

Pion Distribution Amplitude from lattice QCD: towards the continuum limit

Michael Gruber, Fabian Hutzler, Philipp Wein, Piotr Korcyl

and the RQCD collaboration



Universität Regensburg



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Pion distribution amplitude

Definition

Pion DA is the quantum amplitude that the pion moving with momentum p is built of a pair of quark and antiquark moving with momentum xp and $(1 - x)p$ respectively.

Relevance

Pion photoproduction: two off-shell photons provide the hard scale necessary for the factorization into the perturbative and non-perturbative parts. Transition form factor measured most recently experimentally by BaBar '09 and Belle '12.

Implementation

2nd moment of the pion DA, $\langle \xi^2 \rangle$, can be obtained numerically from two-point correlation functions.

Pion distribution amplitude

Definition

$$\begin{aligned}\langle 0 | \bar{d}(z_2 n) \not{\gamma}_5 [z_2 n, z_1 n] u(z_1 n) | \pi(p) \rangle &= \\ &= i f_\pi(p \cdot n) \int_0^1 dx e^{-i(z_1 x + z_2(1-x)) p \cdot n} \phi_\pi(x, \mu^2)\end{aligned}$$

Neglecting isospin breaking effects, $\phi_\pi(x)$ is symmetric under the interchange of momentum fraction $x \rightarrow (1 - x)$

$$\phi_\pi(x, \mu^2) = \phi_\pi(1 - x, \mu^2)$$

Moments of the momentum fraction difference $\xi = x - (1 - x)$ can be estimated on the lattice

$$\langle \xi^{2n} \rangle = \int_0^1 dx (2x - 1)^{2n} \phi_\pi(x, \mu^2)$$

$$\phi_\pi(x, \mu^2) = 6x(1-x) \left[1 + \sum_n a_{2n}^\pi(\mu) C_{2n}^{3/2}(2x-1) \right]$$

Pion distribution amplitude

Local operators

The nonlocal operator can be Taylor expanded and expressed in terms of local operators with derivatives

$$\bar{d}(z_2 n) \not{d} \gamma_5 [z_2 n, z_1 n] u(z_1 n) = \sum_{k,l=0}^{\infty} \frac{z_2^k z_1^l}{k! l!} n^\rho n^{\mu_1} \dots n^{\mu_{k+l}} \mathcal{M}_{\rho, \mu_1, \dots, \mu_{k+l}}^{(k,l)}$$

where

$$\mathcal{M}_{\rho, \mu_1, \dots, \mu_{k+l}}^{(k,l)} = \bar{d}(0) \overleftarrow{D}_{(\mu_1} \dots \overleftarrow{D}_{\mu_k} \overrightarrow{D}_{\mu_{k+1}} \dots \overrightarrow{D}_{\mu_{k+l}}} \gamma_\rho \gamma_5 u(0)$$

Consequently,

$$i^{k+l} \langle 0 | \mathcal{M}_{\rho, \mu_1, \dots, \mu_{k+l}}^{(k,l)} | \pi(p) \rangle = i f_\pi p_\rho p_{\mu_1} \dots p_{\mu_{k+l}} \langle x^l (1-x)^k \rangle$$

Pion distribution amplitude on the lattice

Lattice operators for the 2nd moment

Two local operators are relevant

$$\mathcal{O}_{\rho\mu\nu}^-(x) = \bar{d}(x) \left[\overleftarrow{D}_{(\mu} \overleftarrow{D}_{\nu)} - 2 \overleftarrow{D}_{(\mu} \overrightarrow{D}_{\nu)} + \overrightarrow{D}_{(\mu} \overrightarrow{D}_{\nu)} \right] \gamma_\rho \gamma_5 u(x)$$

and

$$\mathcal{O}_{\rho\mu\nu}^+(x) = \bar{d}(x) \left[\overleftarrow{D}_{(\mu} \overleftarrow{D}_{\nu)} + 2 \overleftarrow{D}_{(\mu} \overrightarrow{D}_{\nu)} + \overrightarrow{D}_{(\mu} \overrightarrow{D}_{\nu)} \right] \gamma_\rho \gamma_5 u(x)$$

Their bare matrix elements between vacuum and a pion state are proportional to

$$\langle 0 | \mathcal{O}_{\rho\mu\nu}^-(x) | \pi \rangle \sim \langle [x - (1-x)]^2 \rangle = \langle \xi^2 \rangle$$

$$\langle 0 | \mathcal{O}_{\rho\mu\nu}^+(x) | \pi \rangle \sim \langle [x + (1-x)]^2 \rangle = \langle 1^1 \rangle$$

Pion distribution amplitude on the lattice

Lattice operators for the 2nd moment

We estimate the following correlation functions

$$C_\rho(t, \mathbf{p}) = a^3 \sum_{\mathbf{x}} e^{-i\mathbf{px}} \langle \mathcal{O}_\rho(\mathbf{x}, t) J_{\gamma_5}(0) \rangle$$

$$C_{\rho\mu\nu}^\pm(t, \mathbf{p}) = a^3 \sum_{\mathbf{x}} e^{-i\mathbf{px}} \langle \mathcal{O}_{\rho\mu\nu}^\pm(\mathbf{x}, t) J_{\gamma_5}(0) \rangle$$

and construct ratios

$$R_{\rho\mu\nu,\sigma}^\pm(t, \mathbf{p}) = \frac{C_{\rho\mu\nu}^\pm(t, \mathbf{p})}{C_\sigma(t, \mathbf{p})} \sim p_\mu p_\nu R^\pm$$

which exhibit plateaux and which we fit to extract the value R^\pm .

Pion distribution amplitude on the lattice

Lattice operators for the 2nd moment

Finally,

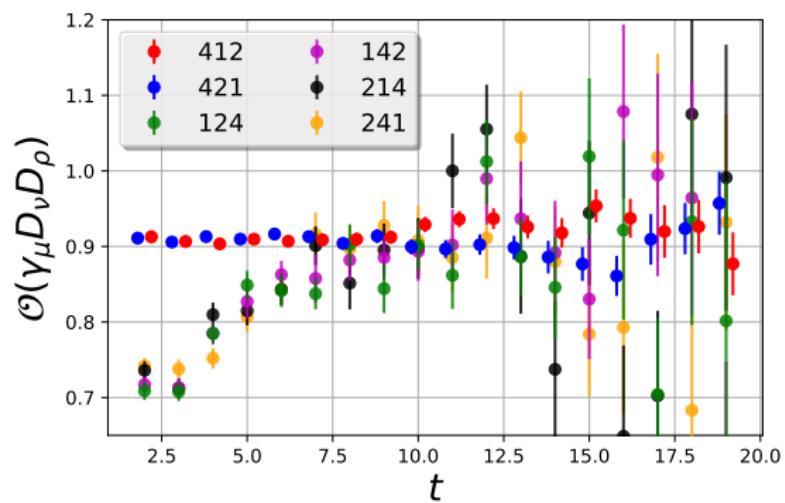
$$\langle \xi^2 \rangle^{\overline{\text{MS}}} = \zeta_{11} R^- + \zeta_{12} R^+,$$

$$a_2^{\overline{\text{MS}}} = \frac{7}{12} \left[5\zeta_{11} R^- + (5\zeta_{12} - \zeta_{22}) R^+ \right]$$

where ζ_{ij} are renormalization constants estimated non-perturbatively in the RI'/SMOM scheme and matched to the MSbar scheme at NLO.

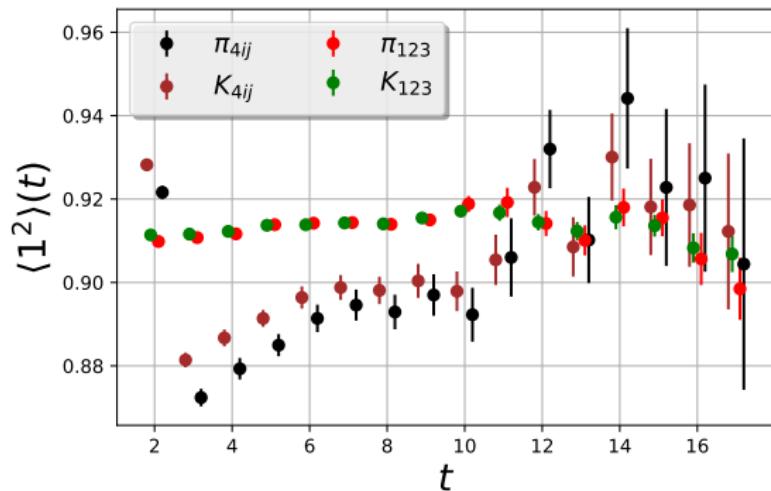
Pion distribution amplitude on the lattice

New momentum combination



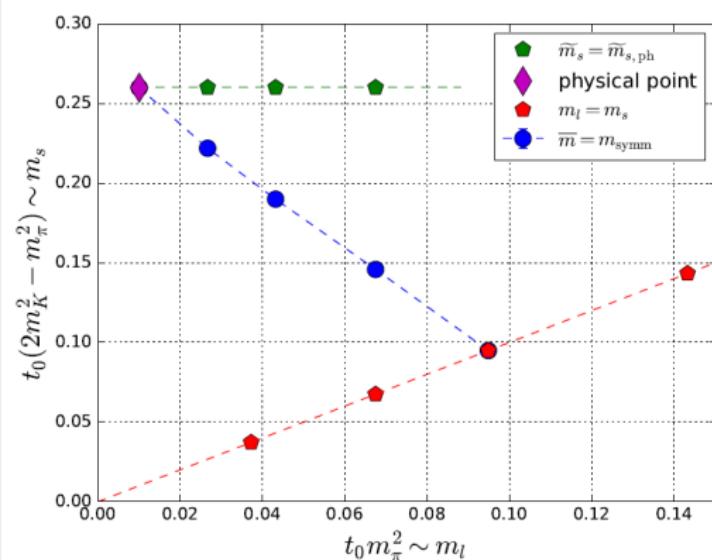
Pion distribution amplitude on the lattice

New momentum combination



Landscape of ensembles

Coordinated Lattice Simulations collaboration

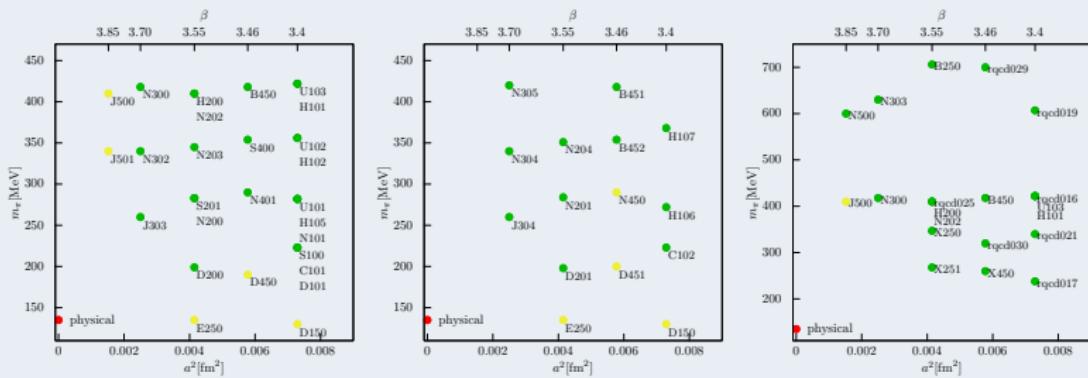


Credit: W. Söldner, Univ. Regensburg

Two trajectories lead to the physical point, a third trajectory has $m_l = m_s$.

Landscape of ensembles

Coordinated Lattice Simulations collaboration



Credit: J. Simeth, Univ. Regensburg

2nd moment of the pion distribution amplitude

Combined fit

We perform a combined fit to all data points (all lattice spacings and all pion/kaon masses along the three trajectories) with continuum ChPT formula (no chiral logs) supplemented with cutoff effects parametrization

$$\langle \xi^2 \rangle_\pi = (1 + c_0 a + c_1 a \bar{M}^2 + c_2^\pi a \delta M^2) \langle \xi^2 \rangle_0 + \bar{A} \bar{M}^2 - 2 \delta A \delta M^2,$$

$$\langle \xi^2 \rangle_K = (1 + c_0 a + c_1 a \bar{M}^2 + c_2^K a \delta M^2) \langle \xi^2 \rangle_0 + \bar{A} \bar{M}^2 + \delta A \delta M^2,$$

with \bar{A} , δA being low energy constants and

$$\bar{M}^2 = \frac{2m_K^2 + m_\pi^2}{3}, \quad \delta M^2 = m_K^2 - m_\pi^2.$$

⇒ 7 fit parameters

2nd moment of the pion distribution amplitude

Extrapolation of $\langle 1^2 \rangle$: check

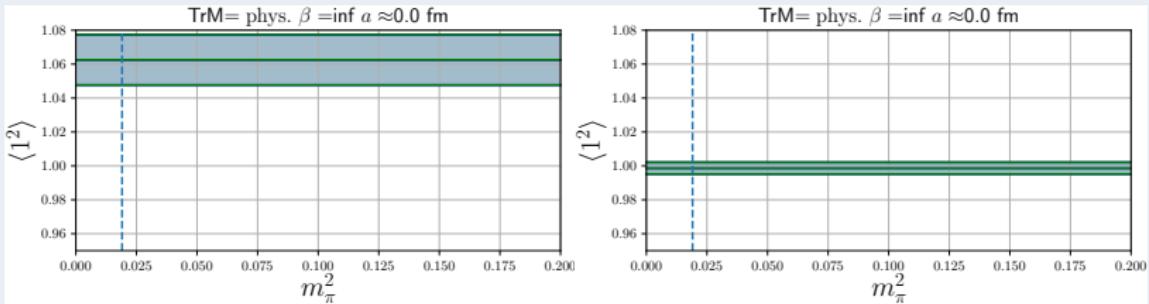
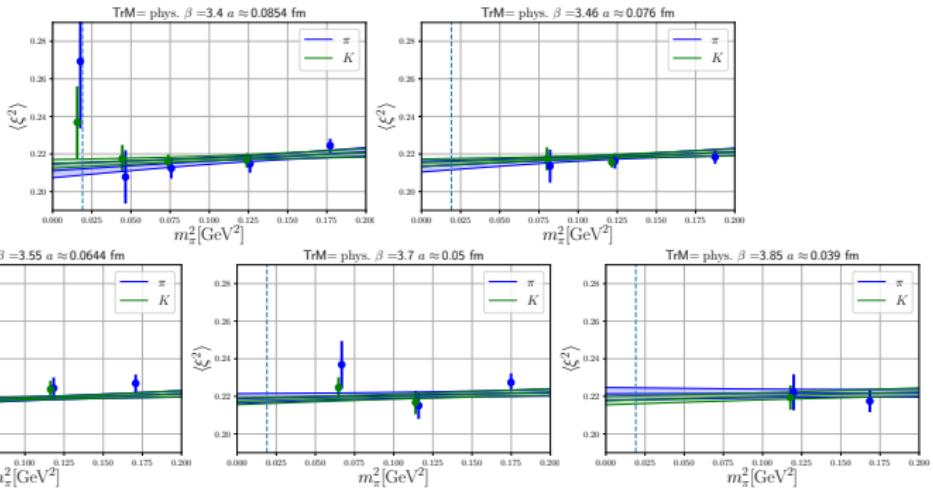
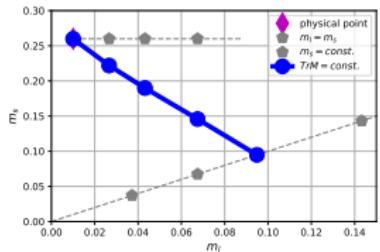
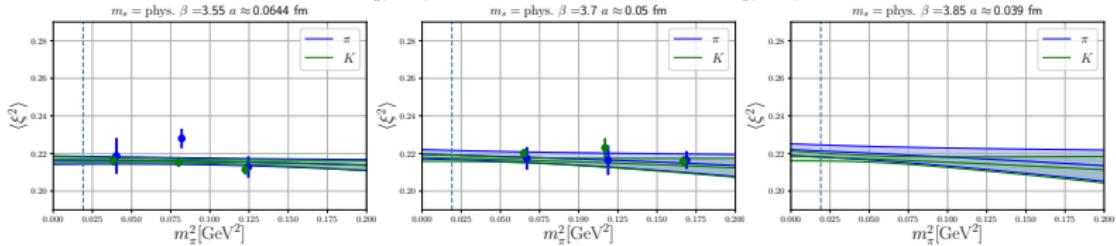
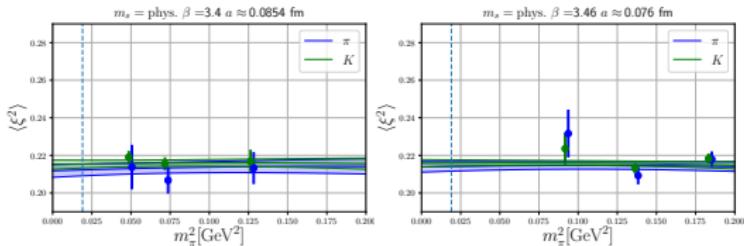
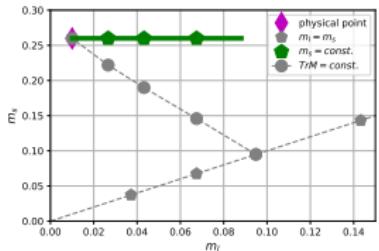


Figure: Old vs. new momentum combination

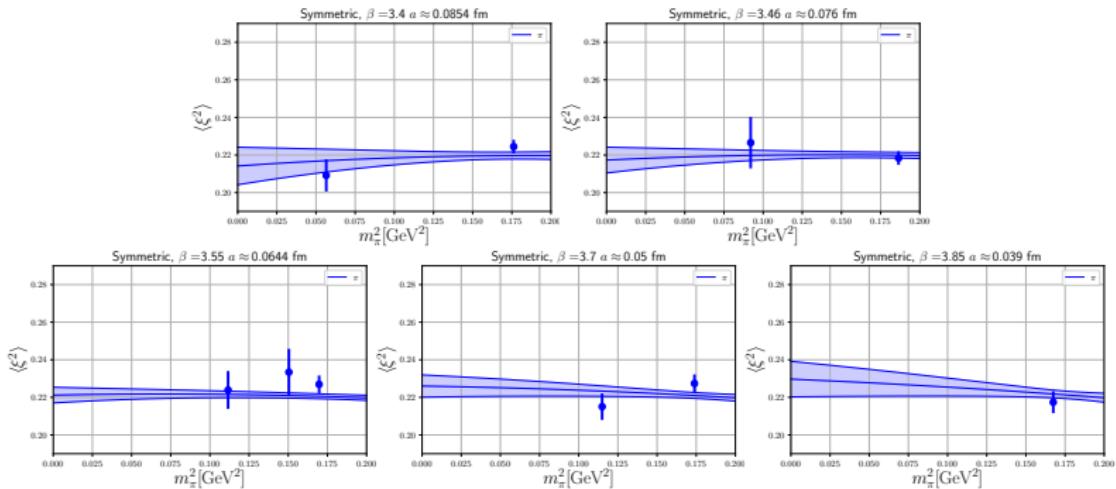
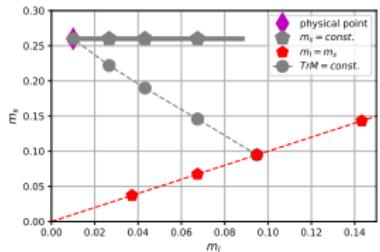
2nd moment of the pion distribution amplitude



2nd moment of the pion distribution amplitude



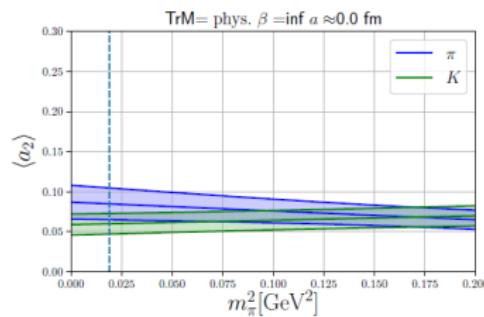
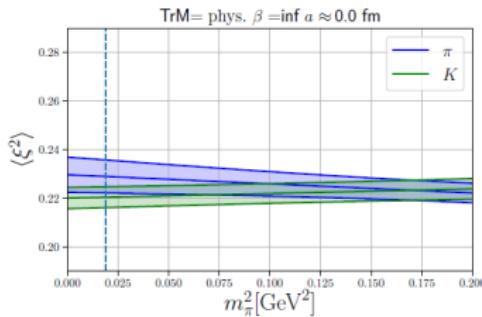
2nd moment of the pion distribution amplitude



2nd moment of the pion distribution amplitude

Continuum extrapolation: preliminary results, systematics under investigation

In the continuum limit $a \rightarrow 0$ we obtain:



- Current values at the physical point:

$$\langle \xi^2 \rangle_\pi = 0.2289(68)$$

$$\langle \xi^2 \rangle_K = 0.2204(42)$$

$$a_2^\pi = 0.0847(198)$$

$$a_2^K = 0.0599(128)$$

Conclusions

Pion distribution amplitude

Our preliminary results (statistical uncertainties only)

$$\langle \xi^2 \rangle_\pi = 0.2289(68) \text{ corresponding to } a_2^\pi = 0.085(20)$$

$$\langle \xi^2 \rangle_K = 0.2204(42) \text{ corresponding to } a_2^K = 0.060(13)$$

Compare with the previous value at finite lattice spacing and for $N_f = 2$:

$$\langle \xi^2 \rangle_\pi = 0.236 \pm 0.008, \text{ (Braun et al. '15).}$$

Full x dependence of the pion DA

In a separate project we are currently estimating non-perturbatively the position space DA (that carries information on the full x dependence)

⇒ Philipp Wein talk: Wednesday, 14:40, lecture hall 106

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