Pion distribution amplitude from Euclidean correlation functions: Exploring universality and higher-twist effects

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Definition of distribution amplitudes

$$|\pi\rangle = |\bar{q}q\rangle + |\bar{q}gq\rangle + \ldots$$

- hard exclusive processes are sensitive to
  - Fock states with smallest number of partons
  - the distribution of the momentum within a Fock state at small transverse distances
- this information is contained in light-cone DAs; leading twist DA ϕ_π

$$\langle 0|\bar{u}(z)[z,-z]\not{z}\gamma_5 u(-z)|\pi(p)\rangle = iF_\pi\, p\cdot z \int_0^1 du\, e^{i(2u-1)p\cdot z}\phi_\pi(u,\mu) \qquad \underline{z^2=0}$$

- quark and antiquark carry the momentum fraction u and $\bar{u} = 1-u$, respectively
- physical information: *complementary* to PDFs
- lattice technique: very *similar* to PDFs

Lattice methods

Problem: on a Euclidean space-time one cannot realize nontrivial lightlike distances

- **traditional solution:** calculate Mellin moments of the DAs ($\hat{=}$ local derivative ops.)
  Hadron structure, Tue 15:20 (P. Korcyl)
  - higher moments $\rightarrow$ problems with renormalization (operator mixing)
- **new approach:** relate DAs to correlation functions at spacelike distance

  $\rightarrow$ requires large hadron momenta
  $\rightarrow$ relies heavily on pQCD

  - **Option 1: use a nonlocal operator** $\langle 0|\bar{q}(z)\Gamma[z,0]q(0)|\pi\rangle$ $\underline{z^2 < 0}$
    Ji arXiv:1305.1539
  - **Option 2: use two local operators** $\langle 0|\bar{q}(z)\Gamma_1 q(z)\bar{q}(0)\Gamma_2 q(0)|\pi\rangle$ $\underline{z^2 < 0}$
    Braun, Müller arXiv:0709.1348
    Ma, Qiu arXiv:1709.03018
  - ...

DA $\longleftrightarrow$ correlation function (schematically & oversimplified)

$$\text{DA} \xleftarrow{\quad\text{LaMET}\quad} \text{quasi-DA} \xleftarrow{\quad\text{FT}\quad} \text{lattice data}$$

Hadron structure, Mon 14:00 (K. Cichy), 14:20 (A. Scapellato), 14:40 (Y. Zhao), 15:00 (Y. Yang), 15:20 (Y. Liu)

Hadron structure, Tue 14:40 (R. Zhang), 16:10 (J. Zhang), 16:30 (N. Karthik), 16:50 (C. Shugert)

or

$$\text{DA} \xleftrightarrow{\quad\text{FT}\quad} \text{Ioffetime-DA} \xleftarrow{\quad\text{pQCD}\quad} \text{lattice data (pseudo-DA)}$$

Hadron structure, Mon 16:10 (S. Zafeiropoulos), 16:30 (J. Karpie)

or

$$\text{DA} \xrightarrow{\quad\text{pQCD (directly in position space)}\quad} \text{lattice data}$$

our ansatz: (also works when using the Wilson-line operator)

- parametrize DA (& higher twist effects) and fit directly to the lattice data
- basic idea very similar to "lattice cross section" talks on PDFs

Hadron structure, Tue 14:00 (R. Sufian), 14:20 (B. Chakraborty)

(feel free to replace DA by PDF on this slide)

Matrix elements $\leftrightarrow$ DAs

$$\mathbb{T}_{\mathrm{XY}}(p\cdot z, z^2) = \langle 0|J_X^\dagger(\tfrac{z}{2})J_Y(-\tfrac{z}{2})|\pi^0(p)\rangle$$

$$J_S = \bar{q}\,u\,,\quad J_P = \bar{q}\gamma_5 u\,,\quad J_V^\mu = \bar{q}\gamma^\mu u \equiv J_{V^\mu}\,,\quad J_A^\mu = \bar{q}\gamma^\mu\gamma_5 u \equiv J_{A^\mu}$$

$$\mathbb{T}_{\mathrm{SP}} = T_{\mathrm{SP}}$$

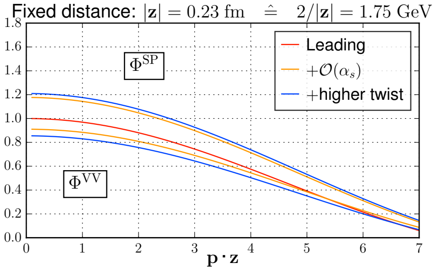
$$\mathbb{T}_{\mathrm{VV}}^{\mu\nu} = \frac{i\varepsilon^{\mu\nu\rho\sigma}p_\rho z_\sigma}{p\cdot z}T_{\mathrm{VV}}$$

$$\mathbb{T}_{\mathrm{VA}}^{\mu\nu} = \frac{p^\mu z^\nu + z^\mu p^\nu - g^{\mu\nu}p\cdot z}{p\cdot z}T_{\mathrm{VA}} + \frac{p^\mu z^\nu - z^\mu p^\nu}{p\cdot z}T_{\mathrm{VA}}^{(2)} + \frac{2z^\mu z^\nu - g^{\mu\nu}z^2}{z^2}T_{\mathrm{VA}}^{(3)} + \frac{2p^\mu p^\nu - g^{\mu\nu}p^2}{p^2}T_{\mathrm{VA}}^{(4)} + g^{\mu\nu}T_{\mathrm{VA}}^{(5)}$$

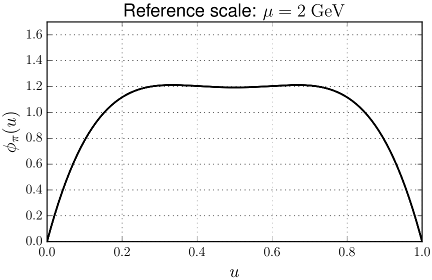
- similar for PS, AA, AV
- q is an auxiliary quark $q \neq u, d$, but $m_q = m_u = m_d$

Matrix elements $\leftrightarrow$ DAs

$$T_{\mathrm{XY}}(p\cdot z, z^2) = F_\pi \frac{p\cdot z}{2\pi^2 z^4} \underbrace{\int_0^1 du\, e^{i(u-1/2)p\cdot z}\phi_\pi(u) + \mathcal{O}(\alpha_s) + \text{higher twist}}_{\equiv\, \Phi^{\mathrm{XY}}(p\cdot z, z^2)}$$



$\longleftrightarrow$



- twist 4 effects estimated using asymptotic shape for chiral-odd twist three DAs $\rightarrow$ one parameter $\delta_2^\pi = 0.17\,\mathrm{GeV}^2$ (at $\mu = 2\,\mathrm{GeV}$, QCD sum rule estimate)

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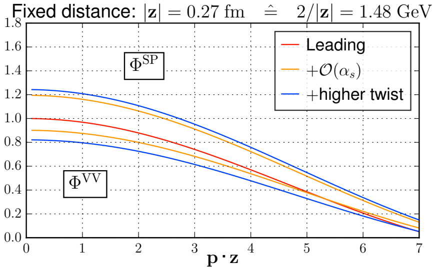

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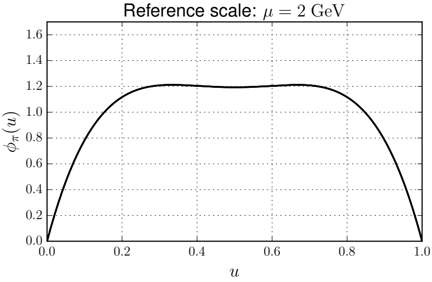

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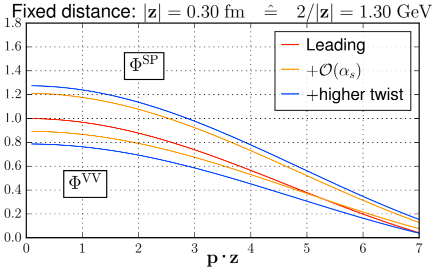

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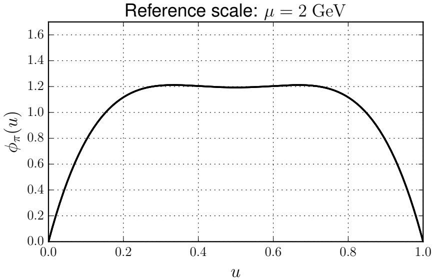

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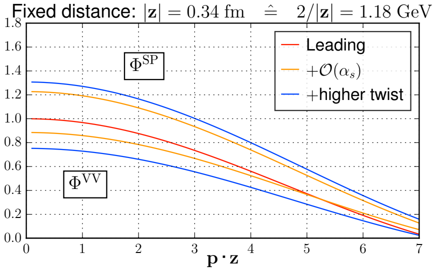

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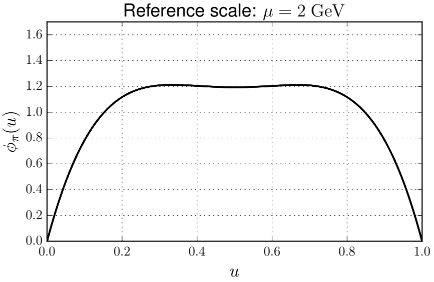

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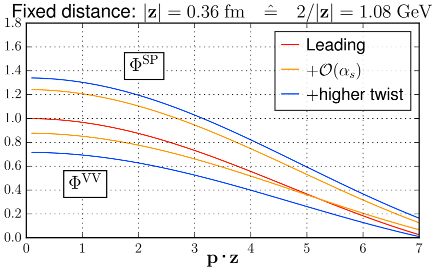

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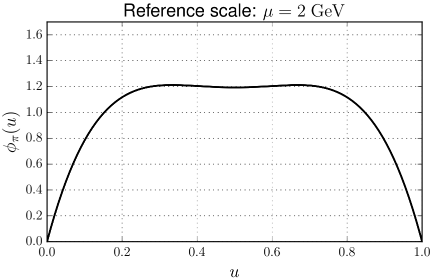

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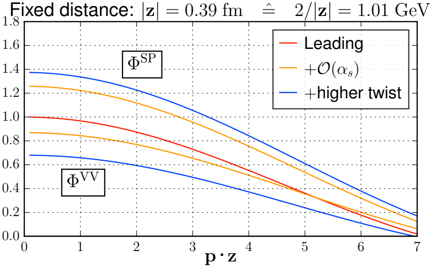

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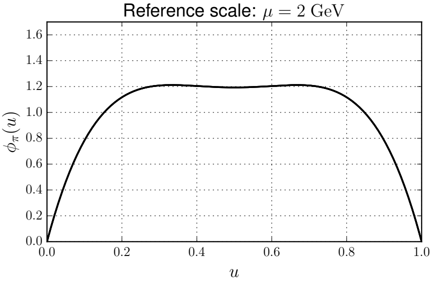

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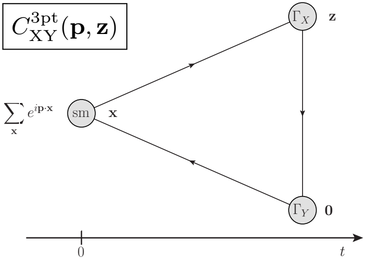
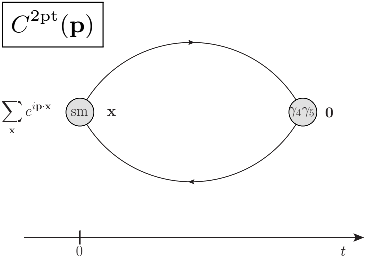
Obtaining the matrix elements from Lattice

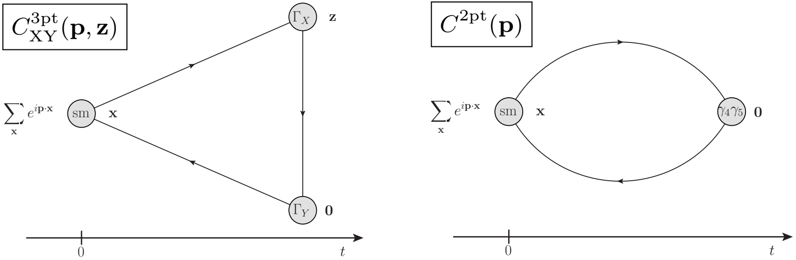




$$\frac{\mathbb{T}_{\mathrm{XY}}(p\cdot z, z^2)}{F_\pi} = \frac{Z_X(\mu)Z_Y(\mu)}{Z_A}\frac{C^{3\text{pt}}_{\mathrm{XY}}(\mathbf{p},\mathbf{z})e^{\frac{i}{2}\mathbf{p}\cdot\mathbf{z}}}{C^{2\text{pt}}(\mathbf{p})}E(\mathbf{p}) + \text{excited states}$$

- the Z_X is the renormalization factor for the respective current (nonperturbatively calculated in RI′-MOM → conversion to $\overline{\text{MS}}$ in 3-loop PT)
- we set both, the renormalization and the factorization scale to $\mu = 2/|\mathbf{z}|$
- phase factor shifts the currents to the symmetric position

Obtaining the matrix elements from Lattice



$$\frac{\mathbb{T}_{\mathrm{XY}}(p\cdot z, z^2)}{F_\pi} = \frac{Z_X(\mu)Z_Y(\mu)}{Z_A}\frac{C^{\mathrm{3pt}}_{\mathrm{XY}}(\mathbf{p},\mathbf{z})e^{\frac{i}{2}\mathbf{p}\cdot\mathbf{z}}}{C^{\mathrm{2pt}}(\mathbf{p})}E(\mathbf{p}) + \text{excited states}$$

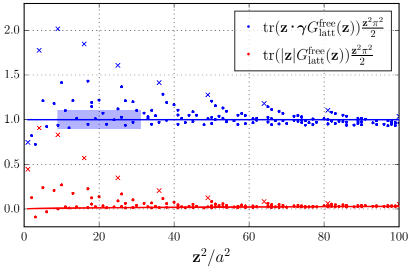
- smearing: momentum smearing
  $\rightarrow$ improved overlap with hadrons at large momentum
- new: we use stochastic estimation
  $\rightarrow$ get a volume average at the cost of some stochastic noise
  $\rightarrow$ much smaller statistical error

Discretization effects of the free Wilson propagator

propagator comparison:

free Wilson vs. free continuum

- large effects in chiral even (blue, $\propto \not{z}$) and chiral odd (red, $\propto \mathbb{1}$) part
- in continuum: chiral odd part strongly suppressed
- **problem on lattice:** large artefacts from terms removing the doublers



solution:

1. use observables, where the chiral odd part does not contribute at tree-level

$$\frac{1}{2}\left(T_{\mathrm{SP}}+T_{\mathrm{PS}}\right), \qquad \frac{1}{2}\left(T_{\mathrm{VA}}+T_{\mathrm{AV}}\right), \qquad \frac{1}{2}\left(T_{\mathrm{VV}}+T_{\mathrm{AA}}\right)$$

2. introduce correction factor for chiral even part
3. most important: ignore distances where the correction $> 10\%$ or $|\mathbf{z}| < 3a$

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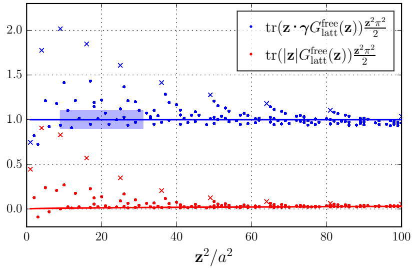


note:

1. upper limit of range determined by $\mu = 2/|\mathbf{z}| \geq 1\,\mathrm{GeV}$
   $\Rightarrow a \to a/2$ shifts the upper limit by a factor 4 to the right
2. discretization effects are strongest along the axes (crosses)
   $\to$ similar for Wilson-line operators?

Numerical study

Simulation details:

- mass-degenerate $N_f = 2$ nonperturbatively improved Wilson (clover) fermions and Wilson gluon action
- $L^3 \times T = 32^3 \times 64$
- coupling parameter $\beta = 5.29 \mathrel{\hat{=}}$ lattice spacing $a \approx 0.071\,\mathrm{fm} = (2.76\,\mathrm{GeV})^{-1}$
- mass parameter $\kappa = 0.13632 \mathrel{\hat{=}}$ pion mass $m_\pi = 0.10675(59)/a \approx 295\,\mathrm{MeV}$
- 12 momenta in different directions with $0.54\,\mathrm{GeV} \leq |\mathbf{p}| \leq 2.03\,\mathrm{GeV}$

DA parametrizations: at the scale $\mu = 2\,\mathrm{GeV}$

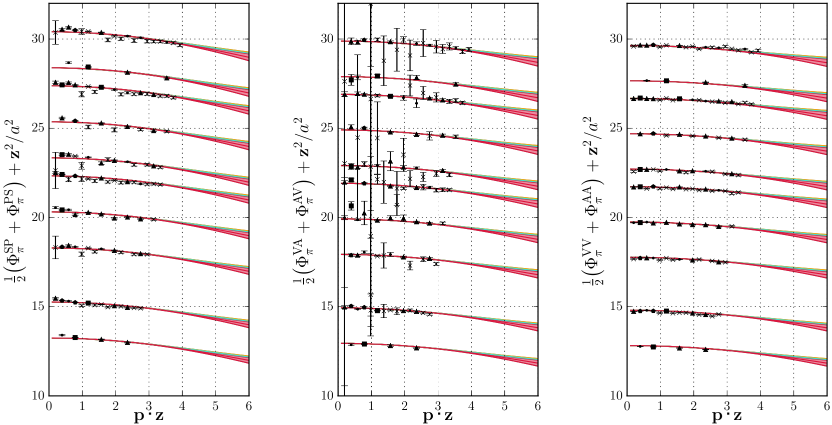
- Expansion in orthogonal (Gegenbauer) polynomials (truncated at $n = 2$ or $n = 4$)

$$\phi_\pi(u, \mu) = 6u(1-u) \sum_{n=0,2,\ldots}^{\infty} a_n^\pi(\mu) C_n^{3/2}(2u-1)\,, \qquad a_0^\pi = 1 \text{ (normalization)}$$

- alternatively we try

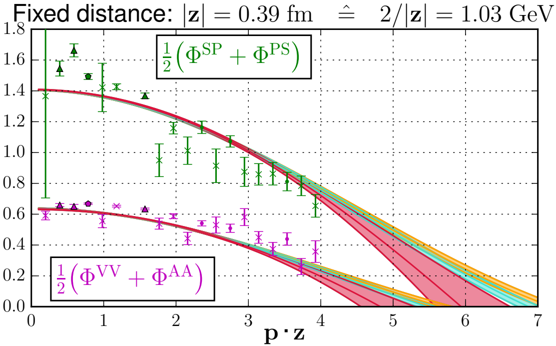
$$\phi_\pi(u, \mu) \propto \Big[u(1-u)\Big]^\alpha\,, \qquad \text{normalized to one}$$

Combined fit to all channels (Legacy Plot)



- two parameters: α, δ_2^π
- two parameters: a_2^π, δ_2^π
- three parameters: a_2^π, a_4^π, δ_2^π $\leftarrow$ yields unreasonable values for a_4^π

Combined fit to all channels



- splitting between SP+PS and VV+AA data is consistent with the pQCD expectation
- “jumping” of the points shows large discretization effects
- probably 2 loop perturbative effects are crucial

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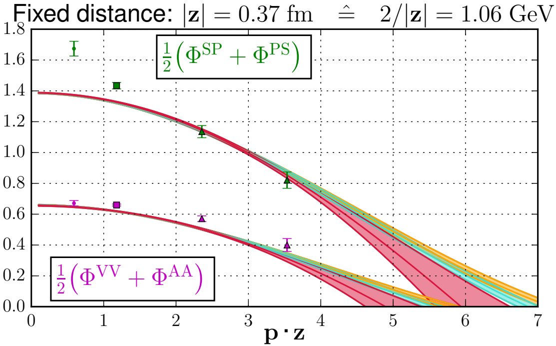

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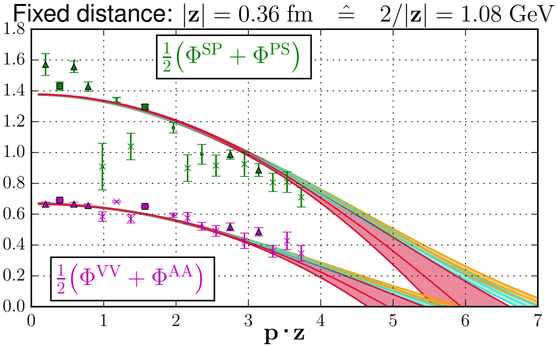

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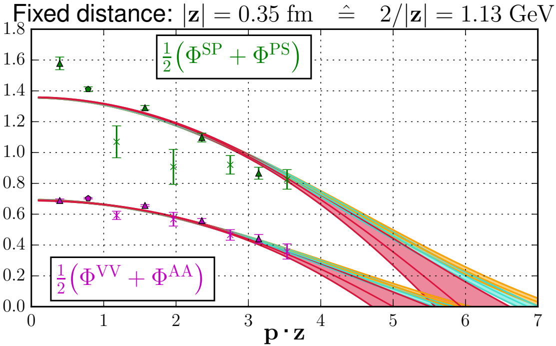

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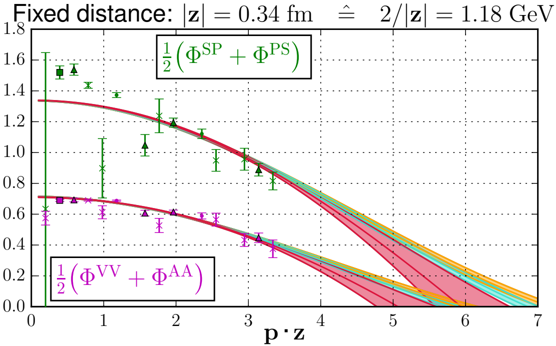

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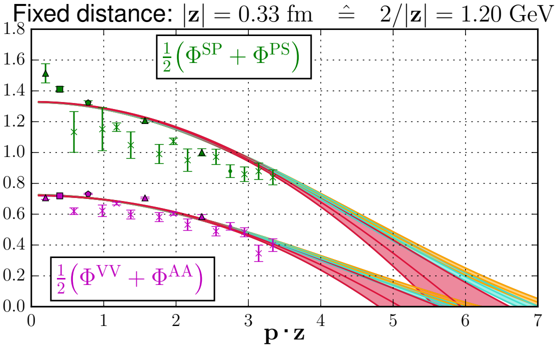

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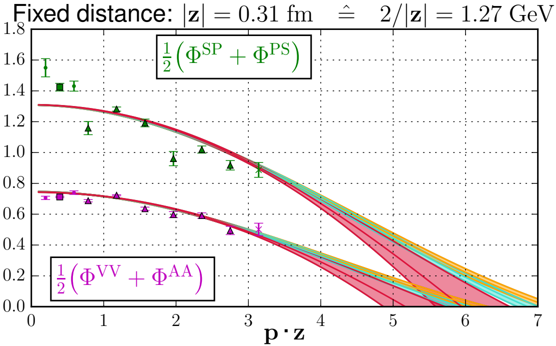

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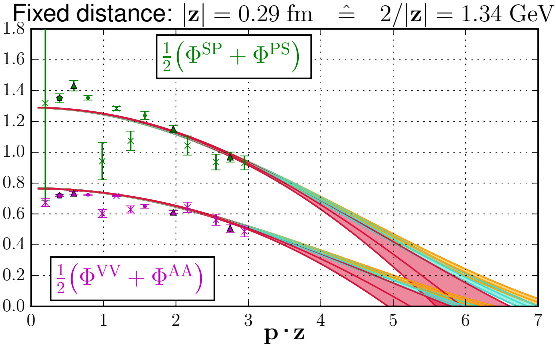

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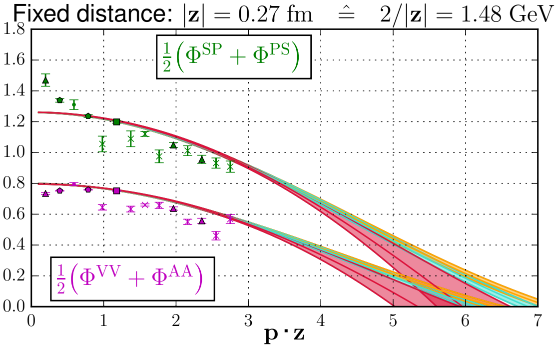

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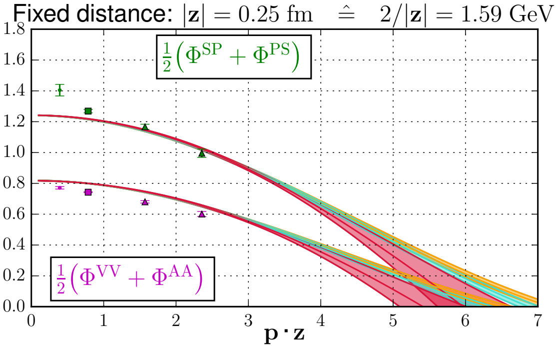

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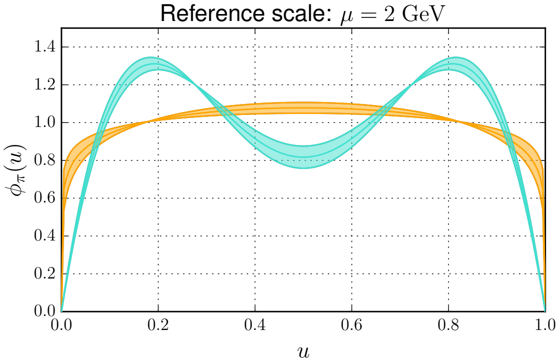
Result for DAs

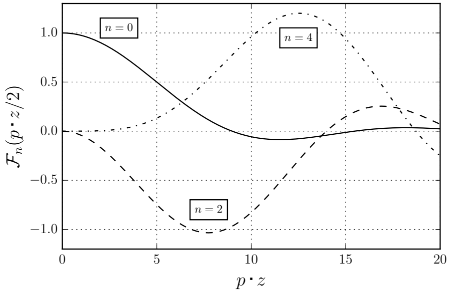


- errorbands show only the statistical error
- parameters: $\alpha = 0.13(5)$, $\delta_2^\pi = 0.223(4)$ GeV^2 $\quad a_2^\pi = 0.30(3)$, $\delta_2^\pi = 0.223(4)$ GeV^2
- both agree perfectly well with our data: **Why?**
- only relevant information from DA for our data points is a_2^π and $a_2^\pi = 0.31(3)$
- **Disclaimer:** current systematic uncertainty for a_2^π, δ_2^π is at least $\approx 50\%$ (fit range variation, estimate for two-loop correction)

Whats the problem with a_4^π?

$$\phi_\pi(u,\mu) = 6u(1-u) \sum_{n=0,2,\dots}^{\infty} a_n^\pi(\mu) C_n^{3/2}(2u-1)$$

$$\Rightarrow \quad \Phi^{\mathrm{XY}} = \sum_{n=0,2,\dots}^{\infty} a_n^\pi(\mu)\mathcal{F}_n(p\cdot z/2) + \mathcal{O}(\alpha_s) + \text{higher twist}$$



Expansion in conformal partial waves $\mathcal{F}_n$

- one needs $|p\cdot z| \gtrsim 5$ to constrain a_4^π to reasonable values
- to discriminate between DAs on last slide: $|p\cdot z| \gtrsim 8$?

Summary

- we have analysed Euclidean correlation functions with two *local* currents
- global fit to multiple channels yields qualitatively reasonable results (universality)
- first determination of HT normalization δ_2^π from lattice QCD (in the ballpark of QCD sum rule estimates)
- statistical accuracy very good for a_2^π and δ_2^π

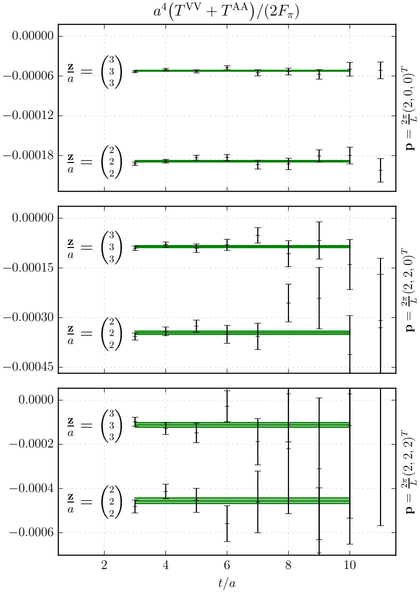
BUT:

- systematic uncertainty for a_2^π and δ_2^π is very large (discretization effects, two-loop perturbative correction not taken into account)
- with current data no determination of a_4^π possible

Next steps:

- goto smaller lattice spacings ($a \approx 0.04\,\text{fm}$ would be nice)
- perturbative two-loop calculation for coefficient functions
- to be sensitive to a_4^π: goto larger momenta ($|\mathbf{p}| > 3\,\text{GeV}$ would be nice)

Plateau Fits



- no sign of excited states (so far)
- for large momentum they might be hidden under bad statistics