Follow-up on $K \to \pi\pi$ in Large N_c

Fernando Romero-López

In collaboration with: A. Donini, P. Hernández & C. Pena

arXiv:1711.10248, arXiv:1607.03262 and on-going work

University of Valencia, IFIC

fernando.romero@uv.es

Michigan State University, July 26, 2018

IFIC INSTITUT DE FÍSICA CORPUSCULAR VNIVERSITAT DE VALÈNCIA "la Caixa" inVisiblesPlus

Overview

Motivation

Non-Leptonic Kaon Decays

- Experimental values for $K \to \pi\pi$ are very well measured in two isospin channels, $I = 0, 2$ and it differs strongly from the Large N_c prediction.

$$\frac{\text{Re } A_0}{\text{Re } A_2} = 22.35 \gg \sqrt{2}\Big|_{\text{Large } N_c},$$

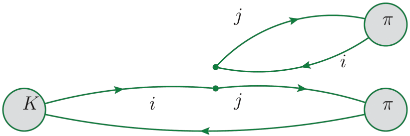
- State of the art result by *RBC-UKQCD, 2015*:

$$\frac{\text{Re } A_0}{\text{Re } A_2} = 31(11),$$

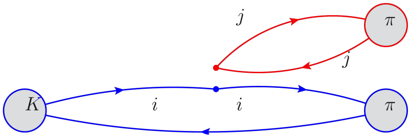
- **Our goal:** combine knowledge from Large N_c, Chiral Perturbation Theory and Lattice QCD to understand the origin of the underlying dynamics.

Current Lattice results for $K \to \pi\pi$

RBC-UKQCD, PRD91 (2015) and PRL115 (2015)

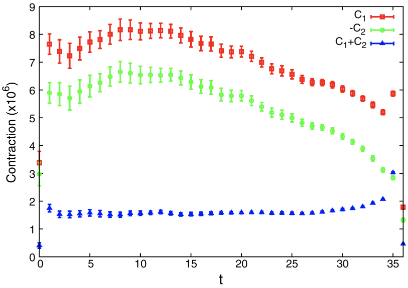


$C_1 \to$ one color trace $\to O(N_c)$



$C_1 \to$ two color traces $\to O(N_c^2)$

- $\frac{\text{Re } A_0}{\text{Re } A_2} = 31(11)$
- It relies on a cancellation
- Large N_c predicts $|C_1| \sim \frac{|C_2|}{3}$
- However $C_1 \sim -0.7 C_2$
- Very big $1/N_c$ corrections?



Open Points

A cancellation between two different diagrams seems to be the source of the enhancement. However, diagrams by themselves are not physical.

A more rigorous way to formulate it would be in terms of **big Large N_c corrections**, and this can be tested non perturbatively.

Even if the result would eventually agree with the experiment, the **origin** of the enhancement in the I=0 channel is **not clear**.

In addition, Large N_c **inspired** approaches are usual in **phenomenology** (*Pich, Buras...*) and understanding the dynamics can be useful.

Framework

Effective $SU(4)_F$ Theory

M_W

$$\mathcal{H}_{SM} \to \mathcal{H}^{N_f=4}_{\Delta S=1} = \sqrt{2} G_F V^*_{us} V_{ud} (C_\pm(M_W) \mathcal{O}^\pm(M_W))$$

$$\mathcal{O}^\pm \equiv (\bar{s}\gamma^L_\mu u)(\bar{u}\gamma^L_\mu d) \pm (\bar{s}\gamma^L_\mu d)(\bar{u}\gamma^L_\mu u) - (u \leftrightarrow c)$$

$$SU(4)_L \times SU(4)_R\text{:}\ \mathcal{O}^+ \to (84,1) \quad \mathcal{O}^- \to (20,1)$$
$$(84,1) \to A_2, A_0, \ (20,1) \to A_0$$

$$\mathcal{H}^{N_f=4}_{\Delta S=1} = \sqrt{2} G_F V^*_{us} V_{ud} (C_\pm(\mu) \mathcal{O}^\pm(\mu))$$

Λ_{ChPT}

$$\mathcal{H}^{N_f=4}_{\Delta S=1} \to \mathcal{H}^{N_f=4}_{ChPT} \propto g_+ \mathcal{O}^+ + g_- \mathcal{O}^-$$

Relating $K \to \pi$ to A_2 and A_0

$$\mathcal{H}_W \propto g^+ \mathcal{O}^+ + g^- \mathcal{O}^-, \tag{1}$$

The tree level result in ChPT for the ratio is:

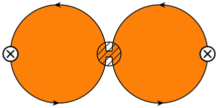
$$\frac{A_0}{A_2} = \frac{1}{2\sqrt{2}} \left(1 + 3\frac{g^-}{g^+}\right) \tag{2}$$

Determine $g^{\pm}$ from Lattice QCD:

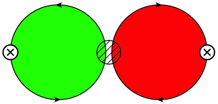
$$\langle K|\mathcal{O}^{\pm}|\pi\rangle \propto g^{\pm} \tag{3}$$

In particular, one can test the scaling with N_c and M_K, M_π.

$g^{\pm} \propto$



$\pm$



Color-connected $O(N_c)$

Color-disconnected $O(N_c^2)$

Lattice Results in the Quenched Approximation

Technical details

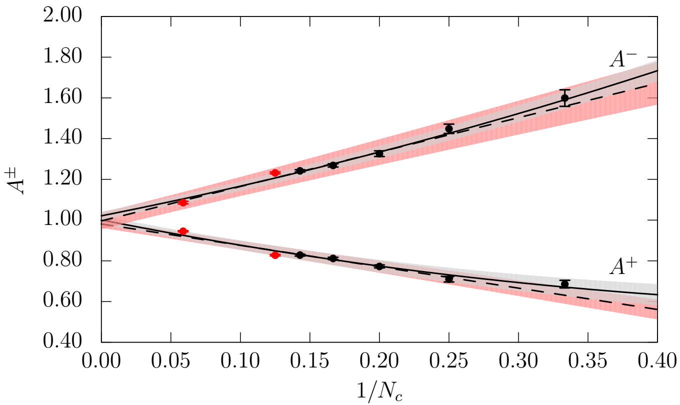
- Test the N_c scaling of:

$$\langle K|\mathcal{O}^{\pm}|\pi\rangle \propto g^{\pm} \tag{4}$$

- Quenched Lattice QCD with twisted mass fermions
- Mass around 570 MeV
- Lattice spacing $a \sim 0.093$ fm
- $N_c = 3 - 7, 8, 17$
- Perturbative one-loop renormalization of the correlation functions
- Running Wilson coefficients to one loop

Results for $K \to \pi$ at Large N_c

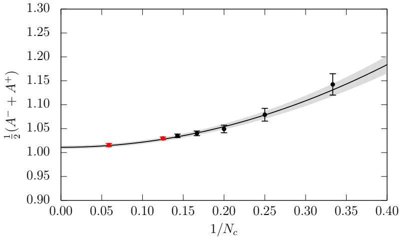
A. Donini, P. Hernández, F. Romero-López, C. Pena, **PRD94 (2016)**



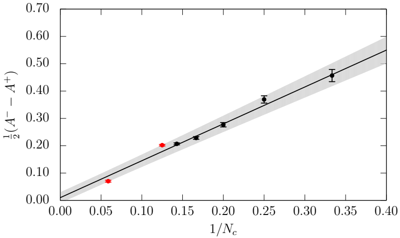
Large N_c corrections for $K \to \pi$ are fully anticorrelated and point towards an enhancement of the ratio!

Results for $K \to \pi$ at Large N_c

A. Donini, P. Hernández, F. Romero-López, C. Pena, **PRD94 (2016)**



Leading part of the Amplitude,
$1 + O(1/N_c^2)$



Subleading part of the Amplitude,
$O(1/N_c)$

Large N_c corrections for $K \to \pi$ are 30-40%

However, *RBC-UKQCD (2015)* obtained $\sim$ 70%

Dynamical Simulations $N_f = 4$ and $N_c \geq 3$

Details of the simulation

Quenching gives the exact Large N_c limit but it can alter subleading $1/N_c$ corrections.
$\Rightarrow$ repeat our study with dynamical fermions

- $N_c = 3$ Parameters inspired from *ETMC*, $N_f = 2 + 1 + 1$: **Iwasaki gauge action**.
- c_{sw} is taken from $N_c = 3$ perturbative result boosted with the plaquette. For $N_c > 3$ is taken as constant. (*Aoki and Kuramashi, 2008*)
- Use **mixed action** setup: Wilson in the sea + TM in valence sector, as in *CLS, arXiv:1711.06017*.
- Lattice spacing $a \simeq 0.075$ fm and lattices with $L \times T = 20 \times 32$, 24×48 and 32×60

Scale Setting

A light charm has a big impact in fermionic observables
$\Rightarrow$ need an observable "independent" of N_f

Scale setting through t_0 depends weakly on N_f at one loop and $N_f = 2, 3$ are available.

$$\langle t^2 E(t) \rangle = \frac{3}{128\pi^2} \frac{N_c^2 - 1}{N_c} \lambda(q) \left(1 + \frac{c_1}{4\pi}\lambda(q) + O(\lambda^2)\right), \qquad (5)$$

with $c_1 = 0.36593 + 0.0075 \frac{N_f}{N_c}$.

For every value of N_c, use for scale setting:

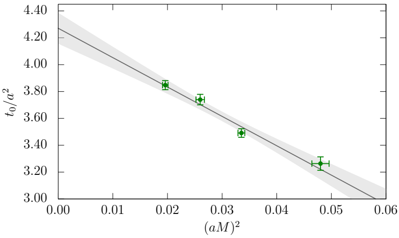
$$(M\sqrt{t_0})\Big|_{M=420\text{ MeV}} = 0.3090(83). \qquad (6)$$

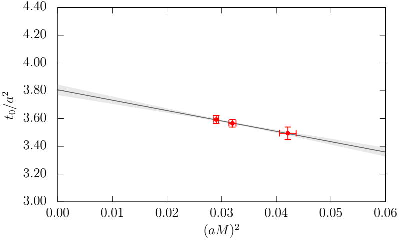
Scale Setting $N_c = 3, 4$

Chiral behaviour of t_0 is known (*Bär and Goltermann, 2014*):

$$t_0(M^2) = t_0^{ch}\left(1 + kM^2\right) + O(M^4) \tag{7}$$

$$\text{Large } N_c \Rightarrow t_0(M^2) = t_0^{ch}$$





Scale setting $N_c = 3$ ($\beta = 1.778$)

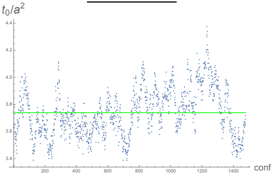
$a = 0.0753(10)^{syst}(4)^{stat}$ fm

Scale Setting $N_c = 4$ ($\beta = 3.570$)

$a = 0.0763(10)^{syst}(2)^{stat}$ fm

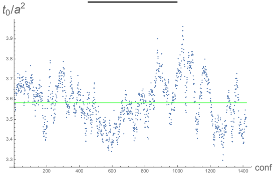
Additional Information to $N_c = 3, 4$

$N_c = 3$

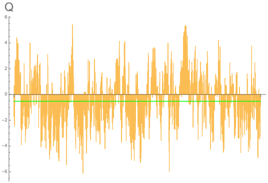


$\tau_{auto}^{t_0} = 35(12)$

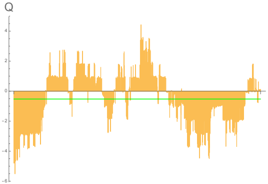
$N_c = 4$



$\tau_{auto}^{t_0} = 34(12)$



$\tau_{auto}^{Q} = 8(2)$



$\tau_{auto}^{Q} = 73(28)$

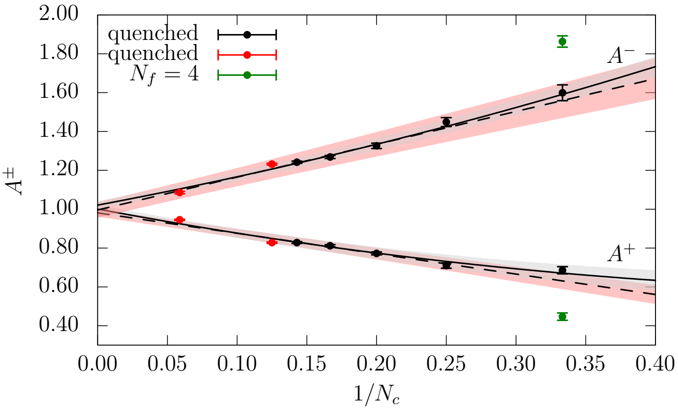
Some remarks:

- Top. Charge is suppressed with N_c.
- τ_{auto}^{Q} blows for the Top. Charge.
- $\tau_{auto}^{t_0}$ does not change.
- Mass dependence in t_0 is suppressed.

Results with $N_f = 4$ and $N_c = 3$

$K \to \pi$ in $N_f = 4$ dynamical fermions

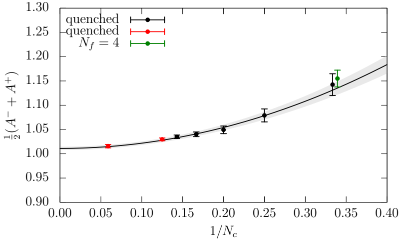
Preliminary Results



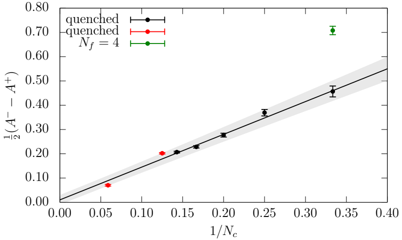
Large N_c corrections for $K \to \pi$ are larger in the unquenched case!

$K \to \pi$ in $N_f = 4$ dynamical fermions

Preliminary Results



Leading part of the Amplitude,
$1 + O(1/N_c^2)$

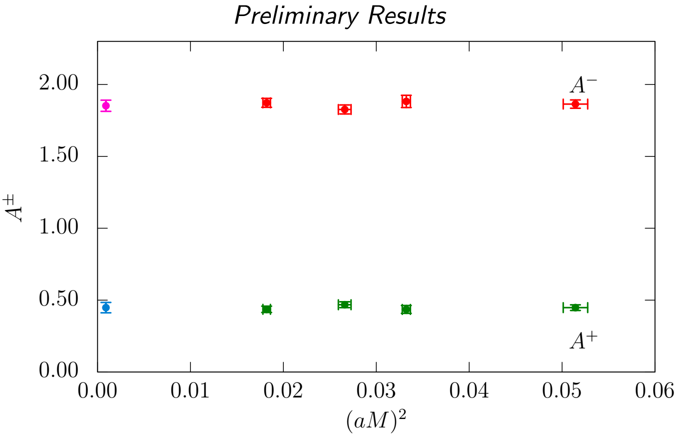


Subleading part of the Amplitude,
$O(1/N_c)$

Large N_c corrections for $K \to \pi$ are $\sim$70%

It seems to agree with *RBC-UKQCD (2015)*.

Mass dependence in $K \to \pi$



Mass dependence for $K \to \pi$ is very flat!

$$\left.\frac{g^-}{g^+}\right|_{N_f=4} = 4.1(3) \quad \text{vs} \quad \left.\frac{g^-}{g^+}\right|_{\text{quenched}} = 2.4(1) \tag{8}$$

Mass Corrections in $N_f = 4$ ChPT

$SU(4)_F$ ChPT Predictions at NLO

$SU(3)_F$ ChPT results are known, but we need an active charm.

With $\mathcal{H}_W \propto g^+\mathcal{O}^+ + g^-\mathcal{O}^-$, we calculate at leading log:

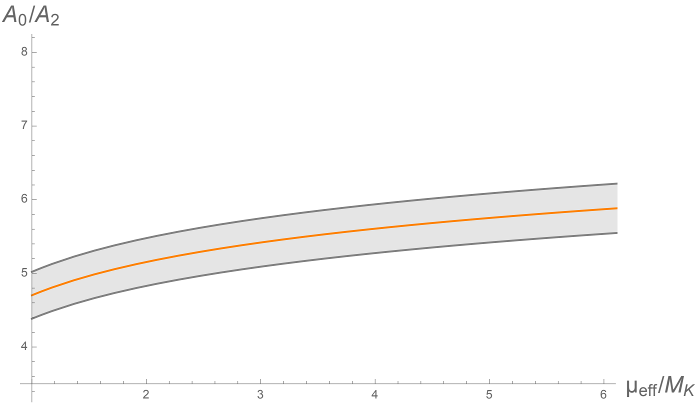
- $M_\pi, M_D \to 0$ and M_K^{phys}

$$\left.\frac{A_0}{A_2}\right|_{M_\pi, M_D \to 0, M_K^{\text{phys}}} = \frac{1}{2\sqrt{2}}\left(1 + 3\frac{g^-}{g^+}\right)$$
$$-\frac{17}{6\sqrt{2}}\left(1 + \frac{1}{17}\frac{g^-}{g^+}\right)\frac{M_K^2}{(4\sqrt{2}\pi F)^2}\log\frac{M_K^2}{\mu_{eff}^2}$$
$$-i\frac{3}{4\sqrt{2}}\pi\left(1 + 5\frac{g^-}{g^+}\right)\frac{M_K^2}{(4\sqrt{2}\pi F)^2},$$

Re (A_0/A_2) is enhanced at NLO!

$SU(4)_F$ ChPT Predictions at NLO

Preliminary Results



Enhancement is still far away from the experiment by a factor 2.
NLO result is **up to 25%**.

Conclusion and Outlook

Summary

- Puzzle of the $\Delta I = 1/2$ still very challenging.
- Explanation for the hierarchy between A_0 and A_2 still unclear.
- Possible factors: $1/N_c$ corrections, breaking of $SU(3)_F$, contribution of the charm
- A dynamical charm quark enables the disentaglement of the effects.
- Lattice QCD can help not only to give the Standard Model prediction, but also to conceptually understand this enhancement.

Outlook

- More dynamical lattice simulations with $N_f = 4$ and $N_c \geq 3$
- Explore the case $N_f = 3 + 1$
- $K \to \pi\pi$ directly on Large N_c Lattice QCD
- *Ab initio* exploration of the N_c dependence of other quantities such as LECs or ϵ'/ϵ ($\to$ Useful for phenomenology.)
- Explore the contribution of other operators BSM and obtain constraints

Thanks for your attention!

This project has received funding through the European Union's Horizon 2020 research and innovation programme under the Marie Sklodowska-Curie grant agreement No. 713673