

Strange Quark Content of Nucleons using the Yang-Mills gradient flow

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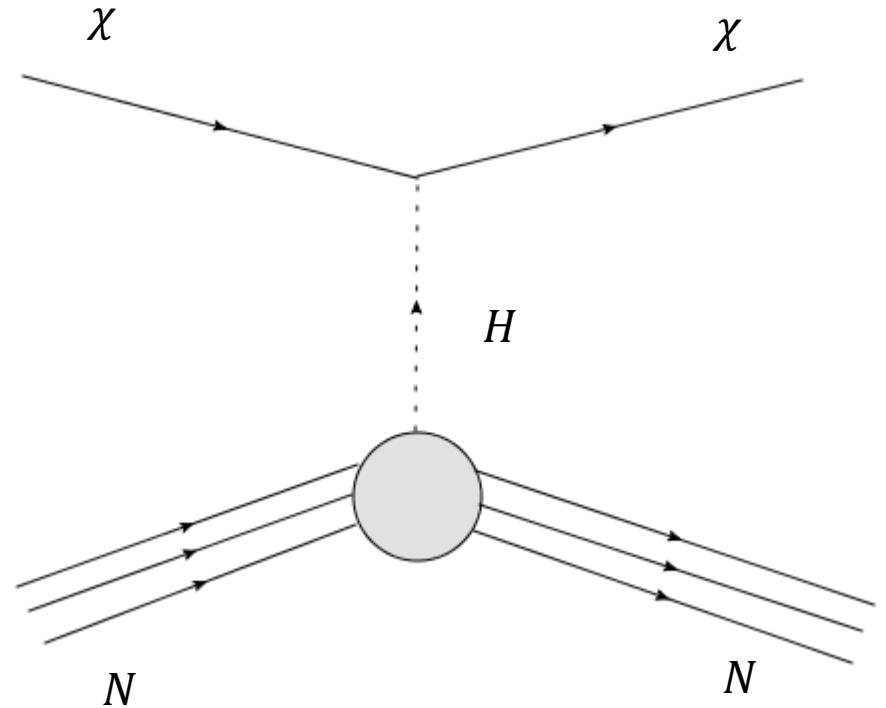
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Dark Matter

$$\sigma_{SI} \sim \left| \sum_f G_f \frac{m_f}{M_N} \langle N | \bar{q}_f q_f | N \rangle \right|^2$$



Quark Content in Nucleons

- Involves calculating disconnected diagrams

$$\mathcal{O}_s(x) = \bar{s}(x)s(x)$$

$$\mathcal{C}^{sub}(x) = \langle \mathcal{N} \mathcal{O}_s(x) \mathcal{N}^\dagger \rangle - \langle \mathcal{O}_s(x) \rangle \langle \mathcal{N} \mathcal{N}^\dagger \rangle$$

Gradient Flow

For Gauge Fields

$$\partial_{t_f} B_\mu(t, x) = D_\nu G_{\mu\nu}(t_f, x) \quad B_\mu(t_f = 0, x) = A_\mu(x)$$

$$D_\mu = \partial_\mu + [B_\mu, \cdot]$$

$$G_{\mu\nu}(t_f, x) = \partial_\mu B_\nu(t_f, x) - \partial_\nu B_\mu(t_f, x) + [B_\mu(t_f, x), B_\nu(t_f, x)]$$

Gradient Flow

For Fermion Fields

$$\partial_{t_f} \chi(t_f, x) = D^2 \chi(t_f, x) \quad \chi(t_f = 0, x) = \psi(x)$$

$$\partial_{t_f} \bar{\chi}(t_f, x) = \bar{\chi} \overleftarrow{D}^2 \quad \bar{\chi}(t_f = 0, x) = \bar{\psi}(x)$$

$$D_\mu = \partial_\mu + B_\mu$$

$$\overleftarrow{D}_\mu = \overleftarrow{\partial}_\mu - B_\mu$$

Small flow time expansion

The small flow time expansion for an operator \mathcal{M} :

$$\bar{\chi}(t_f, x) \mathcal{M} \chi(t_f, x) \stackrel{t_f \rightarrow 0}{\sim} \zeta_0(t_f) \mathbb{I} + \zeta_1(t_f) \bar{\psi}(x) \mathcal{M} \psi(x) + \mathcal{O}(t_f)$$

Wilson coefficients can be calculated in a loop expansion:

$$\zeta_0(t) = \sum_{\ell=1}^{\infty} \zeta_0^\ell(t) \quad \zeta_1(t) = 1 + \sum_{\ell=1}^{\infty} \zeta_1^\ell(t)$$

Or from a lattice

Small flow time expansion

$$\zeta_0^1(t_f) = -\frac{1}{(4\pi)^2} \dim(R) \operatorname{tr} \left(\mathcal{M} M \left\{ \frac{1}{2t_f} + M^2 [\gamma + \ln(2M^2 t_f)] + \mathcal{O}(t_f) \right\} \right)$$

$$\zeta_1^1(t_f) = \frac{g^2}{(4\pi)^2} C_2(R) \left\{ (-6) \left[\frac{1}{\epsilon} + \ln(8\pi\mu^2 t_f) \right] - 10 \right\} + \mathcal{O}(t_f) \quad \text{for } \mathcal{M} \propto \gamma_5$$

$$\zeta_1^1(t_f) = \frac{g^2}{(4\pi)^2} C_2(R) \left\{ (-6) \left[\frac{1}{\epsilon} + \ln(8\pi\mu^2 t_f) \right] - 2 \right\} + \mathcal{O}(t_f) \quad \text{for } \mathcal{M} \propto 1$$

$$\mu = \frac{1}{\sqrt{8t_f}}$$

Strange Content in Nucleons

$$\mathcal{O}_s(t_f, x) = \bar{s}(t_f, x)s(t_f, x)$$

$$\mathcal{O}_s(t_f, x) = \frac{c_0(t_f)}{t_f} m_s + c_1(t_f) m_s(m_u^2 + m_d^2 + m_s^2) + c_2(t_f) m_s^3 + c_3(t_f) \mathcal{O}_s(0, x) + O(t_f)$$

Consider

$$\mathcal{C}^{sub}(t_f, x) = \langle \mathcal{N} \mathcal{O}_s(t_f, x) \mathcal{N}^\dagger \rangle - \langle \mathcal{O}_s(t_f, x) \rangle \langle \mathcal{N} \mathcal{N}^\dagger \rangle$$

$$\mathcal{C}^{sub}(t_f, x) = c_3(t_f) \mathcal{C}^{sub}(0, x) + O(t_f)$$

$$\mathcal{C}^{sub}(0, x) = c_3^{-1}(t_f) [\langle \mathcal{N} \mathcal{O}_s(t_f, x) \mathcal{N}^\dagger \rangle - \langle \mathcal{O}_s(t_f, x) \rangle \langle \mathcal{N} \mathcal{N}^\dagger \rangle] + O(t_f)$$

RG determination of $c_3(t)$

$$c_3(t) = (2b_0\bar{g}^2)^{-\frac{8}{9}} \{1 + O(\bar{g}^2)\}$$

$$c_3(t_f) \sim \left(-\frac{1}{2} \ln(8t_f\Lambda^2) \right)^{\frac{8}{9}}$$

Where b_0 is the one loop coefficient of the QCD β -function. With $N = 3, N_f = 3$:

$$\beta(g) = -g^3 \sum_{k=0}^{\infty} b_k g^{2k}$$

$$b_0 = \frac{1}{(4\pi)^2} \left[\frac{11}{3}N - \frac{2}{3}N_f \right] = \frac{9}{(4\pi)^2}$$

$$d_0 = \frac{1}{(4\pi)^2} \frac{3(N^2 - 1)}{N} = \frac{8}{(4\pi)^2}$$

Pseudo-scalar Density

$$P^{rs}(t_f, x) = \bar{\chi}_r(t_f, x) \gamma_5 \chi_s(t_f, x)$$

$$P^{rs}(0, x) = \bar{\psi}_r(x) \gamma_5 \psi_s(x)$$

$$P^{rs}(t_f, x) = c_3(t_f) P^{rs}(0, x) + \mathcal{O}(t_f)$$

Euclidean Meson Correlator

$$\langle \mathcal{O}_1(t_f, x) \mathcal{O}_2(0, 0) \rangle_T = \frac{1}{Z_T} \sum_{m,n} \langle m | \hat{\mathcal{O}}_1 | n \rangle \langle n | \hat{\mathcal{O}}_2 | m \rangle e^{-x_0 E_n} e^{-(\tau - x_0) E_m}$$

$$C(t_f, x) = 2A_0(t_f) e^{\frac{\tau}{2} E_\pi} \cosh \left(E_\pi \left(\frac{\tau}{2} - x_0 \right) \right)$$

$$C(t_f, x) = \langle P(t_f, x) P(0, x) \rangle = A(t_f) \langle P(0, x) P(0, x) \rangle$$

Time x_0 must be much larger than the flow smearing radius $r_f = \sqrt{8t_f}$

$$P^{rs}\big(t_f,x\big)=c_3\big(t_f\big)P^{rs}\big(0,t_f\big)+\mathcal{O}(t)$$

$$c_3\left(t\right)=\frac{A\big(t_f\big)}{A(0)}=\frac{G_{\pi,t_f}}{G_\pi}+\mathcal{O}(t_f)$$

$$\mathcal{C}^{sub}(0,x)=\frac{G_\pi}{G_{\pi,t_f}}\big[\langle \mathcal{NO}_s(t_f,x)\mathcal{N}^\dagger\rangle-\langle \mathcal{O}_s(t_f,x)\rangle\langle \mathcal{NN}^\dagger\rangle\big]+\mathcal{O}(t_f)$$

Parameters

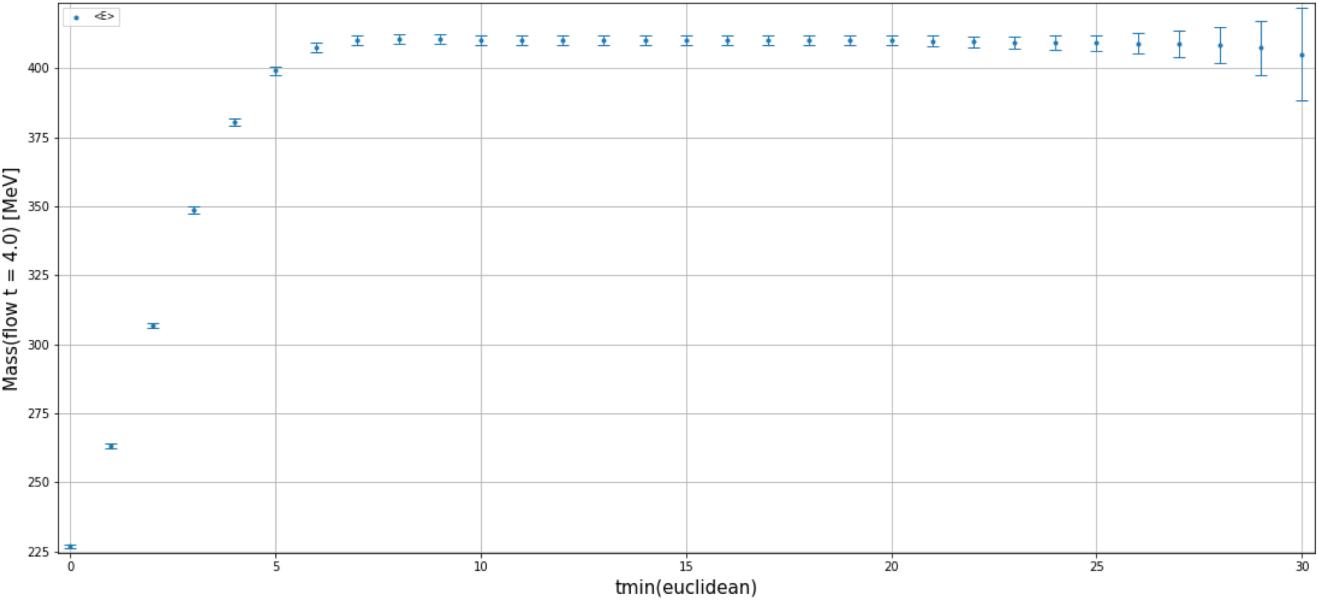
$L^3 \times T$	$16^3 \times 32$	$20^3 \times 40$	$28^3 \times 56$
a	0.1215 fm	0.0980 fm	0.0685 fm
aL	1.944 fm	1.960 fm	1.918 fm
G. Fields	800	800	800

For $32^3 \times 64$

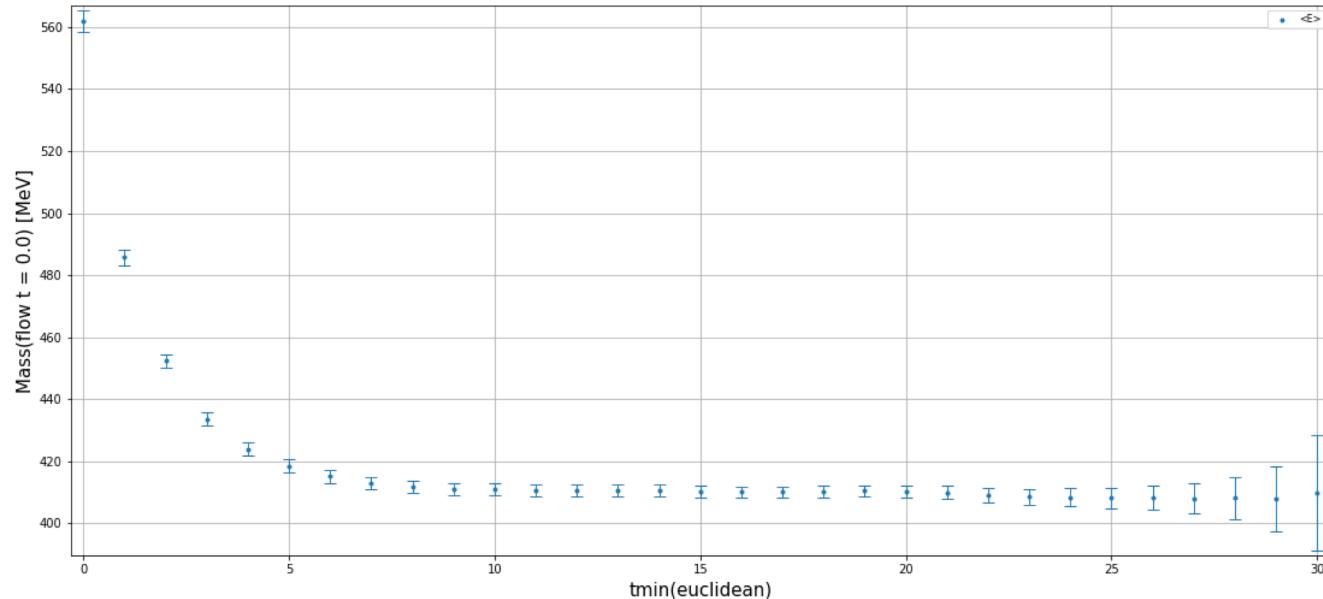
κ	0.13700	0.13727	0.13754
G. Fields	400	400	400

Results

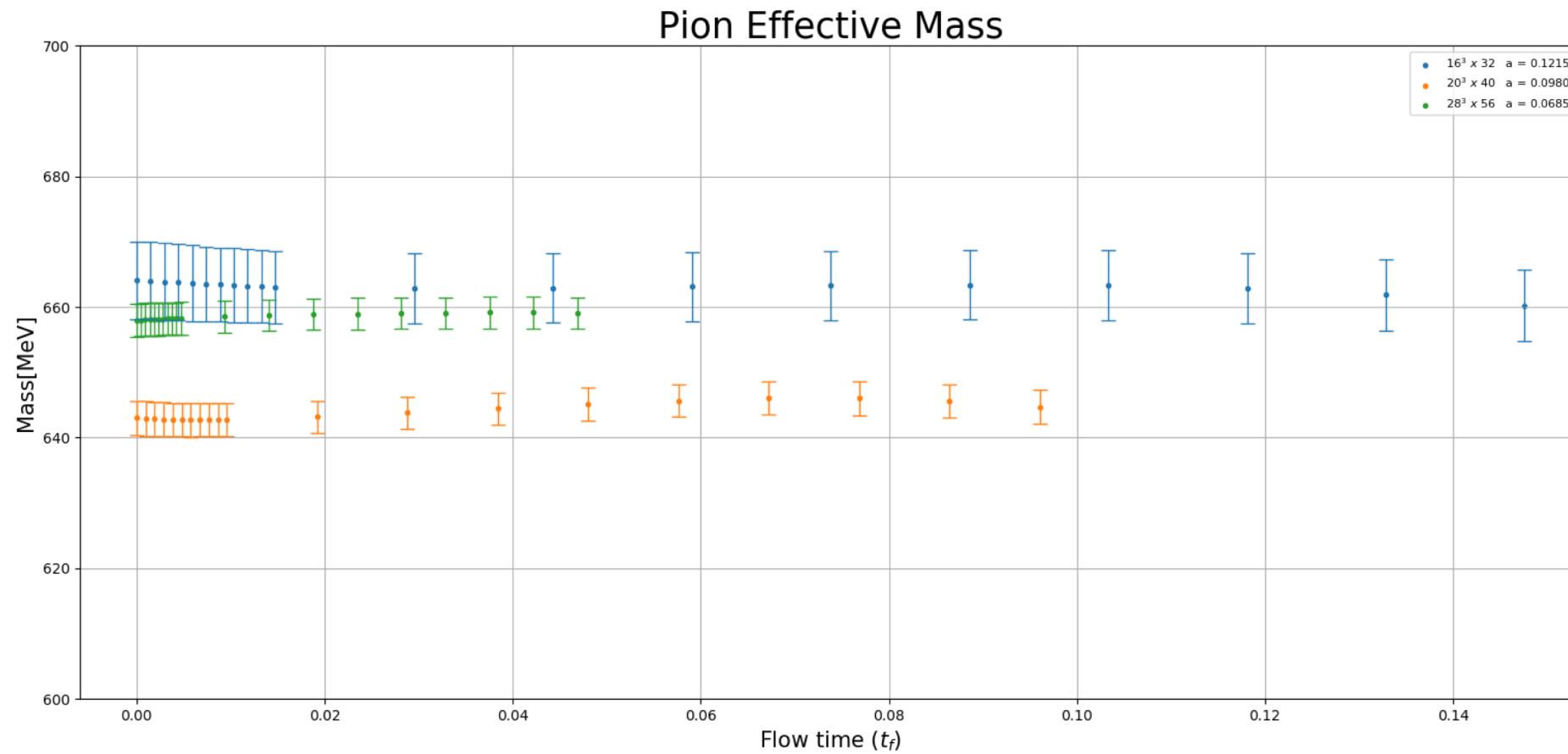
Pion Effective Mass



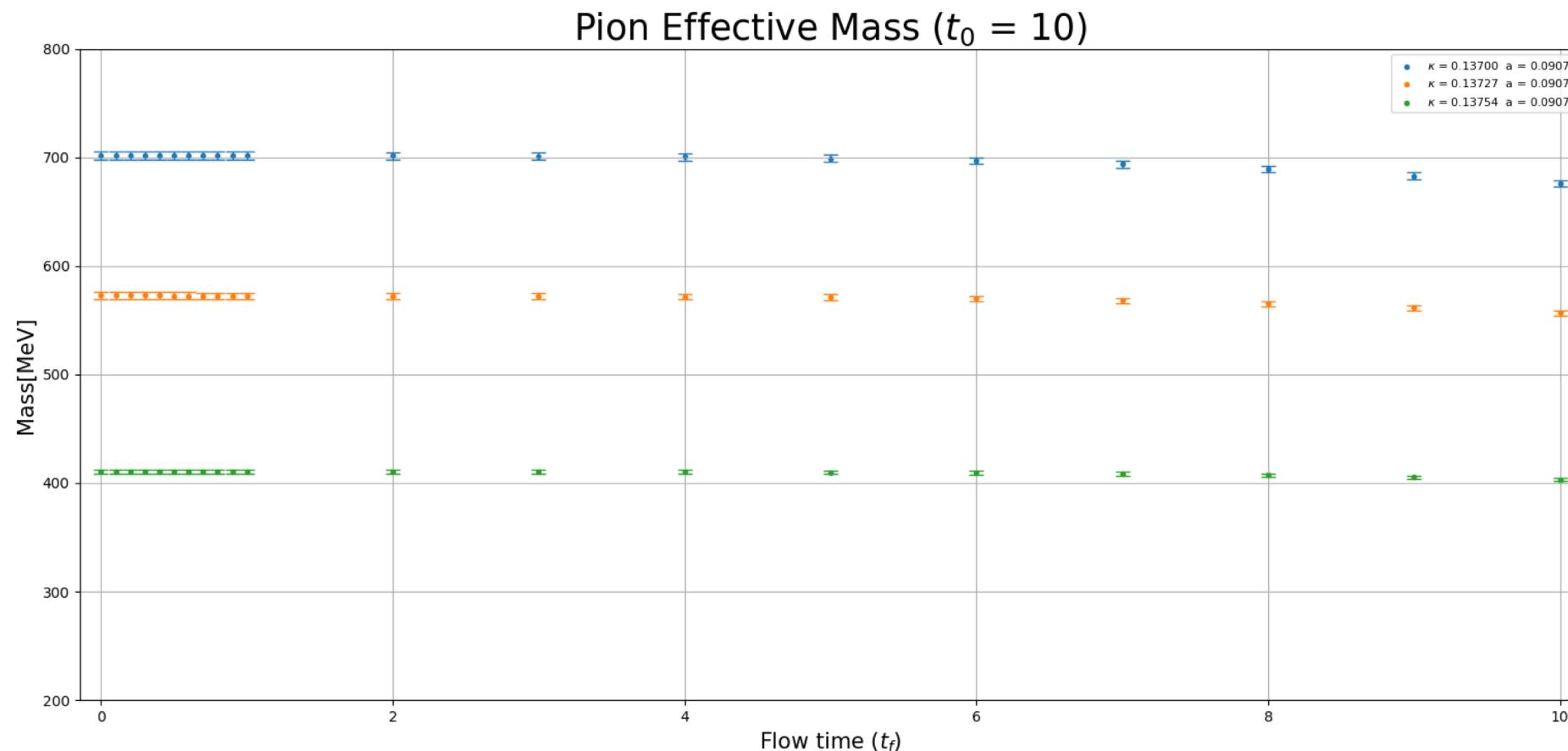
Pion Effective Mass



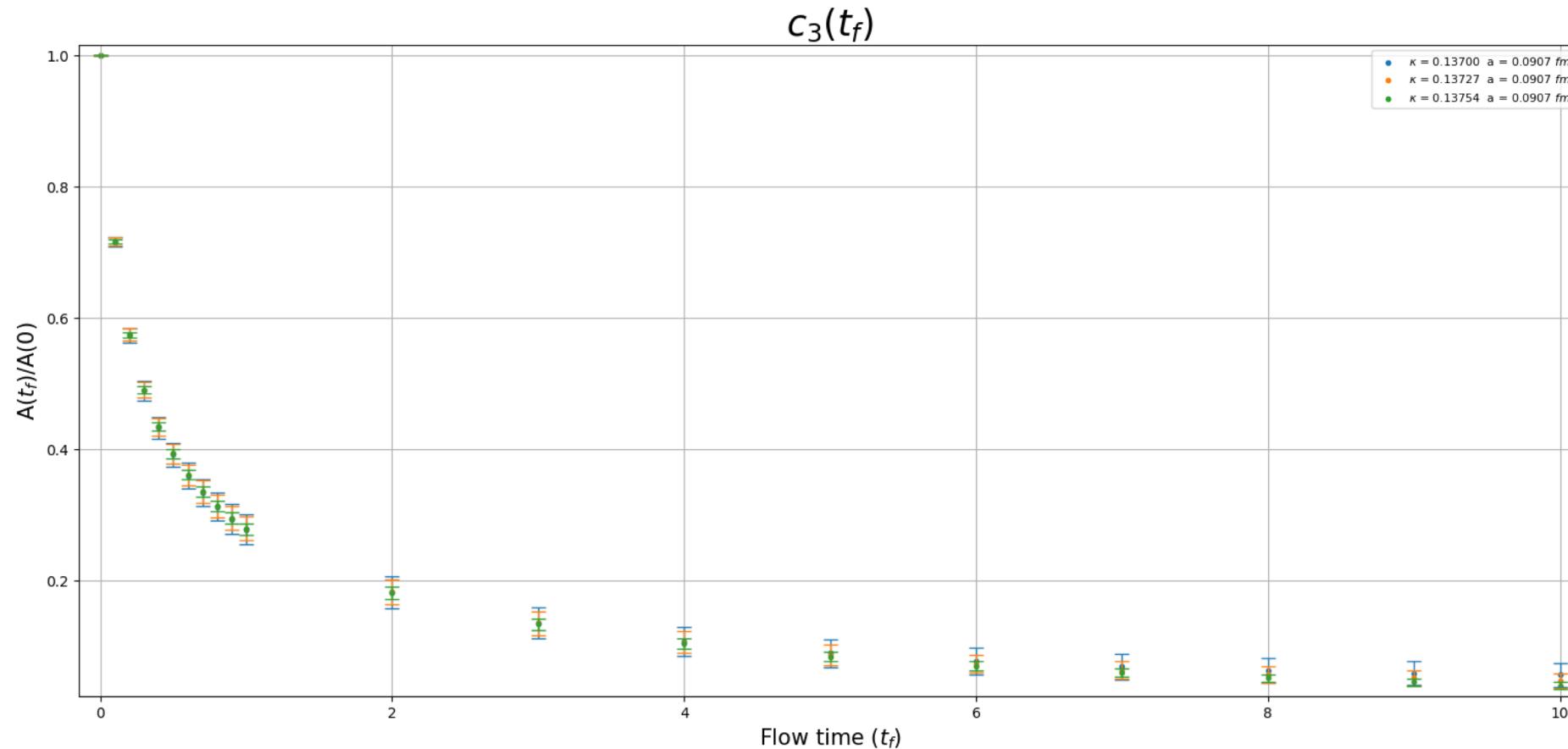
m_π for different lattice spacings



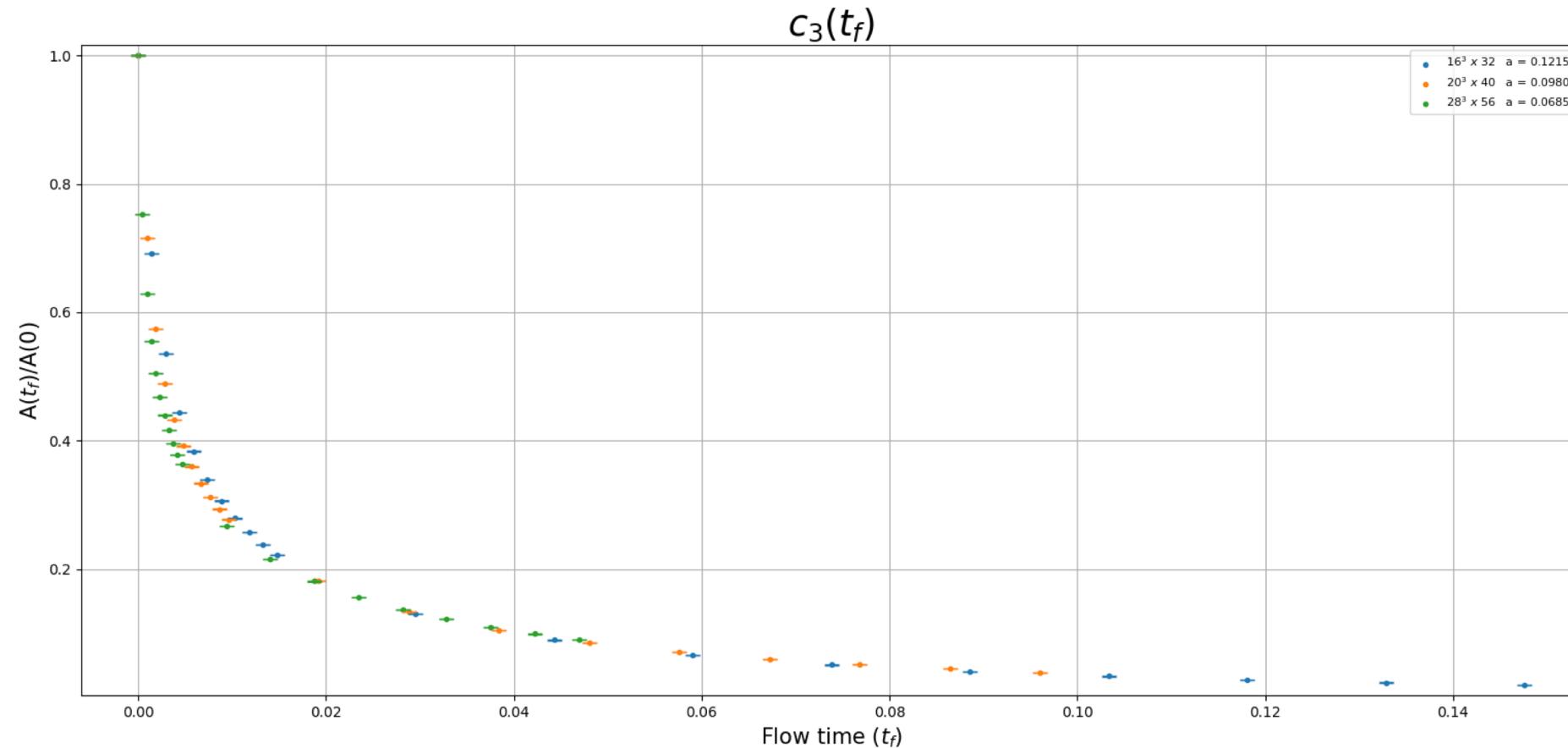
m_π for different Hopping parameters on the $32^3 \times 64$



$c_3(t_f)$ for different Hopping parameters on the $32^3 \times 64$



$c_3(t_f)$ for different lattice spacings



Conclusions and Next Steps

Numerical determination of $c_3(t_f)$ suggests:

- Quark mass independent
- No discretization effects

Next:

Compare the lattice result of $c_3(t)$ with the perturbative calculation

$$\mathcal{C}^{sub}(0, x) = c_3^{-1}(t_f) [\langle \mathcal{N} \mathcal{O}_s(t_f, x) \mathcal{N}^\dagger \rangle - \langle \mathcal{O}_s(t_f, x) \rangle \langle \mathcal{N} \mathcal{N}^\dagger \rangle] + \mathcal{O}(t_f)$$

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