$N\pi$ - state contamination in lattice calculations of the axial form factors of the nucleon

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Introduction

- Progress over the last years: Physical point simulations have become feasible

- Advantage

  No chiral extrapolation needed, i.e. one systematic error eliminated

- Problems

  - Numerically demanding

    need large volumes too

  - Signal-to-noise problem

  - Significant impact in correlation functions of multi-particle-states involving light pions

- ChPT can be used to estimate this multi-particle-state contamination

  ➥ Nucleon axial, scalar, tensor charge; pdf moments   OB, Lattice 2017

$N\pi$ contamination in axial form factors of the nucleon

- In the following:
  $N\pi$ contamination in axial form factors $G_A(Q^2)$ and $G_P(Q^2)$ of the nucleon
- Expectations: $N\pi$ contamination is
  - in $G_A(Q^2)$ of same order as in $g_A = G_A(0)$
  - significantly larger in $G_P(Q^2)$
- Calculational setup is the same as for $N\pi$ contamination in axial charge g_A
  - ➔ OB, Phys. Rev. D 94 (2016) 054505

The axial form factors $G_A(Q^2)$ and $G_P(Q^2)$

Matrix element of local isovector axial vector current

isospin symmetry assumed

$$\langle N(p')|A^a_\mu(0)|N(p)\rangle = \bar{u}(p')\left(\underbrace{\gamma_\mu\gamma_5 G_{\mathrm{A}}(Q^2)}_{\text{axial ff}} - i\gamma_5\frac{Q_\mu}{2M_N}\underbrace{G_{\mathrm{P}}(Q^2)}_{\text{induced pseudo scalar ff}}\right)\frac{\sigma^a}{2}u(p)$$

Momentum transfer $\quad Q_\mu = (iE_{\vec{p}\,'} - iE_{\vec{p}}, \vec{q}) \qquad \vec{q} = \vec{p}\,' - \vec{p} \qquad \vec{p}\,' = 0$

euclidean space time

chosen here

Lattice determination

Standard procedure:

- Compute 3-pt function $$C_{3,A^3_\mu}(\vec{q},t,t') = \sum_{\vec{x},\vec{y}} e^{i\vec{q}\vec{y}}\,\Gamma_{\beta\alpha}\langle N_\alpha(\vec{x},t)A^3_\mu(\vec{y},t')\overline{N}_\beta(0,0)\rangle$$

  Axial vector current at t'
  Nucleon interpolating fields at $t, 0$
  Projector Γ

- Ratio with 2-pt function $$R_\mu(\vec{q},t,t') = \frac{C_{3,A^3_\mu}(\vec{q},t,t')}{C_2(0,t)}\sqrt{\frac{C_2(\vec{q},t-t')}{C_2(0,t-t')}\frac{C_2(\vec{0},t)}{C_2(\vec{q},t)}\frac{C_2(\vec{0},t')}{C_2(\vec{q},t')}}$$

- Consider asymptotically large time separations: $t,\ t',\ t\text{-}t' \to \infty$

$$R_k(\vec{q},t,t') \to \quad \Pi_k(\vec{q}) = \frac{i}{\sqrt{2E_{N,\vec{q}}(M_N+E_{N,\vec{q}})}}\left((M_N+E_{N,\vec{q}})G_{\mathrm{A}}(Q^2)\delta_{3k} - \frac{G_{\mathrm{P}}(Q^2)}{2M_N}q_3q_k\right)$$

- Solve a linear system and extract the form factors $\Pi_k(\vec{q}) \;\to\; G_{\mathrm{A}}(Q^2)\,,\ G_{\mathrm{P}}(Q^2)$

Lattice determination

- In practice: finite time separations t and t'

$$R_k(\vec{q},t,t') \;\to\; G_{\rm A}^{\rm eff}(Q^2,t,t')\,,\; G_{\rm P}^{\rm eff}(Q^2,t,t')$$

- The <u>effective form factors</u> contain excited-state contributions and depend on t, t'

$$G_{\rm A,P}^{\rm eff}(Q^2,t,t') \;=\; G_{\rm A,P}(Q^2)\Big[1+\Delta G_{\rm A,P}(Q^2,t,t')\Big]$$

- Dominant excited state for physical pion mass and large time separations:

2-particle $N\pi$ states

ChPT including nucleons

- SU(2) ChPT at LO

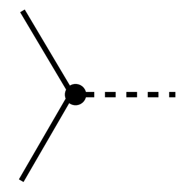
  Gasser, Sainio, Švarc 1988

  isospin symmetry, euclidean space time

  contains the three pions and the nucleon doublet $\Psi = \begin{pmatrix} p \\ n \end{pmatrix}$

  $$\mathcal{L}^{(1)}_{\mathrm{int},1\pi} = \frac{ig_A}{2f}\overline{\Psi}\gamma_\mu\gamma_5\sigma^a\Psi\,\partial_\mu\pi^a \quad \rightarrow$$

  nucleon-pion vertex

  g_A axial charge $\qquad f$ pion decay constant

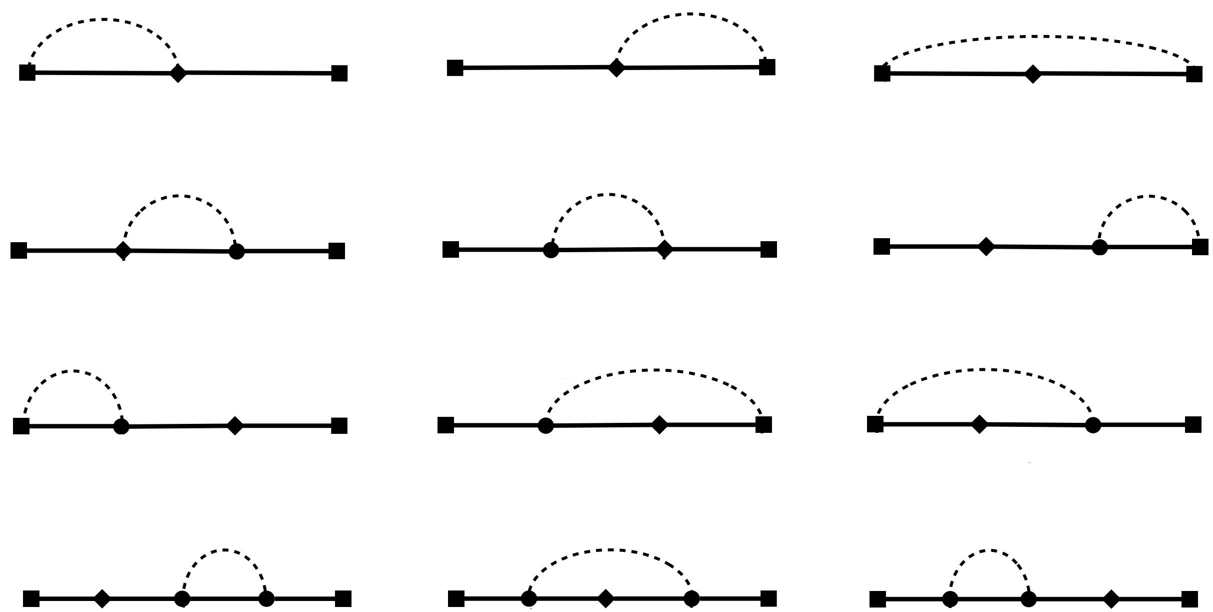
- Low energy constants at this order: g_A, f, M_N, M_π
  experimentally well-known

- Also known: chiral expressions for
  - axial vector current
    Gasser, Sainio, Švarc 1988, Fettes *et al* 2000
  - nucleon interpolating fields (local and smeared)
    Nagata et al 2008; Wein, Bruns, Hemmert, Schäfer 2011; OB 2015

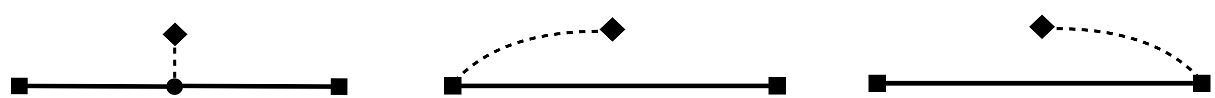
$N\pi$ contribution to the form factors

- To do: Compute 2-pt and 3-pt functions and the ratio R_k in ChPT
- Example: Feynman diagrams for the 3-pt function

Loop diagrams:



Tree diagrams:



Note: Tree diagrams are expected to give large $N\pi$ contribution
vanish for $Q^2 = 0$ ➔ do not contribute to g_A

- Status: Leading contribution in $1/M_\mathrm{N}$-expansion computed
  Work in progress: $1/M_\mathrm{N}$-correction

$N\pi$ contribution to the form factors

- Consider plateau estimates* for the form factors

$$G_A^{\rm plat}(Q^2,t) = \min_{0<t'<t} G_A^{\rm eff}(Q^2,t,t')$$

$N\pi$ contamination leads to <u>overestimation in G_A</u>

$$G_P^{\rm plat}(Q^2,t) = \max_{0<t'<t} G_P^{\rm eff}(Q^2,t,t')$$

$N\pi$ contamination leads to <u>underestimation in G_P</u>

Comment: $t' \approx t/2$. Midpoint estimate is equally good

- ChPT requires $t \gtrsim 2\mathrm{fm}$ such that t and $t-t' \gtrsim 1\mathrm{fm}$

(Experience from nucleon charges and pdf moments)

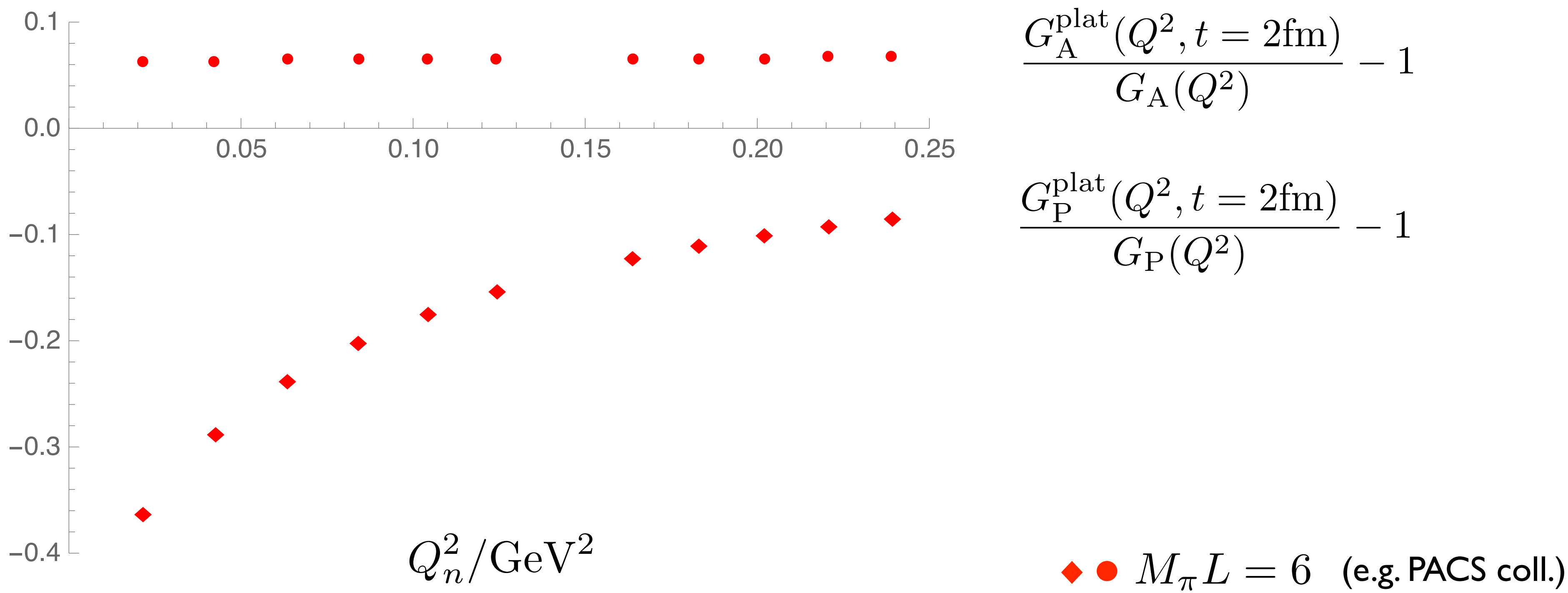
- ChPT calculation in finite spatial volume, box length L, periodic BC

➥ discrete momenta $\vec{q}_n$ and momentum transfer Q_n^2

* I have nothing to say about the summation method

Results

for t = 2 fm



- G_A^plat overestimates by ≈ 5% (no visible Q^2 dependence)
  ➥ agrees with result for g_A in previous calculation
- G_P^plat underestimates by ≈10% - 40% depending on momentum transfer

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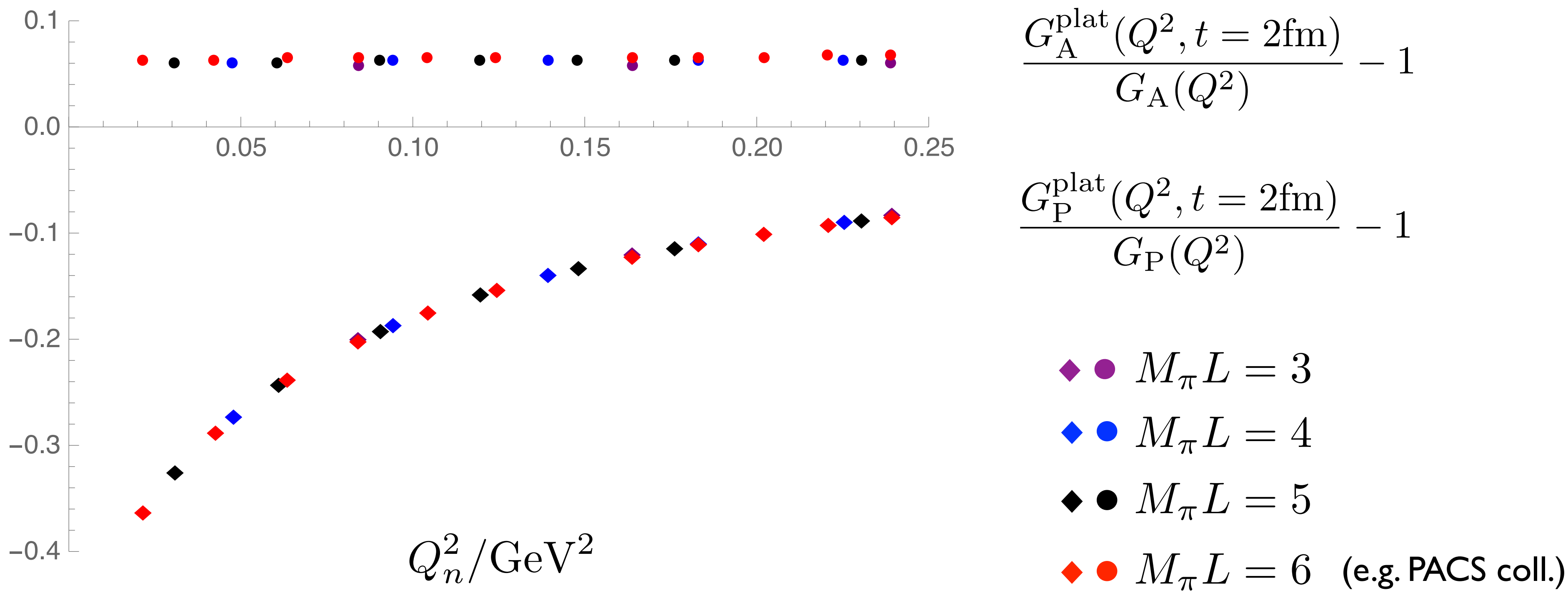

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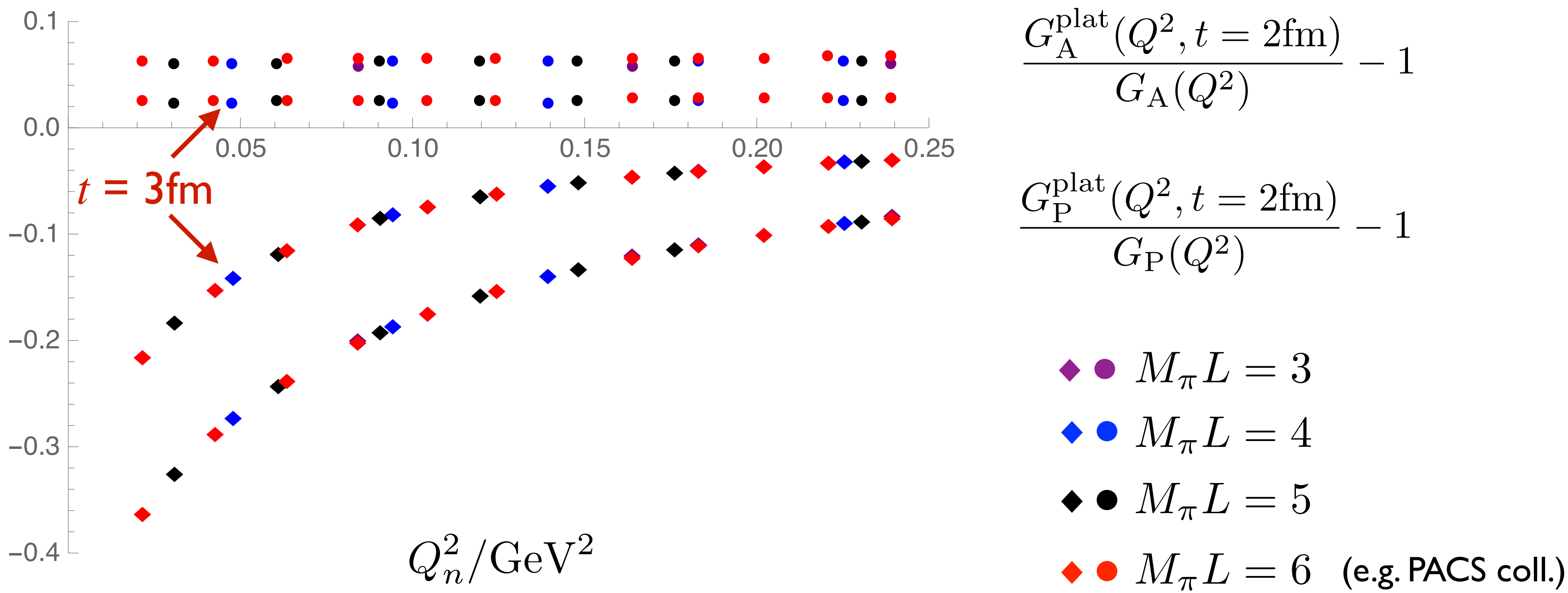

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- G_P^plat underestimates by ≈10% - 40% depending on momentum transfer
- Small FV effect for $M_\pi L \geqq 3$
- Increasing t to 3 fm reduces $N\pi$ contribution roughly by a factor $1/2$

Observation 1

For some momentum transfer the extraction of the eff. form factors

$$R_k(\vec{q},t,t') \to G_{\rm A}^{\rm eff}(Q^2,t,t')\,,\ G_{\rm P}^{\rm eff}(Q^2,t,t')$$

can be done in various ways.

Example: $\vec{q}_A = \frac{2\pi}{L}(1,0,1)$ $\quad \vec{q}_B = \frac{2\pi}{L}(1,1,0)$ lead to the same Q^2

Extract effective form factors using

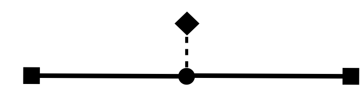
1. $R_3(\vec{q}_A,t,t')$ $\quad R_3(\vec{q}_B,t,t')$
2. $R_1(\vec{q}_A,t,t')$ $\quad R_3(\vec{q}_B,t,t')$
3. $R_1(\vec{q}_A,t,t')$ $\quad R_3(\vec{q}_A,t,t')$

The three choices give practically the same effective form factors !
Deviations of $O(10^{-4})$

Observation 2

Recall: $$G_{\rm P}^{\rm eff}(Q^2,t,t') \;=\; G_{\rm P}(Q^2)\left[1+\Delta G_P^{N\pi}(Q^2,t,t')\right]$$

Observation: The loop diagram contribution to $\Delta G_P^{N\pi}$ is tiny
$\Delta G_P^{N\pi}$ is dominated by one tree diagram!



Consequences

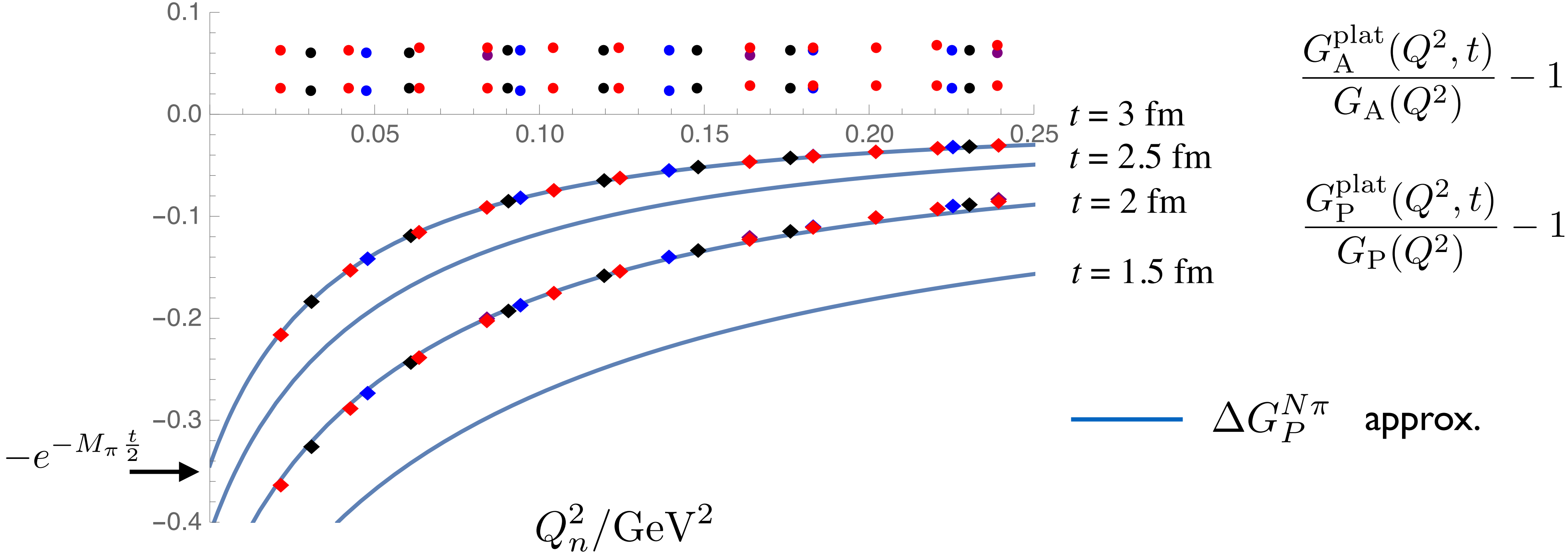
- ChPT is expected to work better for $\Delta G_{\rm P}^{N\pi}$ than for $\Delta G_{\rm A}^{N\pi}$ (no tower of narrowly spaced $N\pi$ states)
  ➡ result expected to be reliable for time separations much less than 2 fm (?)

- Excellent approximation:

$$\Delta G_P^{N\pi}(\vec{q},t,t'=t/2) \;\approx\; -\exp\left[-E_{\pi,\vec{q}}\frac{t}{2}\right]\cosh\left[\frac{\vec{q}^{\,2}}{2M_N}\frac{t}{2}\right] \xrightarrow{\;\vec{q}\to 0\;} -e^{-M_\pi \frac{t}{2}}$$

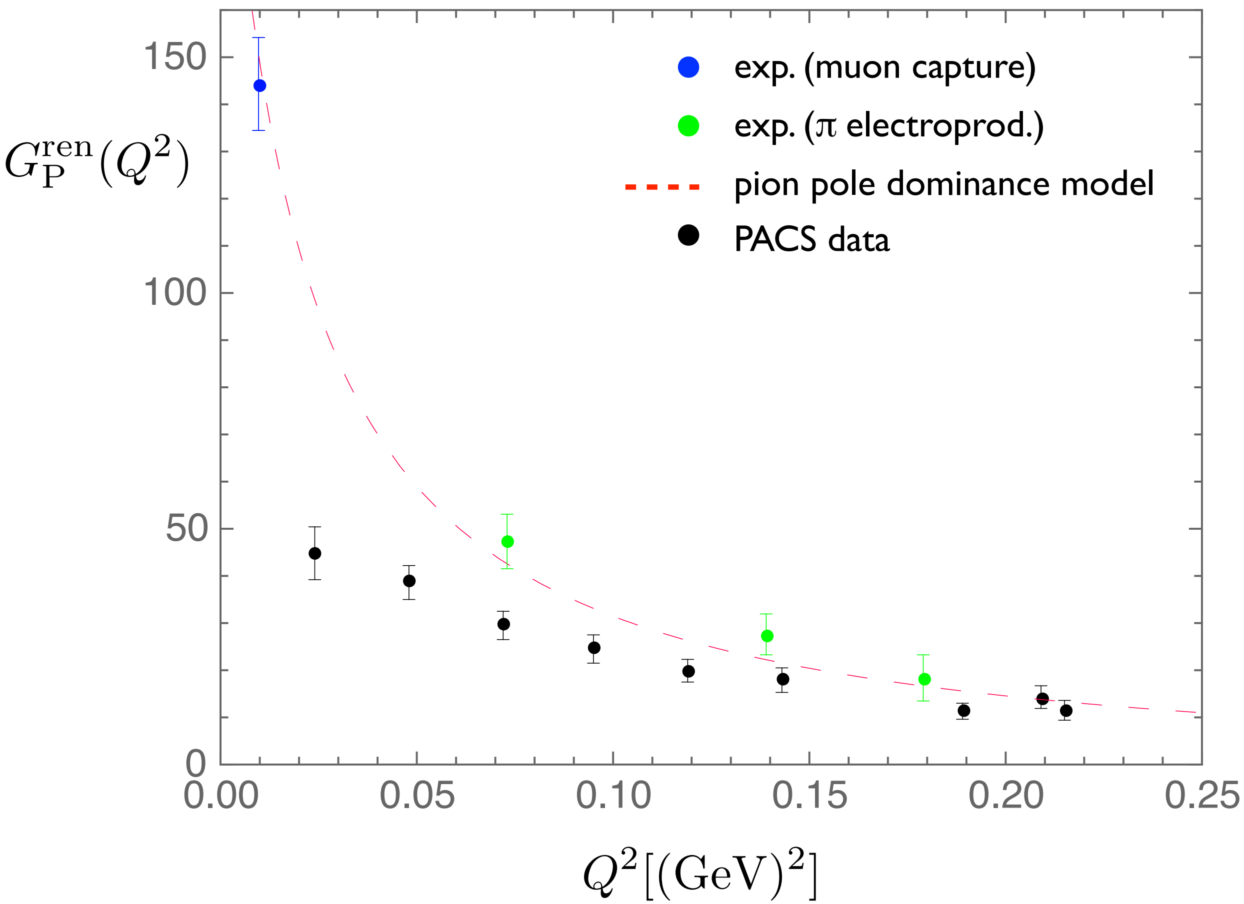
Note: Determined mainly by M_π and the source-sink separation t

Results



$N\pi$ contribution for small Q^2 is governed by $-e^{-E_{\pi,\vec{q}}\frac{t}{2}}$

Impact on lattice calculations of $G_P(Q^2)$



Recent preprint by PACS coll.
arXiv:1807.03974

$$a \approx 0.085\,\mathrm{fm}$$

$$M_\pi \approx 146\,\mathrm{MeV}$$

$$M_\pi L \approx 6$$

$$t \approx 1.3\,\mathrm{fm}$$

- Data underestimate experimental results and pion pole dominance model

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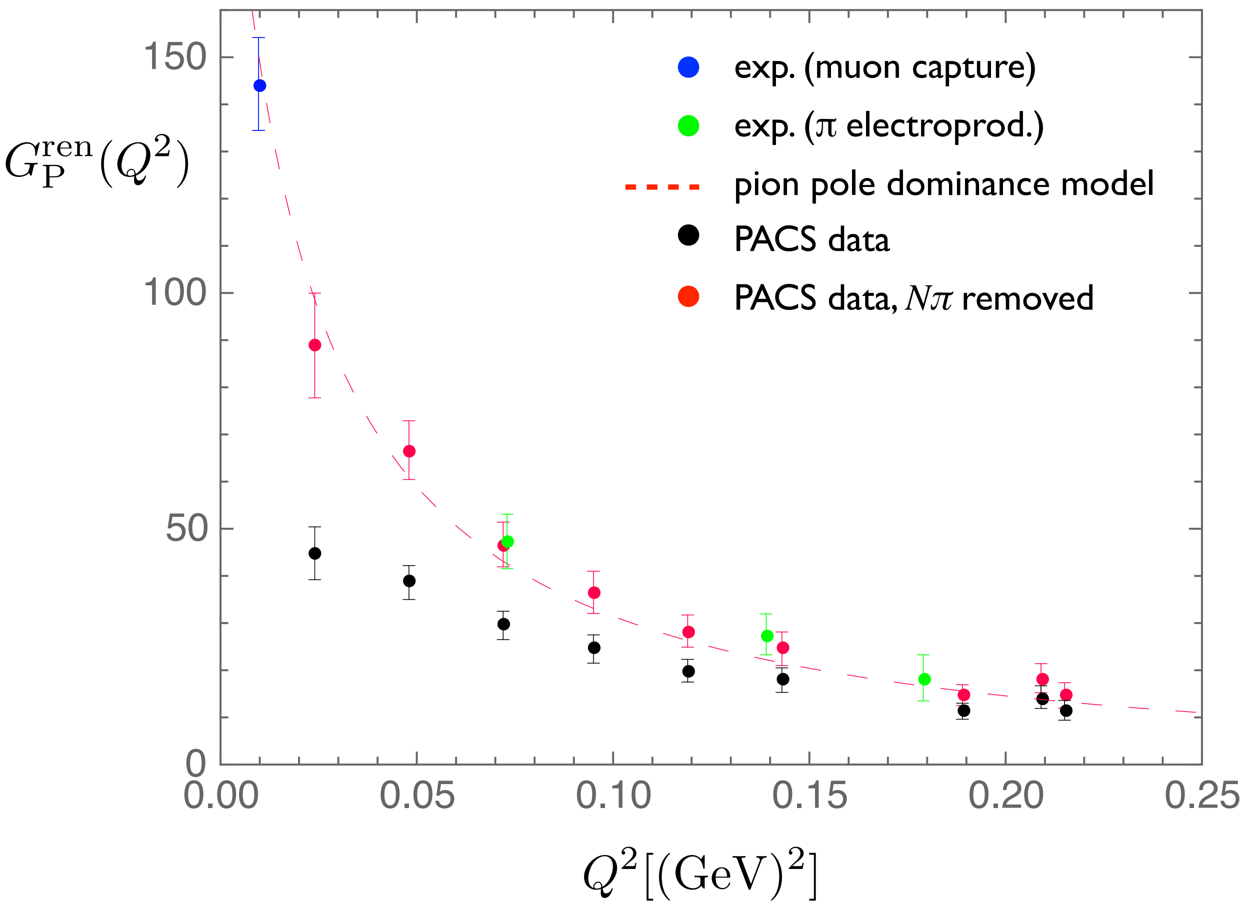

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Remove $N\pi$ contamination from the data by

$$G_P(Q^2) = \frac{G_P^{\mathrm{plat}}(Q^2, t)}{1 + \epsilon_P^{\mathrm{app}}(Q^2, t)}$$

- Data underestimate experimental results and pion pole dominance model
- Corrected data agree much better with pion pole dominance model
  To do: Check for various t, continuum limit, …

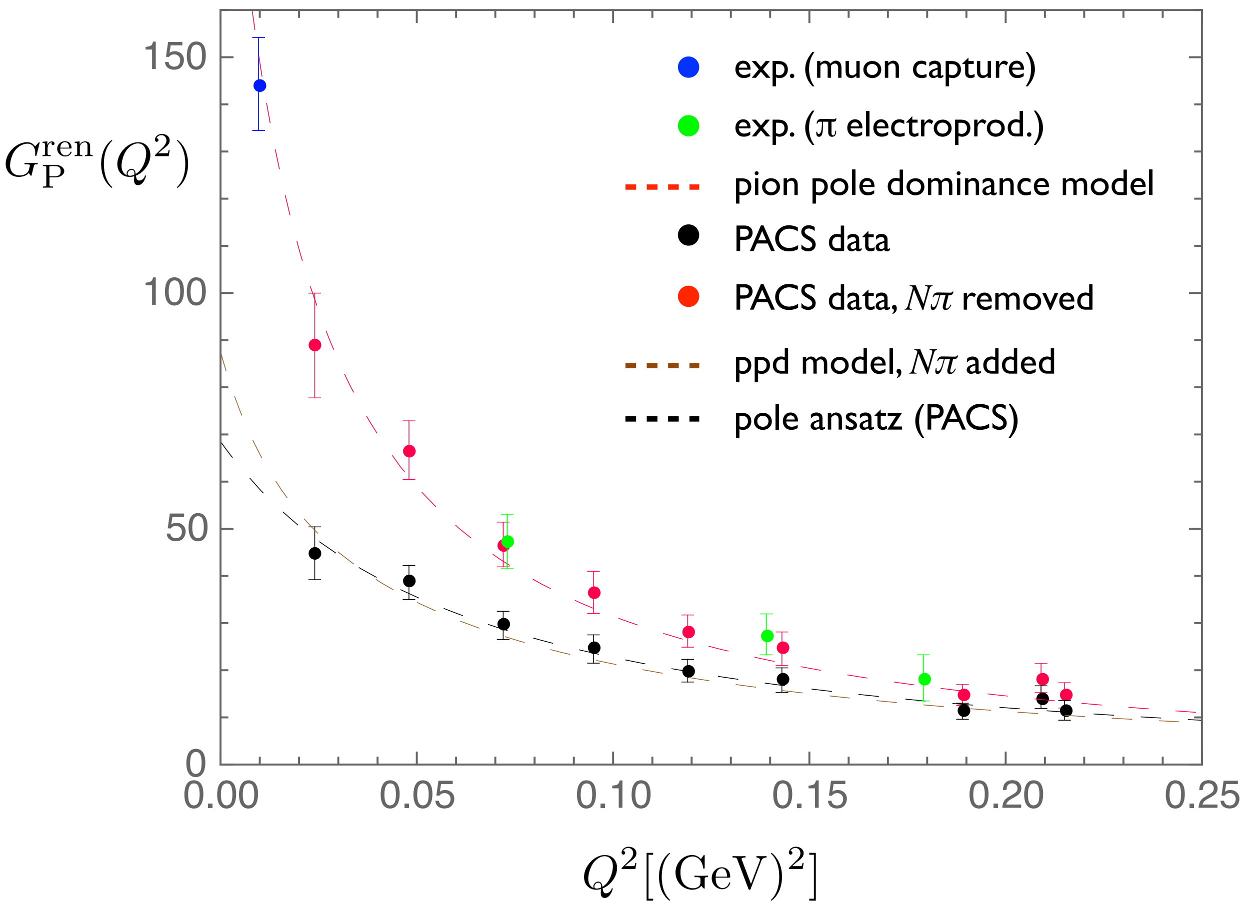
Summary and outlook

- Presented here: LO ChPT results for the $N\pi$ excited-state contamination in the plateau estimates for the axial form factors of the nucleon
  - Overestimation for G_A, flat Q^2 dependence
  - Underestimation for G_P, strong Q^2 dependence for low Q^2
    ➡ can qualitatively explain the distortion observed in lattice data for G_P

- Outlook: Analogous calculation for
  - pseudo scalar form factor
  - form factors for the vector current

Backup slides

Impact on lattice calculations of $G_P(Q^2)$



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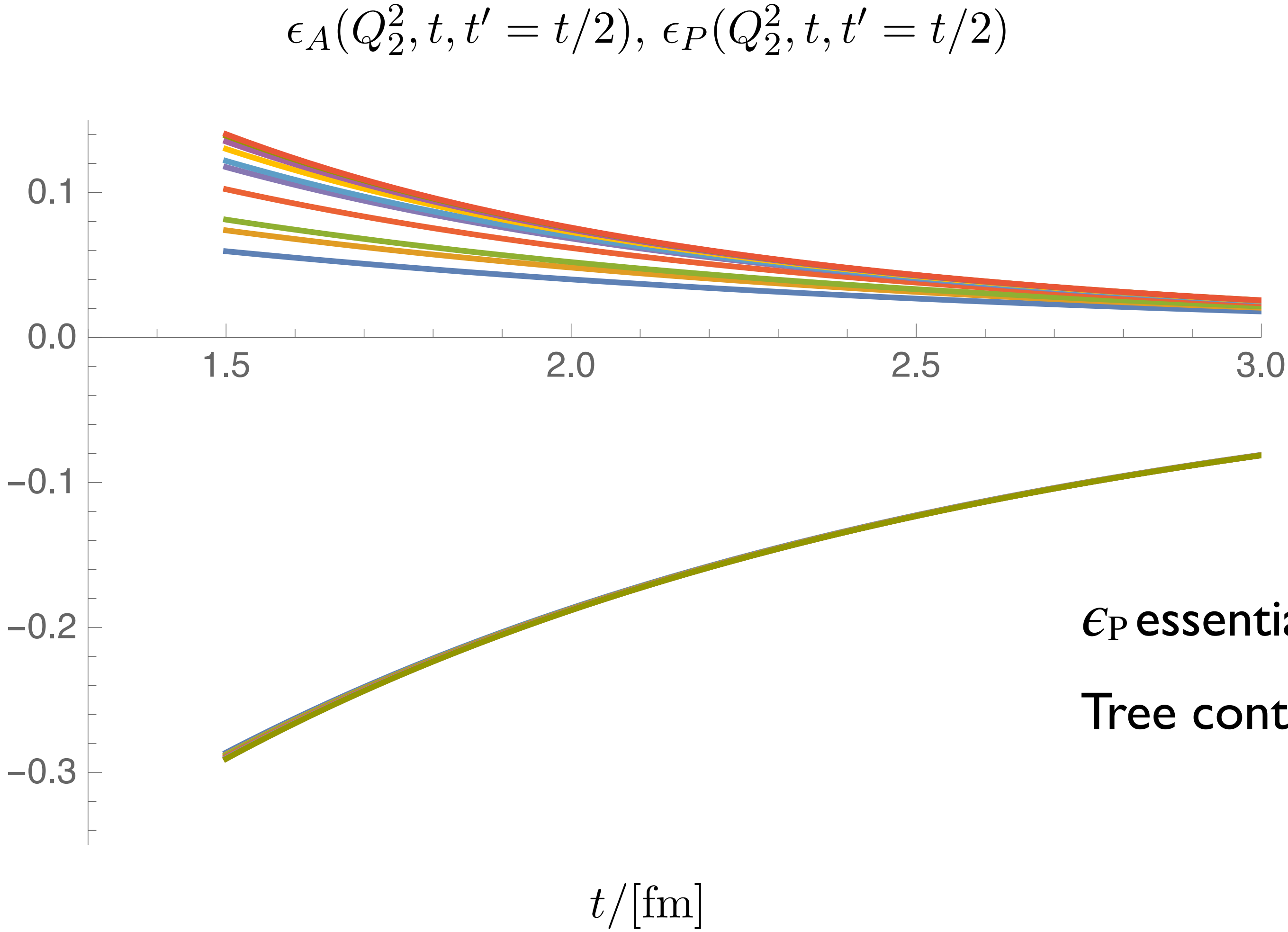
Pole ansatz by PACS:

$$G_{\rm P}(Q^2) = \frac{4M_N^2 G_{\rm A}(Q^2)}{Q^2 + M_{\rm pole}^2}$$

Fit to data:

$$M_{\rm pole} = 256(17)\,\mathrm{MeV}$$

$N\pi$ contamination as a function of source-sink separation



$M_\pi L = 4$

$n_q = 2$

$n_{p,\max} = 1 \ldots 12$

"tower of $N\pi$ states"

ϵ_{P} essentially independent of $n_{p,\mathrm{max}}$

Tree contribution dominates ϵ_{P}

$N\pi$ contamination in the correlation functions

3-pt function:

$$C_{3,\mu}(\vec{q},t,t') = C^{N}_{3,\mu}(\vec{q},t,t') + C^{N\pi}_{3,\mu}(\vec{q},t,t')$$
$$= C^{N}_{3,\mu}(\vec{q},t,t')\left(1 + Z_\mu(\vec{q},t,t')\right)$$

computable in ChPT

2-pt function: analogously

Ratios:

$$R_\mu(\vec{q},t,t') = \Pi_\mu(\vec{q})\left(1 + Z_\mu(\vec{q},t,t') + \frac{1}{2}Y(\vec{q},t,t')\right)$$

from 2-pt functions

$N\pi$ contamination in the correlation functions

$$Z_\mu(\vec{q},t,t') = \quad a_\mu(\vec{q})e^{-\Delta E(0,\vec{q})(t-t')} + \tilde{a}_\mu(\vec{q})e^{-\Delta E(\vec{q},-\vec{q})t'}$$ ← tree diagrams

$$+ \sum_{\vec{p}} b_\mu(\vec{q},\vec{p})e^{-\Delta E(0,\vec{p})(t-t')} + \tilde{b}_\mu(\vec{q},\vec{p})e^{-\Delta E(q,\vec{p})t'}$$

loop diagrams

$$+ \sum_{\vec{p}} c_\mu(\vec{q},\vec{p})e^{-\Delta E(0,\vec{p})(t-t')}e^{-\Delta E(\vec{q},\vec{p})t'}$$

Energy gaps:

$$\Delta E(0,\vec{q}) = E_{\pi,\vec{q}} + E_{N,q} - M_N$$
$$\Delta E(0,\vec{p}) = E_{\pi,\vec{p}} + E_{N,p} - M_N$$
$$\Delta E(\vec{q},-\vec{q}) = E_{\pi,\vec{q}} + M - E_{N,q}$$

Non-trivial results of the ChPT calcultion: The coefficients in Z_μ

$N\pi$ contamination in the correlation functions

Example: Coefficients a_k from the tree-level diagrams

$$a_k(\vec{q}) = a_k^{\infty}(\vec{q}) + \frac{E_{\pi,q}}{M_N} a_k^{\mathrm{corr}}(\vec{q}) + \mathrm{O}\left(\frac{1}{M_N^2}\right)$$

NR Limit: $$a_{k=1,2}^{\infty}(\vec{q}) = -\frac{1}{2} \qquad a_{k=3}^{\infty}(\vec{q}) = \frac{1}{2}\frac{q_3^2}{E_{\pi,q}^2 - q_3^2}$$

Relevant result for approximate $\Delta G_P^{N\pi}$

Correction:

$$a_{k=1,2}^{\mathrm{corr}}(\vec{q}) = -\frac{1}{4}\left(\frac{M_\pi^2}{E_{\pi,\vec{q}}^2} - \frac{1}{g_A}\right) \qquad a_{k=3}^{\mathrm{corr}}(\vec{q}) = \frac{1}{4}\left(\frac{M_\pi^2}{E_{\pi,\vec{q}}^2} - \frac{1}{g_A}\right)\frac{q_3^2}{E_{\pi,q}^2 - q_3^2}$$

ChPT: Single nucleon contribution

2-pt function

3-pt function

➡ $$G_{\rm A}(Q^2) = g_A \qquad G_{\rm P}(Q^2) \, = \, 4M_N^2 \frac{g_A}{Q^2 + M_\pi^2}$$