

Nucleon charges and quark momentum fraction with $N_f = 2 + 1$ Wilson fermions

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Theory

We consider nucleon forward matrix elements of **isovector** operator insertions \mathcal{O}

$$\langle N(p', s') | \mathcal{O} | N(p, s) \rangle.$$

- “Standard” charges (g_A , g_T , g_S) require **local** operators:

$$\mathcal{O}_\mu^A = \bar{q} \gamma_\mu \gamma_5 q, \quad \mathcal{O}^S = \bar{q} q, \quad \mathcal{O}_{\mu\nu}^T = \bar{q} i \sigma_{\mu\nu} q.$$

- $\langle x \rangle_{u-d}$ (and helicity, transversity moments) from **one-derivative, dimension-four** operators:

$$\mathcal{O}_{\mu\nu}^{vD} = \bar{q} \gamma_{\{\mu} \overleftrightarrow{D}_{\nu\}} q, \quad \mathcal{O}_{\mu\nu}^{aD} = \bar{q} \gamma_{\{\mu} \gamma_5 \overleftrightarrow{D}_{\nu\}} q, \quad \mathcal{O}_{\mu\nu\rho}^{tD} = \bar{q} \sigma_{[\mu\{\nu} \overleftrightarrow{D}_{\rho\}} q,$$

- Including spin-projection with $\Gamma_0 = \frac{1}{2}(1 + \gamma_0)$ and $\Gamma_z = \Gamma_0(1 + i\gamma_5\gamma_3)$ we compute the ratio

$$R_{\mu_1, \dots, \mu_n}^{\mathcal{O}}(t_f, t, t_i) \equiv \frac{C_{\mu_1, \dots, \mu_n}^{\mathcal{O}, 3\text{pt}}(\vec{q} = 0, t_f, t_i, t; \Gamma_z)}{C^{2\text{pt}}(\vec{q} = 0, t_f - t_i; \Gamma_0)}.$$

- In practice $t_{\text{sep}} \equiv t_f - t_i \lesssim 1.5 \text{ fm}$:

\Rightarrow Ground state convergence not guaranteed, fitting the ratio (“plateau method”) not good enough...

Computation of two- and three-point functions

We use the truncated solver method:

$$\langle \mathcal{O} \rangle = \langle \frac{1}{N_{LP}} \sum_{i=1}^{N_{LP}} \mathcal{O}_n^{LP} \rangle + \langle \mathcal{O}_{\text{bias}} \rangle, \quad \mathcal{O}_{\text{bias}} = \frac{1}{N_{HP}} \sum_{i=1}^{N_{HP}} (\mathcal{O}_n^{HP} - \mathcal{O}_n^{LP}).$$

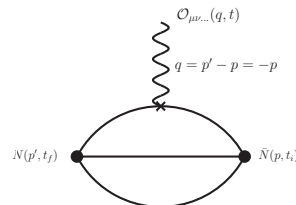
Comput. Phys. Commun. 181 (2010) 1570-1583
Phys. Rev. D 91 (2015) no.11, 114511

Typically, per configuration:

- $N_{HP} = 1$ high-precision inversion(s)
- $N_{LP} = 16 \dots 48$ low-precision inversions

→ **Gain of factor 2-3 in compute time**

- For 3pt functions we use sequential inversions through the sink, setting $p' = 0$.
- **Isvector matrix elements require only quark-connected 3pt functions**
- For isoscalar matrix elements we work on adding disconnected diagrams



Gauge ensembles

ID	β	T/a	L/a	aM_π	M_π /GeV	$M_\pi L$	N_{HP}	N_{LP}	twist-2	t_{sep} /fm
C101	3.40	96	48	0.0976(09)	0.223(3)	4.68	1908	15264	no	1.0, 1.2, 1.4
H102	3.40	96	32	0.1541(06)	0.352(4)	4.93	7988	0	no	1.0, 1.2, 1.4
H105	3.40	96	32	0.1219(10)	0.278(4)	3.90	4076	48912	yes	1.0, 1.2, 1.4
N401	3.46	128	48	0.1118(06)	0.289(4)	5.37	701	11216	yes	1.1, 1.2, 1.4, 1.5, 1.7
S400	3.46	128	32	0.1352(06)	0.350(4)	4.33	1725	27600	yes	1.1, 1.2, 1.4, 1.5, 1.7
D200	3.55	128	64	0.0661(03)	0.203(3)	4.23	1021	32672	yes	1.0, 1.2, 1.3, 1.4
N200	3.55	128	48	0.0920(03)	0.283(3)	4.42	1697	20364	yes	1.0, 1.2, 1.3, 1.4
N203	3.55	128	48	0.1130(02)	0.347(4)	5.42	1540	24640	yes	1.0, 1.2, 1.3, 1.4, 1.5
J303	3.70	192	64	0.0662(03)	0.262(3)	4.24	531	8496	yes	1.0, 1.1, 1.2, 1.3
N302	3.70	128	48	0.0891(03)	0.353(4)	4.28	1177	18832	yes	1.0, 1.1, 1.2, 1.3, 1.4

- $N_f = 2 + 1$ flavors of non-perturbatively improved Wilson clover fermions. *JHEP 1502 (2015) 043*
- Lüscher-Weisz gauge action *Commun.Math.Phys. 97 (1985)*
- Exceptional configurations are suppressed by a twisted mass regulator. *PoS LATTICE2008 (2008) 049*
- Generated with open boundary conditions in time. *Comput. Phys. Commun. 184 (2013)*
- Up to five source-sink separations; typically 1.0fm to 1.4fm.

Renormalization

Non-perturbative renormalization has been performed in collaboration with Regensburg for the **three lower values of β** using the Rome-Southampton method:

β	Z_A	$Z_S^{\overline{\text{MS}}}$	$Z_T^{\overline{\text{MS}}}$	$Z_{v2a}^{\overline{\text{MS}}}$	$Z_{v2b}^{\overline{\text{MS}}}$	$Z_{r2a}^{\overline{\text{MS}}}$	$Z_{r2b}^{\overline{\text{MS}}}$	$Z_{h1a}^{\overline{\text{MS}}}$	$Z_{h1b}^{\overline{\text{MS}}}$
3.40	0.75328(21)	0.6506(19)	0.83359(12)	1.10492(7)	1.11662(6)	1.09696(7)	1.13422(07)	1.13805(7)	1.14732(07)
3.46	0.76043(17)	0.6290(17)	0.84754(08)	1.12233(4)	1.12868(4)	1.11514(4)	1.14753(04)	1.15746(4)	1.16673(04)
3.55	0.77060(14)	0.6129(14)	0.86656(07)	1.15690(4)	1.16072(4)	1.15014(5)	1.17916(05)	1.19552(5)	1.20471(05)
3.70	0.78788(18)	0.5758(16)	0.89943(12)	1.21051(8)	1.20719(7)	1.20472(9)	1.22591(10)	1.25457(9)	1.26365(10)

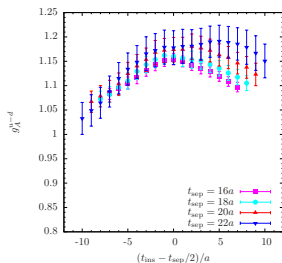
- Each of the derivative operators falls into two different irreps of $H(4)$.
- Matrix elements agree in the continuum limit.
- Blue irreps are required for the vD , aD and tD operators used in our calculation.
- Values at $\beta = 3.70$ extrapolated.
- (Relative) effects of renormalization are of similar size as found in other studies.
- Errors are statistical only; irrelevant for total error budget.
- Results are given in $\overline{\text{MS}}$ at $Q^2 = 4 \text{ GeV}^2$.

Phys. Rev. D54 (1996) 5705
Phys. Rev. D82 (2010) 114511

Excited states

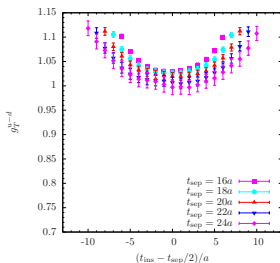
Nucleon structure calculations are notoriously hampered by **excited state contaminations**:

- Need large t_{sep} for plateau method → **signal-to-noise problem**
- Lattice determinations of g_A use to approach exp. value from below (at few percent level).



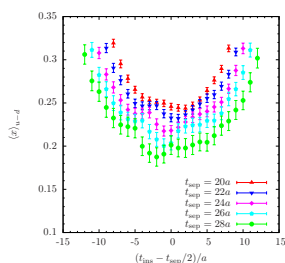
g_A on D200

($M_\pi = 203$ MeV, $a = 0.064$ fm)



g_T on N203

($M_\pi = 347$ MeV, $a = 0.064$ fm)



$\langle x \rangle_{u-d}$ on N302

($M_\pi = 353$ MeV, $a = 0.049$ fm)

- We observe excited state contamination on all ensembles.
- Contamination generally worse for twist-2 (and at smaller pion masses).
- No ground state convergence observed up to $t_{\text{sep}} = 1.5$ fm.

Simultaneous fits

The effective charge / formfactor $R^{\mathcal{O}}(t_f, t, t_i, Q^2)$ can be described by a tower of states ($t_i = 0$)

$$R^{\mathcal{O}}(t_f, t, Q^2) = G_{\mathcal{O}}(Q^2) + \sum_n \left(a_n^{\mathcal{O}}(Q^2) e^{-\Delta_n t} + b_n^{\mathcal{O}}(Q^2) e^{-\Delta'_n(t_f - t)} + c_n^{\mathcal{O}}(Q^2) e^{-\Delta'_n t_f - (\Delta_n - \Delta'_n)t} \right).$$

- $G_{\mathcal{O}}(Q^2)$ denotes the actual form factor,
- Δ_n, Δ'_n are energy gaps,
- $a_n^{\mathcal{O}}(Q^2), b_n^{\mathcal{O}}(Q^2)$ and $c_n^{\mathcal{O}}(Q^2)$ denote amplitudes.

Assuming symmetric plateaux for $Q^2 = 0$ we have ($g_{\mathcal{O}} \equiv G_{\mathcal{O}}(0)$)

$$R^{\mathcal{O}}(t_f, t, 0) = g_{\mathcal{O}} + \sum_n A_n^{\mathcal{O}} \left(e^{-\Delta_n t} + e^{-\Delta_n(t_f - t)} \right) + C_n^{\mathcal{O}} e^{-\Delta_n t_f}.$$

- $A_n^{\mathcal{O}} \equiv a_n^{\mathcal{O}}(0) = b_n^{\mathcal{O}}(0), C_n^{\mathcal{O}} \equiv c_n^{\mathcal{O}}(0)$ depend on observable \mathcal{O} .
- Δ_n are the same for different \mathcal{O} .

\Rightarrow **Simultaneous fit with common, free gap for all observables.**

Simultaneous fits

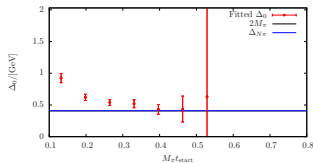
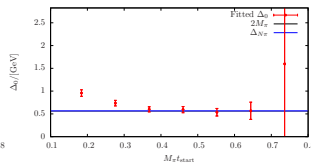
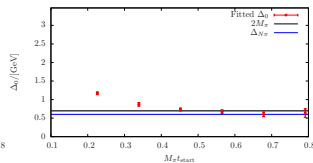
We investigated several fit models:

- 1 Fit to individual observables; one fixed gap $\Delta_0 = 2M_\pi$.
- 2 Fit to individual observables; one free gap.
- 3 **Simultaneous fit to all observables; one free gap**

Fits are subject to the following procedures / constraints:

- Data are explicitly symmetrized around $(t_f - t_i)/2 = t_f/2$.
- Fits use data from range $[t_{\text{start}}, t_f/2]$ for all available values of t_f .
- “Simultaneous” fits use local and twist-2 data **if available**, otherwise the three local charges
- **Final results from fits with same, fixed $M_\pi t_{\text{start}}$ for ALL ensembles.**
- **Can track convergence of free gap as function of $M_\pi t_{\text{start}} \rightarrow$ choice for fixing $M_\pi t_{\text{start}}$.**

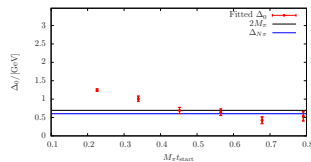
Gap convergence – simultaneous fits

D200 ($M_\pi = 203$ MeV, $a = 0.064$ fm)N200 ($M_\pi = 283$ MeV, $a = 0.064$ fm)N203 ($M_\pi = 347$ MeV, $a = 0.064$ fm)

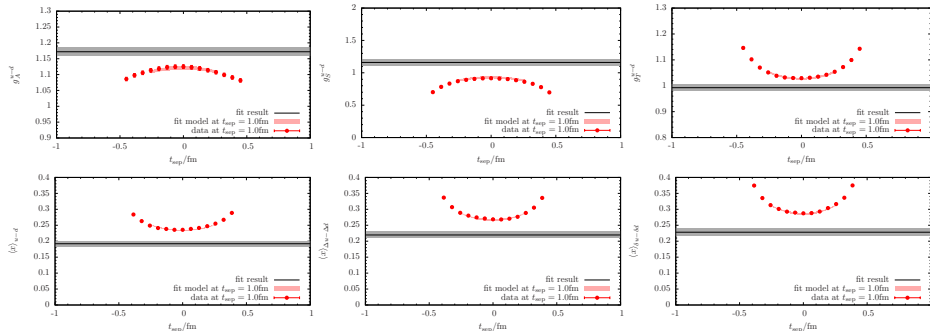
- Results from fitting $R^\mathcal{O}(t_f, t, 0) = g_\mathcal{O} + \sum_n A_n^\mathcal{O} (e^{-\Delta_n t} + e^{-\Delta_n(t_f-t)})$
- Clear convergence on most ensembles; no observable dependence.
- Trade-off between statistical error and systematics due to excited states

→ We choose $M_\pi t_{\text{start}} = 0.4$ for our final analysis.

- Convergence still visible for high statistics ensembles, including term $\sim C_n^\mathcal{O} e^{-\Delta_n t_f}$.
- But errors on Δ_0 much larger.
- Single observable fits tend to become unstable.

N203 ($M_\pi = 347$ MeV, $a = 0.064$ fm)

Simultaneous fit example

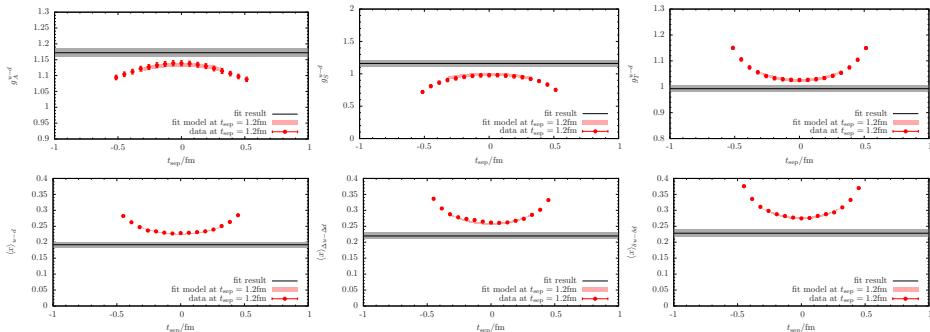


Results for all six observables from simultaneous (free Δ_0) fit on N203 ($M_\pi = 347$ MeV, $a = 0.064$ fm).

Results for $t_{sep} = 16a$ are shown.

- Data described well across observables, values of t_{sep} .
- Corrections can be sizable compared to plateau method at e.g. $t_{sep} = 1.3fm$
- Simultaneous fits supersede fixed gap / single observable fits (less ambiguity, better signal).

Simultaneous fit example

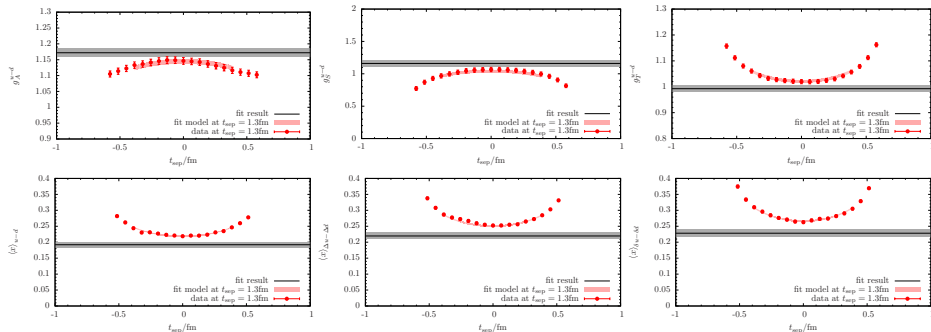


Results for all six observables from simultaneous (free Δ_0) fit on N203 ($M_\pi = 347$ MeV, $a = 0.064$ fm).

Results for $t_{\text{sep}} = 18a$ are shown.

- Data described well across observables, values of t_{sep} .
- Corrections can be sizable compared to plateau method at e.g. $t_{\text{sep}} = 1.3$ fm
- Simultaneous fits supersede fixed gap / single observable fits (less ambiguity, better signal).

Simultaneous fit example

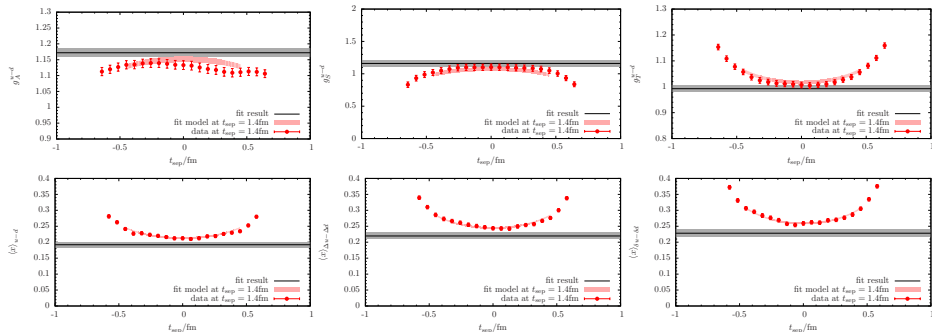


Results for all six observables from simultaneous (free Δ_0) fit on N203 ($M_\pi = 347 \text{ MeV}$, $a = 0.064 \text{ fm}$).

Results for $t_{\text{sep}} = 20a$ are shown.

- Data described well across observables, values of t_{sep} .
- Corrections can be sizable compared to plateau method at e.g. $t_{\text{sep}} = 1.3 \text{ fm}$
- Simultaneous fits supersede fixed gap / single observable fits (less ambiguity, better signal).

Simultaneous fit example

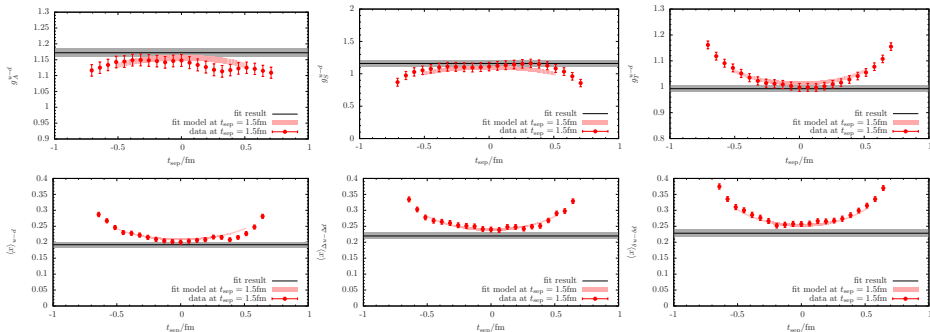


Results for all six observables from simultaneous (free Δ_0) fit on N203 ($M_\pi = 347\text{ MeV}$, $a = 0.064\text{ fm}$).

Results for $t_{\text{sep}} = 22a$ are shown.

- Data described well across observables, values of t_{sep} .
- Corrections can be sizable compared to plateau method at e.g. $t_{\text{sep}} = 1.3\text{ fm}$
- Simultaneous fits supersede fixed gap / single observable fits (less ambiguity, better signal).

Simultaneous fit example



Results for all six observables from simultaneous (free Δ_0) fit on N203 ($M_\pi = 347\text{ MeV}$, $a = 0.064\text{ fm}$).

Results for $t_{\text{sep}} = 24a$ are shown.

- Data described well across observables, values of t_{sep} .
- Corrections can be sizable compared to plateau method at e.g. $t_{\text{sep}} = 1.3\text{ fm}$
- Simultaneous fits supersede fixed gap / single observable fits (less ambiguity, better signal).

Chiral, continuum and finite size (CCF) fit models

The general version of our CCF fit model reads

$$O(M_\pi, a, L) = A_O + B_O M_\pi^2 + C_O M_\pi^2 \log M_\pi + D_O a^{n(O)} + E_O M_\pi^2 e^{-M_\pi L},$$

where

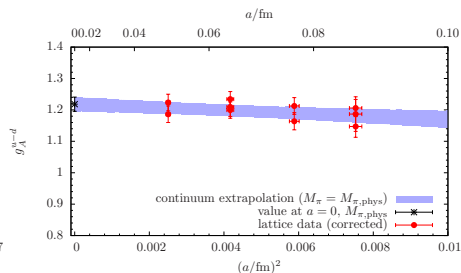
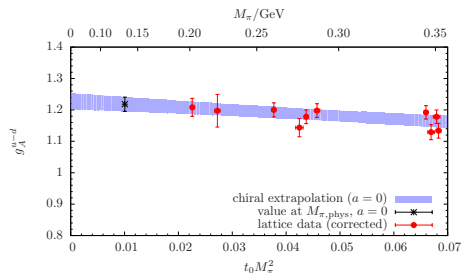
- $n(O) = \begin{cases} 2 & \text{if } O = g_A, g_S \\ 1 & \text{else} \end{cases}$,
- $A \equiv A_O$, $B \equiv B_O$, $C \equiv C_O$, $D \equiv D_O$ and $E \equiv E_O$ are free fit parameters,
- We denote fit models by the letters of the fit parameters of included terms, e.g. “ABD”.
- For $O = g_A$ the coefficient C is known analytically, i.e.

$$C_{g_A} = \frac{-\hat{g}_A}{(2\pi f_\pi)^2} (1 + 2\hat{g}_A^2).$$

- Fitting free parameter C unstable; large χ^2/dof .
- We perform fits with and w/o cut of $M_\pi < 290 \text{ MeV}$.
- We use t_0 to set the scale, with

$$\sqrt{8t_0^{\text{phys}}} = 0.415(4)_{\text{stat}}(2)_{\text{sys}} \text{ fm}.$$

Chiral + continuum fit – g_A

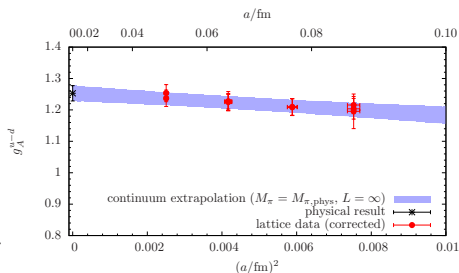
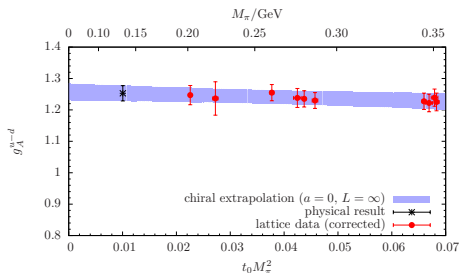


- Chiral and continuum corrections are mild and of similar size.
- Result from combined chiral and continuum fit (model ABD):

$$g_A^{ABD} = 1.218(23)$$

- Result 3σ away from experimental value.
- Fit seems to give reasonable description of data.
- However: Some “scattering” in the data remains ...

Chiral + continuum + FS fit – g_A



- Including a finite size term (model **ABDE**) further improves fit.
- $\mathcal{O}(3\%)$ shift in physical value: $g_A^{ABDE} = 1.253(24)$.
- Data not yet sensitive to **chiral log**:

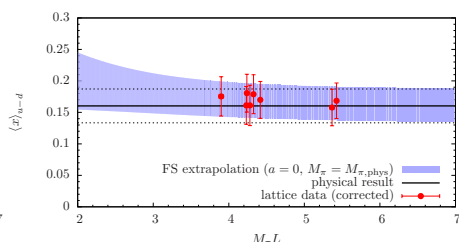
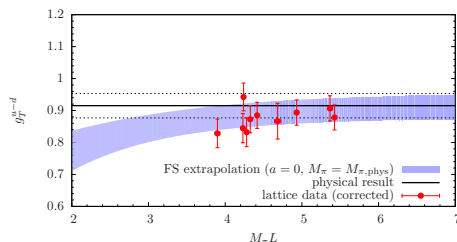
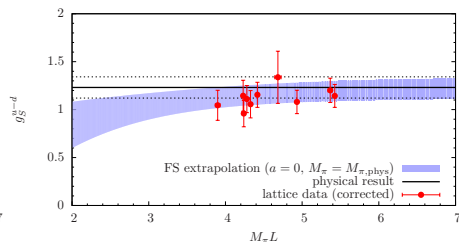
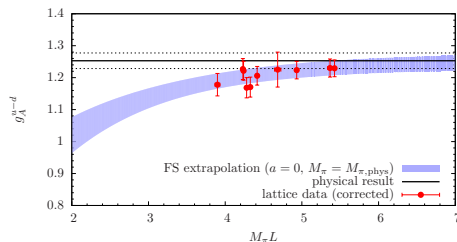
$$g_A^{ABDE} = 1.253(24) \quad \text{vs.} \quad g_A^{ABCDE} = 1.249(26)$$

- However, significant effect on fit parameter B (in units of the scale):

$$B_{g_A}^{ABDE} = 11(11) \quad \text{vs.} \quad B_{g_A}^{ABCDE} = 34(11)$$

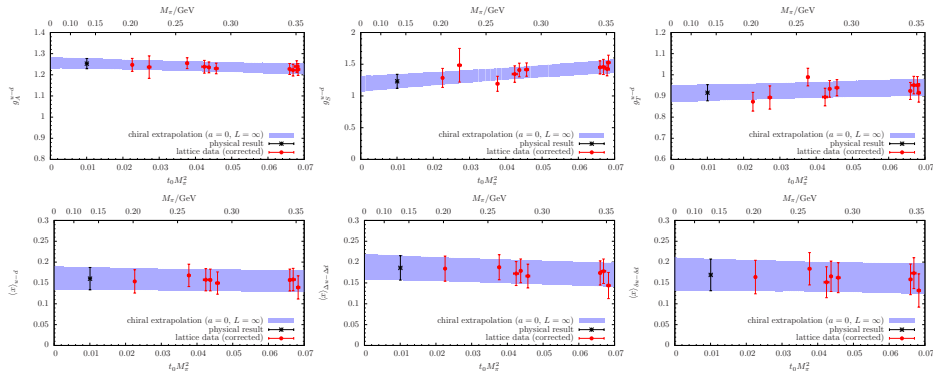
→ Fit cannot distinguish between term $\sim M_\pi^2$ and $\sim M_\pi^2 \log M_\pi$.

Finite size effects



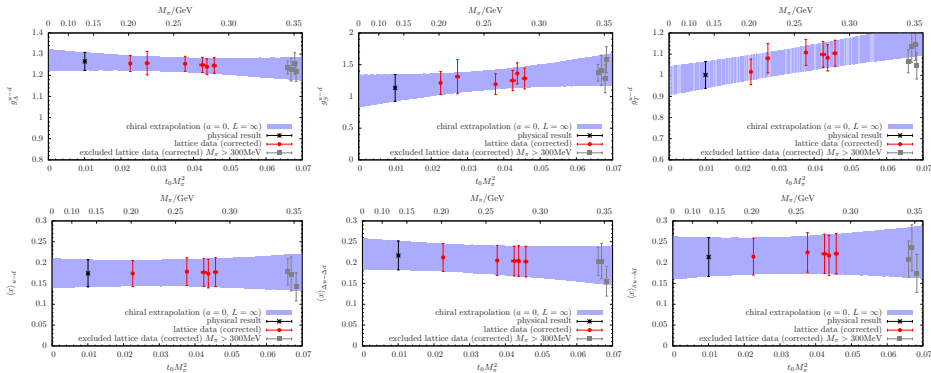
- FS correction very significant for g_A : $E_{g_A} = 328(61)$ (in units of the scale)
- Also relevant for g_S, g_T .
- FS-term almost not constrained for twist-2 data.

Final CCF fits (ABDE) – no M_π -cut: Chiral extrapolation



- Lattice data is corrected for continuum + FS extrapolation.
- Errors are highly correlated!
- Apart from g_T fits describe the data very well.
- Chiral extrapolation generally mild; basically flat for twist-2 observables (within errors).

Final CCF fits (ABDE) – $M_{\pi,\text{cut}} = 300\text{ MeV}$: Chiral extrapolation



- Larger errors (as expected).
- For twist-2 almost no d.o.f. left.
- Again, data well described by fits; some improvement for g_T .
- However: subtle interplay between M_{π} -cut and different terms in the fit!

Results (still preliminary!)

Results for nucleon charges:

Fit		g_A^{u-d}	g_S^{u-d}	g_T^{u-d}
ABDE	all M_π	1.253(24)	1.23(11)	0.915(38)
ABDE	$M_\pi < 290 \text{ MeV}$	1.266(43)	1.13(21)	1.002(63)
ABD	all M_π	1.218(23)	1.17(10)	0.894(37)
ABD	$M_\pi < 290 \text{ MeV}$	1.231(38)	0.99(19)	0.948(58)

Results for lowest moments of dim-4 operators:

Fit		$\langle x \rangle_{u-d}$	$\langle x \rangle_{\Delta u - \Delta d}$	$\langle x \rangle_{\delta u - \delta d}$
ABDE	all M_π	0.160(27)	0.186(29)	0.169(38)
ABDE	$M_\pi < 290 \text{ MeV}$	0.174(33)	0.217(35)	0.213(47)
ABD	all M_π	0.163(27)	0.186(29)	0.170(38)
ABD	$M_\pi < 290 \text{ MeV}$	0.182(32)	0.221(35)	0.213(45)

- Finite size and continuum corrections are non-negligible (and of similar absolute size).
- Results from fit to data with $M_\pi = 200 \dots 350 \text{ MeV}$.
- Our data are not yet sensitive to the chiral log.
- Some tension for g_T ; only case for which CCF fit has rather large $\chi^2_{\text{red}} = 2.7$ fitting all data.

Summary and outlook

- We computed isovector charges and moments for local and twist-2 operators.
- Results for $M_\pi \approx 200 \dots 350$ MeV, $M_\pi L \approx 3.9 \dots 5.4$ and four values of $a \approx 0.050 \dots 0.086$ fm.
- Simultaneous fits promising for controlling excited states at reasonable stat. error.
- **Consistent and observable-independent analysis / treatment of excited states.**
- **Combined chiral + continuum + finite size extrapolation for all observables.**
- All three corrections are of similar size.

Ongoing work / future plans:

- Add ensemble with $M_\pi L \approx 3$ for better control of FS effects. (short term)
- Include ensemble $((T/a) \times (L/a)^3 = 192 \times 96^3, a = 0.064 \text{ fm})$ at physical M_π .
- Additional ensemble(s) / higher statistics on selected ensembles (?)
- Q^2 -dependence, e.g. el.-mag. FF, axial FF, GFFs, ...
- Include quark disconnected diagrams. \rightarrow isoscalar observables, el.-strange FF.