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Long-distance contributions to $0\nu 2\beta$ decay

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Introduction

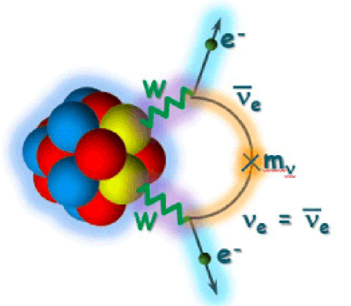
Fundamental question: are the neutrinos Dirac or Majoranna-type fermions?

- A Dirac fermion consists of a pair of mass degenerate Majorana fermions
- Electric charge conservation forces charged fermion to be Dirac type
- As a neutral fermion, a neutrino can be Dirac, Majorana or the mixed type

$0\nu 2\beta$ decay

- Easiest way to confirm neutrino as Majorana fermion
- Direct evidence of lepton number violation
- Provide information about the absolute neutrino mass

$\Rightarrow \sim 20$ experiments proposed (2 operating, 4 under construction)



Minimal extension of SM – exchange of three light Majorana neutrinos

- Long-distance contribution dominated
- Critical to control the uncertainty of nuclear matrix elements

$\Rightarrow$ Lattice QCD interplays with χEFT

Light-neutrino exchange in $0\nu 2\beta$ decay

- $\Delta I = 1$ effective Lagrangian for β decay

$$\mathcal{L}_{eff}^{\Delta I=1} = 2\sqrt{2} G_F V_{ud} (\bar{u}_L \gamma_\mu d_L)(\bar{e}_L \gamma_\mu \nu_{eL})$$

- $\Delta I = 2$ effective Hamltionian for 2β decay

$$\mathcal{H}_{eff}^{\Delta I=2} = \frac{1}{2!}\int d^4x\, \mathcal{L}_{eff}^{\Delta I=1}(x)\mathcal{L}_{eff}^{\Delta I=1}(0)$$

- Neutrino flavor eigenstate mixes with three mass eigenstates

$$\bar{e}_L\gamma_\mu\nu_{eL} \quad \rightarrow \quad \sum_k \bar{e}_L\gamma_\mu U_{ek}\nu_{kL}$$

- Assume that $0\nu 2\beta$ is mediated by exchange of light Majorana neutrinos

$$\sum_k \bar{e}_L(x)\gamma_\mu U_{ek}\underline{\nu_{kL}(x)\,\bar{e}_L(0)\gamma_\nu U_{ek}\nu_{kL}}(0)$$

$$= -\sum_k \bar{e}_L(x)\gamma_\mu U_{ek}\underline{\nu_{kL}(x)\,\overline{\nu^c_k}_L}(0)\gamma_\nu U_{ek} e^c_L(0)$$

$$= -\sum_k \bar{e}_L(x)\gamma_\mu U_{ek}P_L\left(\int \frac{d^4q}{(2\pi)^4}\frac{-i\not{q}+m_k}{q^2+m_k^2}e^{iqx}\right)P_L\gamma_\nu U_{ek}e^c_L(0)$$

$$\approx -m_{\beta\beta}\int \frac{d^4q}{(2\pi)^4}\frac{e^{iqx}}{q^2}\bar{e}_L(x)\gamma_\mu\gamma_\nu e^c_L(0)$$

In the last step, $\not{q}$ vanishes and m_k enters into the effective mass $m_{\beta\beta}$

$$m_{\beta\beta} = \sum_k m_k U_{ek}^2$$

$0\nu 2\beta$ decay amplitude is proportional to the absolute neutrino mass

Decay amplitude

- Decay amplitude of $\mathcal{A} = \langle f, e_1, e_2 | \mathcal{H}_{eff}^{\Delta I=2} | i\rangle$ is given by

$$\mathcal{A} \propto \int d^4x\, \langle f|J_\mu^L(x)J_\nu^L(0)|i\rangle \int \frac{d^4q}{(2\pi)^4}\frac{e^{iqx}}{q^2}\langle e_1, e_2|\bar{e}_L(x)\gamma_\mu\gamma_\nu e^c_L(0)|0\rangle$$

$$= \int \frac{d^3\vec{q}}{(2\pi)^3}\sum_n\left[\frac{\langle f|J_\mu^L|n\rangle\langle n|J_\nu^L|i\rangle}{|\vec{q}|(E_n+|\vec{q}|+E_{e2}-E_i)}\bar{u}_L(\vec{p}_{e1})\gamma_\mu\gamma_\nu u^c_L(\vec{p}_{e2}) - \{e_1\leftrightarrow e_2\}\right]$$

- $\mathcal{A}$ can be split into two parts: $\mathcal{A} \propto \mathcal{A}_1 + \mathcal{A}_2$

$$\mathcal{A}_1 = \int \frac{d^3\vec{q}}{(2\pi)^3}\sum_n\left[\frac{\langle f|J_\mu^L|n\rangle\langle n|J_\mu^L|i\rangle}{|\vec{q}|(E_n+|\vec{q}|+E_{e2}-E_i)} + \frac{\langle f|J_\mu^L|n\rangle\langle n|J_\mu^L|i\rangle}{|\vec{q}|(E_n+|\vec{q}|+E_{e1}-E_i)}\right]\times\bar{u}_L(\vec{p}_{e1})u^c_L(\vec{p}_{e2})$$

$$\mathcal{A}_2 = \int \frac{d^3\vec{q}}{(2\pi)^3}\sum_n\left[\frac{\langle f|J_\mu^L|n\rangle\langle n|J_\nu^L|i\rangle}{|\vec{q}|(E_n+|\vec{q}|+E_{e2}-E_i)} - \frac{\langle f|J_\mu^L|n\rangle\langle n|J_\nu^L|i\rangle}{|\vec{q}|(E_n+|\vec{q}|+E_{e1}-E_i)}\right]\times\bar{u}_L(\vec{p}_{e1})\frac{[\gamma_\mu,\gamma_\nu]}{2}u^c_L(\vec{p}_{e2})$$

Here $\mathcal{A}_2$ may be suppressed by a factor of

$$\frac{|E_{e1}-E_{e2}|}{|E_n+|\vec{q}|+E_{e1}-E_i|} \sim \frac{|E_{e1}-E_{e2}|}{E_F} \sim O\left(\frac{1}{40}\right)$$

where $|E_{e1}-E_{e2}| \sim 1$ MeV and $E_F \sim 40$ MeV is the Fermi energy

Thus one can focus on the dominant contribution – $\mathcal{A}_1$

Lattice setup

- In the finite volume, the hadronic part of the decay amplitude is given by

$$\mathcal{A}_1^{\rm had} = \frac{1}{L^3}\sum_{\vec{q}\neq\vec{0}}\sum_n\left[\frac{\langle f|J_\mu^L|n\rangle\langle n|J_\mu^L|i\rangle}{|\vec{q}|(E_n+|\vec{q}|+E_{e2}-E_i)} + \frac{\langle f|J_\mu^L|n\rangle\langle n|J_\mu^L|i\rangle}{|\vec{q}|(E_n+|\vec{q}|+E_{e1}-E_i)}\right]$$

  - Zero mode of the neutrino propagator has been removed
  - No exponentially growing contamination when increasing Euclidean time
  - Finite volume correction can be evaluated

$$\Delta_{\rm FV} = \left(\frac{1}{L^3}\sum_{\vec{q}\neq 0} - \int\frac{d^3\vec{q}}{(2\pi)^3}\right)[\cdots] \approx -\frac{\kappa}{4\pi L}\cdot 2\langle f|J_\mu^L|n\rangle\langle n|J_\mu^L|i\rangle, \quad \kappa = 2.837$$

- Neutrino propagator can be implemented in a stochastic way

$$S_\nu(x,y) \approx \frac{1}{N_r}\sum_{r=1}^{N_r}\phi_r(x)\phi_r^*(y), \quad \phi_r(x) = \frac{1}{\sqrt{L^3T}}\sum_{q_0,\vec{q}\neq\vec{0}}\frac{\xi_r(q)}{\sqrt{\hat{q}^2}}e^{iqx}$$

where $\hat{q}^2 = \sum_i (2\sin(q_i/2))^2$ and $\xi_r(q)$ satisfies $\frac{1}{N_r}\sum_r \xi_r(q)\xi_r^*(p) \approx \delta_{p,q}$

- No short-distance divergence

$$\int d^4x\, e^{i\Lambda x}\mathcal{L}_{eff}^{\Delta I=1}(x)\mathcal{L}_{eff}^{\Delta I=1}(0) \sim 8G_F^2V_{ud}^2\frac{m_{\beta\beta}}{\Lambda^2}(\bar{u}_L\gamma_\mu d_L)(\bar{u}_L\gamma_\mu d_L)\bar{e}_L e^c_L$$

Simplest $0\nu 2\beta$ processes: $\pi^-\to\pi^+e^-e^-$ and $\pi^-\pi^-\to e^-e^-$

- $\Delta I = 2$ transitions relevant for experiments

$$^{136}\mathrm{Xe}\to{}^{136}\mathrm{Ba}\, e^-e^-, \quad {}^{76}\mathrm{Ge}\to{}^{76}\mathrm{Se}\, e^-e^-, \quad \cdots$$

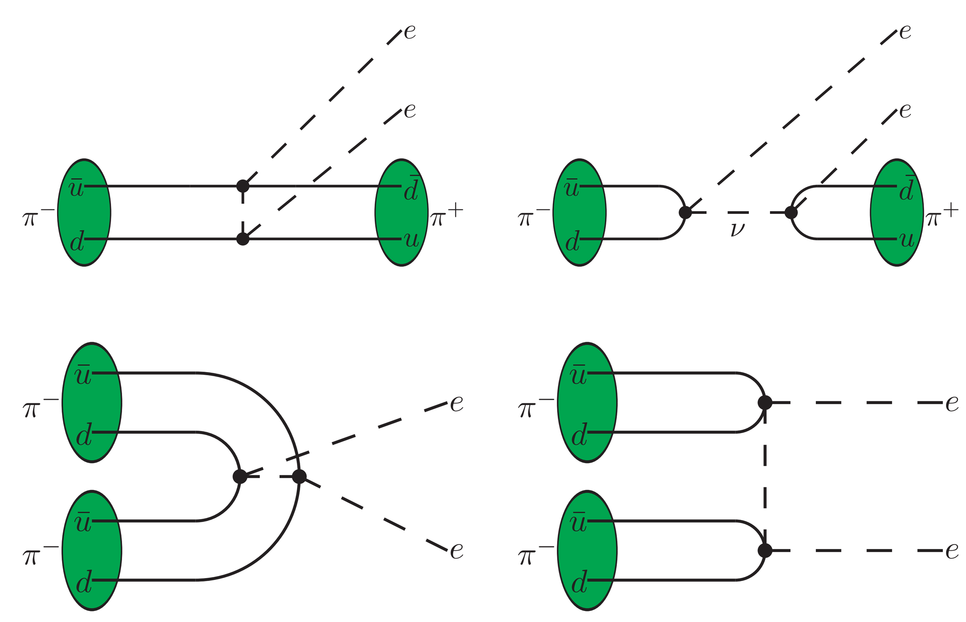
$\Rightarrow$ nuclei too heavy, beyond the capability of LQCD

- Light hadron systems

$$\pi^-\pi^-\to e^-e^-, \quad n\to p\pi^+e^-e^-, \quad nn\to ppe^-e^-, \quad \cdots$$

$\Rightarrow$ Lattice calculation can provide LECs for χEFT

- Simplest $0\nu 2\beta$ processes
  - $\pi^-\to\pi^+e^-e^-$
  - $\pi^-\pi^-\to e^-e^-$

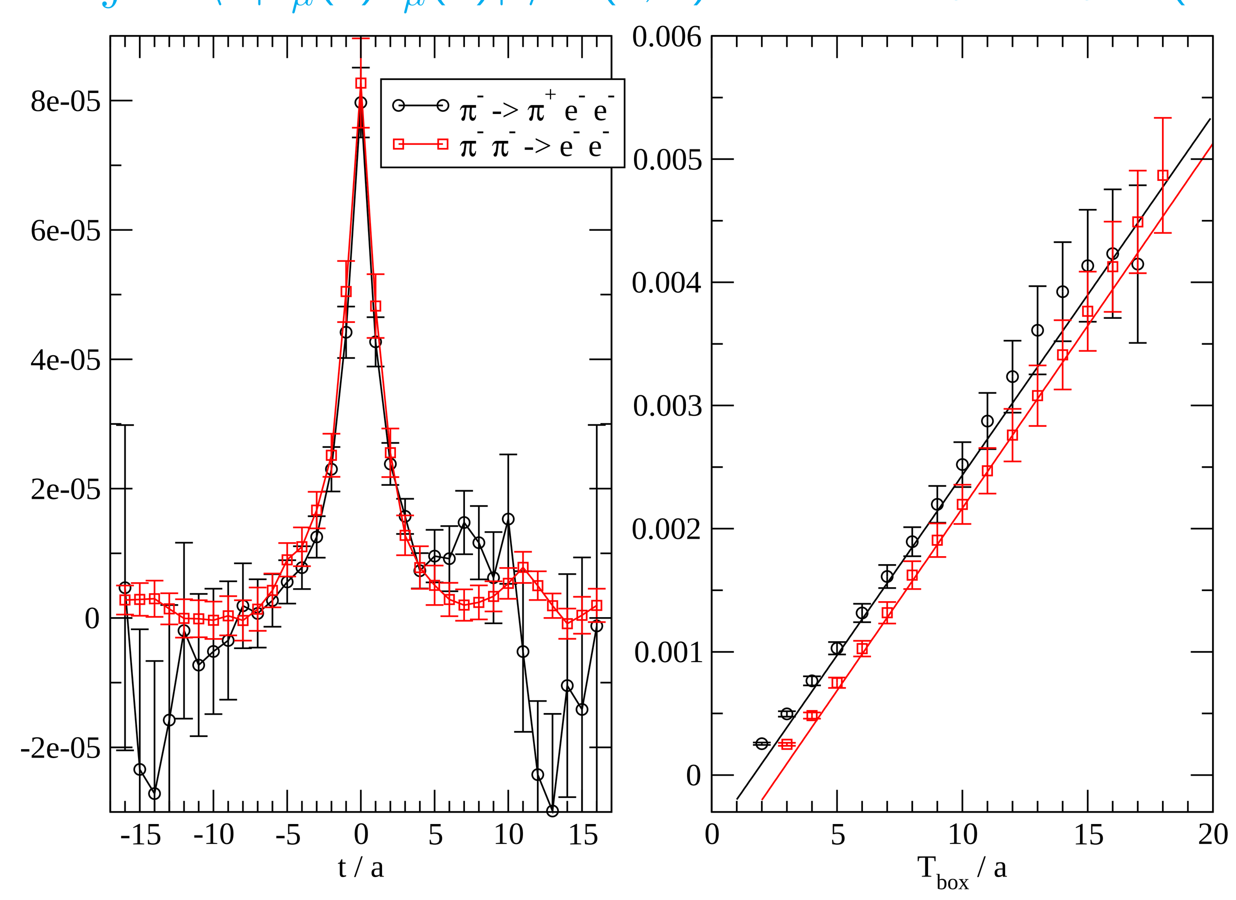


Preliminary results

- Ensemble information: $2+1$ flavor DWF with Iwasaki gauge action

$$m_\pi = 420\ \text{MeV}, \quad a^{-1} = 1.73\ \text{GeV}, \quad L = 1.83\ \text{fm}, \quad N_{\rm cfg} = 50$$

- We evaluate $\int dt\, \langle f|J_\mu^L(t)J_\mu^L(0)|i\rangle S_\nu(t,0)e^{E_e t}$ with $E_e = m_e = (E_i - E_f)/2$



  - t-dep. of unintegrated amplitude (left); $T_{\rm box}$-dep. of integrated amplitude (right)

$$A(\pi^-\to\pi^+e^-e^-)/f_\pi^2 = 1.89(15)\times 10^{-2}, \quad A(\pi^-\pi^-\to e^-e^-)/f_\pi^2 = 1.92(14)\times 10^{-2}$$

Crossing symmetry is confirmed between $\pi^-\to\pi^+e^-e^-$ and $\pi^-\pi^-\to e^-e^-$

- For $\pi^-\pi^-\to e^-e^-$, we can also set $E_e = |\vec{p}_e| = (E_i - E_f)/2$, $m_e = 0$

Conclusion

- We perform an exploratory study on $0\nu 2\beta$ decay
- Next step: calculation at physical pion mass + control of systematic effects
- Future direction: move to more complicated $0\nu 2\beta$ system