SYM flow equation in N=1 SUSY

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In collaboration with Naoya Ukita

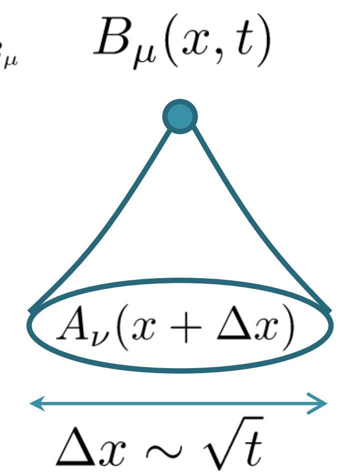
Yang-Mills gradient flow

- Gradient flow equation with flow time t

[Luscher, 2010]

$$A_\mu(x) \;\to\; B_\mu(x,t) \qquad B_\mu(x,t=0)=A_\mu(x)$$

$$\partial_t B_\mu(x,t) = D_\nu G_{\nu\mu}(x,t) \;\propto -\left.\frac{\partial S_{YM}}{\partial A_\mu(x)}\right|_{A_\mu \to B_\mu}$$



- Various studies in lattice QCD are done based on

  - smoothing effects for fields

  - UV-finiteness of flowed field correlators

  [Luscher 2010, 2013, Luscher-Weisz 2011, …]

Gradient flow in SYM

- Motivation

  - same applications as with QCD

    $T_{\mu\nu}$ energy momentum tensor [Suzuki, 2013]

    Numerical test of AdS/CFT $\quad \eta/s = \dfrac{1}{4\pi}$

  - flowed supercurrent is used to construct supersymmetric continuum limit from lattice theory

    [Hieda-Kasai-Makino-Suzuki, 2017]

- In this talk, we construct SYM-flow equation in Wess-Zumino gauge and study UV-finiteness of flowed field correlators

Plan of this talk

1. N=1 SYM and SYM-gradient flow

2. SYM-flow in WZ gauge and finiteness of correlators for flowed N=1 multiplet

3. Summary

1. N=1 SYM and SYM gradient flow

N=1 SYM in superfield formalism

- SYM action

$$S = \frac{1}{16g^2}\int d^4x\,\mathrm{tr}\Big(W^\alpha(V)W_\alpha(V)|_{\theta\theta} + h.c.\Big)$$

$$W_\alpha = -\frac{1}{4}\bar{D}\bar{D}e^{-2gV}D_\alpha e^{2gV}$$

vector superfield

$\theta_\alpha, \bar{\theta}_{\dot{\alpha}}$: two component Grassmann

$$V = C + i\theta\chi + \frac{i}{2}\theta\theta(M+iN) - \theta\sigma^\mu\bar{\theta}A_\mu$$
$$+ i\theta\theta\bar{\theta}(\lambda + \frac{i}{2}\bar{\sigma}^\mu\partial_\mu\chi) + \frac{1}{2}\theta\theta\bar{\theta}\bar{\theta}(D + \frac{1}{2}\Box C) + \cdots$$

- invariant under linear supersymmetry transformation

$$V \to V + \delta^0_\xi V$$

but C, χ, M, N are unwanted fields

Wess-Zumino gauge fixing

- S is invariant under extended gauge transf.

$$e^{2gV'} = e^{\Phi^\dagger} e^{2gV} e^{\Phi}$$

chiral superfield $\Phi = A + \sqrt{2}\theta\psi + \theta\theta F$

- taking the component fields of Φ as

$$A = \frac{C}{2}, \ \psi = -i\frac{\chi}{\sqrt{2}} \cdots \quad \Rightarrow \quad C' = \chi' = M' = N' = 0$$

Wess-Zumino gauge

- supersymmetry is modified as

$$\delta_\xi = \delta_\xi^0 + \delta_\Lambda^{gauge}$$

δ_ξ^0 -supersymmetry is broken in Wess-Zumino gauge

N=1 SYM in Wess-Zumino gauge

- SYM action

$$S = \frac{1}{16g^2}\int d^4x\,\mathrm{tr}\Big(W^\alpha(V)W_\alpha(V)|_{\theta\theta} + h.c.\Big)$$

$$= \int d^4x\mathrm{tr}\left\{-\frac{1}{4}F_{\mu\nu}F^{\mu\nu} - 2i\bar\lambda\bar\sigma^\mu D_\mu\lambda + \frac{1}{2}D^2\right\}$$

- supersymmetry transformation $\delta_\xi = \delta^0_\xi + \delta^{gauge}_\Lambda$

$$\delta_\xi A_\mu = i\xi\sigma_\mu\bar\lambda - i\lambda\sigma_\mu\bar\xi$$

$$\delta_\xi\lambda = i\xi D + \sigma^{\mu\nu}\xi F_{\mu\nu}$$

$$\delta_\xi D = -\xi\sigma^\mu D_\mu\bar\lambda - D_\mu\lambda\sigma^\mu\bar\xi$$

SYM-flow in superfield formalism

[Kikuchi-Onogi, 2014]

$$\partial_t V^a = -g^{ab}(V)\frac{\partial S_{SYM}}{\partial V^b} \qquad V = \sum_{a=1}^{N_c^2-1} V^a T^2$$

- SUSY and extended gauge invariant norm

$$||\delta V||^2 = \frac{1}{2g^2}\int d^4x d^2\theta d^2\bar{\theta}\ \mathrm{tr}\left(e^{-2gV}\delta e^{2gV} e^{-2gV}\delta e^{2gV}\right)$$
$$= g_{ab}(V)\delta V^a \delta V^b$$

$$g^{ab}(V) = 4g^2 \mathrm{tr}\left\{T^a \frac{{\mathcal{L}_V}^2}{\cosh(2g\mathcal{L}_V)-1} T^b\right\} \qquad \mathcal{L}_V X \equiv [V, X]$$

- invariant under δ_ξ^0-super and extended gauge transformations

2. SYM flow in Wess-Zumino gauge

SYM flow with a gauge fixing term

- gauge fixing term

[Kikuchi-Onogi,2014]

$$\partial_t V^a = -g^{ab} \frac{\partial S_{SYM}}{\partial V^b} + \delta_\Phi V^a$$

- choosing Φ as

D.K. and Ukita

$$\Phi = A + \sqrt{2}\theta\psi + \theta\theta F$$

$$A = 2D + i\omega \qquad \omega = \alpha \partial_\mu A_\mu$$

$$\psi = -2\sqrt{2}\sigma^\mu D_\mu \bar{\lambda}$$

$$F = 0$$

Wess-Zumino gauge is kept at non-zero flow time

SYM-flow equation in component fields

D.K. and Ukita

$$\partial_t B_\mu = D_\nu G_{\nu\mu} + \alpha D_\mu(\partial_\nu B_\nu) \quad -i\bar{\chi}\gamma_\mu\chi$$

$$\partial_t \chi = D_\mu D_\mu \chi + i\alpha[\chi, \partial_\nu B_\nu] \quad +i\frac{g}{2}\gamma_\mu\gamma_\nu[G_{\mu\nu}, \chi] + i[H, \gamma_5\chi]$$

$$\partial_t H = D_\mu D_\mu H + i\alpha[H, \partial_\mu B_\mu] \quad +i(D_\mu\bar{\chi}\gamma_5\gamma_\mu\chi + \bar{\chi}\gamma_\mu\gamma_5 D_\mu\chi)$$

$$B_\mu(x, t=0) = A_\mu(x)$$
$$H(x, t=0) = D(x)$$
$$\chi(x, t=0) = \psi(x)$$

new terms originated from SYM gradient

- all fields mix with each other
- invariant under t-independent gauge transformations for $\alpha = 0$

five dimensional SUSY

- SUSY for four dimensional fields

$$\delta_\xi A_\mu(x) = \bar{\xi}\gamma_\mu \psi(x)$$

$$\delta_\xi \psi(x) = \frac{1}{2}\gamma_\mu\gamma_\nu \xi F_{\mu\nu}(x) + \gamma_5 \xi D(x)$$

$$\delta_\xi D(x) = \bar{\xi}\gamma_5\gamma_\mu D_\mu \psi(x)$$

- an extension to d+1 dimensions

$$\delta_\xi B_\mu(x,t) = \bar{\xi}\gamma_\mu \chi(x,t)$$

$$\delta_\xi \chi(x,t) = \frac{1}{2}\gamma_\mu\gamma_\nu \xi G_{\mu\nu}(x,t) + \gamma_5 \xi H(x,t)$$

$$\delta_\xi H(x,t) = \bar{\xi}\gamma_5\gamma_\mu D_\mu \chi(x,t)$$

Consistency between SYM-flow and SUSY

D.K. and Ukita

- The commutator of ∂_t and δ_ξ

$$\frac{\partial}{\partial t}\delta_\xi B_\mu - \delta_\xi \frac{\partial}{\partial t} B_\mu = D_\mu \omega \qquad \omega \equiv -D_\nu(\bar{\xi}\gamma_\nu \chi) - \delta_\xi(\alpha \partial_\nu B_\nu)$$

$$\frac{\partial}{\partial t}\delta_\xi \chi - \delta_\xi \frac{\partial}{\partial t}\chi = i[\chi, \omega]$$

$$\frac{\partial}{\partial t}\delta_\xi H - \delta_\xi \frac{\partial}{\partial t} H = i[H, \omega]$$

$$\frac{\partial}{\partial t}\delta_\xi - \delta_\xi \frac{\partial}{\partial t} = \delta^g_\omega \quad \text{(gauge transformation)}$$

- For gauge-invariant operators, the SYM flow and SUSY in WZ gauge are consistent

Renormalization of four dimensional SYM

- N=1 SYM action

$$S_{SYM} = \frac{1}{g_0^2}\int d^D x \,\mathrm{tr}\left\{\frac{1}{2}F_{\mu\nu}^2 - \bar{\psi}\gamma_\mu D_\mu \psi + D^2 + \frac{1}{\xi_0}(\partial_\mu A_\mu)^2 + 2\partial_\mu \bar{c} D_\mu c\right\}$$

- Renormalization (dimensional reg., MS-bar)

$$g_0^2 = \mu^{2\epsilon} g^2 Z_g \qquad \xi_0 = \xi Z_A$$

$$A_\mu = Z_g^{1/2} Z_A^{1/2} A_{R,\mu} \qquad \psi = Z_g^{1/2} Z_\psi^{1/2} \psi_R \qquad D = Z_g^{1/2} D_R$$

$$c = Z_c Z_g^{1/2} Z_A^{1/2} c_R \qquad \bar{c} = Z_g^{1/2} Z_A^{1/2} \bar{c}_R$$

$$Z_g = 1 - 3\frac{g^2 C_2(G)}{16\pi^2\epsilon} + \mathcal{O}(g^4)$$

$$Z_A = 1 + \frac{3-\xi}{2}\frac{g^2 C_2(G)}{16\pi^2\epsilon} + \mathcal{O}(g^4)$$

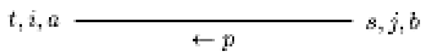
$$Z_\psi = 1 - \xi\frac{g^2 C_2(G)}{16\pi^2\epsilon} + \mathcal{O}(g^4)$$

$$Z_c = 1 + \frac{3-\xi}{4}\frac{g^2 C_2(G)}{16\pi^2\epsilon} + \mathcal{O}(g^4)$$

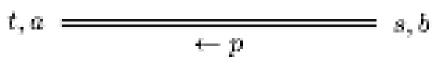
Feynman diagrams for flowed fields

- Propagators and vertices

$$= \frac{g_0^2\delta_{ab}}{p^2}\left[\left(\delta_{\mu\nu}-\frac{p_\mu p_\nu}{p^2}\right)+\xi_0\frac{p_\mu p_\nu}{p^2}\right]e^{-(t+s)p^2}$$

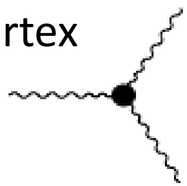


$$= \frac{g_0^2 i(\not{p}C)_{ij}\delta_{ab}}{p^2}e^{-(t+s)p^2}$$



$$= g_0^2\delta_{ab}e^{-(t+s)p^2}$$

vertex



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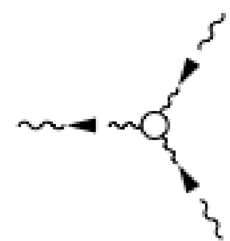
- Flow propagators and flow vertices

flow propagator

$$\propto \theta(t-s)e^{-(t-s)p^2}$$

flow vertex



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1-loop calculations using SYM flow

- SYM flow equation

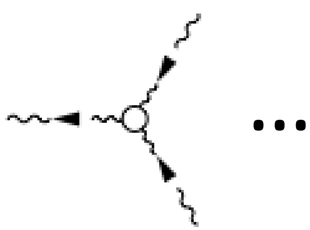
$$\partial_t B_\mu = D_\nu G_{\nu\mu} + \alpha D_\mu(\partial_\nu B_\nu) -i\bar{\chi}\gamma_\mu\chi$$

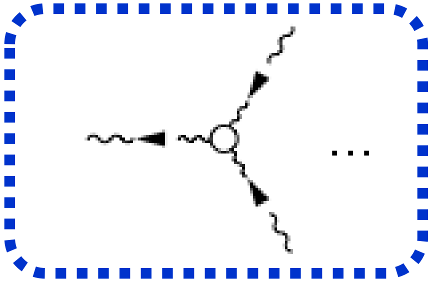
$$\partial_t \chi = D_\mu D_\mu \chi + i\alpha[\chi, \partial_\mu B_\mu] + i\frac{g}{2}\gamma_\mu\gamma_\nu[G_{\mu\nu}, \chi] + i[H, \gamma_5\chi]$$

$$\partial_t H = D_\mu D_\mu H + i\alpha[H, \partial_\mu B_\mu] + i(D_\mu\bar{\chi}\gamma_5\gamma_\mu\chi + \bar{\chi}\gamma_\mu\gamma_5 D_\mu\chi)$$

- New terms (blue ones) yield new flow vertices in perturbative calculations

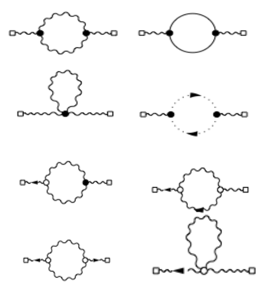
flow vertex





Flowed gauge field (non-SUSY flow)

$$\langle B^a_\mu(t,p)B^b_\nu(s,-p)\rangle\Big|_{\mathrm{pole}} = \left[c_1\left(\delta_{\mu\nu}-\frac{p_\mu p_\nu}{p^2}\right)+c_2\xi\frac{p_\mu p_\nu}{p^2}\right]\frac{\delta_{ab}e^{-(t+s)p^2}}{p^2}\frac{g^4C_2(G)}{16\pi^2\epsilon}$$

		c_1	c_2
tree		-3	$-\frac{3+\xi}{2}$
1-loop		$+3$	$+\frac{3+\xi}{2}$
total		0	0

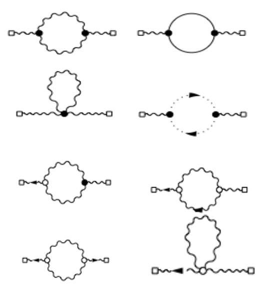
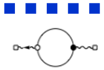
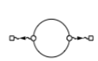
$$\boxed{B^R_\mu = B_\mu}$$

UV-finite

no extra Z-factor

Flowed gauge field (SYM flow)

$$\langle B^a_\mu(t,p)B^b_\nu(s,-p)\rangle\Big|_{\text{pole}} = \left[c_1\left(\delta_{\mu\nu}-\frac{p_\mu p_\nu}{p^2}\right)+c_2\xi\frac{p_\mu p_\nu}{p^2}\right]\frac{\delta_{ab}e^{-(t+s)p^2}}{p^2}\frac{g^4C_2(G)}{16\pi^2\epsilon}$$

		c_1	c_2
tree		-3	$-\frac{3+\xi}{2}$
1-loop		$+3$	$+\frac{3+\xi}{2}$
		$+1$	$+1$
		-1	-1
		0	0
total		0	0

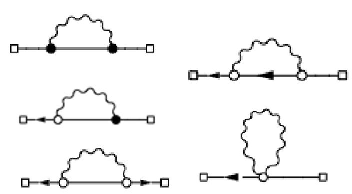
$$\boxed{B^R_\mu = B_\mu}$$

UV-finite

no extra Z-factor

Flowed fermion (non-SUSY flow)

$$\langle \chi^a(t,p)\chi^b(s,-p)\rangle\Big|_{\mathrm{pole}} = c_1 \frac{i\delta_{ab}(\gamma_\mu p_\mu C)e^{-(t+s)p^2}}{p^2}\frac{g^4 C_2(G)}{16\pi^2\epsilon}$$

		c_1
tree		-3
1-loop		-3
total		-6

$$\boxed{\chi^R = Z_\chi^{1/2}\chi}$$

UV-div.

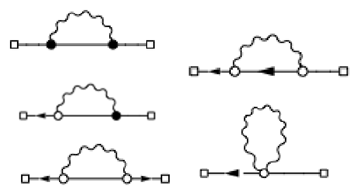
extra Z-factor is needed

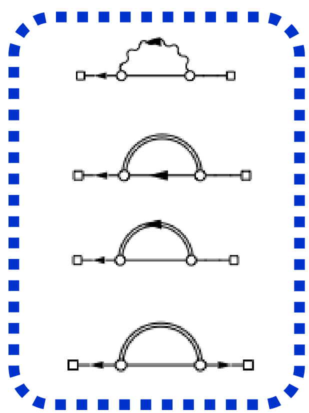
Flowed fermion (SYM flow)

$$\langle \chi^a(t,p)\chi^b(s,-p)\rangle\Big|_{\text{pole}} = c_1 \frac{i\delta_{ab}(\gamma_\mu p_\mu C)e^{-(t+s)p^2}}{p^2}\frac{g^4 C_2(G)}{16\pi^2\epsilon}$$

c_1

tree  -3

1-loop  -3 $+9/2$



$+5/2$

$-1/2$

$-1/2$

0

total -6

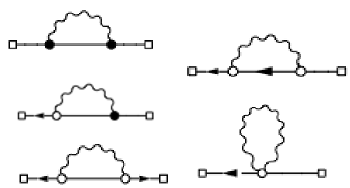
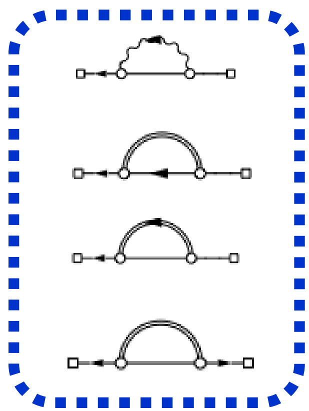
$$\boxed{\chi^R = Z_\chi^{1/2}\chi}$$

UV-div.

extra Z-factor is needed

Flowed fermion (SYM flow)

$$\langle \chi^a(t,p)\chi^b(s,-p)\rangle\Big|_{\mathrm{pole}} = c_1 \frac{i\delta_{ab}(\gamma_\mu p_\mu C)e^{-(t+s)p^2}}{p^2}\frac{g^4 C_2(G)}{16\pi^2\epsilon}$$

		c_1
tree		-3
1-loop		-3 $+9/2$
		$+5/2$ $-1/2$ $-1/2$ 0
	total	0

$$\boxed{\chi^R = \chi}$$

UV-finite
no extra Z-factor

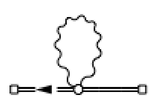
Flowed auxiliary field (SYM flow)

$$\langle H^a(t,p)H^b(s,-p)\rangle\Big|_{\text{pole}} = c_1\delta_{ab}e^{-(t+s)p^2}\frac{g^4C_2(G)}{16\pi^2\epsilon}$$

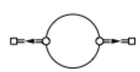
c_1

tree  -3

 -3



1-loop

 $+3$

 $+3$

total 0

$$\boxed{H^R = H}$$

UV-finite

no extra Z-factor

5. Summary

Conclusion

- SYM gradient flow can be defined in terms of component fields (in Wess-Zumino gauge)

- consistency relation $\partial_t \delta_\xi - \delta_\xi \partial_t = \delta^g_{\omega(\xi)}$ implies that

  SYM gradient flow = supersymmetric flow

- 2-point function of flowed N=1 gauge multiplet is UV-finite, at least, at 1-loop level

  $$B^R_\mu = B_\mu \qquad \chi^R = \chi \qquad H^R = H$$

- We are now tackling a proof of UV-finiteness of any correlators for all order of PT