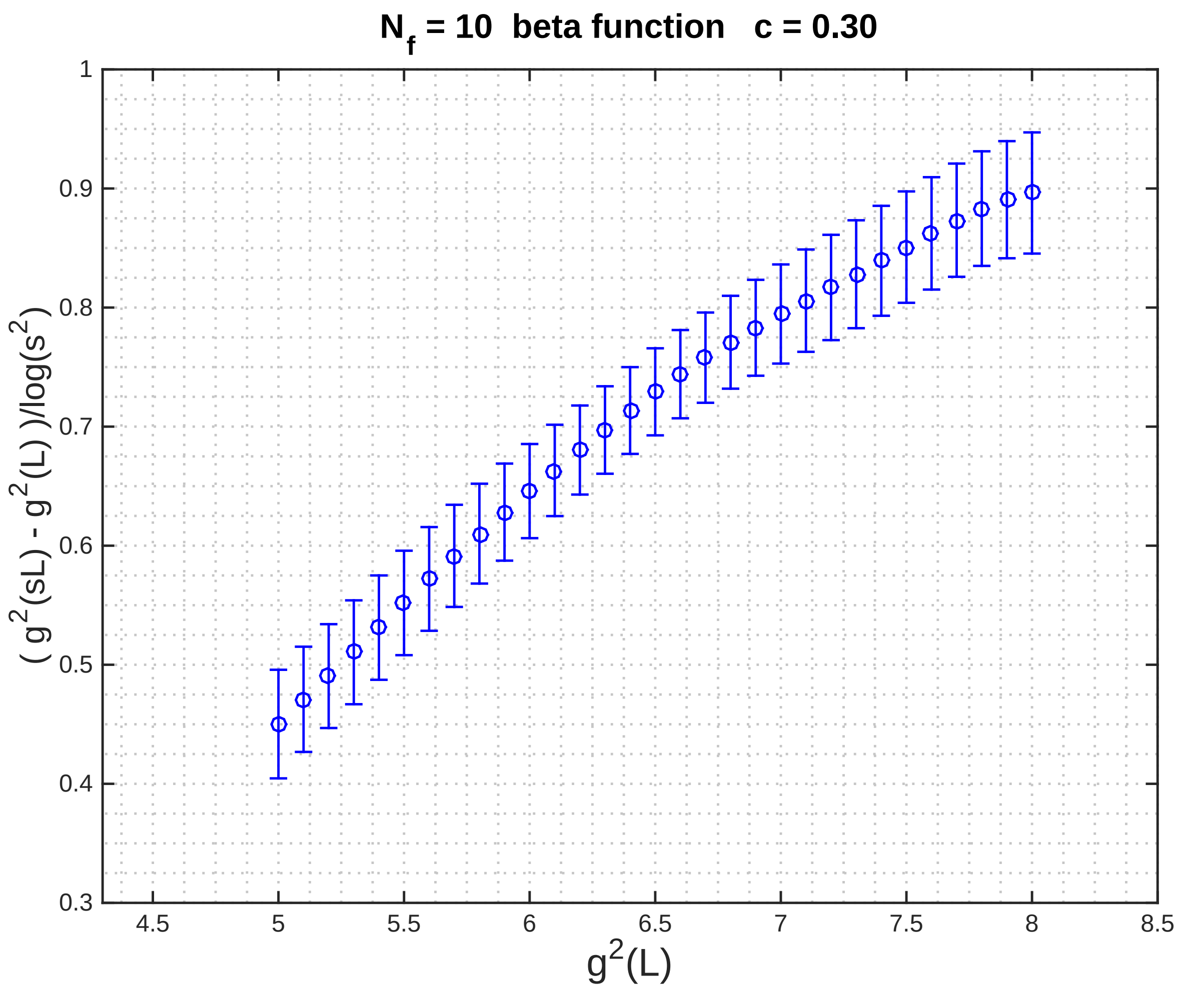
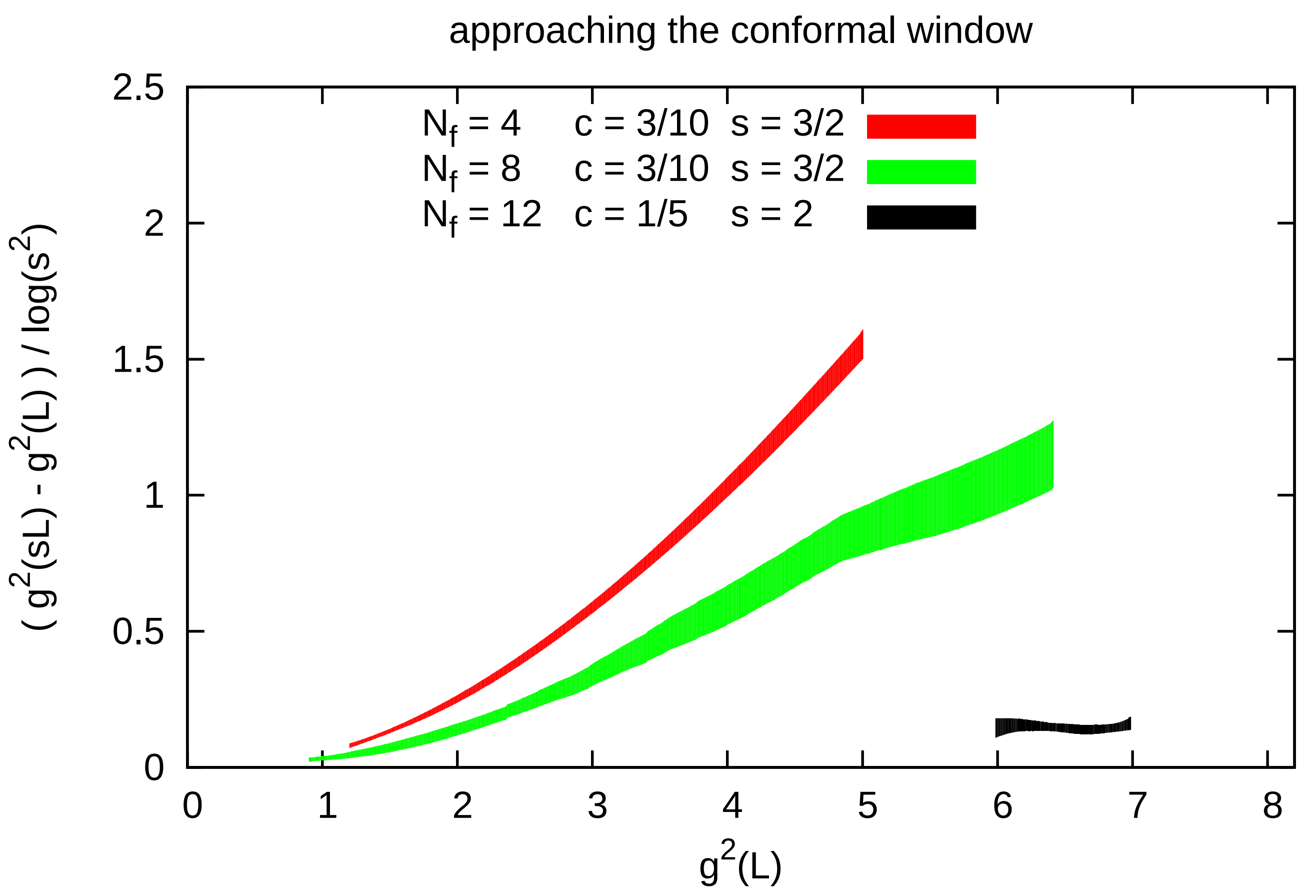


$c = 3/10$

$s = 2$

Final result



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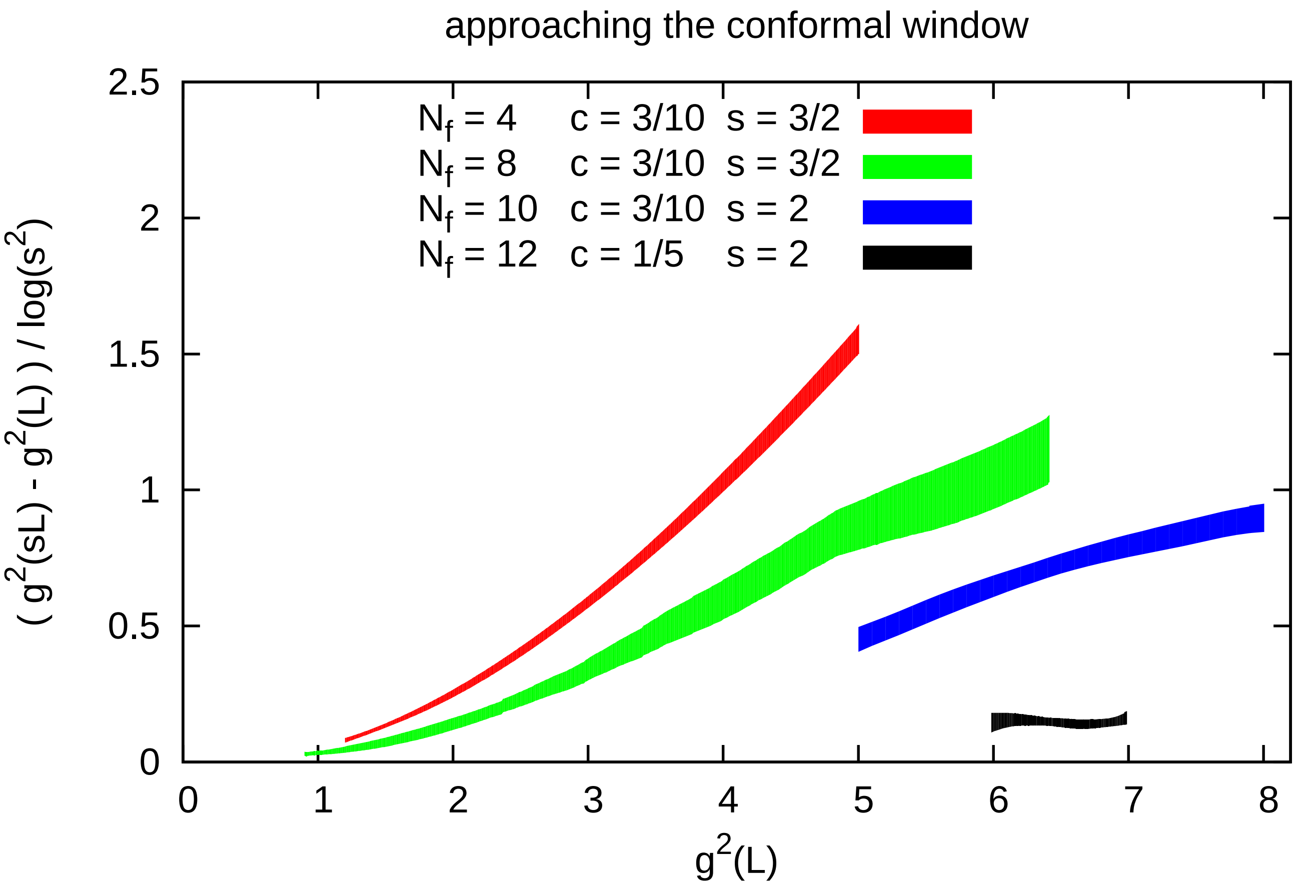

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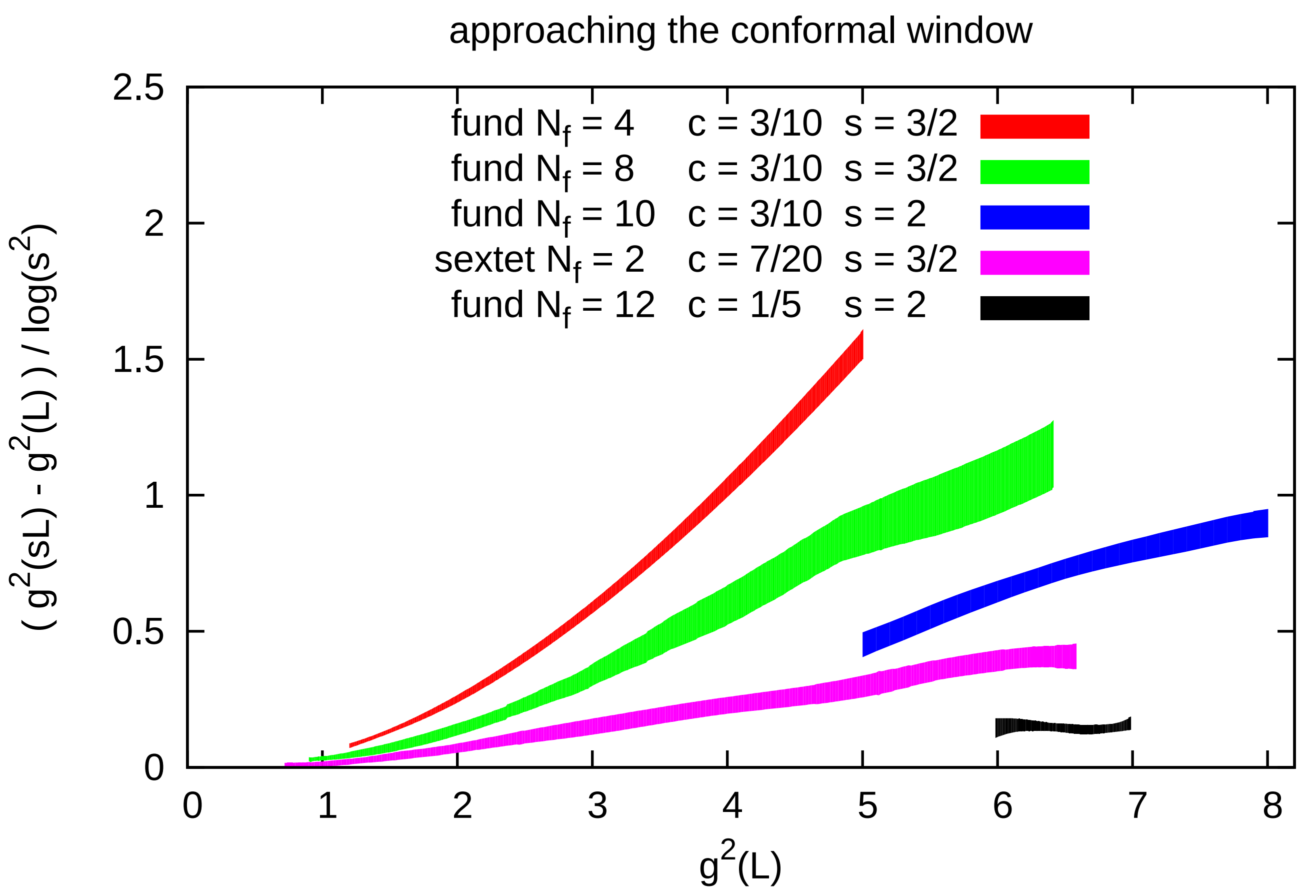
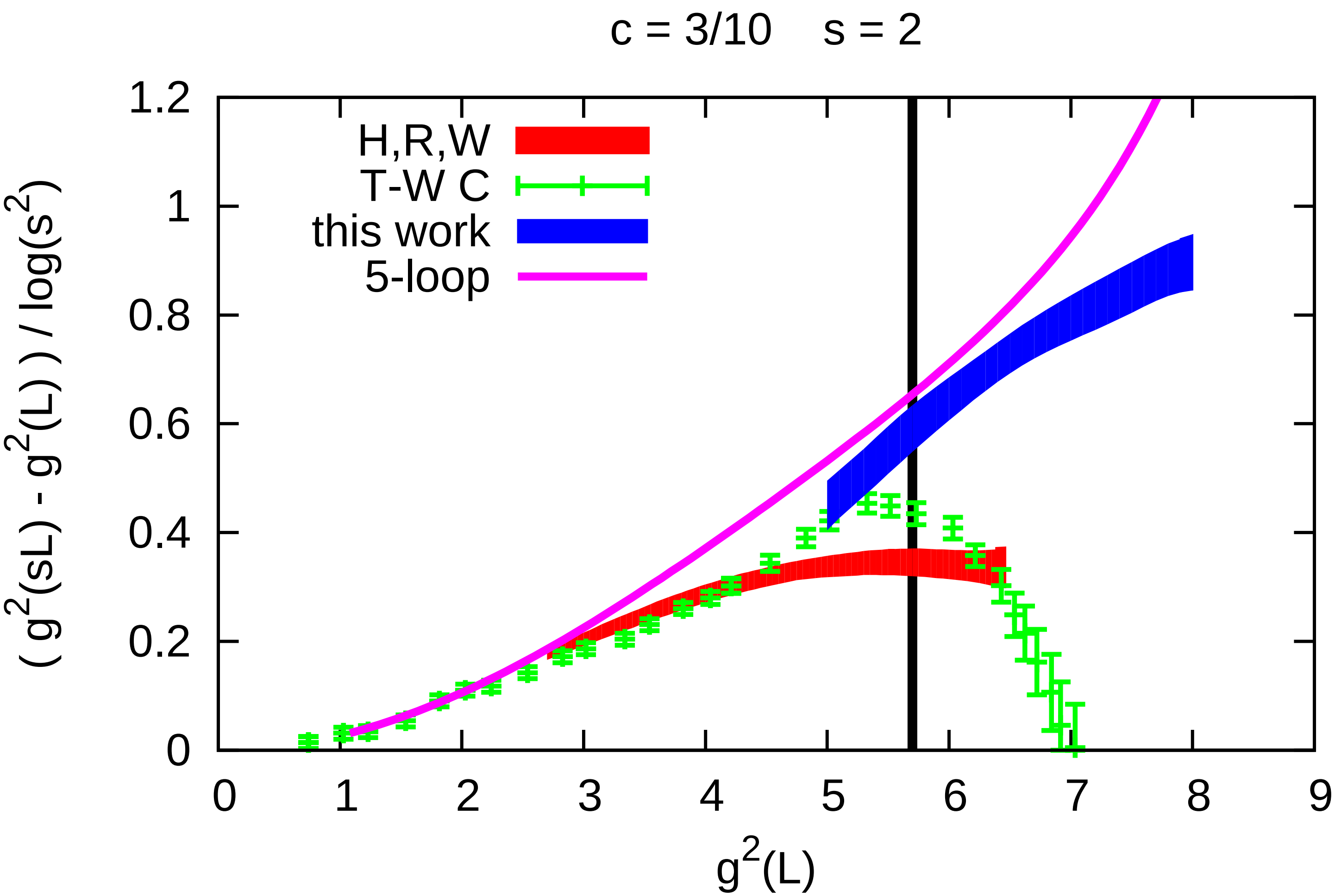


Comparison with existing literature



Why the disagreement?

- Domain wall - too small volumes?
- Domain wall - residual mass non-zero?
- Non-universality (???)

Non-universality?

Outside conformal window: β-function positive for all $0 < g^2(L)$

Only Gaussian UV fixed point, governs continuum limit, $g_0 \to 0$

Bare perturbation theory (i.e. perturbation theory on cut-off scale) reliable close to continuum

Various discretizations can be judged to be in the right universality class by perturbation theory

Anything $=$ continuum $+ \ O(a)$ is okay
(dimension, symmetries, locality)

Staggered, Wilson, domain wall, overlap, etc. all okay

Non-universality?

Inside conformal window: β-function has simple zero at g_*^2 and is positive for $0 < g^2(L) < g_*^2$

Gaussian UV fixed point still there (only these 2)

Non-trivial RG flow between UV fixed point $g^2 = 0$ and IR fixed point $g^2 = g_*^2$ as $L = 0, \ldots, \infty$

In particular, running is via dimensionless quantity ΛL

Small ΛL: $g^2(L) \sim \frac{1}{\log\left(\frac{1}{\Lambda L}\right)}$ Large ΛL: $g^2(L) \sim g_*^2 - \frac{const}{(\Lambda L)^\alpha}$

Non-universality?

For $0 < g^2(L) < g_*^2$ the volume L is finite in physical units (Λ)

At finite L, i.e. $g^2(L) < g_*^2$, continuum limit is governed by Gaussian UV fixed point

Bare perturbation theory still reliable close to the continuum limit

Various discretizations can be judged to be in the right universality class by perturbation theory

For $g^2(L) < g_*^2$ exactly the same story as outside conformal window

Non-universality?

Other example: $T^3 \times R$ Hamiltonian formulation, $g^2(L) < g_*^2$

- Non-trivial finite masses $M_i = C_i/L$
- Well-defined ratios $C_{ij} = M_i/M_j = C_i/C_j$
- Lattice corrections: $O(a^2)$
- Continuum limit $g_0 \to 0$ or $\beta \to \infty$
- Same as outside conformal window

Non-universality?

Only $g^2(L = \infty) = g_*^2$ is tricky (but not needed for $g^2(L) < g_*^2$) both on T^4 and $T^3 \times R$

Staggered, Wilson, domain wall, overlap, etc, MUST give the same result for $g^2(L) < g_*^2$

If not, they would disagree in QCD too

Non-universality?

Proposal: fix $g^2(L) = 5.7$

What is $(g^2(sL) - g^2(L))/\log(s^2)$?

All 3 results give positive β-function

What happens for $g^2(L) > 5.7$ is irrelevant

There MUST be agreement once continuum limit is carefully/correctly done via $g_0 \to 0$ or $\beta \to \infty$

All 3 groups should agree (before going to higher $g^2(L)$)

Non-zero domain wall residual mass

Finite 5th dimension: H,W,R: mostly $L_s = 12$, T-W C: $L_s = 16$

Bare mass zero, but residual mass non-zero

Most severe: large $g^2(L) \sim 6 - 7$

Finite mass effect: $g^2(\beta, L/a, am) = g^2(\beta, L/a) + \Delta(L/a)$

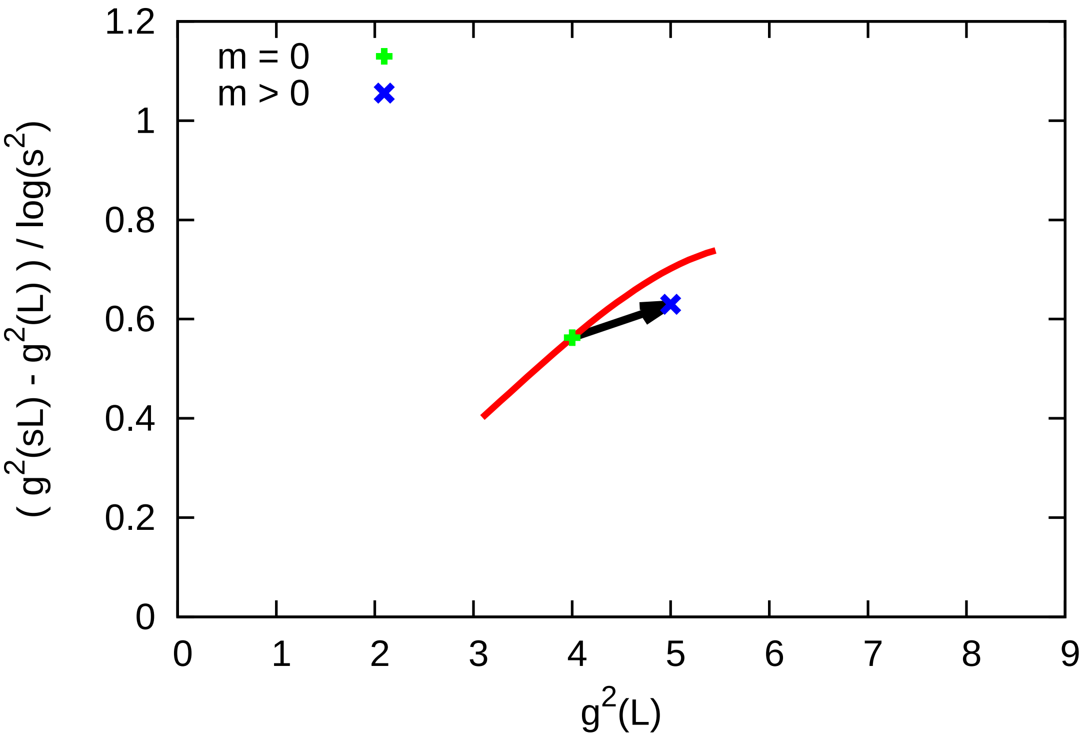
Introduce: $x = g^2(L)$, $\qquad y = (g^2(sL) - g^2(L))/\log(s^2)$

Mass effect in x direction: $\Delta x = \Delta(L/a)$

Mass effect in y direction: $\Delta y = \frac{\Delta(sL/a) - \Delta(L/a)}{\log(s^2)}$

Volume-independent mass-dependence completely cancels, remaining effect: volume-dependent mass-dependence $\rightarrow \Delta y$ expected to be small

Non-zero domain wall residual mass (cartoon sketch)



Is the shifted $b(g^2)\Big|_{m>0}$ curve above or below $b(g^2)\Big|_{m=0}$?

Depends on $\frac{db(g^2)}{dg^2}\Big|_{m=0}$ more or less than $\frac{\Delta y}{\Delta x} = \frac{\Delta(sL/a) - \Delta(L/a)}{\Delta(L/a)\log(s^2)}$

H,W,R: $m > 0$ is always above $m = 0$???

Conclusions and outlook

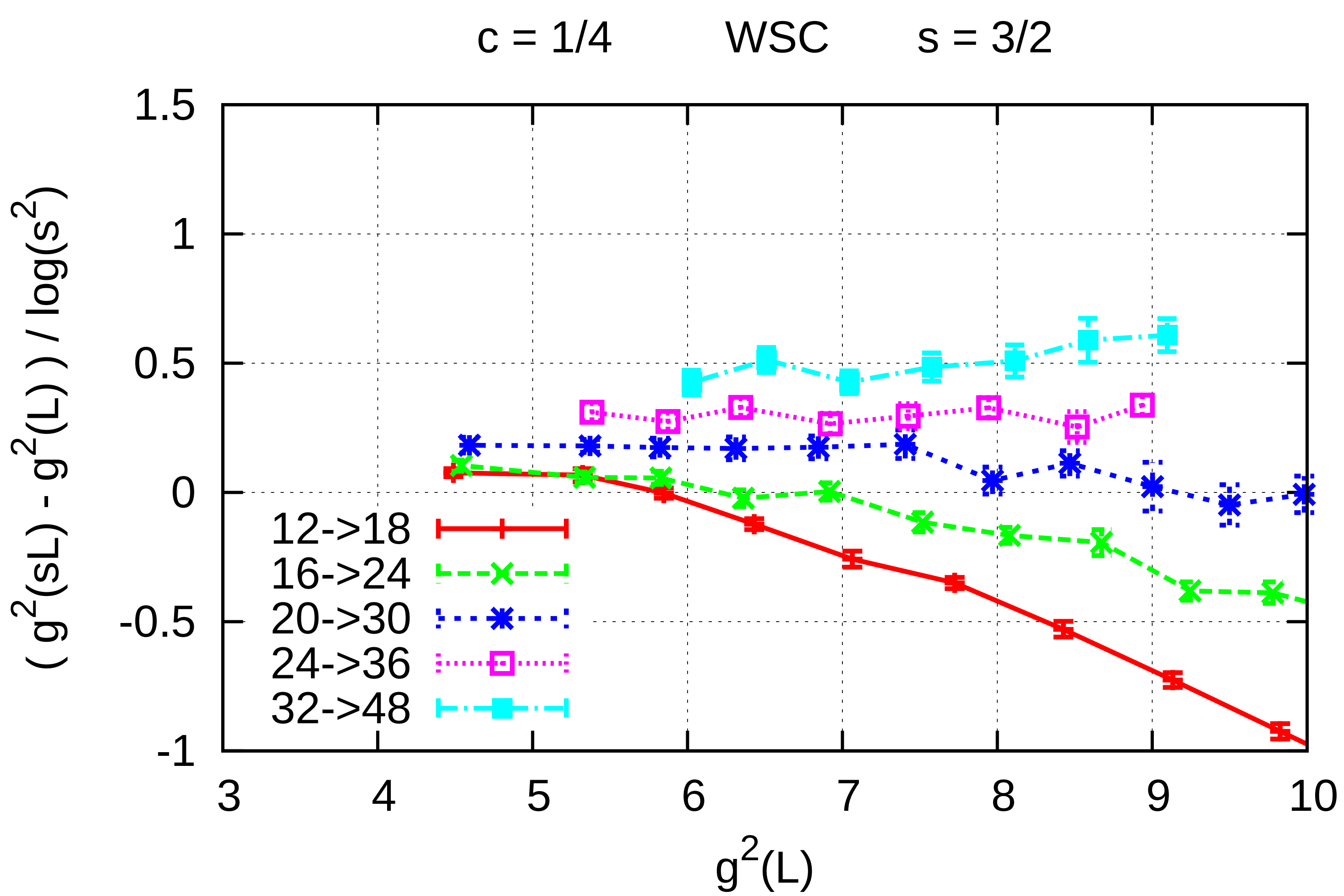
- No IR fixed point for $5 < g^2(L) < 8$
- Potential reasons for disagreements: too small lattice volumes and/or too large residual mass for H,W,R and T-W C
- Even with IRFP: $g^2(L) < g_*^2$ is universal (in usual sense)
- 3-way discrepancy should be resolved, e.g. at $g^2(L) = 5.7$

- Would be good: low energy observables in p-regime
- Hadron/glueball spectrum, chiral condensate, etc.
- Running coupling in p-regime

Thank you for your attention!

Backup slides

Results



c = 1/4 WSC s = 3/2
(g^2(sL) - g^2(L)) / log(s^2)
g^2(L)
1.5
1
0.5
0
-0.5
-1
3
4
5
6
7
8
9
10
12->18
16->24
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24->36
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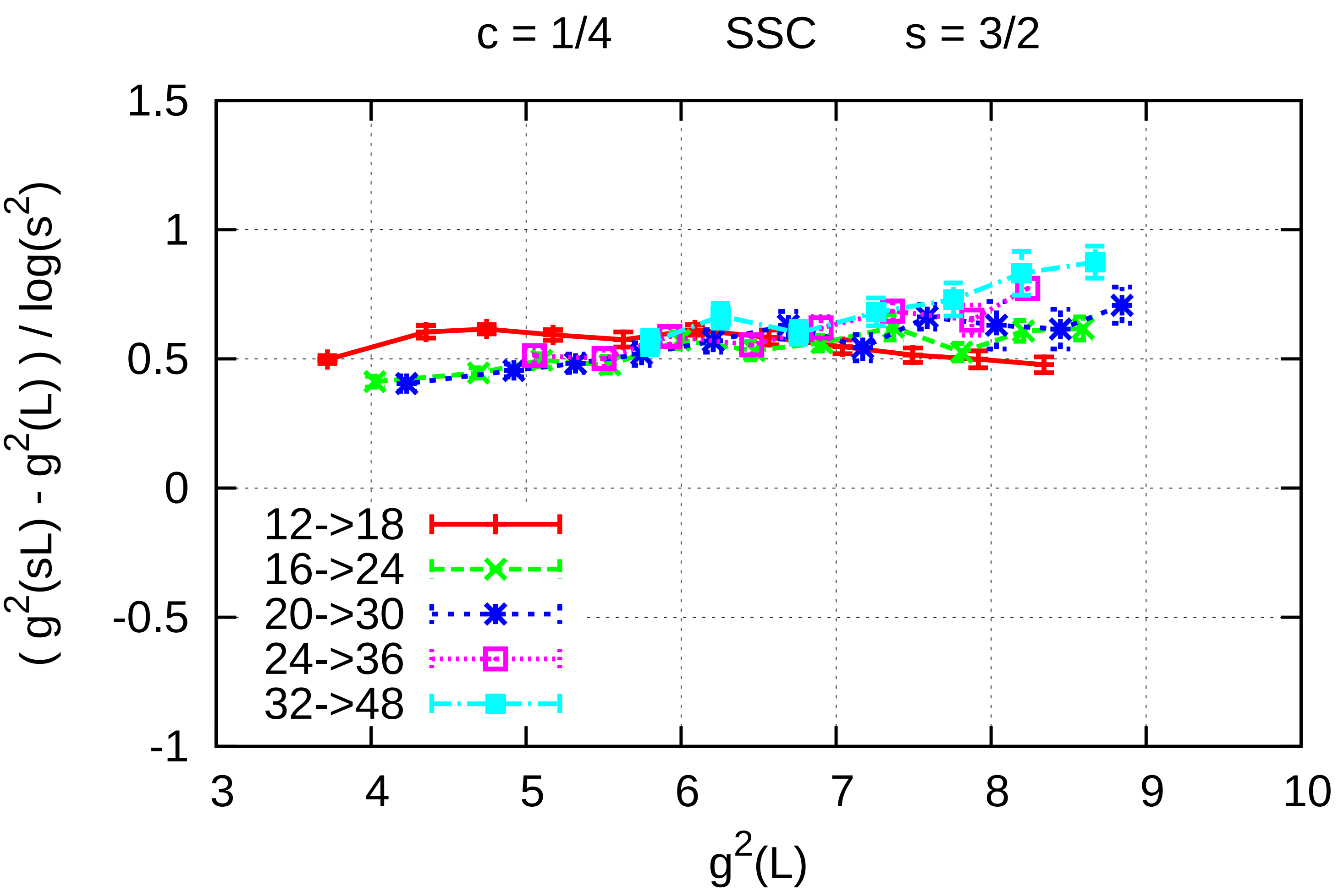

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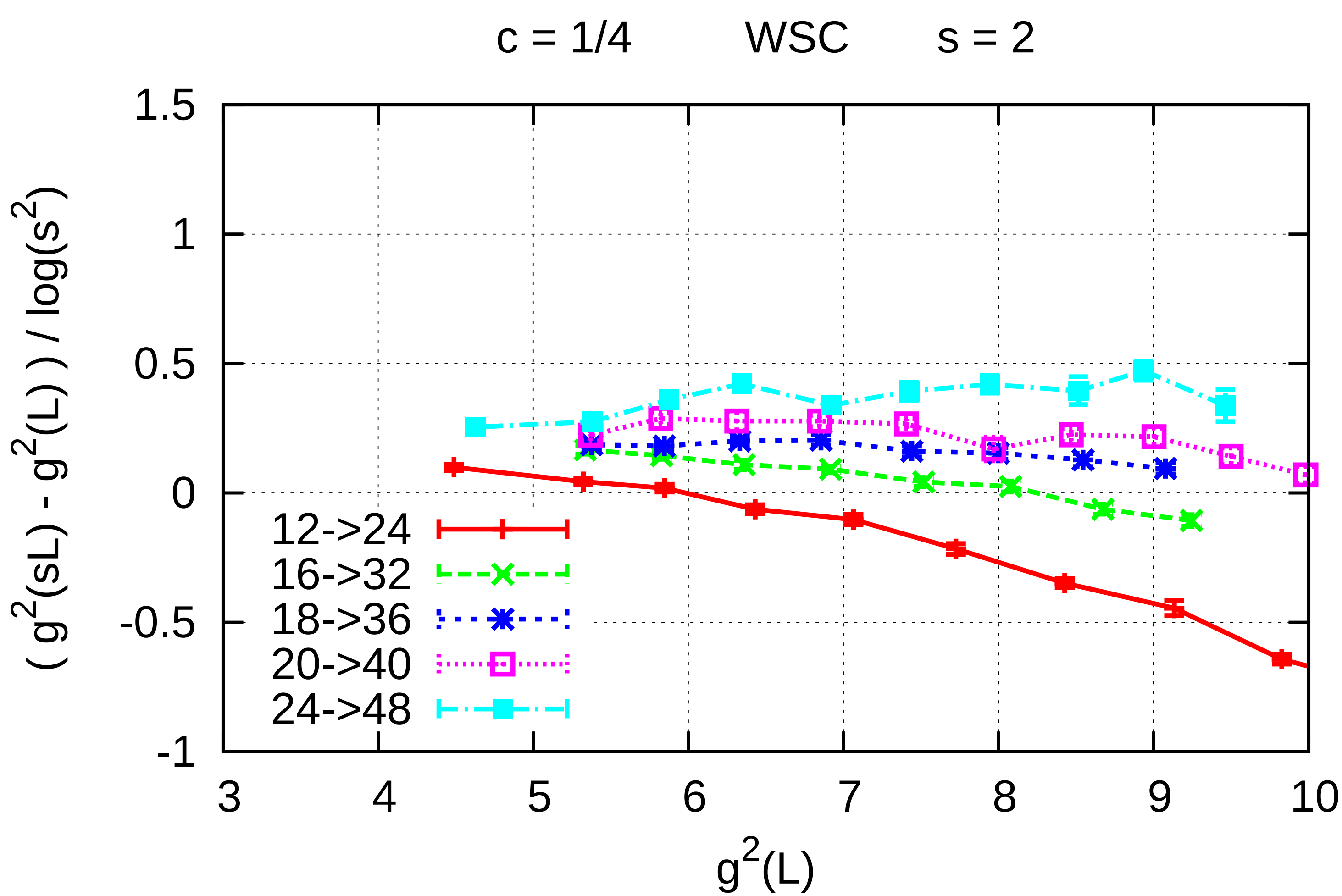

c = 1/4 WSC s = 2
(g^2(sL) - g^2(L)) / log(s^2)
g^2(L)
12->24
16->32
18->36
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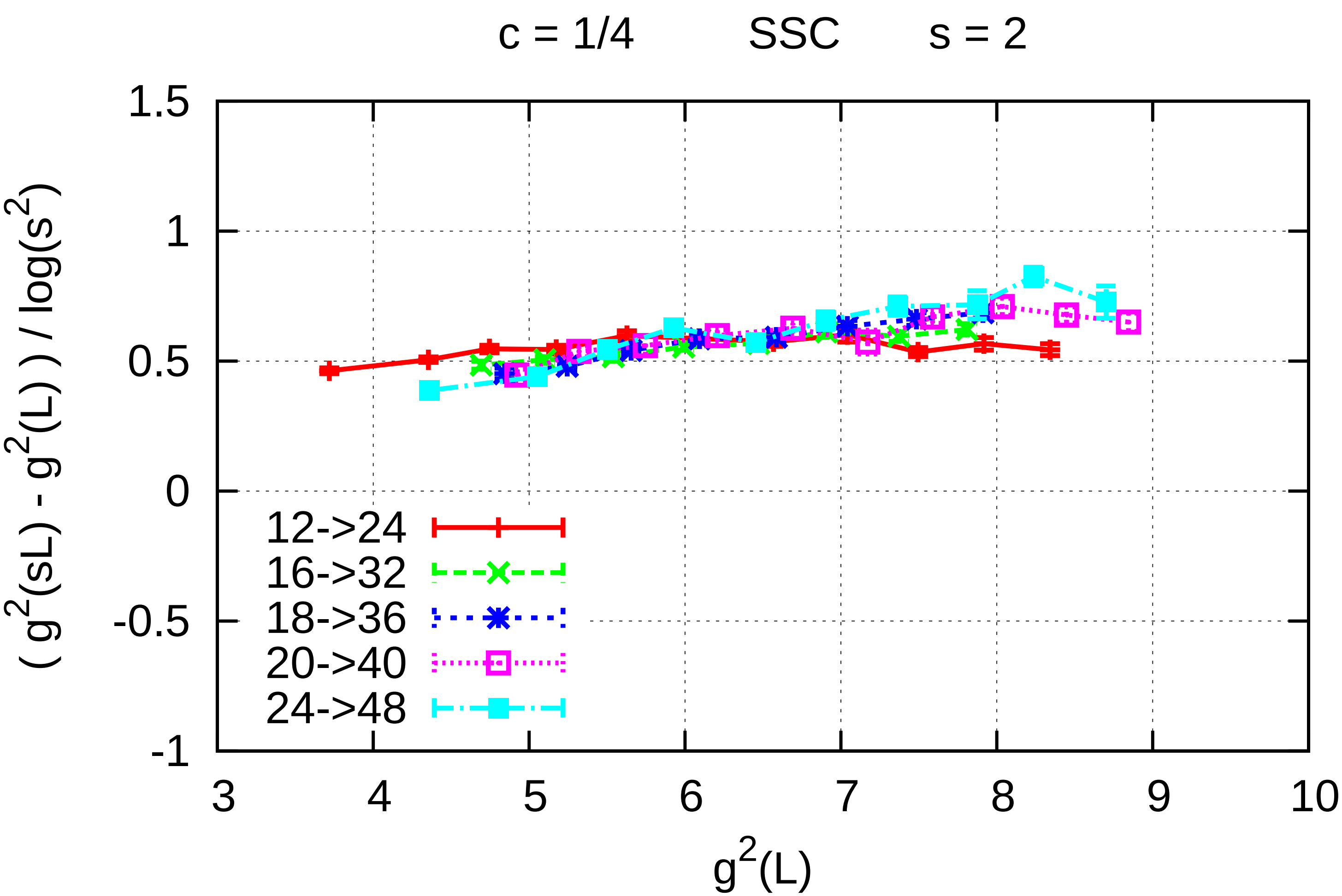
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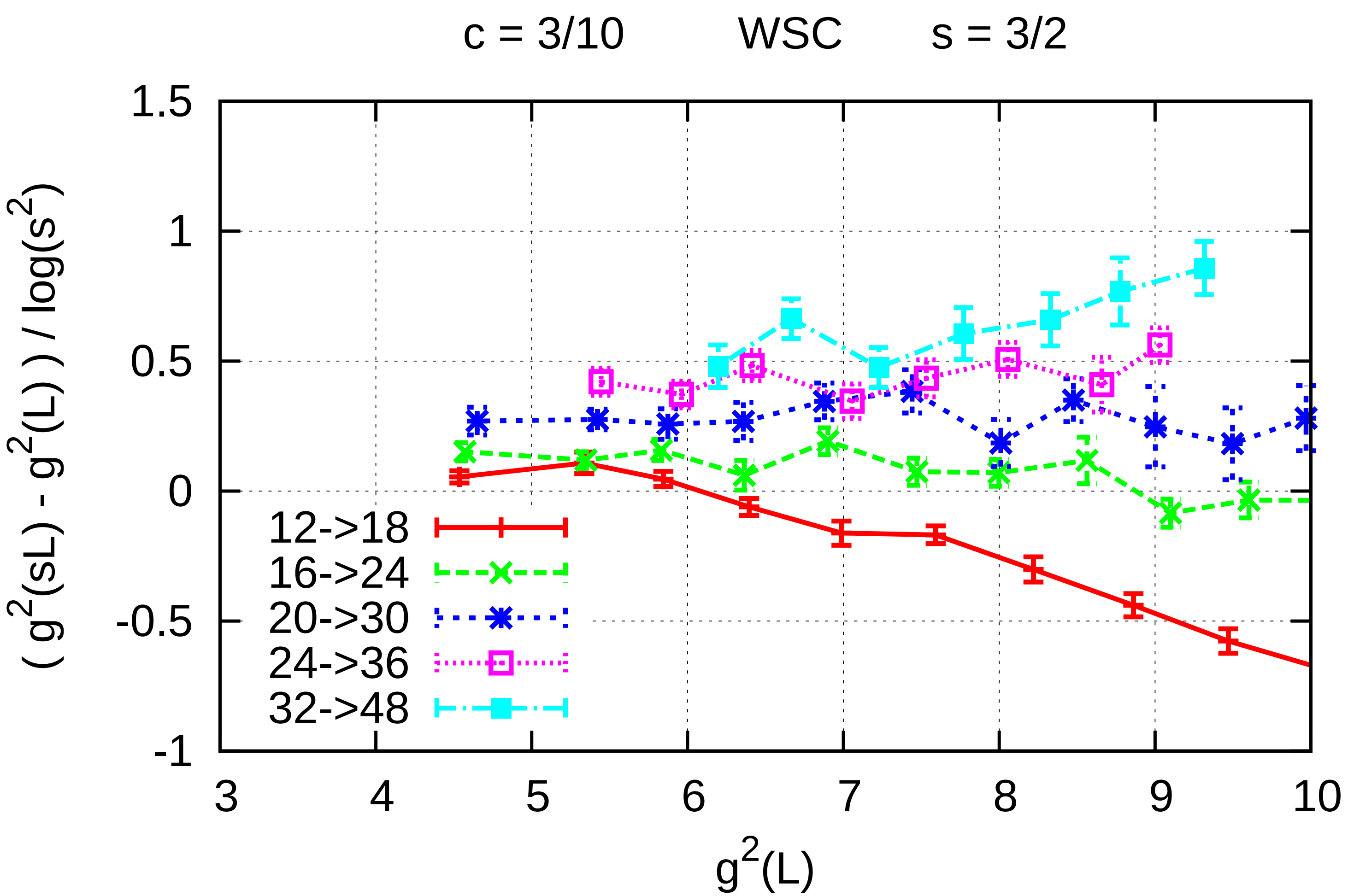

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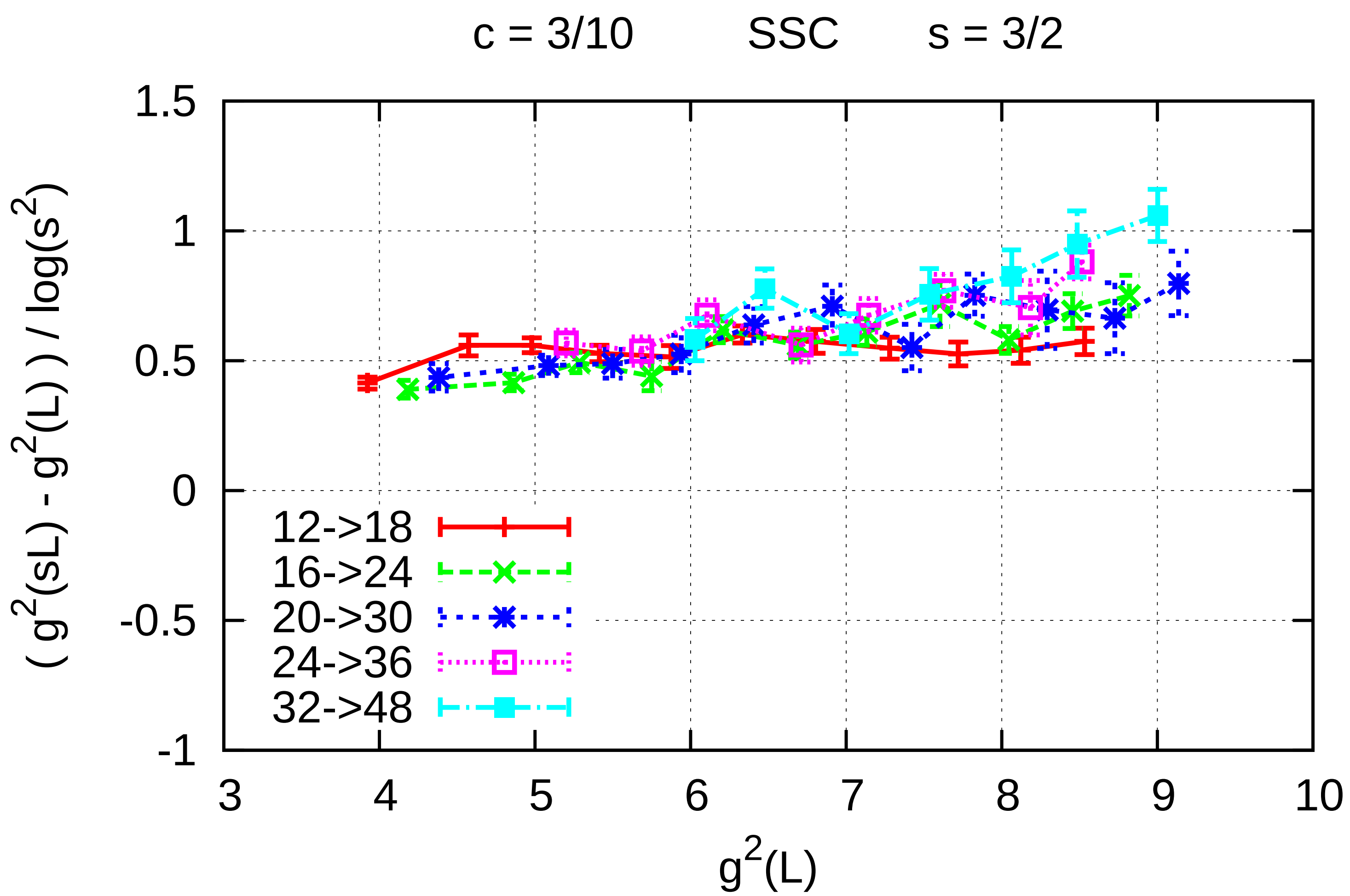

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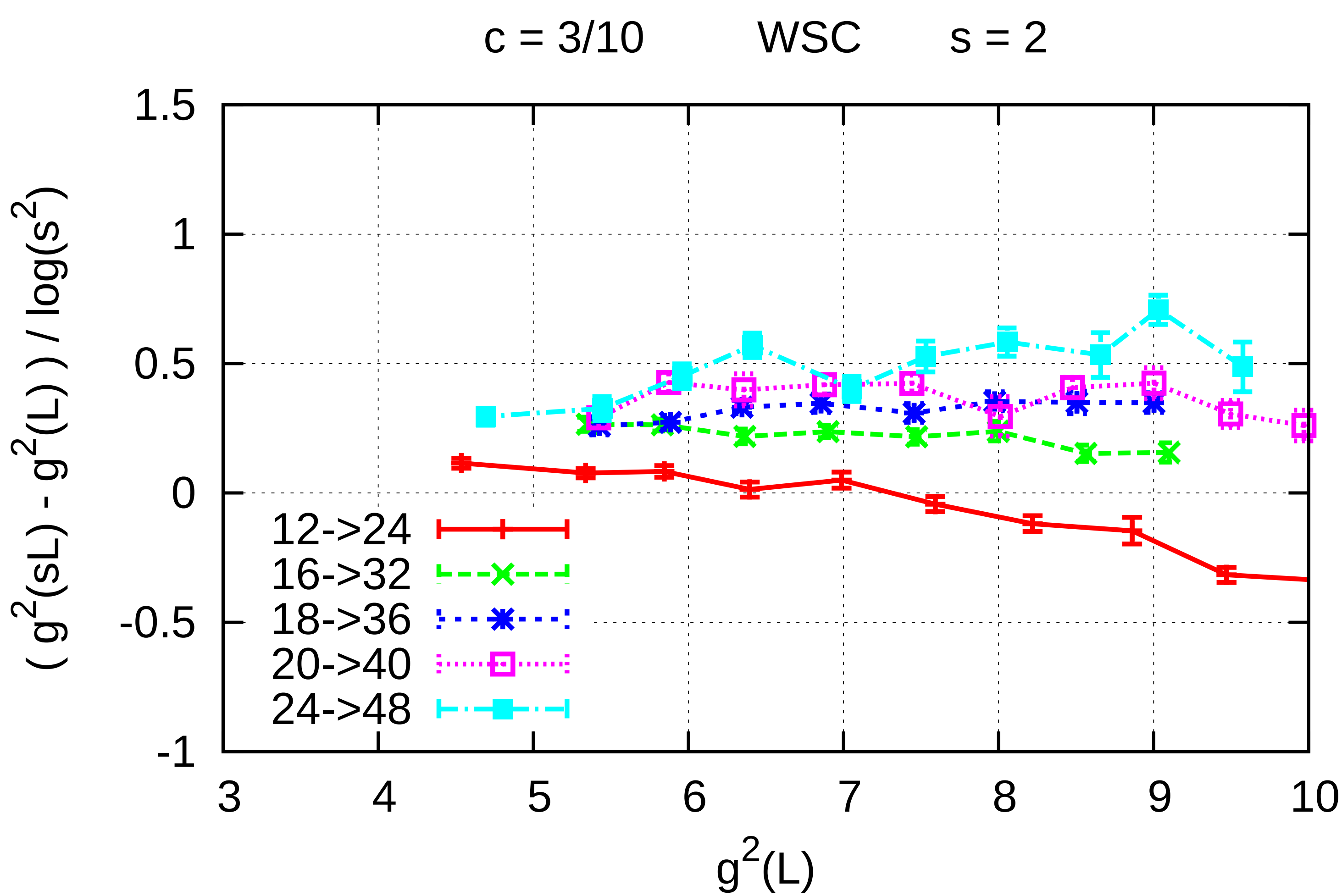

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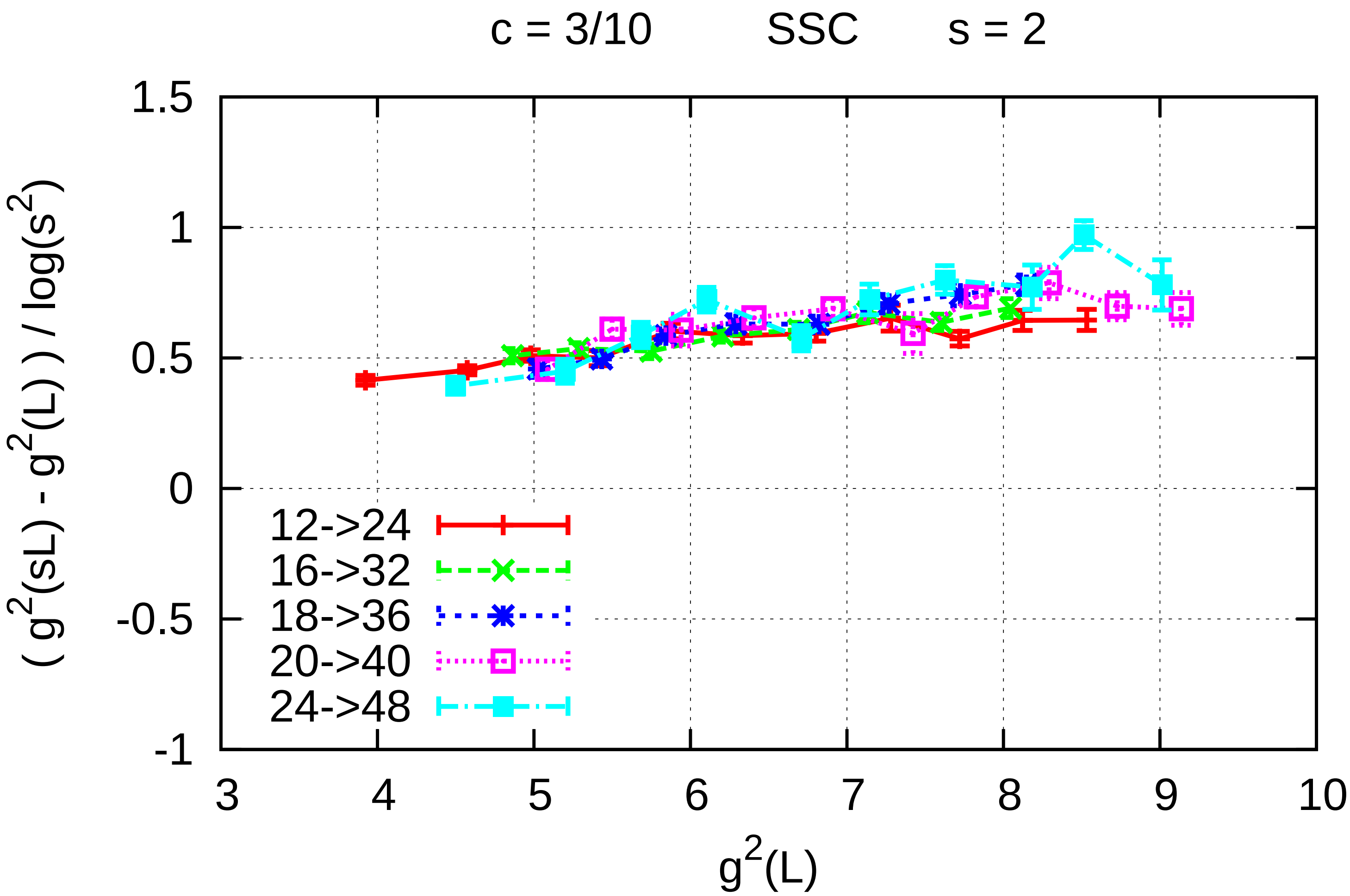

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