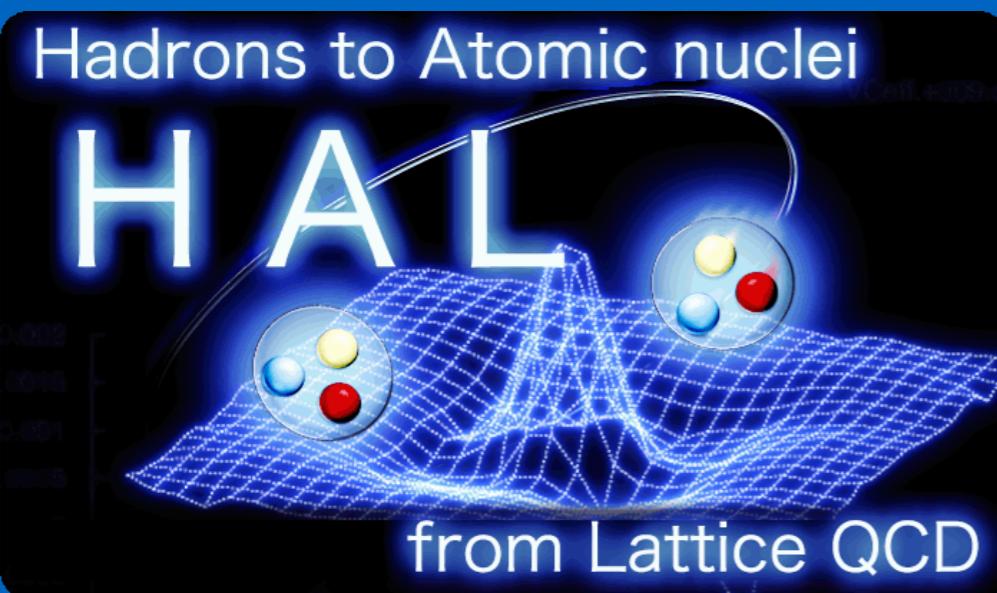


# HAL QCD method and Nucleon-Omega Interaction with Physical Quark Masses

Takumi Iritani (RIKEN)

LATTICE2018, July 22-28, 2018, East Lansing, MI, USA



for HAL QCD Collaboration

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F. Etminan (Univ. of Birjand)

1. Difficulties of two-baryons in lattice QCD  
& HAL QCD method
2. Nucleon-Omega Interaction
3. Summary

[\*\*>> Backup\*\*](#)

[\*\*>> Supplemental Material\*\*](#)

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# Challenges in Multi-Baryon Systems

**ground state** from temporal correlation  $\rightarrow$  long time sep.

$$C_{NN}(t) = c_0 \exp(-E_0^{NN} t) + c_1 \exp(-E_1^{NN} t) + \dots \simeq c_0 \exp(-E_0^{NN} t)$$

- **S/N Problem** (A: mass num.)

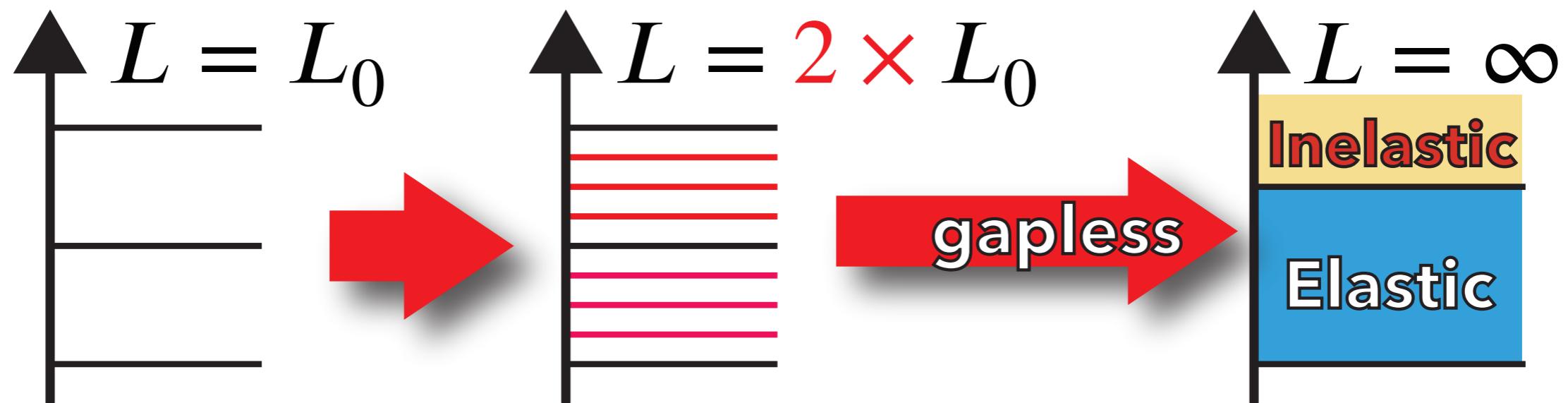
Parisi '84, Lepage '89

$$S/N \sim \exp[-A \times (m_N - (3/2)m_\pi) \times t]$$

- **Scattering state contamination**

$$\Delta E \sim p^2/m_N \sim \mathcal{O}(m_N^{-1} L^{-2}) \ll \mathcal{O}(\Lambda_{\text{QCD}})$$

$$\xrightarrow{\quad} t_{\text{elastic}} \gtrsim \mathcal{O}(m_N L^2) \gg \mathcal{O}(\Lambda_{\text{QCD}}^{-1})$$



# 2 Hadrons in Lattice QCD (1) Direct Method

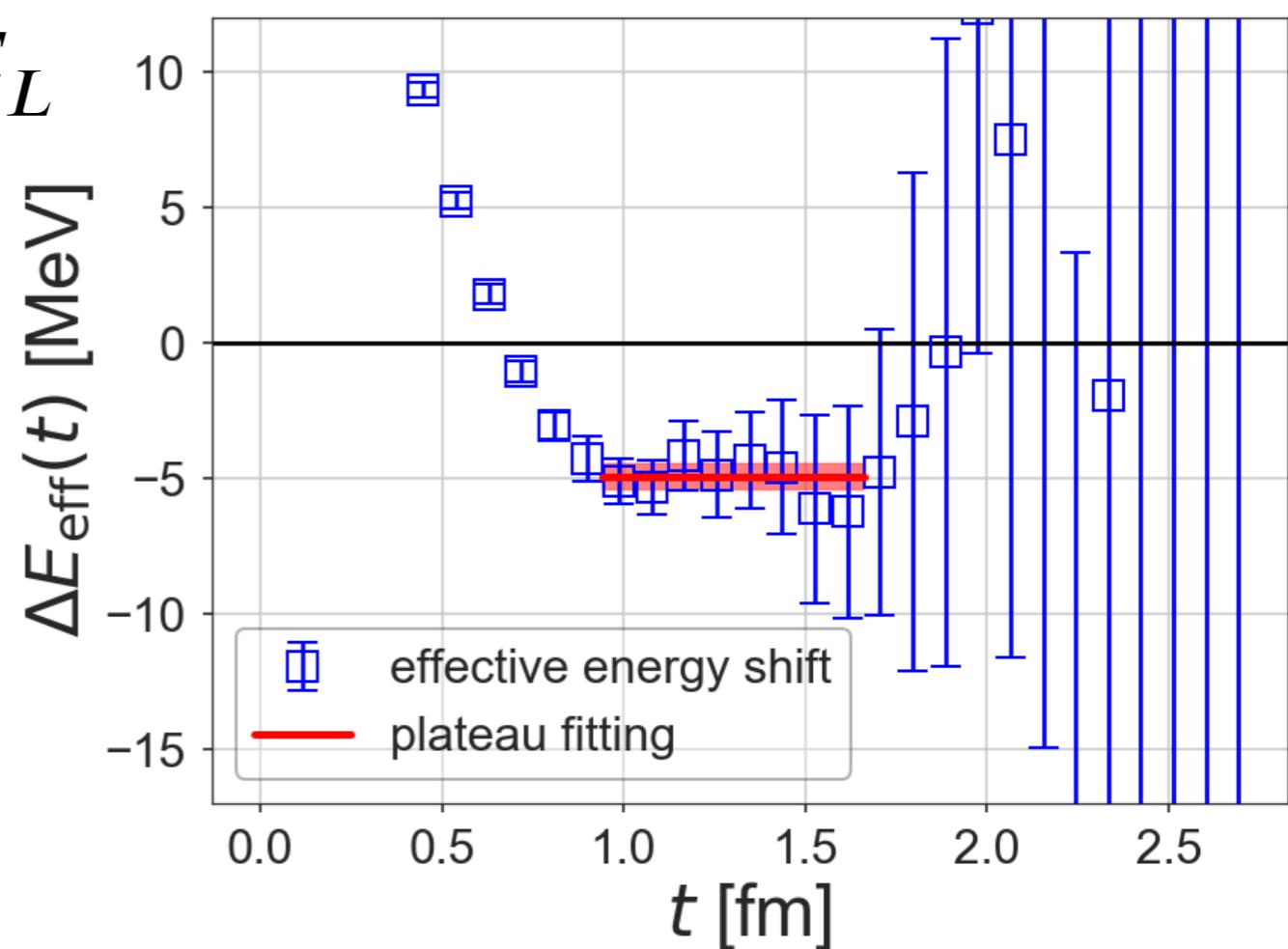
- **temporal correlation  $\rightarrow$  ground state energy**

$$R(t) \equiv \langle B(t)^2 \bar{B}(0)^2 \rangle / \{ \langle B(t) \bar{B}(0) \rangle \}^2 \longrightarrow \exp[-(E_{BB}^L - 2m_B)t]$$

- **Plateau of “Effective energy shift”**

$$\Delta E_{\text{eff}}(t) \equiv \frac{1}{a} \log \frac{R(t)}{R(t+a)} \rightarrow \Delta E_L$$

**g.s. saturation is mandatory!**



- **Finite volume energy shift  
 $\rightarrow$  Binding Energy**

$$\lim_{L \rightarrow \infty} \Delta E_L = -\text{B.E.}$$

# Scattering State Contamination in “Direct Method”

$$R(t) = b_0 e^{-\Delta E_{BB} t} \left( 1 + \frac{b_1}{b_0} e^{-\delta E_{el} t} + \frac{c_0}{b_0} e^{-\delta E_{inel} t} \right)$$

$$\delta E_{el} = 50 \text{ MeV} \sim \mathcal{O}(1/L^2), \quad \delta E_{inel} = 500 \text{ MeV} \sim \mathcal{O}(\Lambda_{QCD})$$

## Effective energy shift

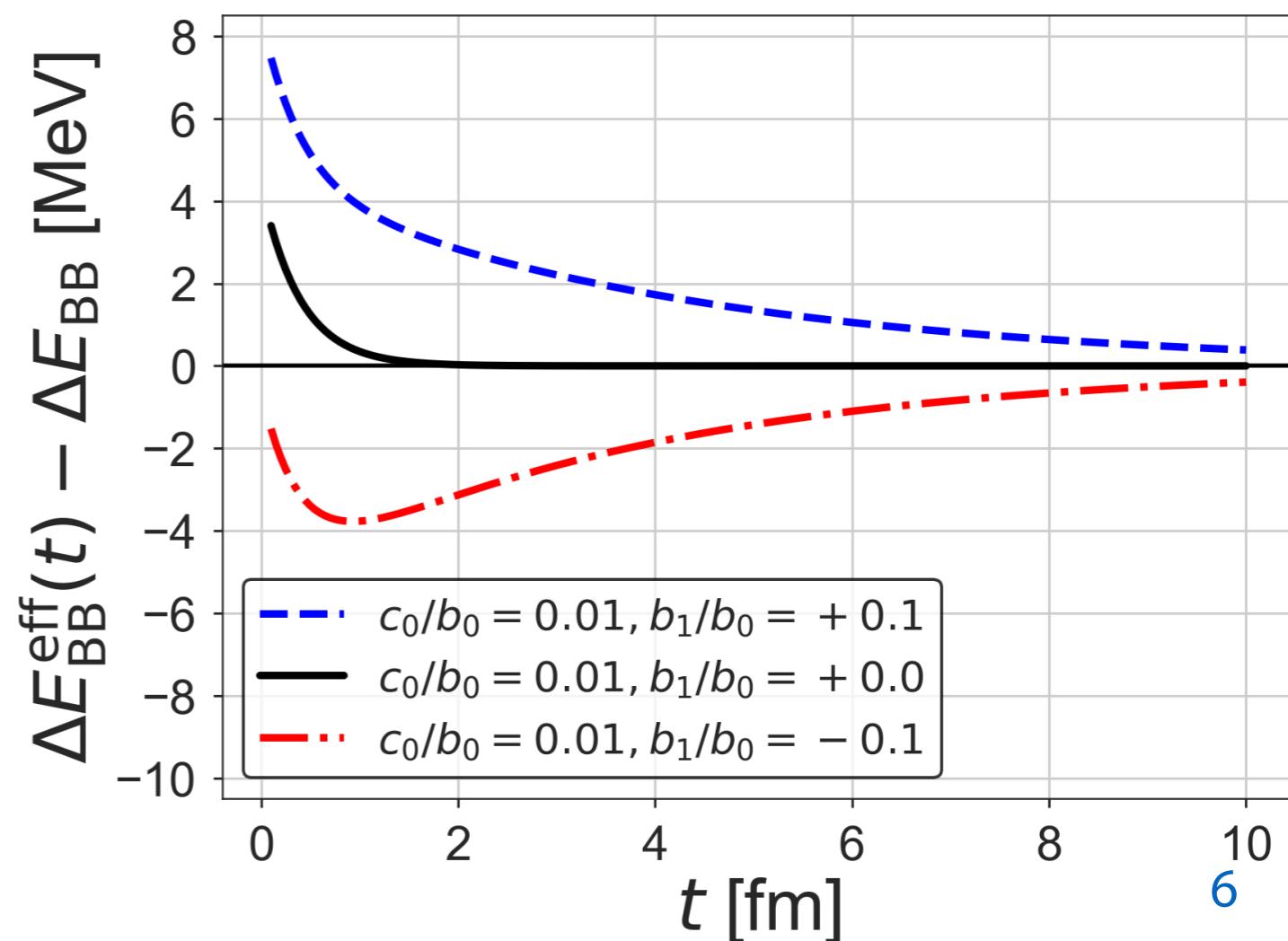
$$\Delta E_{\text{eff}}(t) \equiv \frac{1}{a} \log \frac{R(t)}{R(t+a)} \rightarrow \Delta E_L$$

elastic state saturation

$$t \sim 1 \text{ fm} \gg 1/\delta E_{inel}$$

ground state saturation

$$t \sim 10 \text{ fm} \gg 1/\delta E_{el}$$



# Scattering State Contamination in “Direct Method”

$$R(t) = b_0 e^{-\Delta E_{BB} t} \left( 1 + \frac{b_1}{b_0} e^{-\delta E_{el} t} + \frac{c_0}{b_0} e^{-\delta E_{inel} t} \right)$$

$$\delta E_{el} = 50 \text{ MeV} \sim \mathcal{O}(1/L^2), \quad \delta E_{inel} = 500 \text{ MeV} \sim \mathcal{O}(\Lambda_{QCD})$$

## Effective energy shift

$$\Delta E_{\text{eff}}(t) \equiv \frac{1}{a} \log \frac{R(t)}{R(t+a)} \rightarrow \Delta E_L$$

ground state saturation

$$t \sim 10 \text{ fm} \gg 1/\delta E_{el}$$

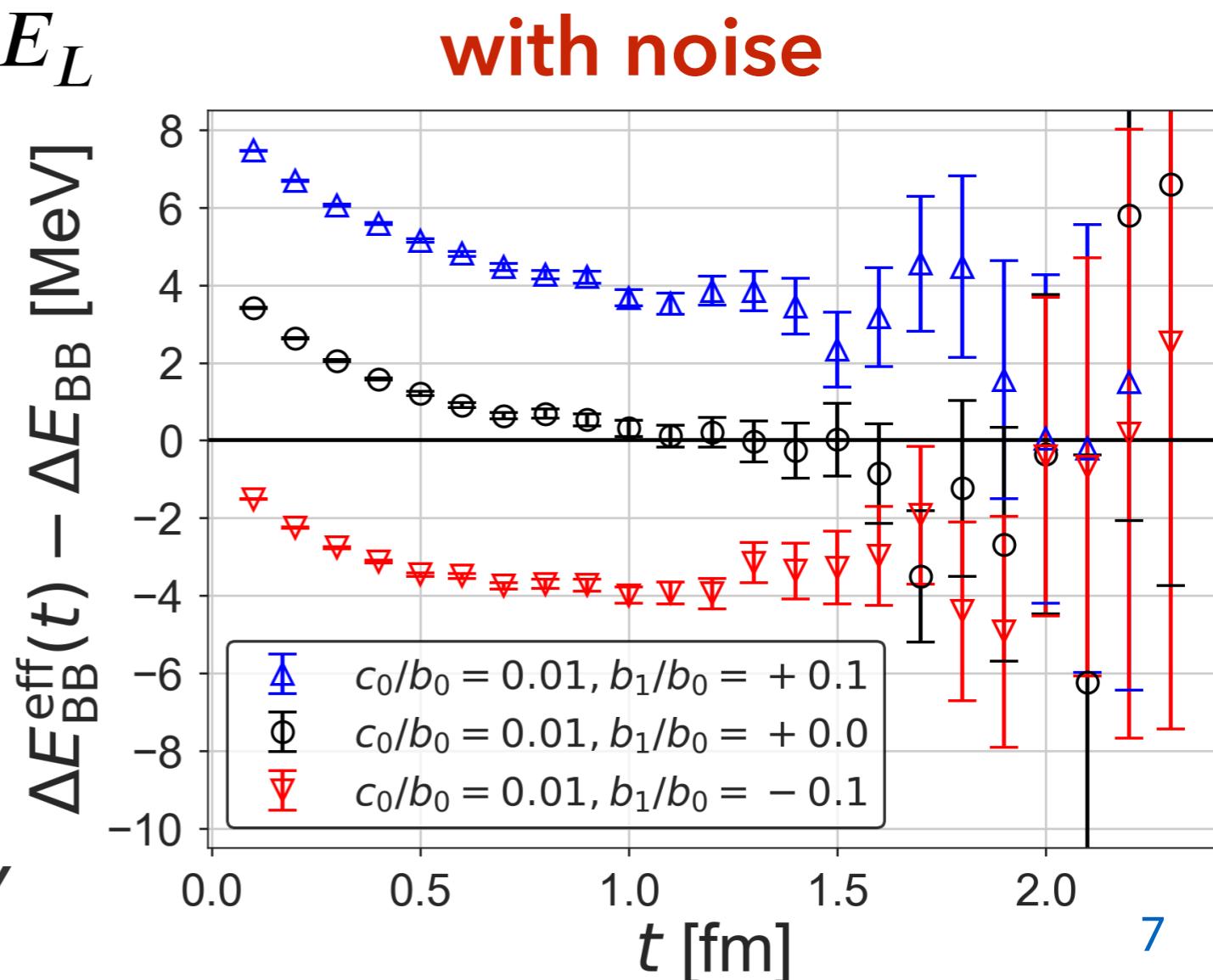
→ “fake plateau” @  $\sim 1.5 \text{ fm}$

**plateaux are unreliable!**

measurement at  $t > 10 \text{ fm}$

or

*variational method is mandatory  
to extract ground state*



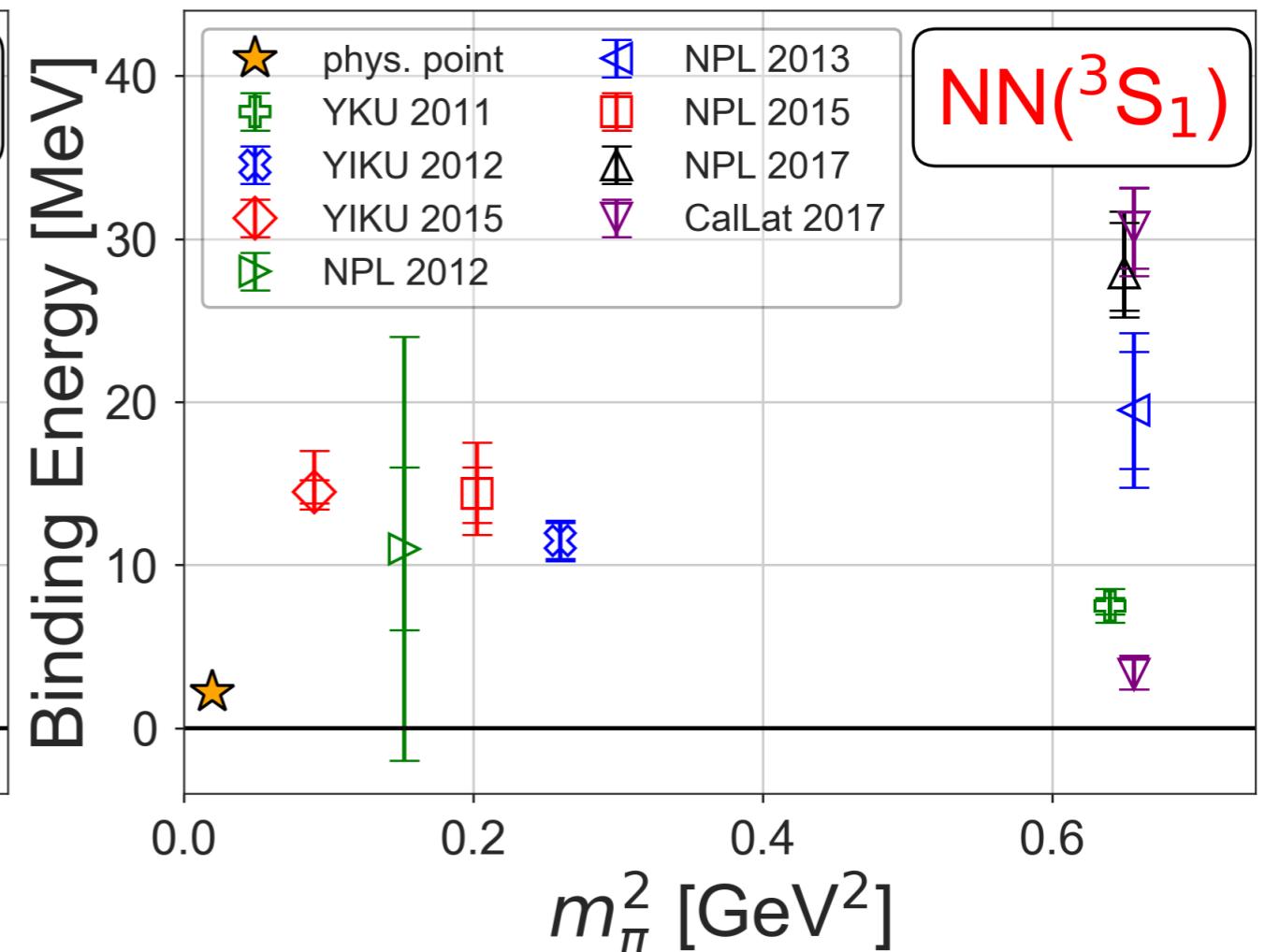
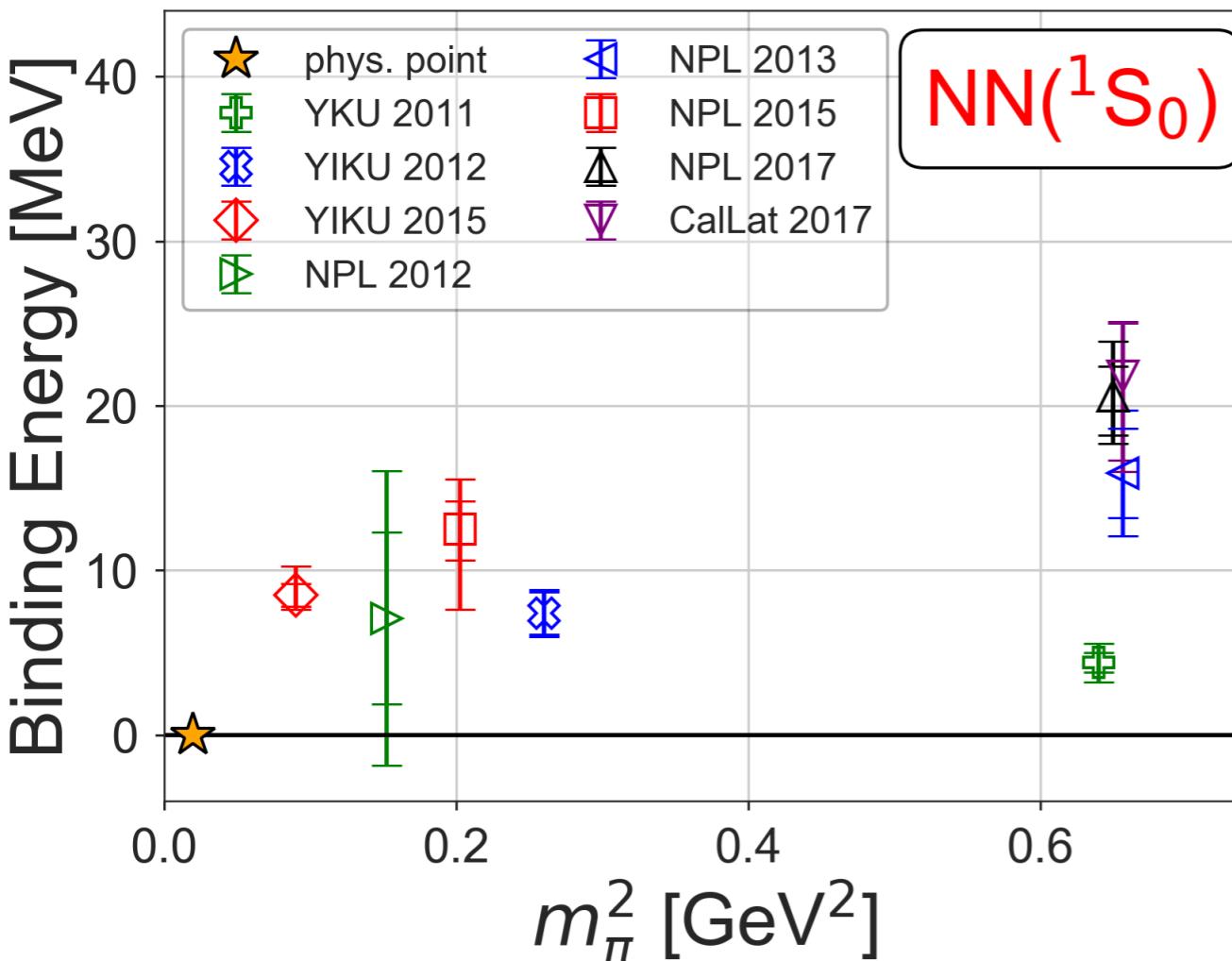
# NN Systems from Direct Method in Lattice QCD

For  $m_\pi > 300$  MeV

deeply bound **dineutron** & **deuteron** are observed

Question: these results are reliable?

these do not use variational method  
& fit plateaux early time around 1.5 fm!

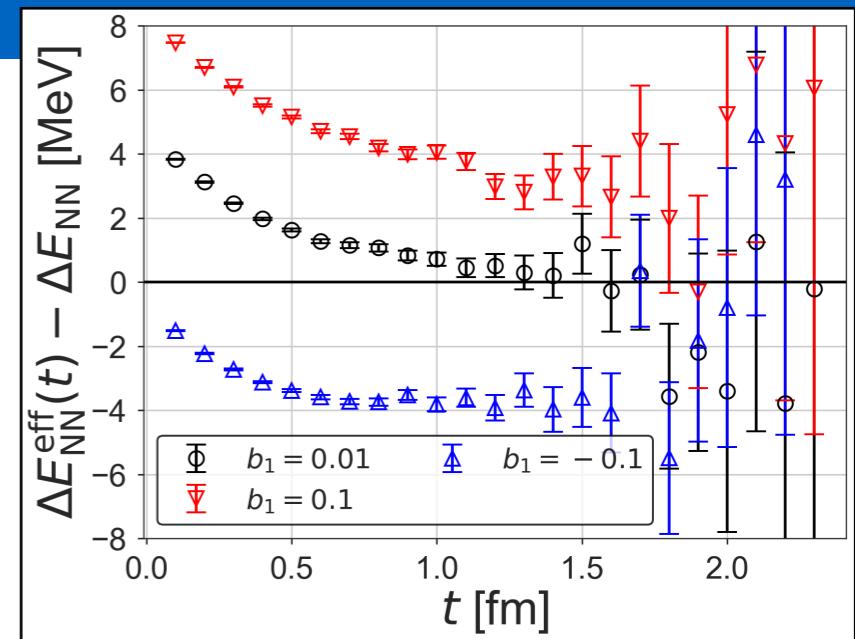


Y(I)KU: Yamazaki, (Ishikawa), Kuramashi, Ukawa  
NPL: NPLQCD Coll  
CalLat: CallLat Coll.

# Fundamental Problem in Direct Method: Fake Plateau

## Simple plateau fitting is challenging.

plateau cannot guarantee g.s. saturation,  
which requires huge temporal correlation



## Consistency/Normality Check

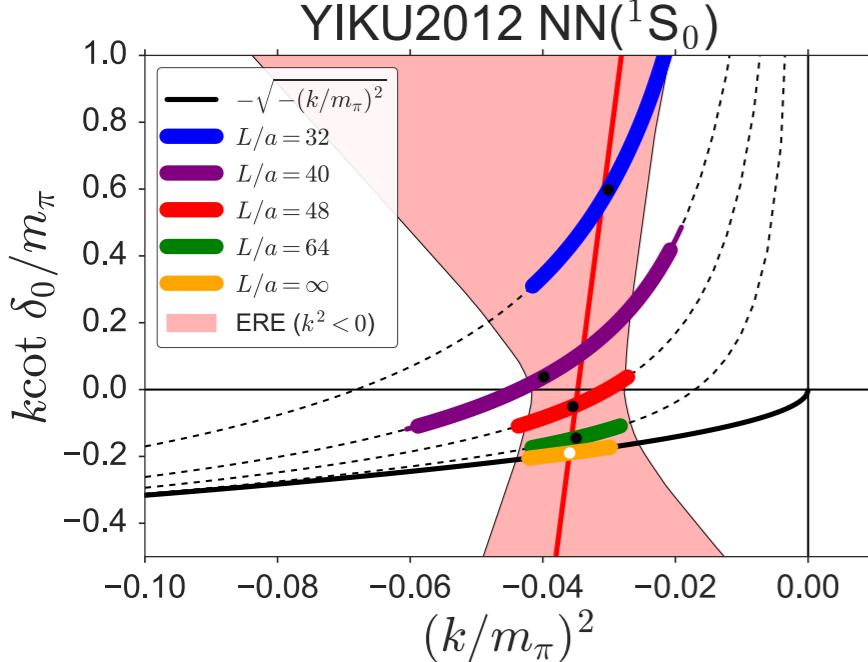
Refs. **TI** for HAL QCD Coll., PRD96.034521(2017)  
S. Aoki, T. Doi, **TI**, LATTICE2017 Proc., arXiv:1707.08800.

all previous studies show “anomalous scattering phase shift”

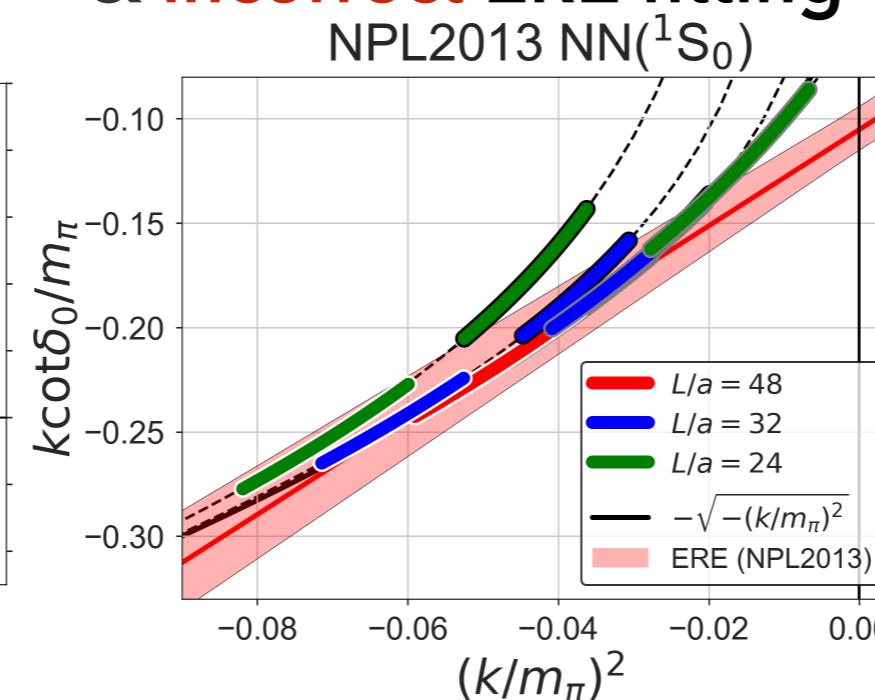
→ fake plateau  
check >> supplemental material

$$k \cot \delta_0(k) \simeq -\frac{1}{a_0} + \frac{1}{2} r_{\text{eff}} k^2 + \dots$$

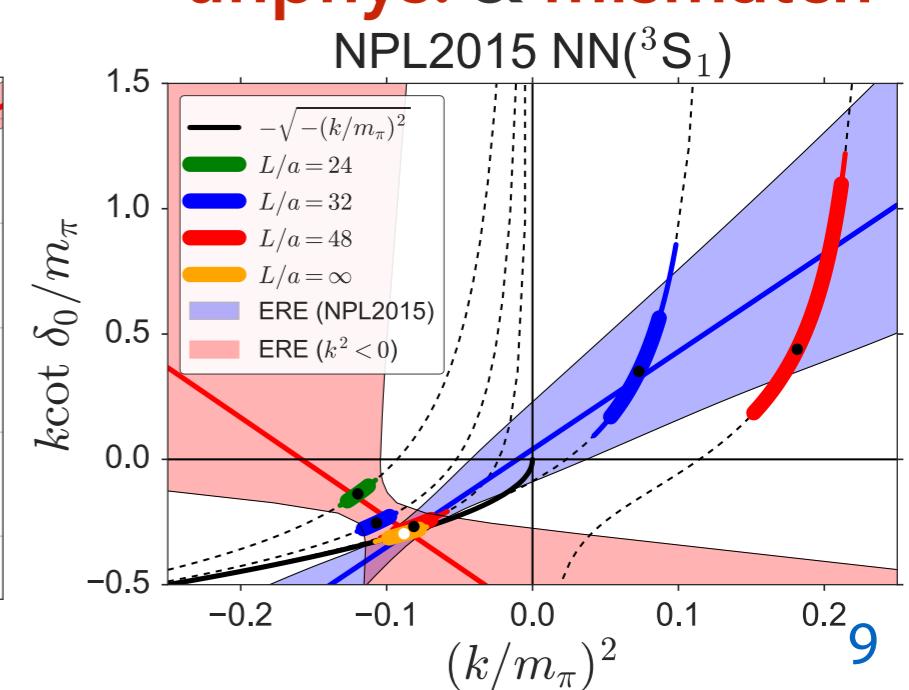
singular ERE



unphysical S-matrix residue  
& incorrect ERE fitting



unphys. & mismatch



# Two-Baryon in Lattice QCD

- 1. Direct Method**
- 2. HAL QCD Method**

# 2 Hadrons in Lattice QCD (2) HAL QCD Method

Ishii-Aoki-Hatsuda '06

Hadrons to Atomic nuclei from Lattice QCD

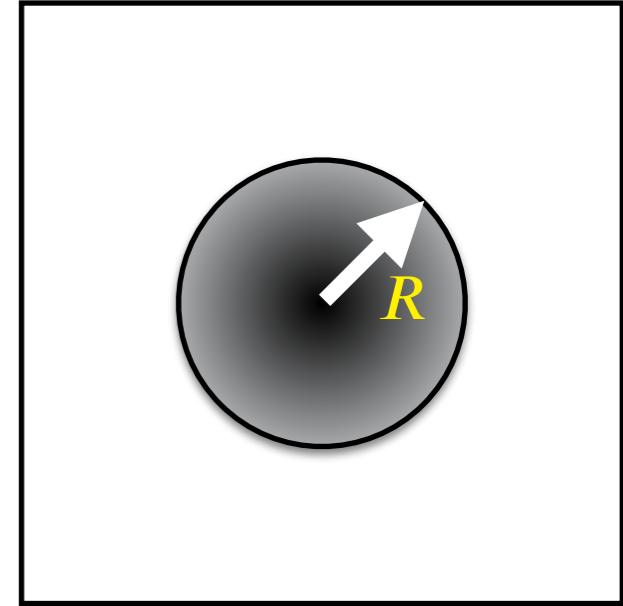
## ♦ spatial correlation

Nambu-Bethe-Salpeter wave func.

$$\psi_k(\vec{r}) = \langle 0 | B(\vec{r}) B(\vec{0}) | B(\vec{k}) B(-\vec{k}); \text{in} \rangle$$

- **asymptotic region**  $r \gg R$

$$\psi_k(\vec{r}) \simeq C \sin(kr - l\pi/2 + \delta(k))/(kr)$$



- **interacting region**  $r < R$

$$(\nabla^2 + k^2)\psi_k(\vec{r}) = m \int d\vec{r}' U(\vec{r}, \vec{r}') \psi_k(\vec{r}')$$



E-indep. & non-local & faithful to phase shifts

$$U(\vec{r}, \vec{r}') = \sum_{|\vec{p}| \leq p_{\text{th}}} [E_p - H_0] \psi_p(\vec{r}) \psi_p^*(\vec{r}')$$

HAL QCD method: **NBS** wave fun.  $\rightarrow$  **U(r,r')**  $\rightarrow$  **observables**

# Time-dependent HAL QCD Method (1)

Ishii for HAL QCD '12

$$R(\vec{r}, t) \equiv \langle 0 | T \{ B(\vec{x} + \vec{r}, t) B(\vec{x}, t) \} \bar{J}_{2B}(0) | 0 \rangle / \{ C_B(t) \}^2$$

$$= \sum_n a_n \underbrace{\psi_{W_n}(\vec{r})}_{\downarrow} e^{-(W_n - 2m_B)t} + \mathcal{O}(e^{-\Delta W_{\text{th}}t})$$

**g.s. & scat. states NBS func.**

*they are signals*  $\rightarrow [E_{W_0} - H_0] \psi_{W_0}(\vec{r}) = \int d\vec{r}' U(\vec{r}, \vec{r}') \psi_{W_0}(\vec{r}')$

$$[E_{W_1} - H_0] \psi_{W_1}(\vec{r}) = \int d\vec{r}' U(\vec{r}, \vec{r}') \psi_{W_1}(\vec{r}')$$

$\vdots$

**with elastic saturation**  $\mathcal{O}(e^{-\delta E_{\text{inel}}t}) \ll \mathcal{O}(e^{-\delta E_{\text{el}}t})$

→  $\left[ \frac{1}{4m_B} \frac{\partial^2}{\partial t^2} - \frac{\partial}{\partial t} - H_0 \right] R(\vec{r}, t) = \int d\vec{r}' U(\vec{r}, \vec{r}') R(\vec{r}', t)$

# Time-dependent HAL QCD Method (2)

$$\left[ \frac{1}{4m_B} \frac{\partial^2}{\partial t^2} - \frac{\partial}{\partial t} - H_0 \right] R(\vec{r}, t) = \int d\vec{r}' U(\vec{r}, \vec{r}') R(\vec{r}', t)$$

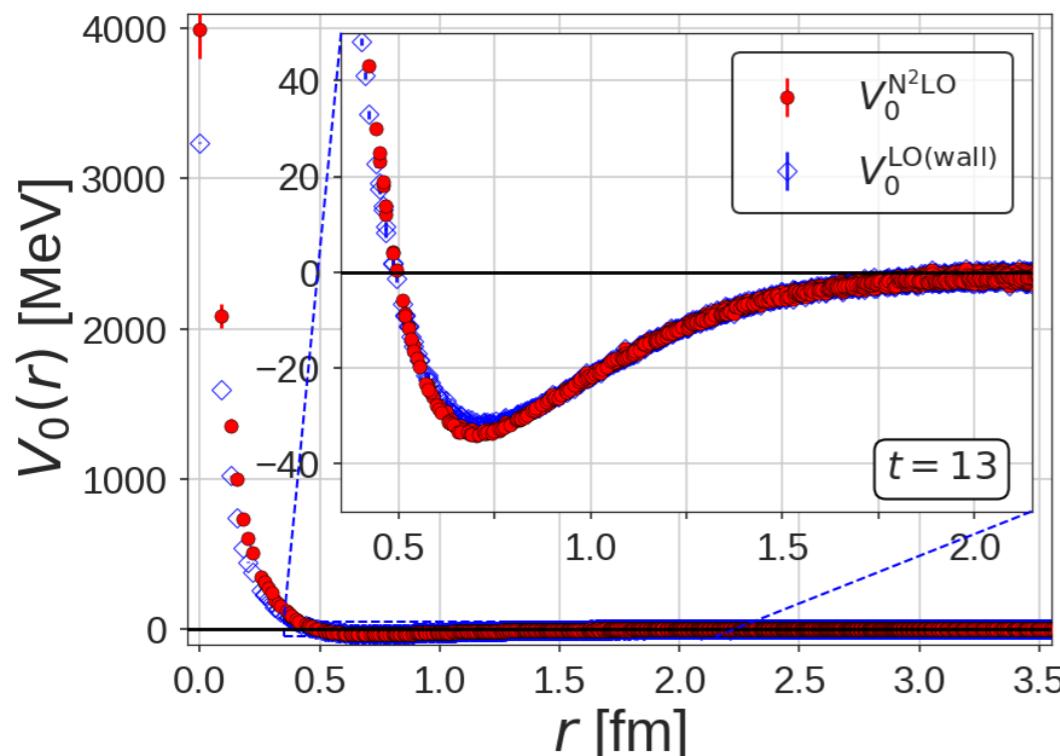
E-indep. & non-local  $U(r, r')$ : **velocity expansion** → local pot.

$$U(\vec{r}, \vec{r}') = [V_0(\vec{r}) + V_1(\vec{r}) \mathbf{L} \cdot \mathbf{S} + V_2(\vec{r}) \nabla^2 + \dots] \delta(\vec{r} - \vec{r}')$$

*what about the convergence?*

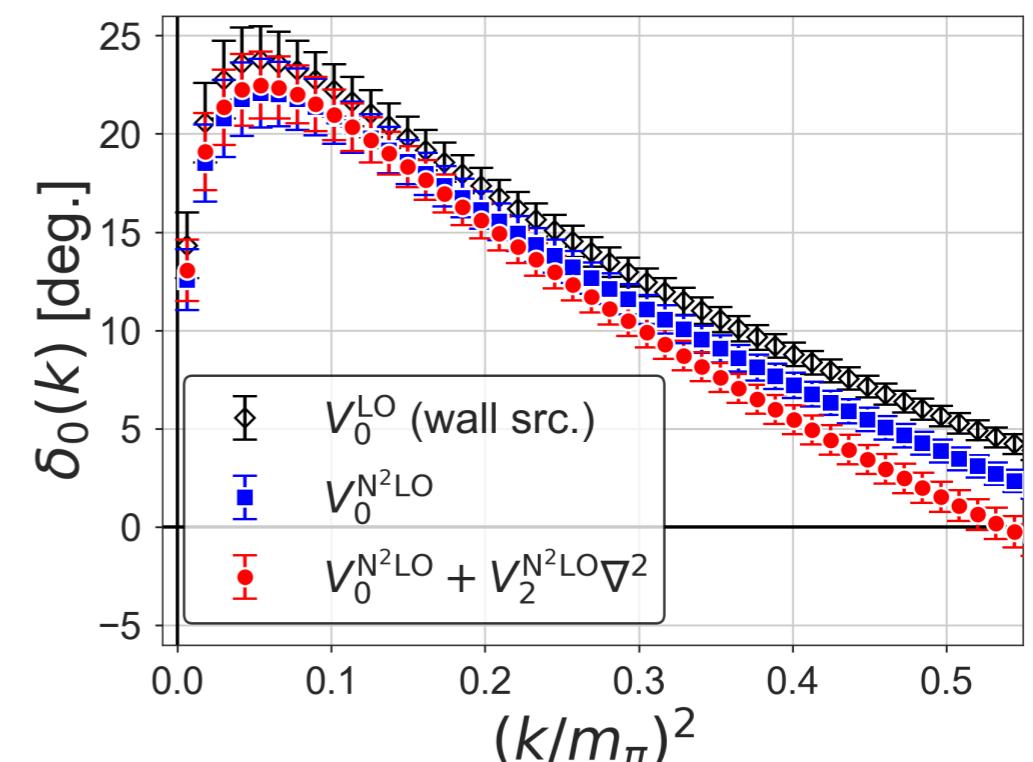
**wall-type quark src. & “Leading order approx.” works well!**

$$U(\vec{r}, \vec{r}') \simeq V_0^{\text{LO}}(\vec{r}) \delta(\vec{r} - \vec{r}')$$



$\Xi\Xi(^1S_0)$  at  $m_\pi = 510$  MeV

convergence of scat. phase shift



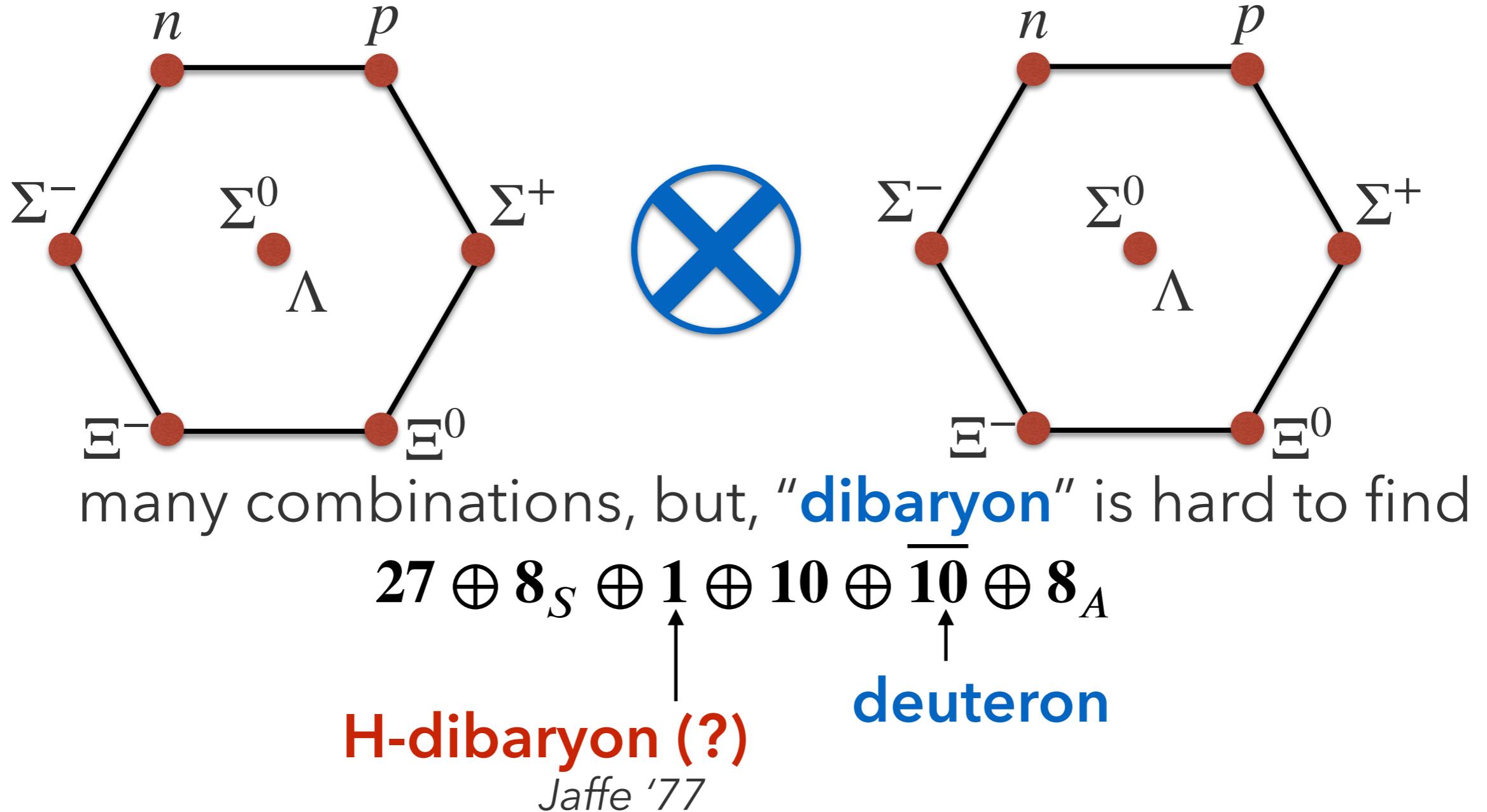
Ref: TI for HAL QCD Coll. >>[arXiv:1805.02365](https://arxiv.org/abs/1805.02365) 13

1. Difficulties of two-baryons in lattice QCD  
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[\*\*>> Backup\*\*](#)

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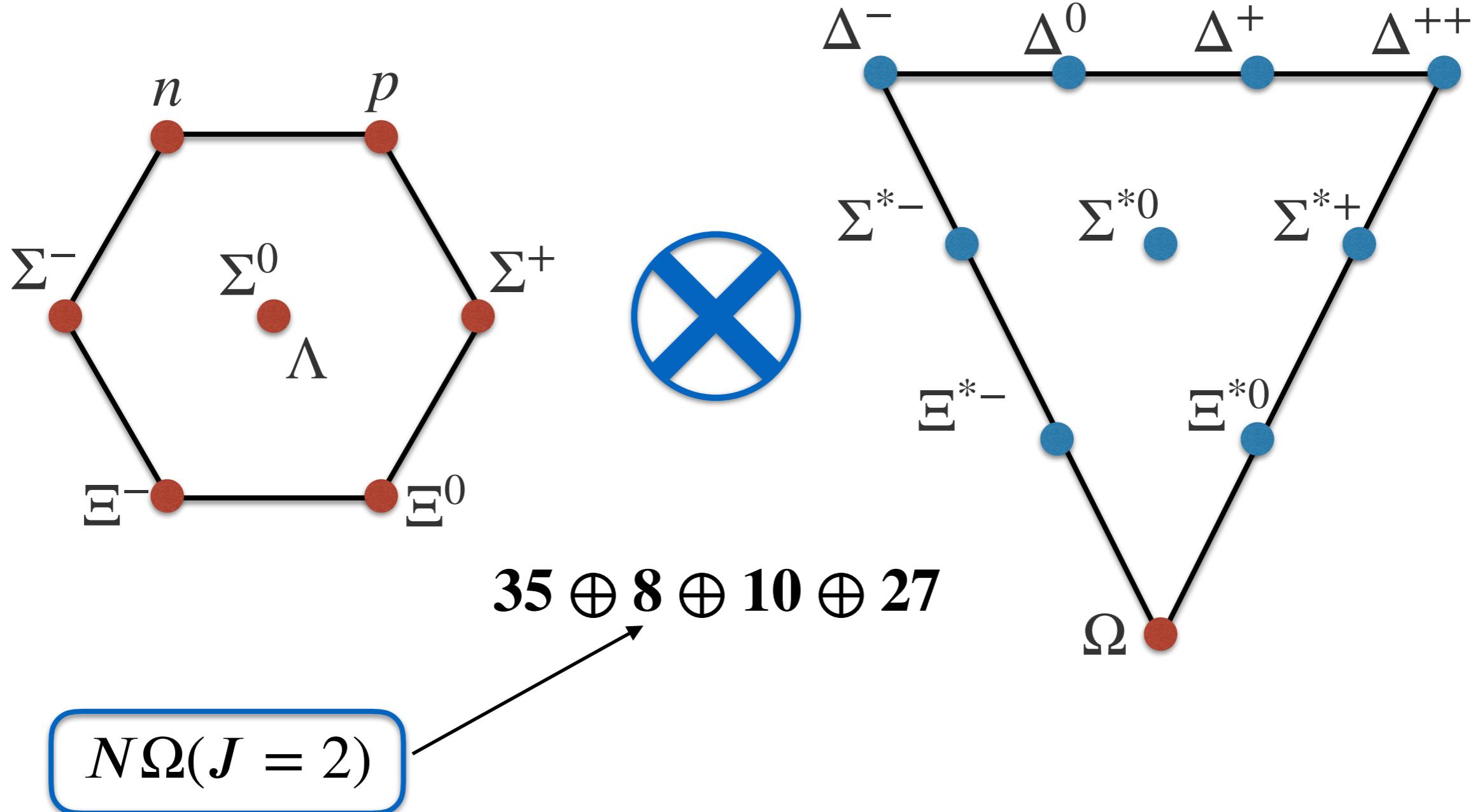
# Octet Baryons and Octet Baryons



Prediction of dibaryons is difficult,  
due to the uncertainties of the baryon interactions.

HAL QCD method is useful to predict “dibaryon states”

# Octet Baryons and Decuplet Baryons



$\Omega$ : only decay via weak interaction  $\rightarrow$  “long” life-time

$N(uud/udd) + \Omega(sss)$ : No Pauli Blocking

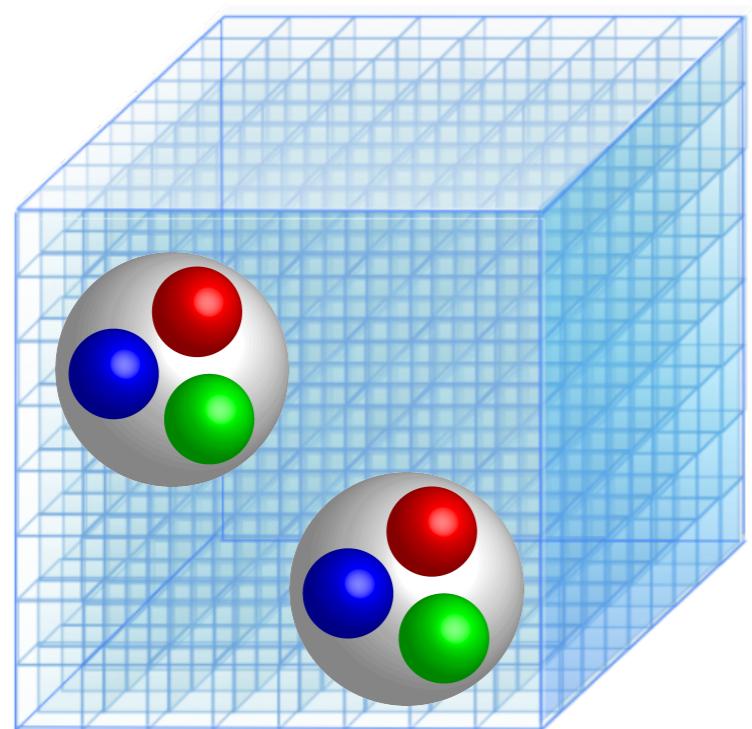
Goldman et al. '87, Oka '88, ...

→ a possible candidate of “**dibaryon**”

# $N\Omega$ Systems at almost Physical Point

## Setups

- $96^4 \sim (8.1 \text{ fm})^4$  at  $a^{-1} = 2.333 \text{ GeV}$  ( $a = 0.0846 \text{ fm}$ )
- $M_\Pi = 146 \text{ MeV}$ ,  $M_N = 964 \text{ MeV}$ ,  $M_\Omega = 1712 \text{ MeV}$
- single channel analysis of  **${}^5S_2(S\text{-wave } J = 2)$** 
  - $\Lambda\Xi$  &  $\Sigma\Xi$  D-wave decay suppressed



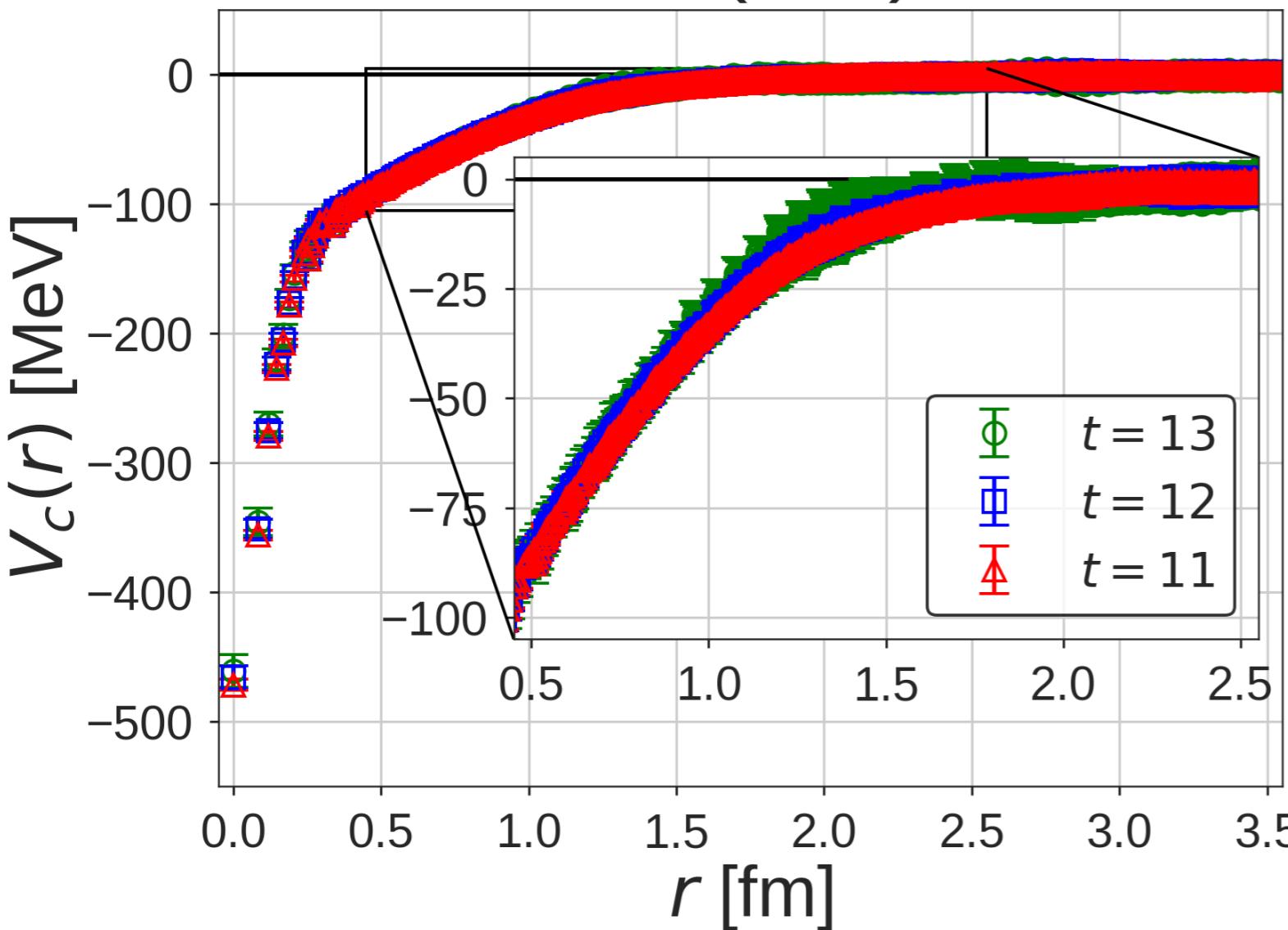
K-computer & FX100 @ RIKEN



Next talk by T. Doi: Octet x Octet channels &  $\Omega\Omega$

# $N\Omega$ potential in $^5S_2$ channel

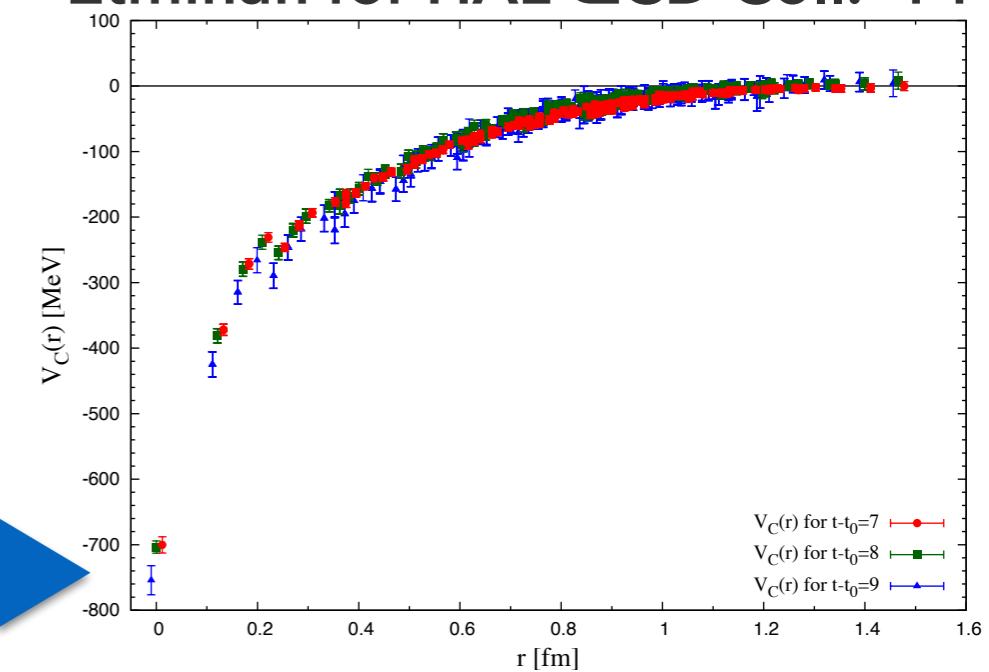
$N\Omega(^5S_2)$



qualitatively the same at  $m_{\pi} = 875$  MeV

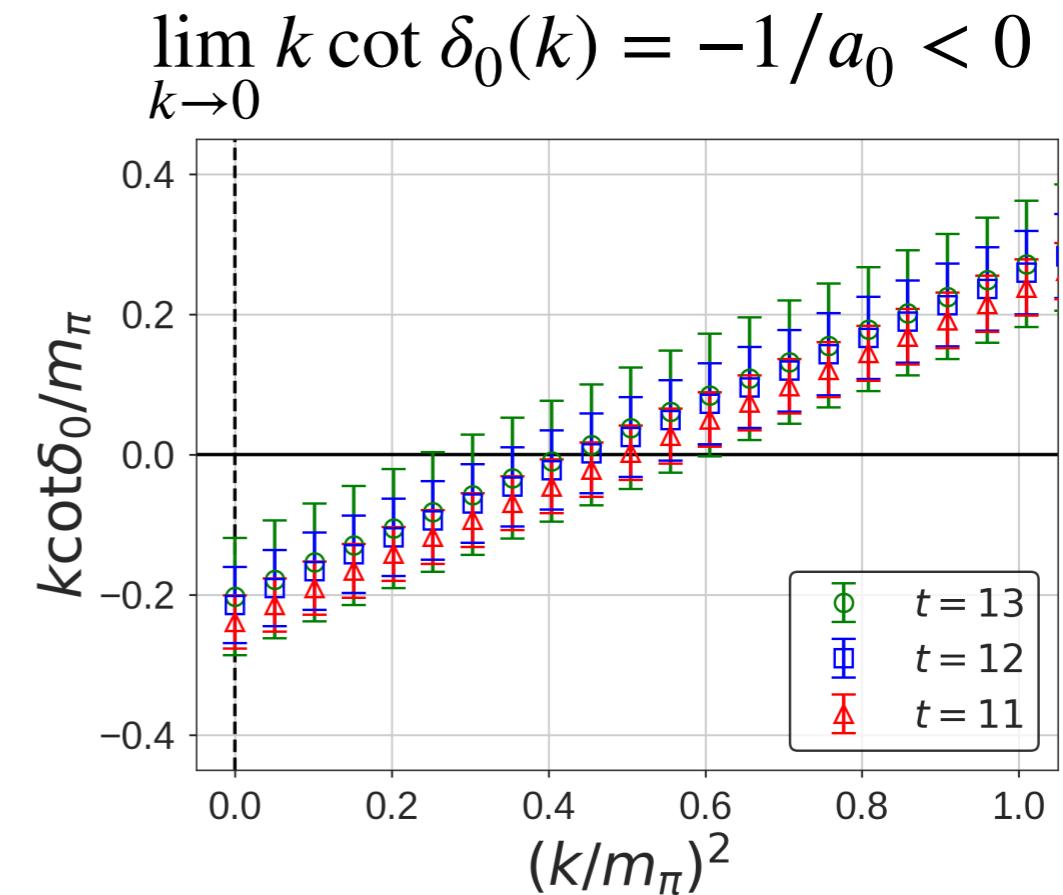
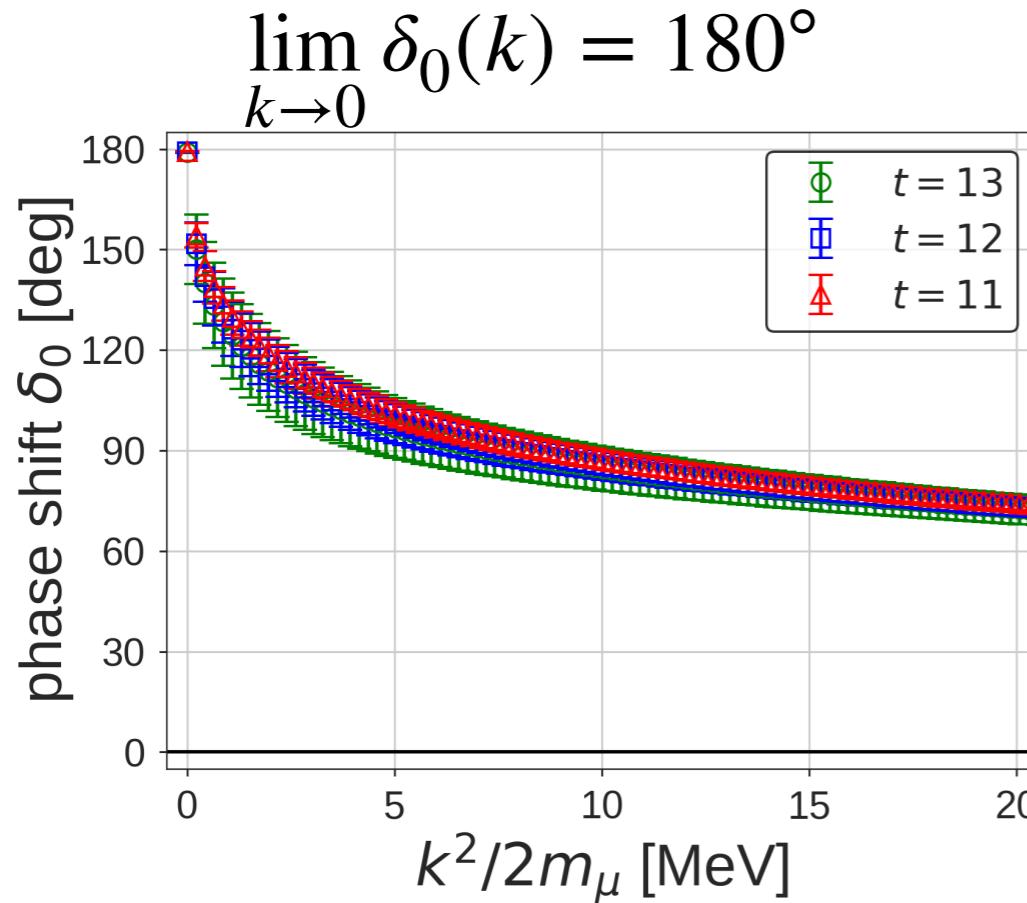


Etminan for HAL QCD Coll. '14

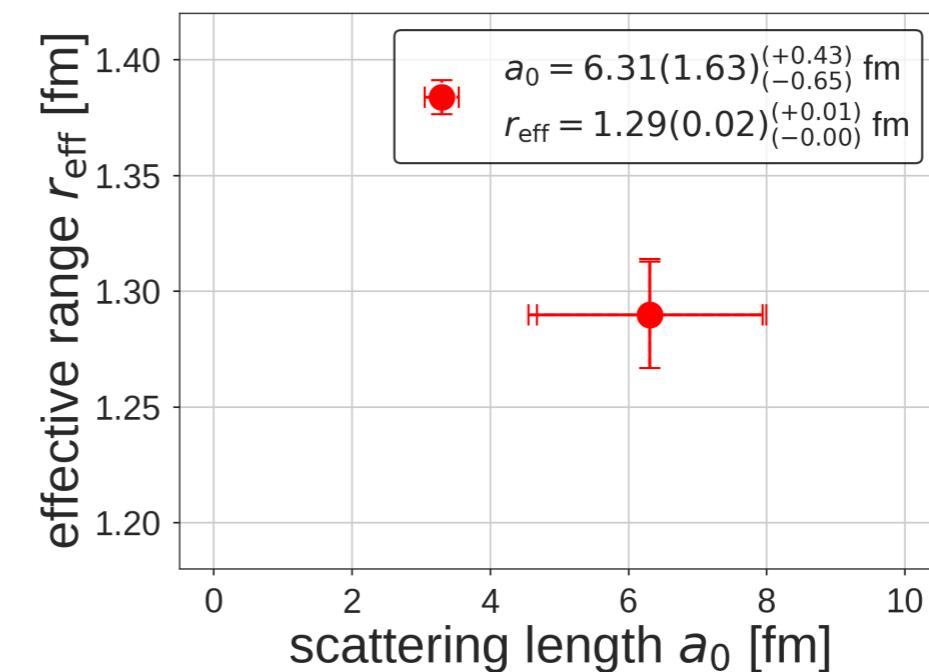


B.E. =  $18.9(5.0)(+12.1)(-1.8)$  MeV 18

# Scattering Phase Shift of $N\Omega(^5S_2)$



Effective Range Expansion  $k \cot \delta_0 = -\frac{1}{a_0} + \frac{1}{2} r_{\text{eff}} k^2$



new "dibaryon" state

- n+p → deuteron
- $\Omega+\Omega \rightarrow$  di-Omega

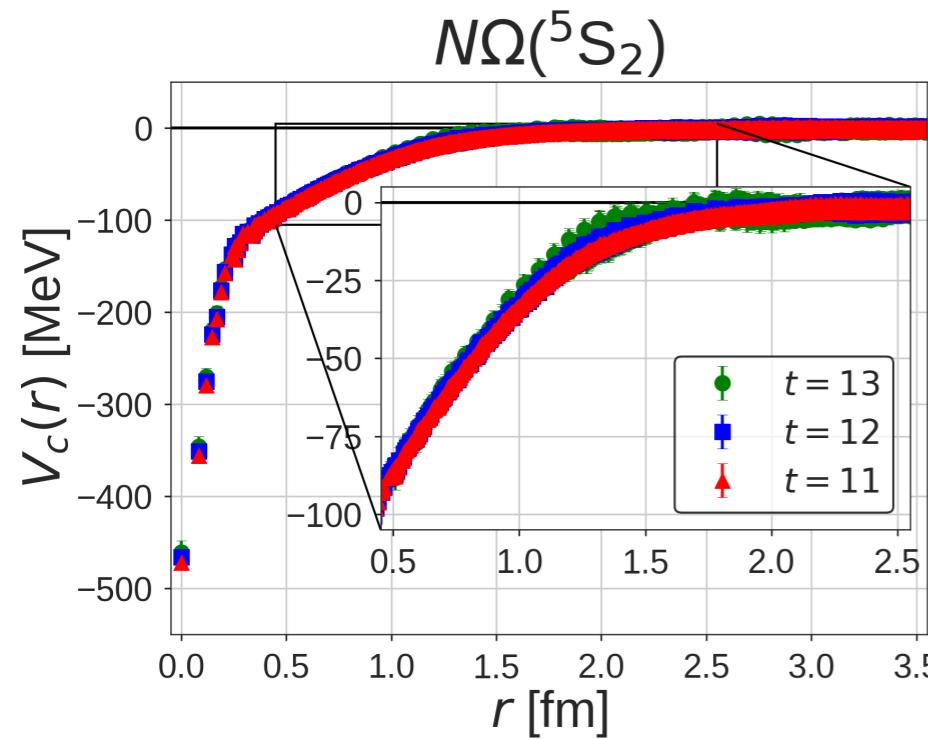
Gongyo for HAL QCD Coll. PRL120,212001(2018)

- N+ $\Omega \rightarrow$  ???

using Gauss + Form Factor \* Yukawa<sup>2</sup>-type form

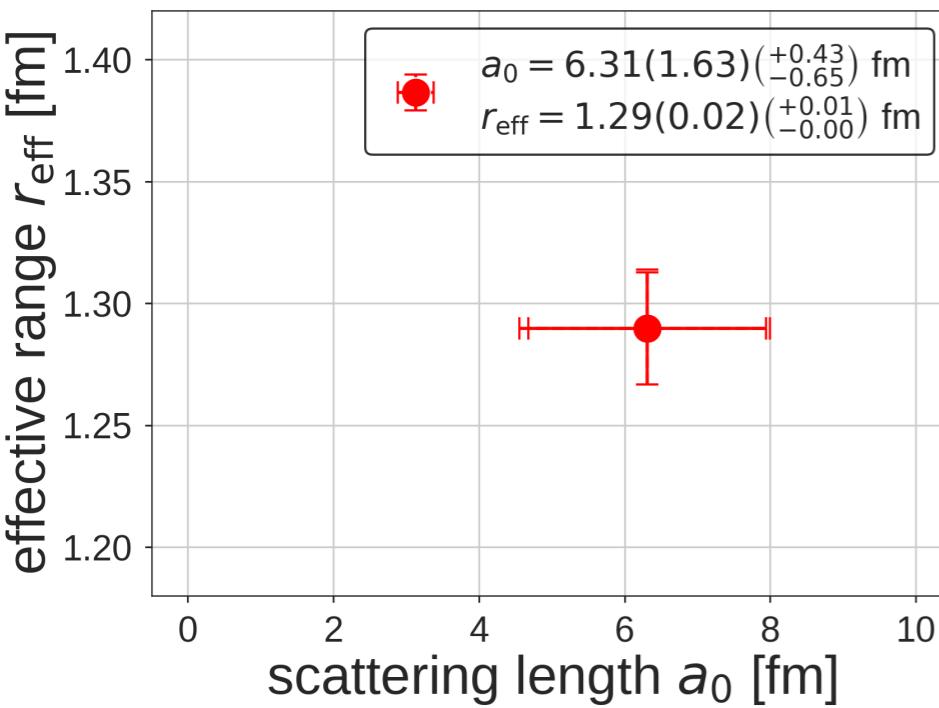
$$V(r) = a_0 e^{-a_1 r^2} + a_2 (1 - e^{-a_3 r^2}) (e^{-m_\pi r}/r)^2$$

# Binding Energy & Root Mean Square Radius

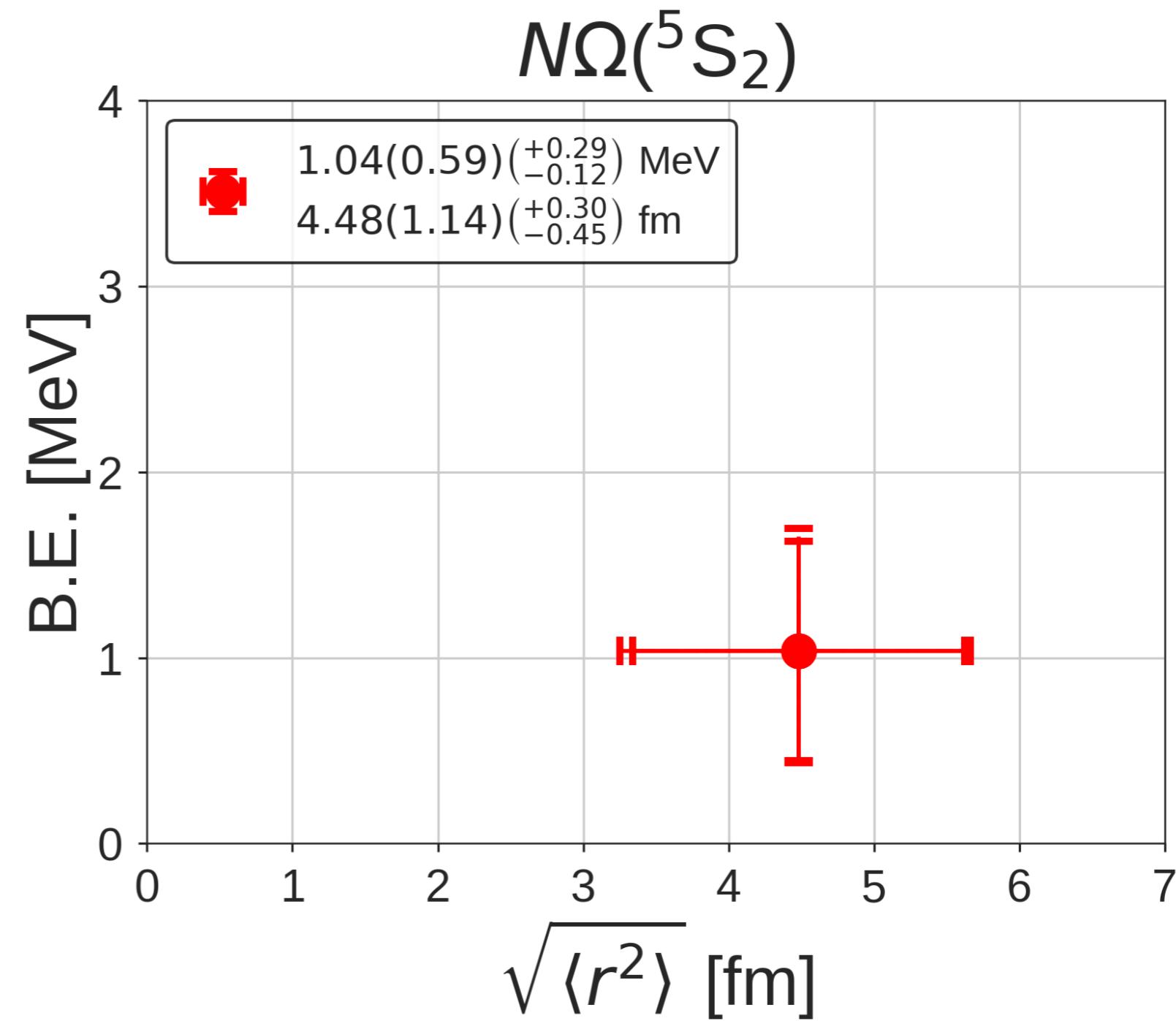


*a loosely bound system*

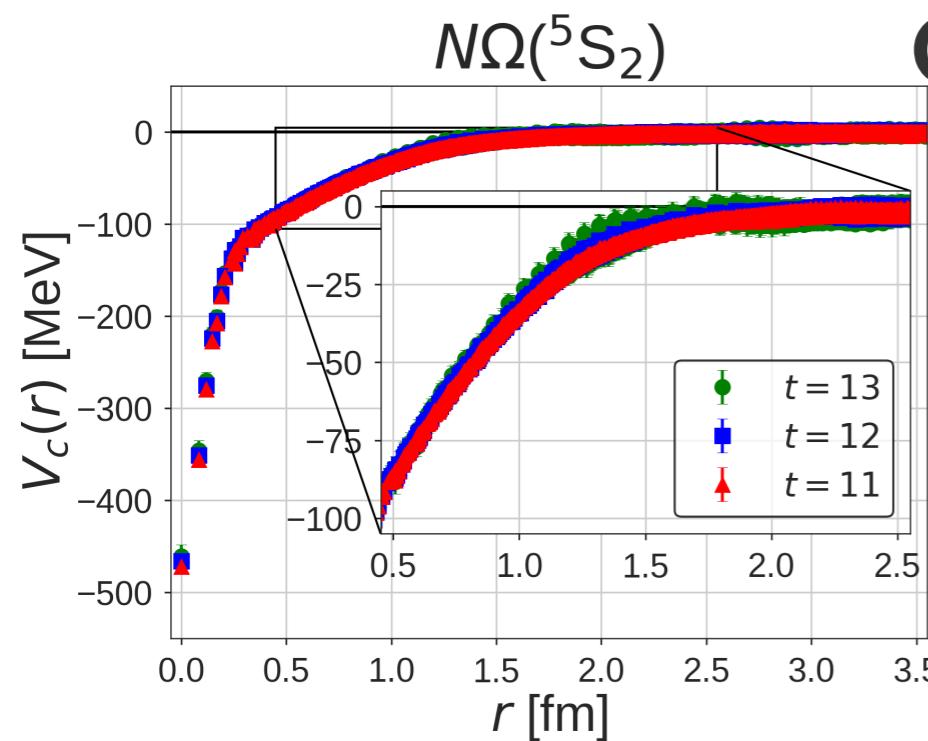
$$\sqrt{\langle r^2 \rangle} \sim a_0$$



formation of "dibaryon" state



# QCD + Coulomb Potential



Coulomb potential

$$+ \left( -\frac{\alpha}{r} \right)$$



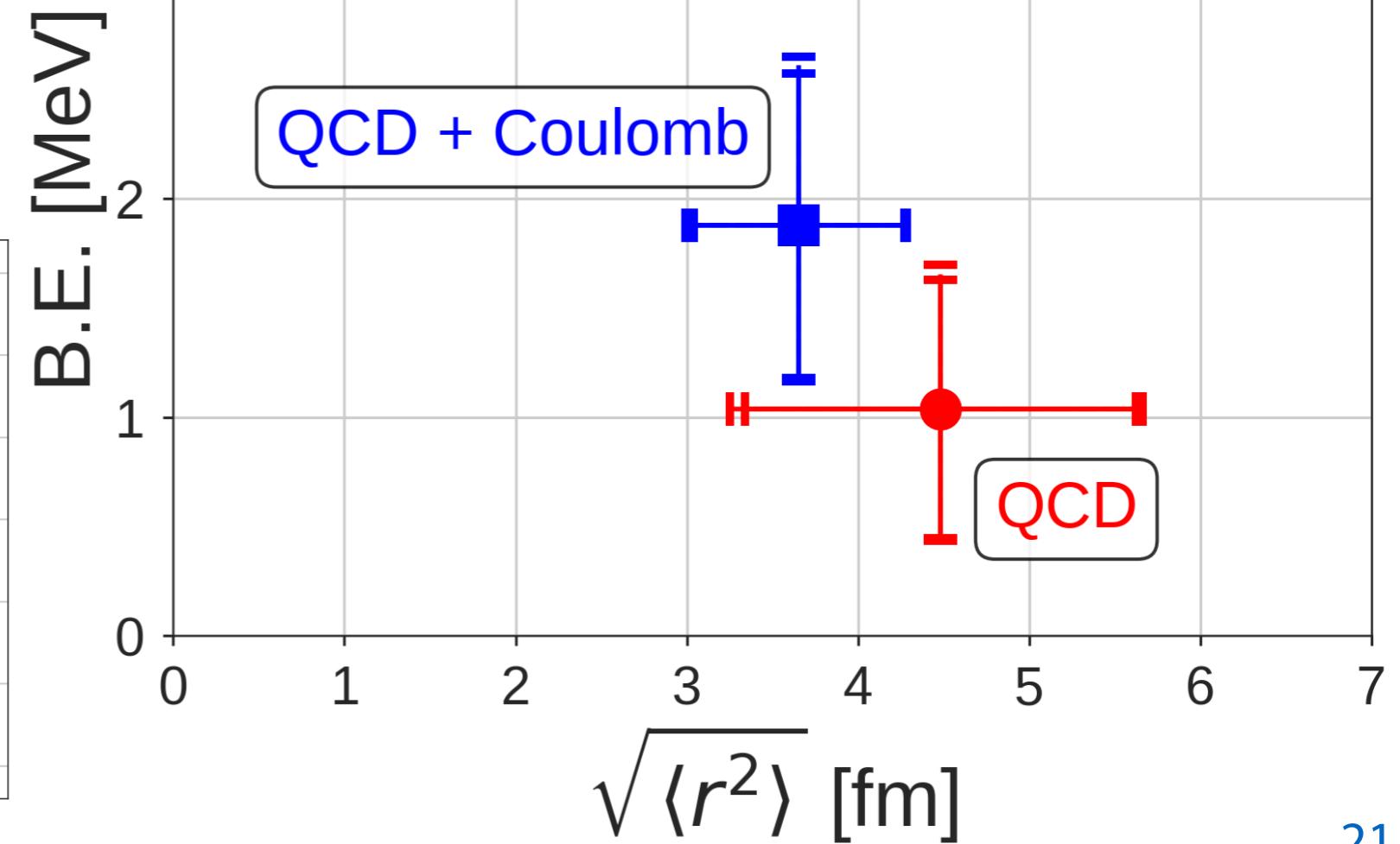
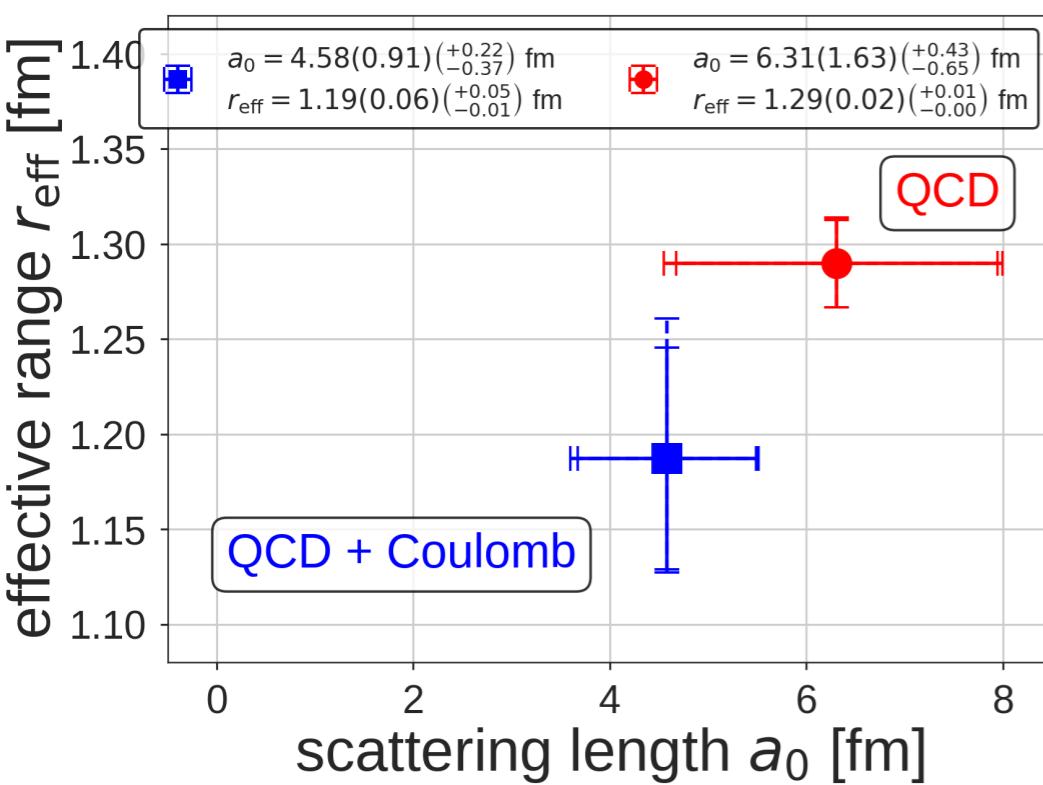
enhancement of B.E.

$p\Omega$ : "attractive"

$N\Omega(^5S_2)$



ERE param.  $\sqrt{\langle r^2 \rangle} \sim a_0$



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# Summary

- Two-baryon systems in lattice QCD is one of the hot topics
  - “**Direct calculation**” suffers from **fake signal due to scattering states** & previous studies without using variational method are unreliable.
    - check >> **supplemental material** of this slide
    - **HAL QCD method** is free from this problem, which extracts the interaction from both g.s. and scattering states.
- **NΩ(<sup>5</sup>S<sub>2</sub>) system** at almost physical point ( $m_{\text{pi}} \sim 146 \text{ MeV}$ )
  - strong attractive potential without a repulsive core
  - a prediction based on lattice QCD
    - a loosely bound state with **B.E. ~ 1 MeV**
    - scattering length & effective range are also estimated  
⇒ search for “*Dibaryon*” in heavy ion collision

# Backups

# Lüscher's Finite Volume Method & Bound State

1. Lüscher's formula

$$k \cot \delta_0(k) = \frac{1}{\pi L} \sum_{\mathbf{n} \in \mathbb{Z}^3} \frac{1}{|\mathbf{n}|^2 - (kL/2\pi)^2}$$

2. Interpolation by **Effective Range Expansion (ERE)**

$$k \cot \delta_0(k) \simeq -\frac{1}{a_0} + \frac{1}{2} r_{\text{eff}} k^2 + \dots$$

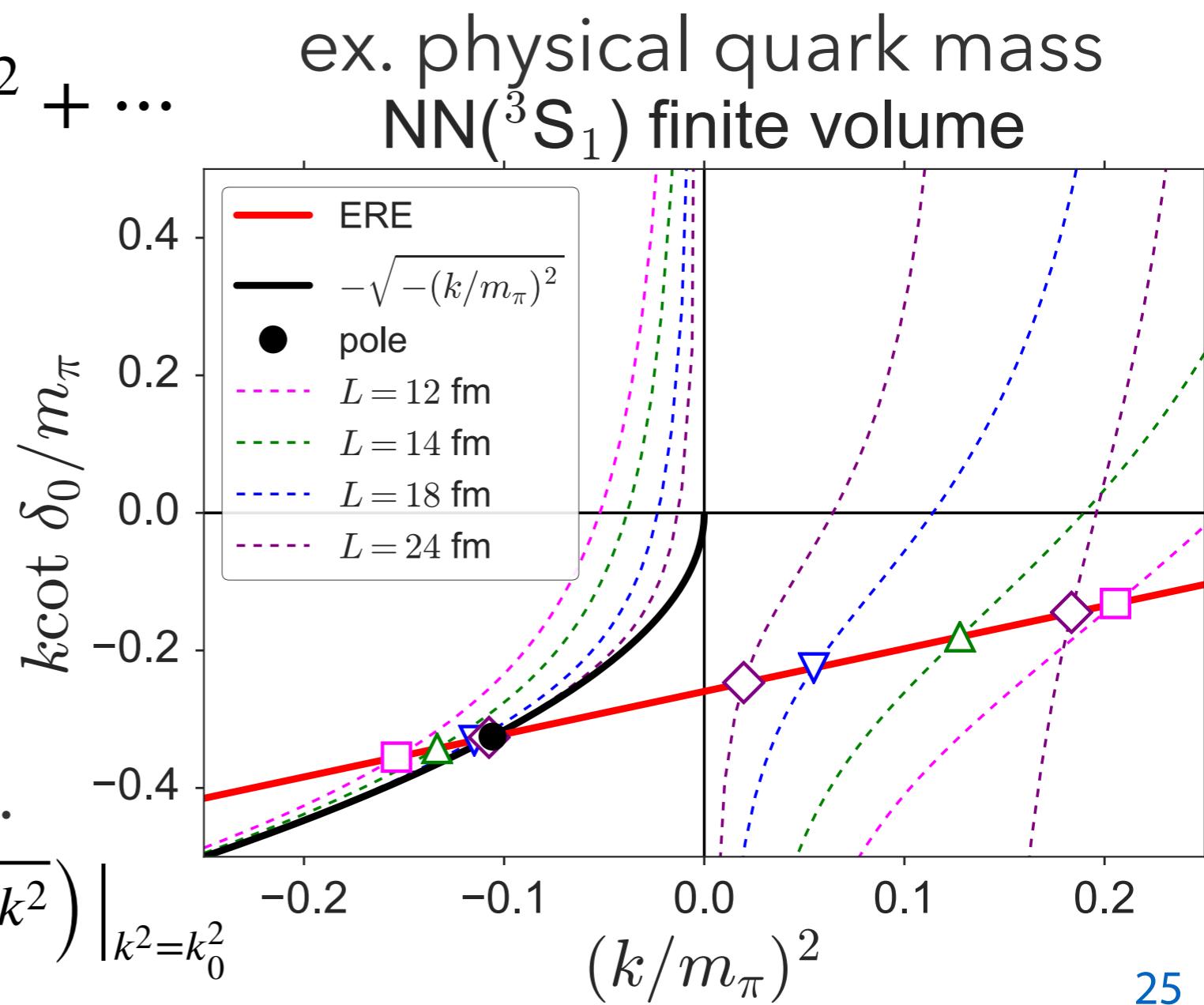
3. Search **bound state**

@ S-matrix's pole

$$k_0 \cot \delta_0(k_0) = -\sqrt{-k_0^2}$$

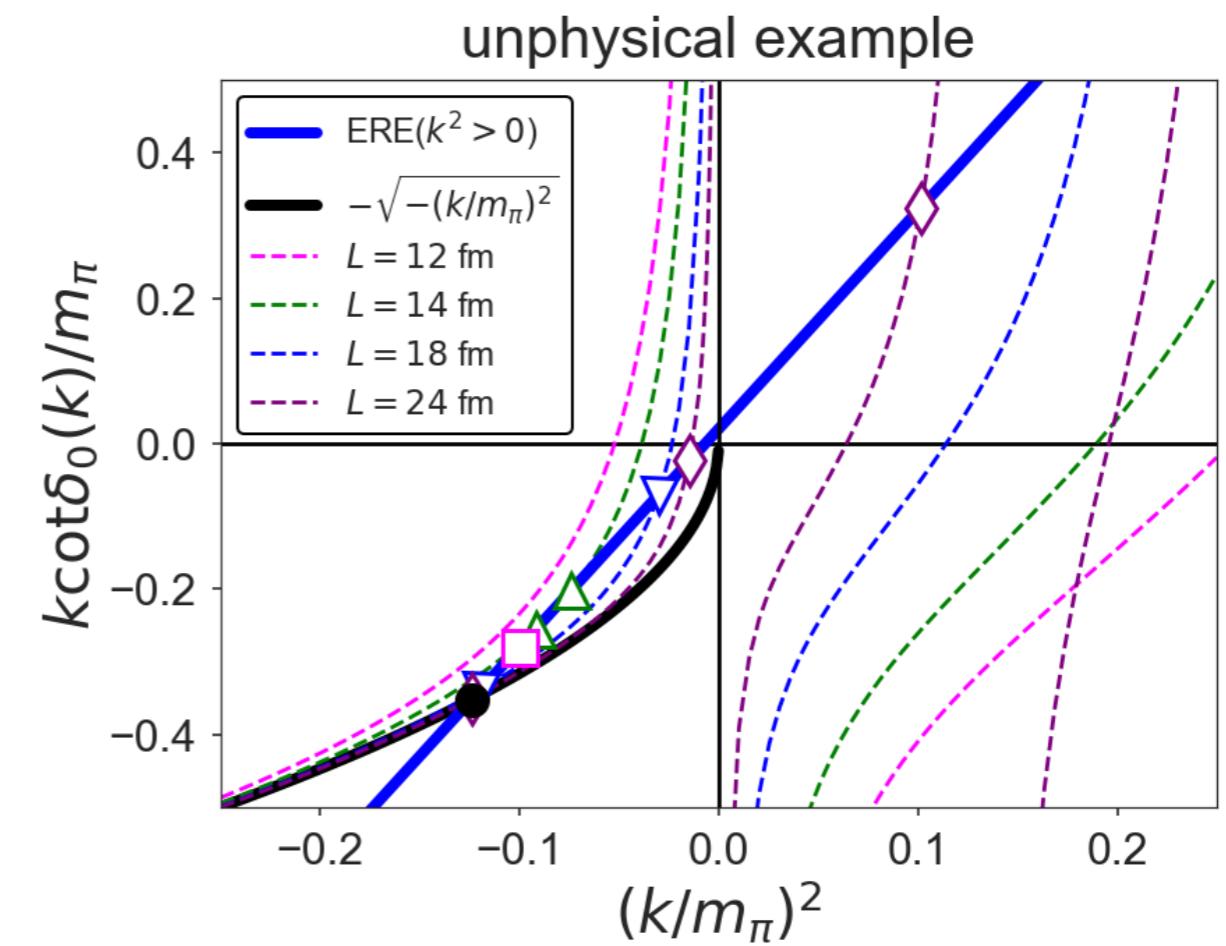
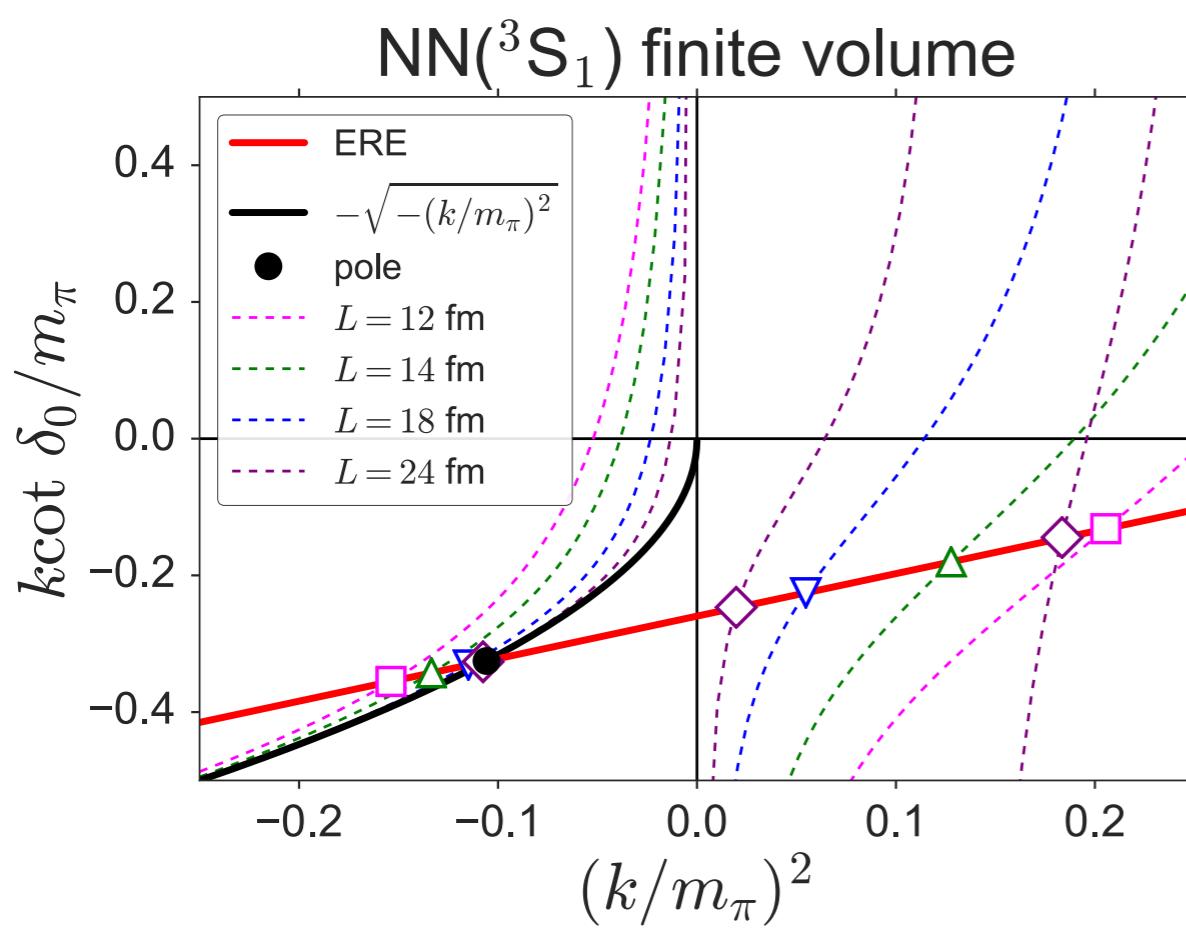
with physical pole cond.

$$\frac{d}{dk^2}(k \cot \delta_0(k)) \Big|_{k^2=k_0^2} < \frac{d}{dk^2} \left( -\sqrt{-k^2} \right) \Big|_{k^2=k_0^2}$$



# Physical Condition of the S-matrix Pole

$$\frac{d}{dk^2} \left[ k \cot \delta_0(k) - \left( -\sqrt{-k^2} \right) \right] \Big|_{k^2 = -\kappa_b^2} = -\frac{1}{\beta_b^2} < 0$$

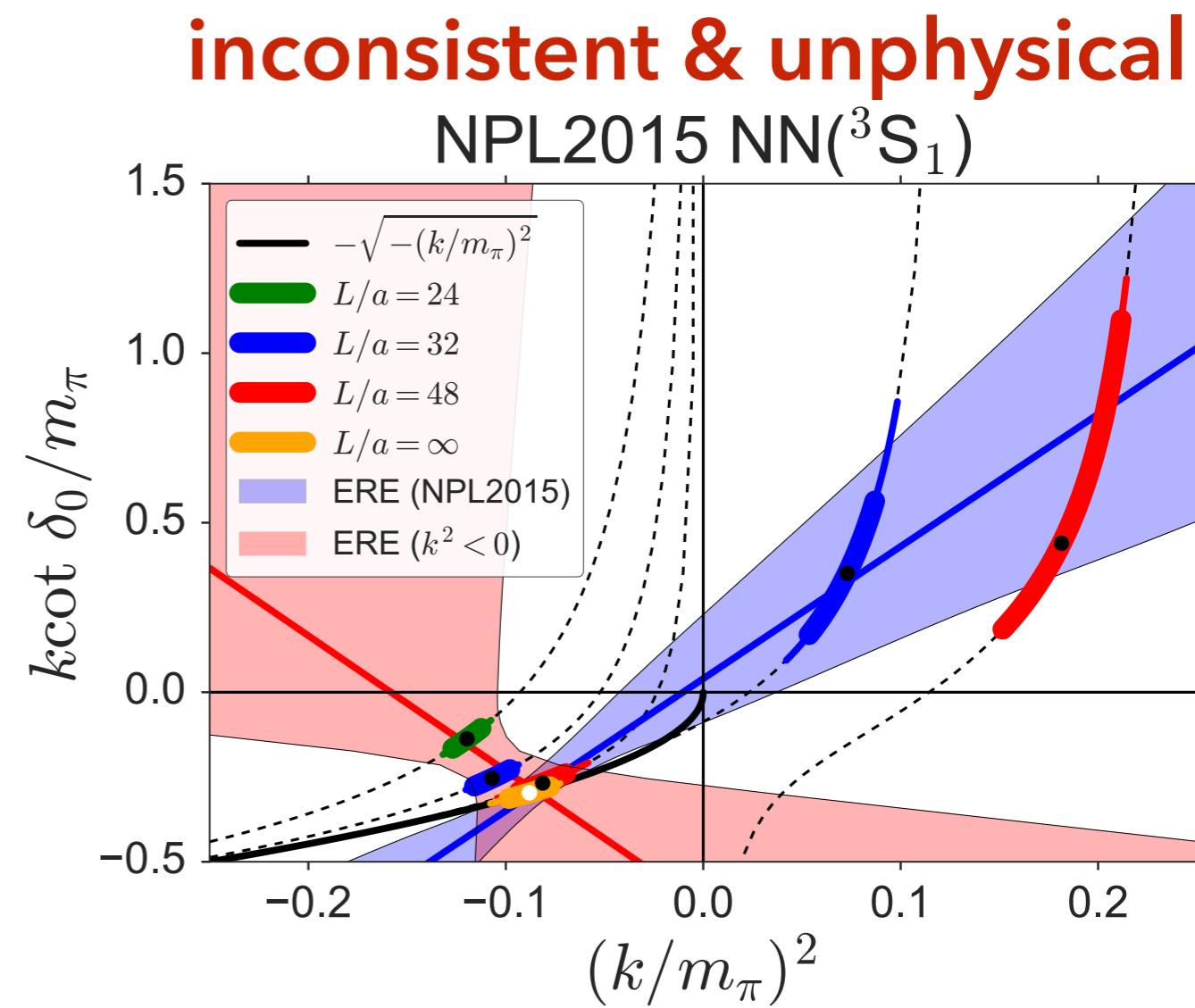
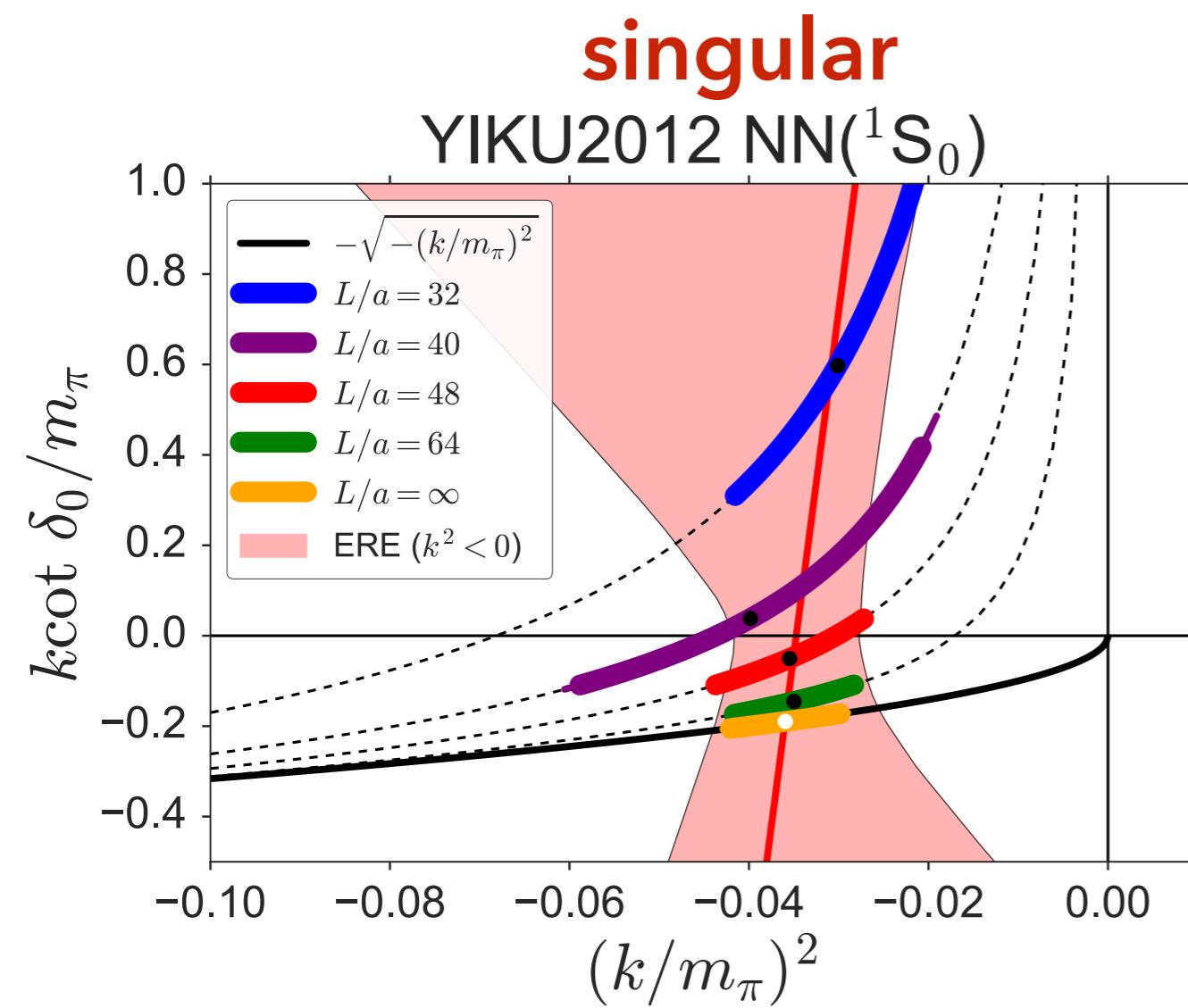


# Consistency Check of Phase Shift by the Direct Method

- All previous studies show anomalous ERE.

$$k \cot \delta_0(k) \simeq -\frac{1}{a_0} + \frac{1}{2} r_{\text{eff}} k^2 + \dots$$

- These imply the fake plateaux.



# Summary of Consistency/Normality Checks

Refs. **TI** for HAL QCD Coll., [PRD96.034521\(2017\)](#)

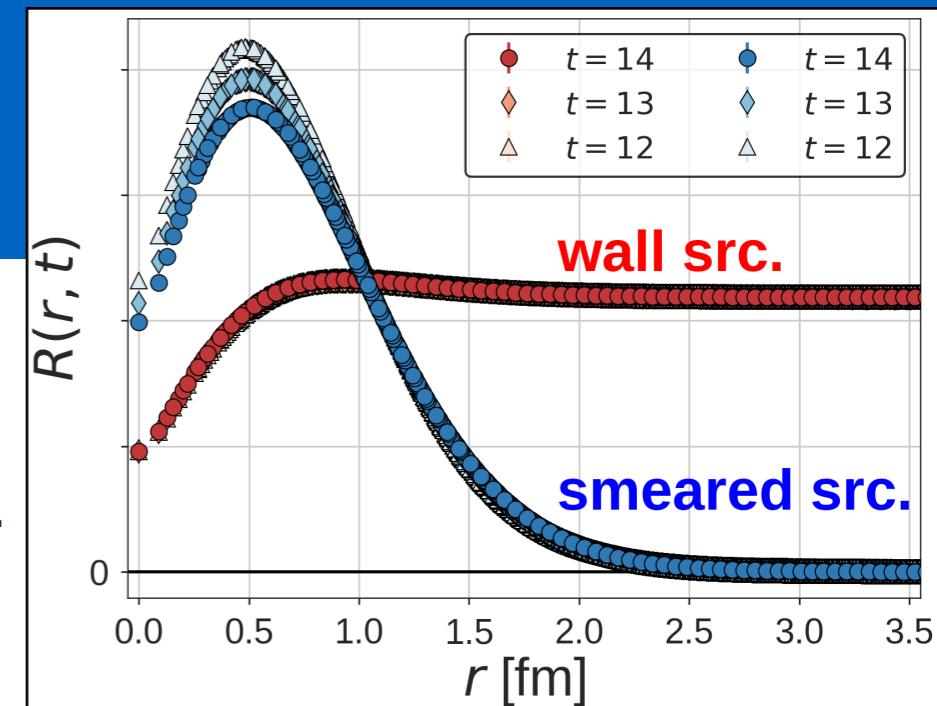
S. Aoki, T. Doi, **TI**, LATTICE2017 Proc., [arXiv:1707.08800](#).

	source independence	consistency of $k^2 < 0$ and $k^2 > 0$	non-singular ERE	physical residue
Yamazaki et al. 2011		No	No	
NPLQCD 2012			No	
Yamazaki et al. 2012	No		No	
NPLQCD 2013	No			No
NPLQCD 2015		No		No
Yamazaki et al. 2015			No	No
CaLat 2017	No			No
NPLQCD 2017				No

These tests are “**necessary condition**”,  
it cannot guarantee the correctness of the results.

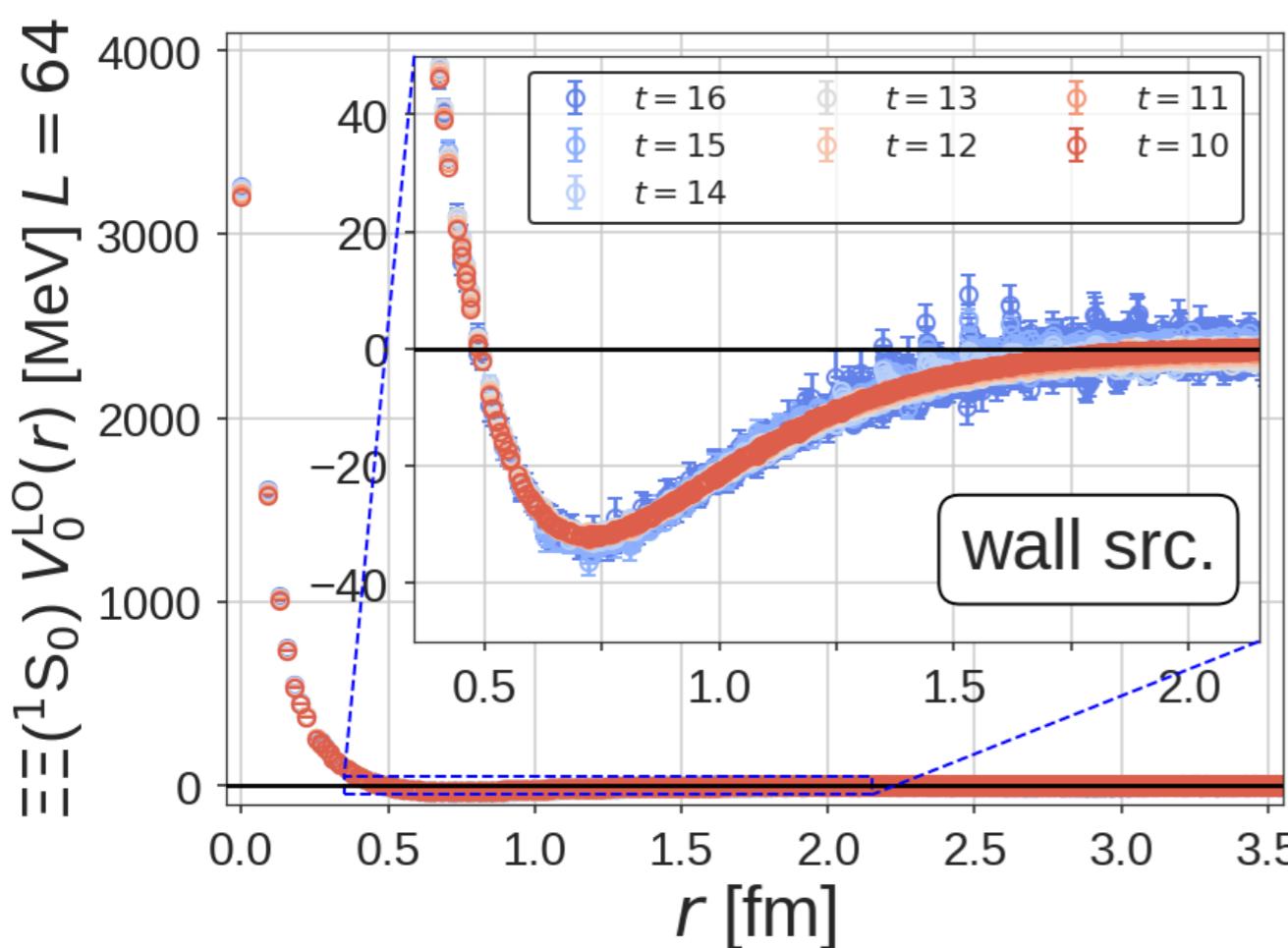
# Potential (Leading Order)

$$V_0^{\text{LO}}(r) = \frac{1}{4m_B} \frac{(\partial^2/\partial t^2)R}{R} - \frac{(\partial/\partial t)R}{R} - \frac{H_0 R}{R}$$

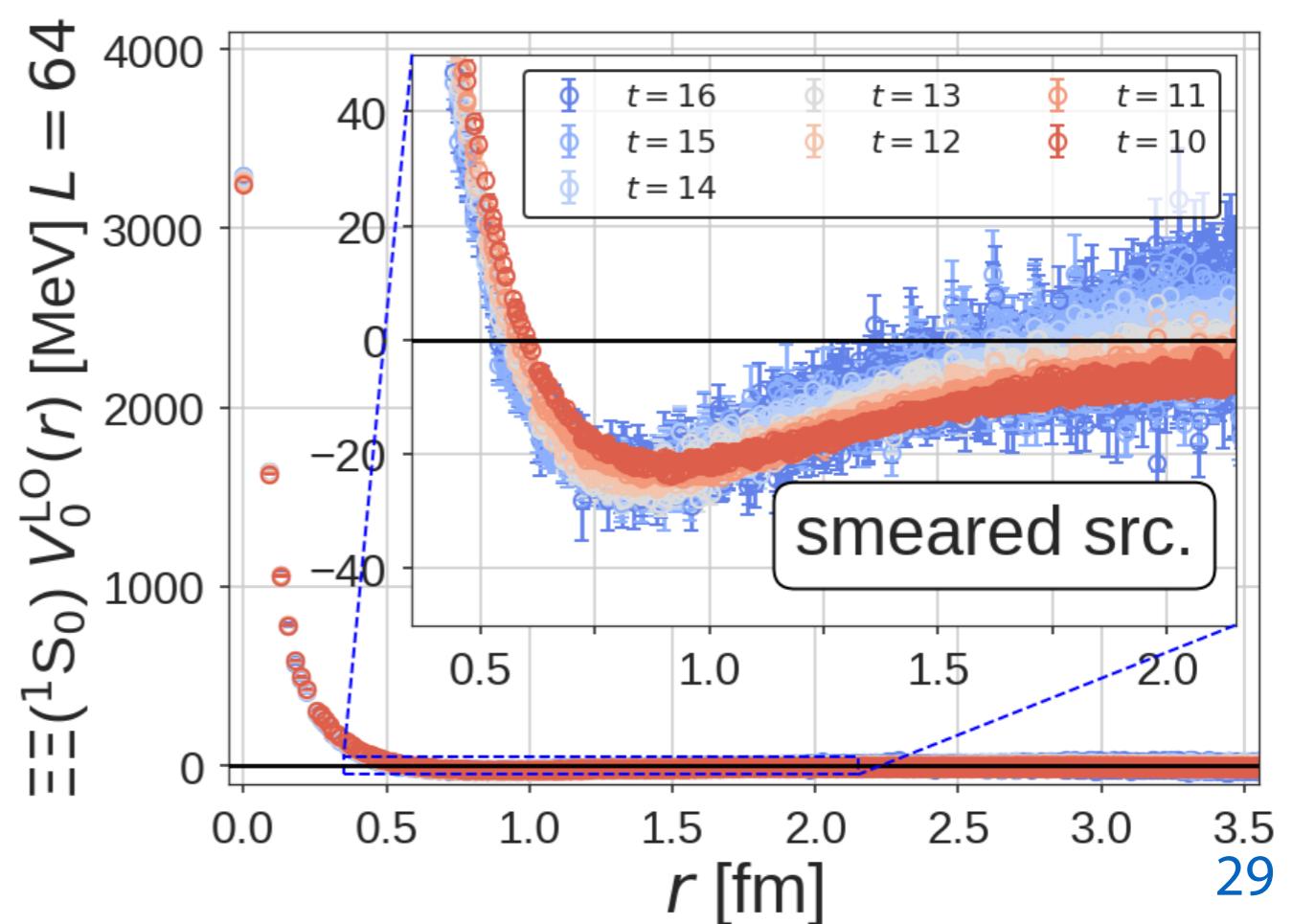


R-corr. differs, but *qualitatively* the same behavior

**wall src.** – t-independent

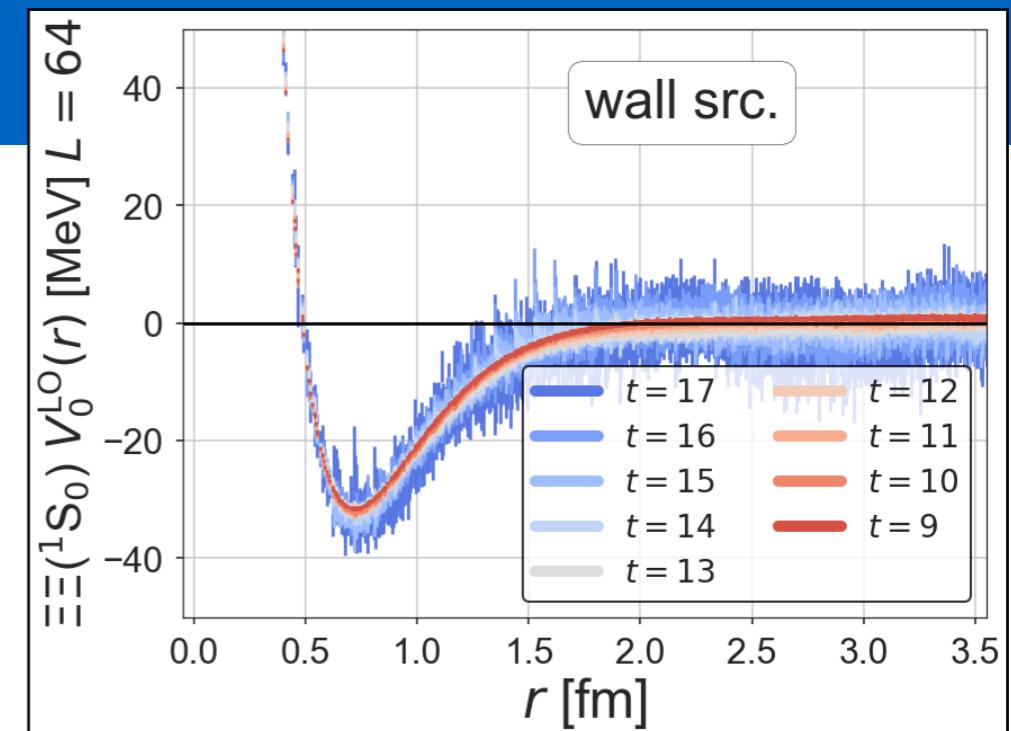


**smeared src.** – t-dependent

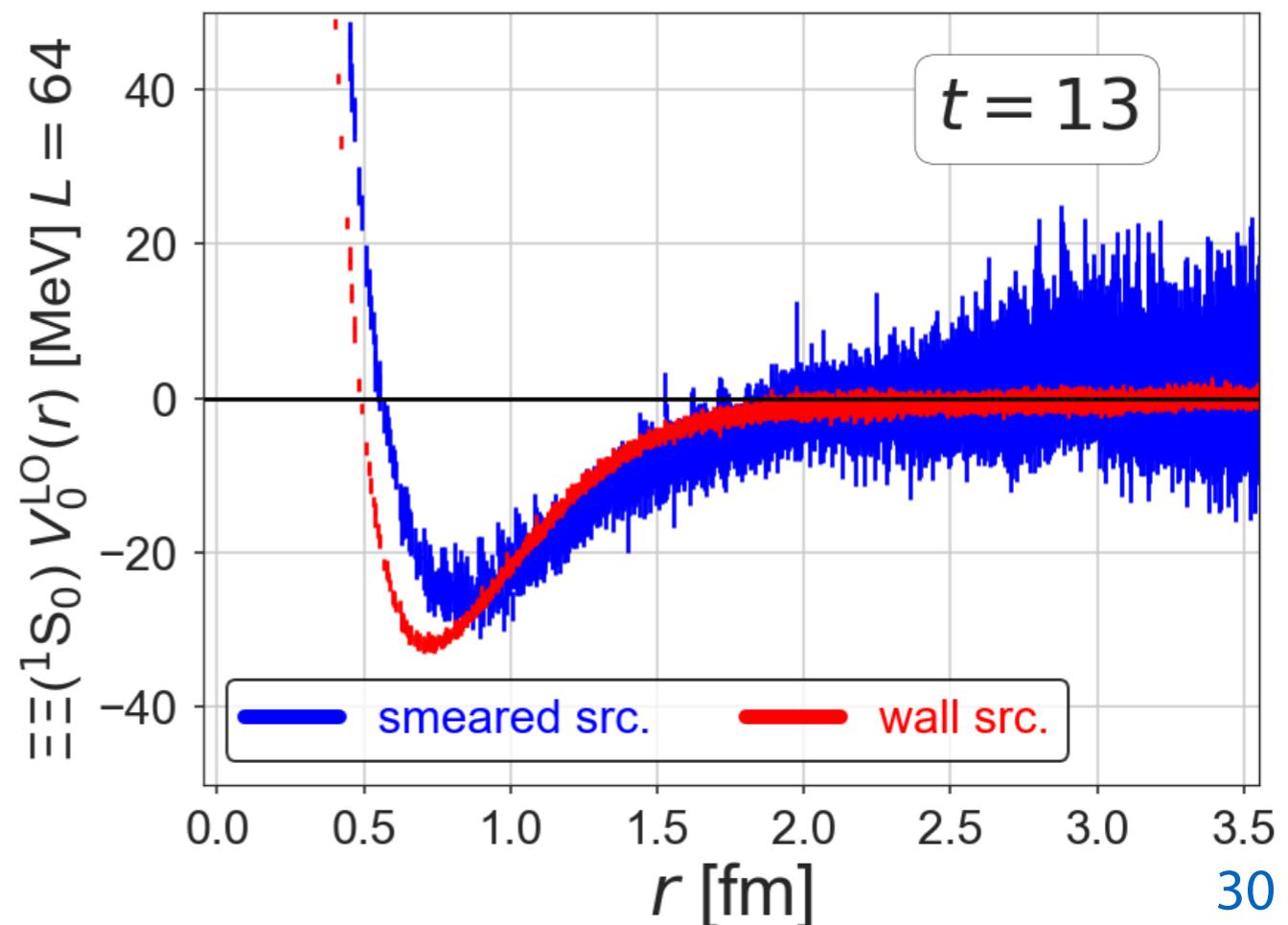
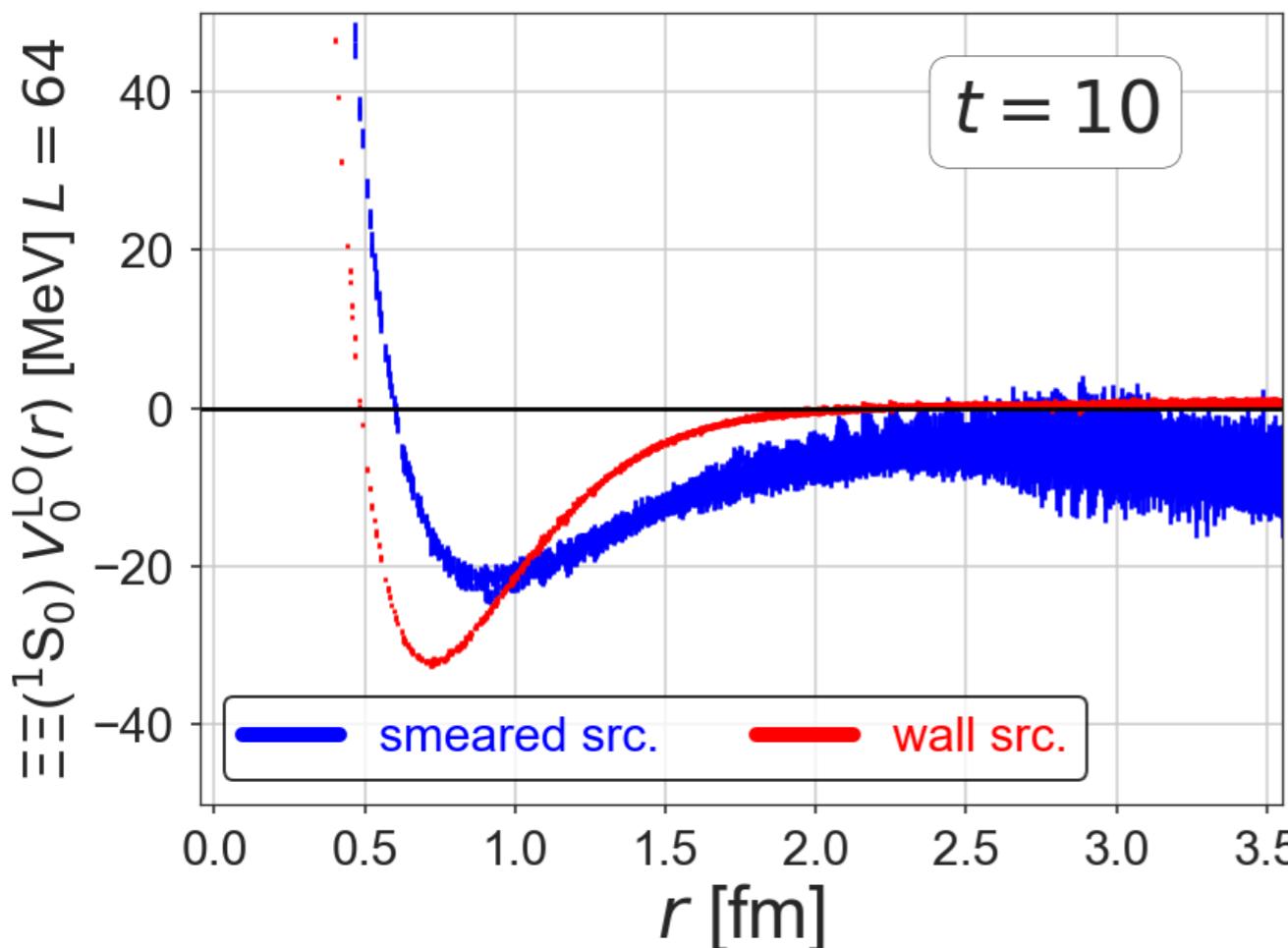


# Systematic in LO Potential

- **Wall** src. = time-indep.
- **Smeared** src.  $\rightarrow$  **Wall** src.



t-dep. implies systematics from truncation



# Higher Order Correction (N<sup>2</sup>LO)

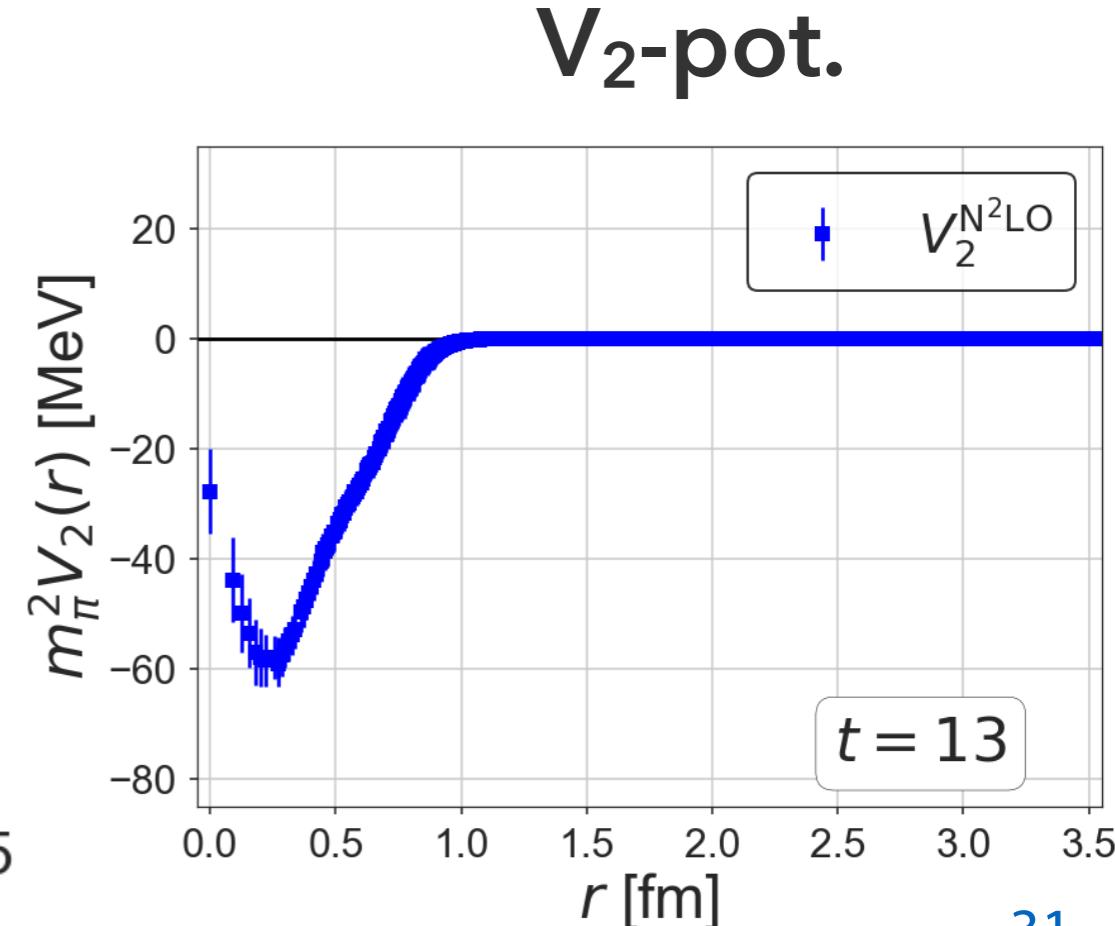
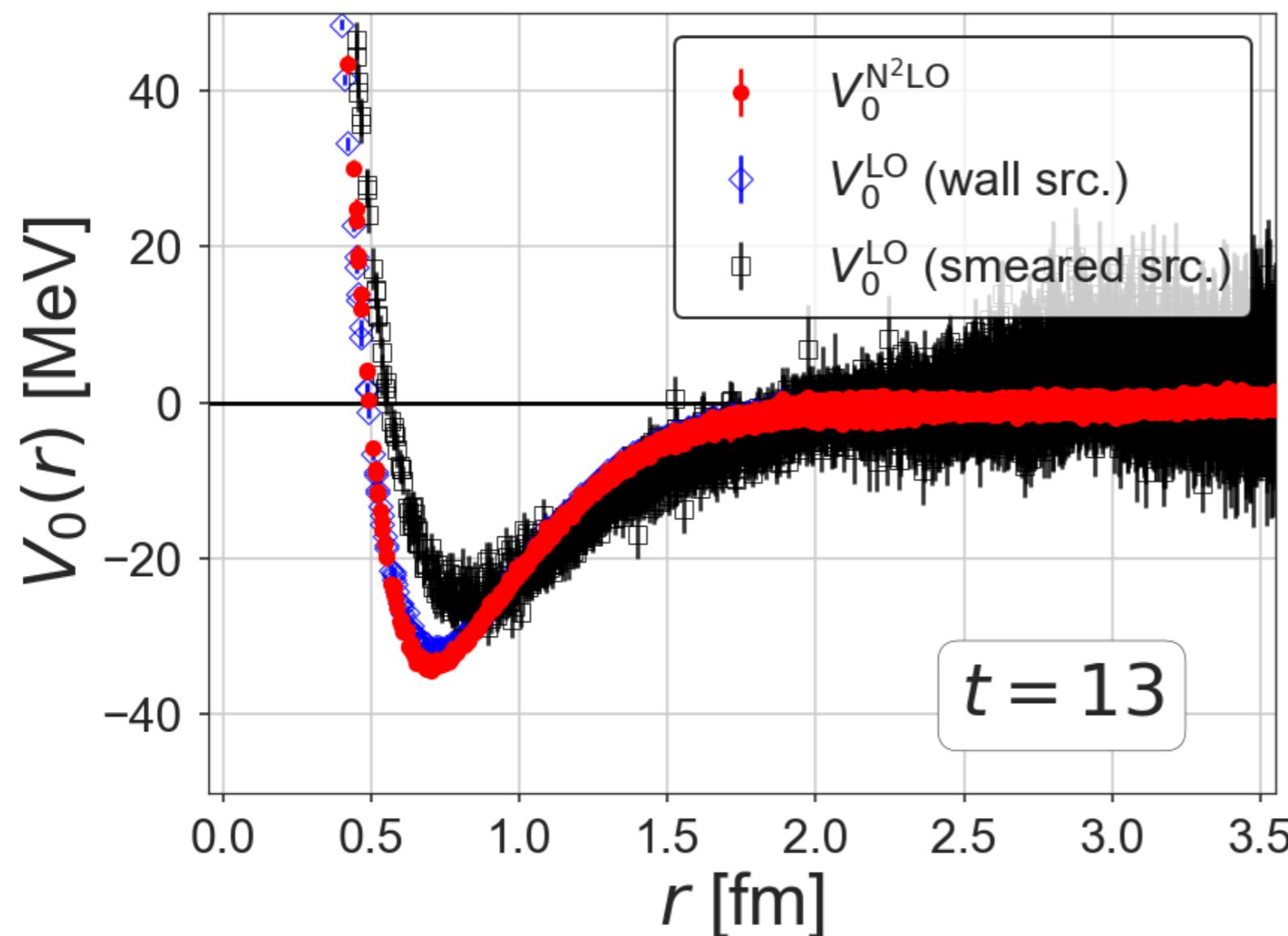
$$U(r, r') \simeq \left[ V_0^{\text{N}^2\text{LO}}(r) + V_2^{\text{N}^2\text{LO}}(r) \nabla^2 \right] \delta(r - r')$$

**wall src.**  $\rightarrow$  small  $V_2 \nabla^2$  correction

**smeared src.**  $\rightarrow$  large  $V_2 \nabla^2$  correction

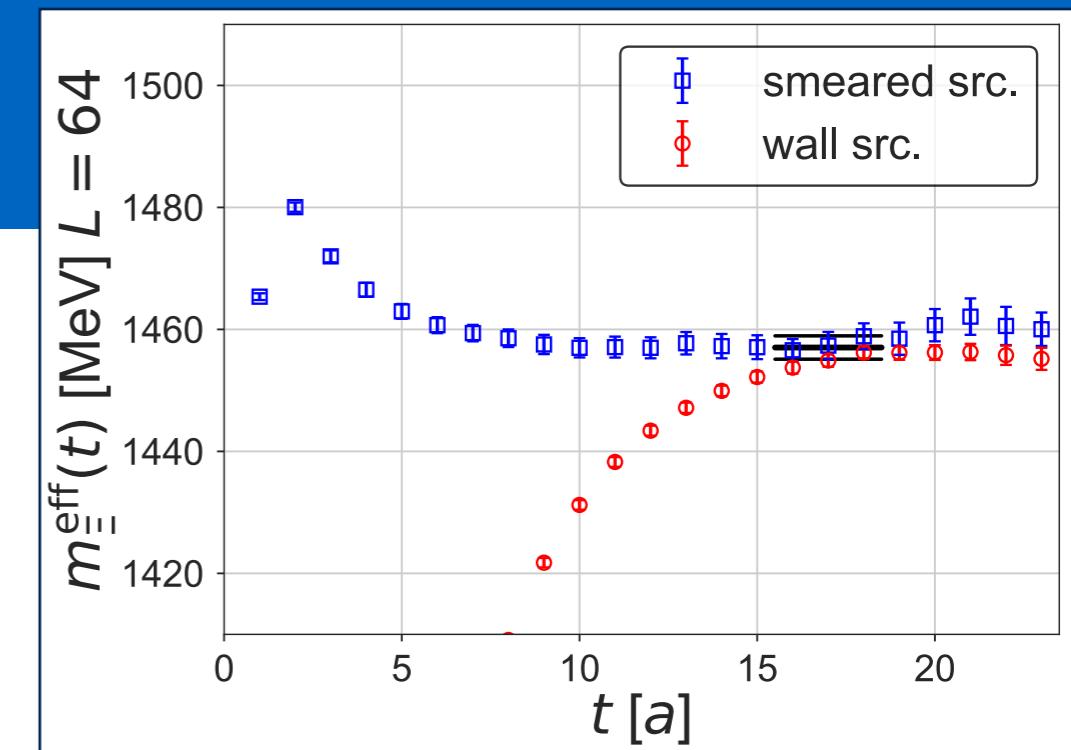
$$\xrightarrow{} V_2(r) \nabla^2 R^{\text{wall/smear}}(r)$$

dep. on shape of  $R$

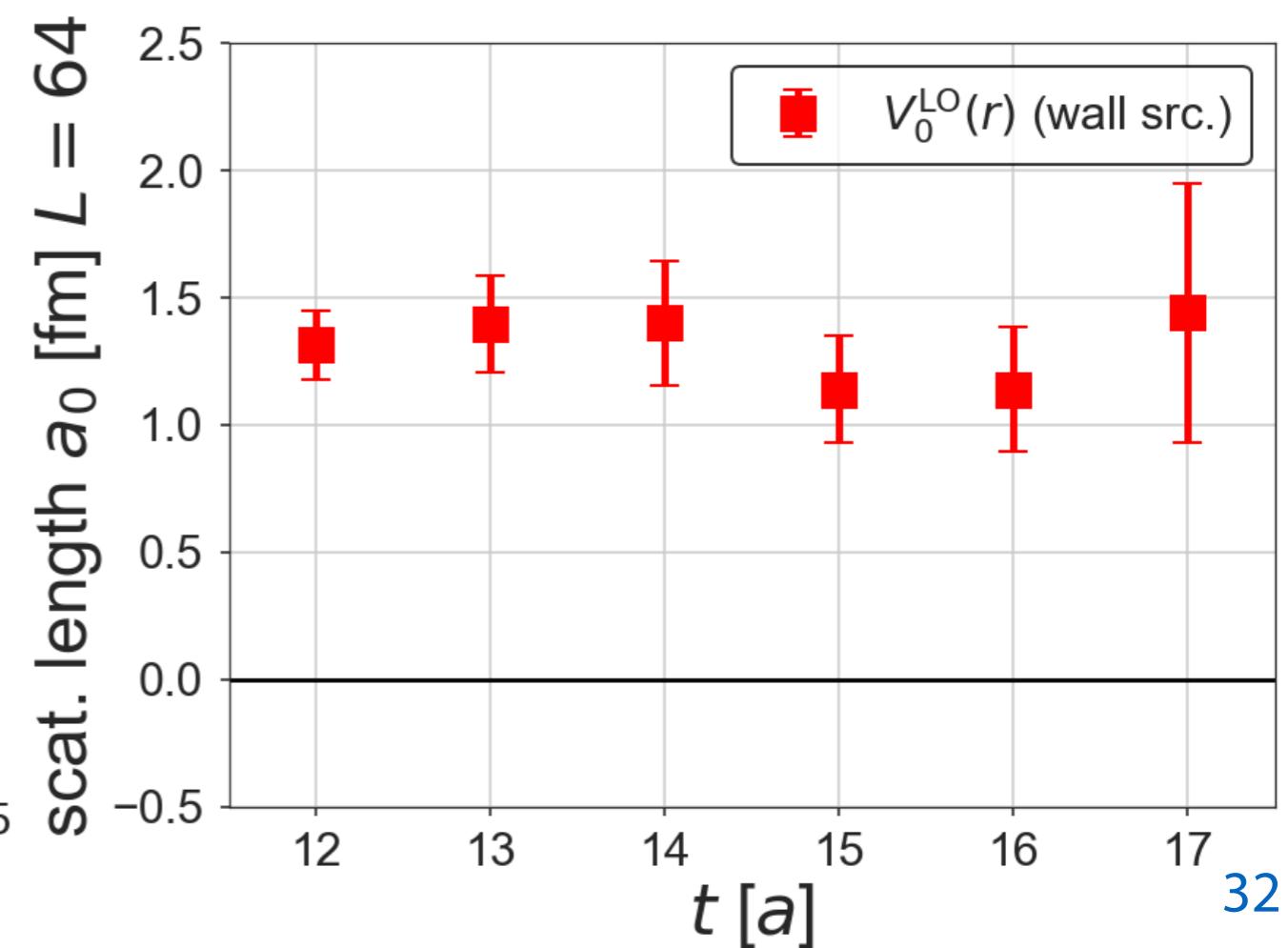
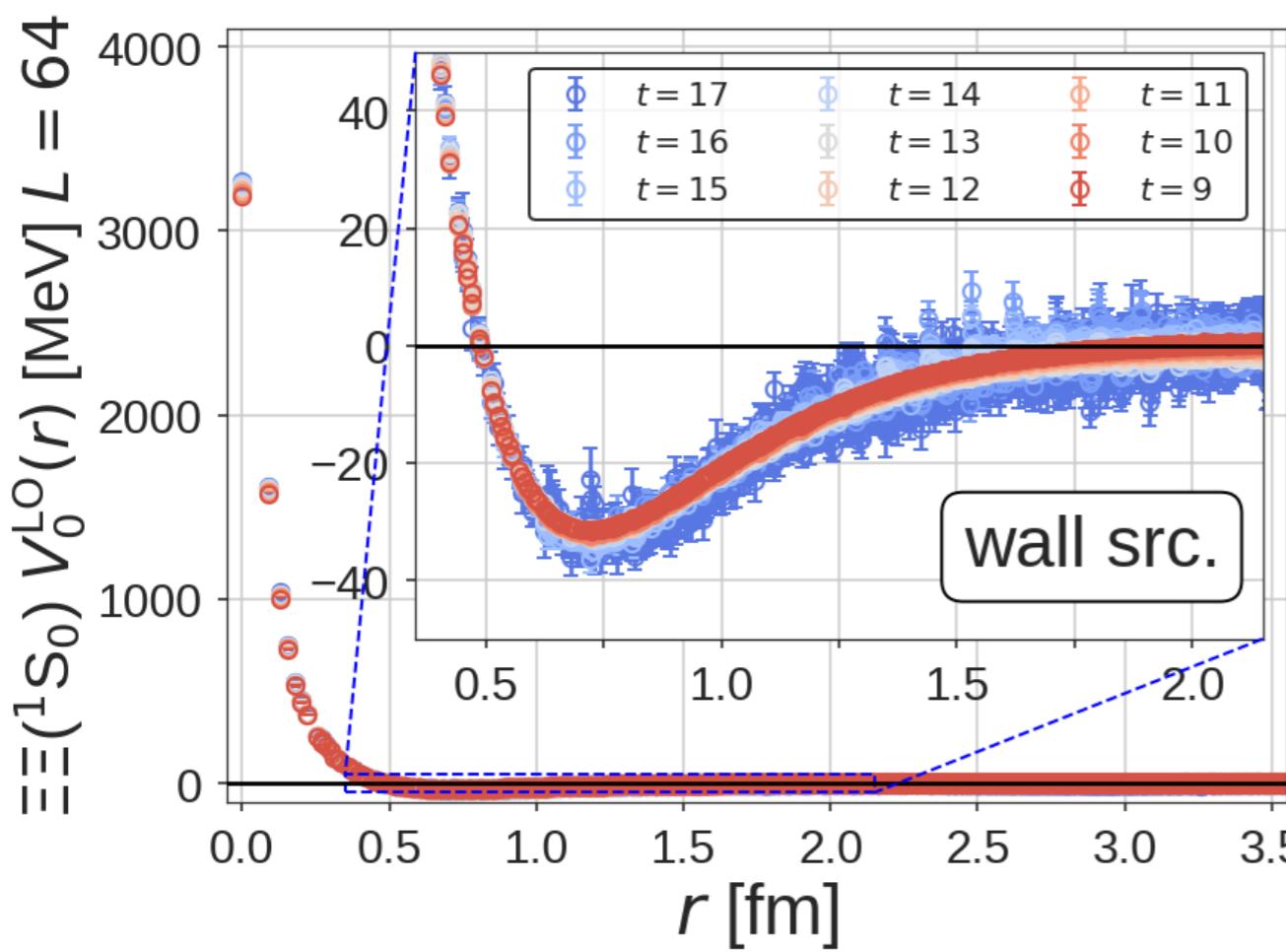


# t-dep. of the Wall src.

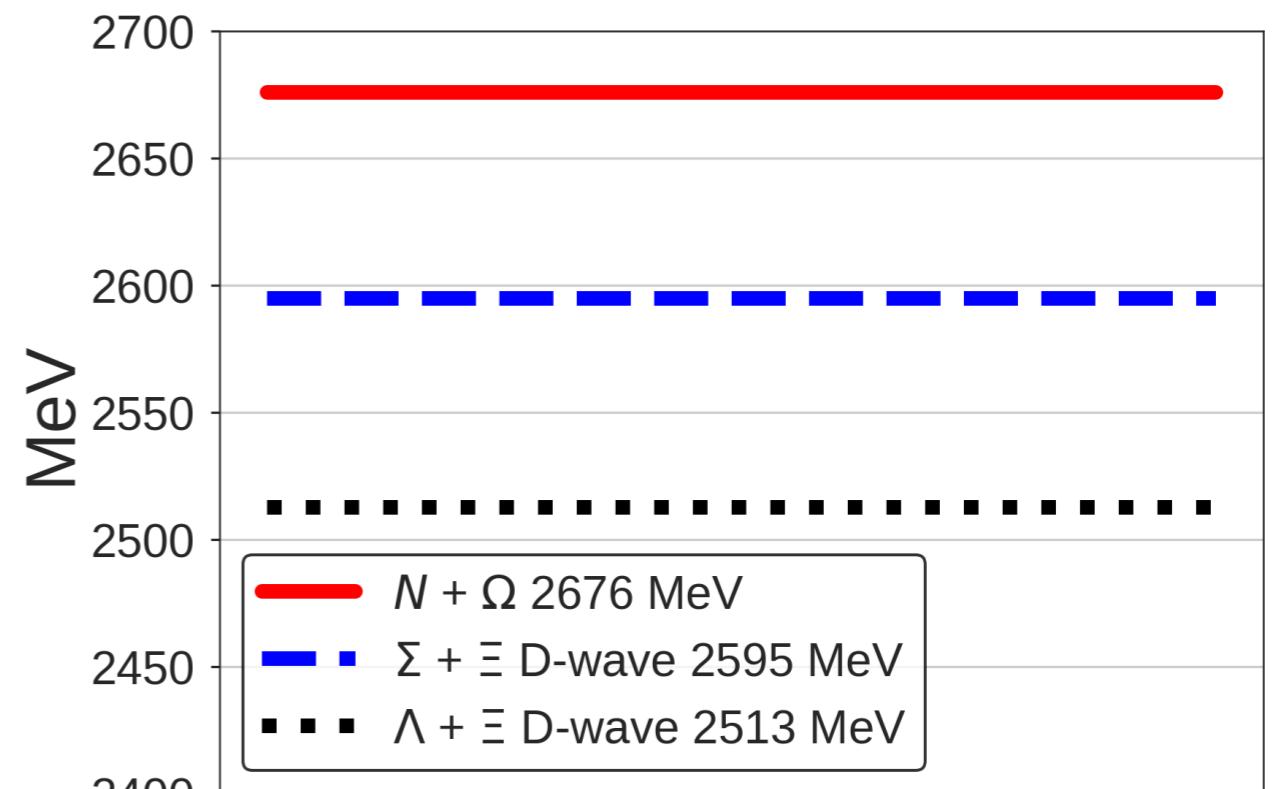
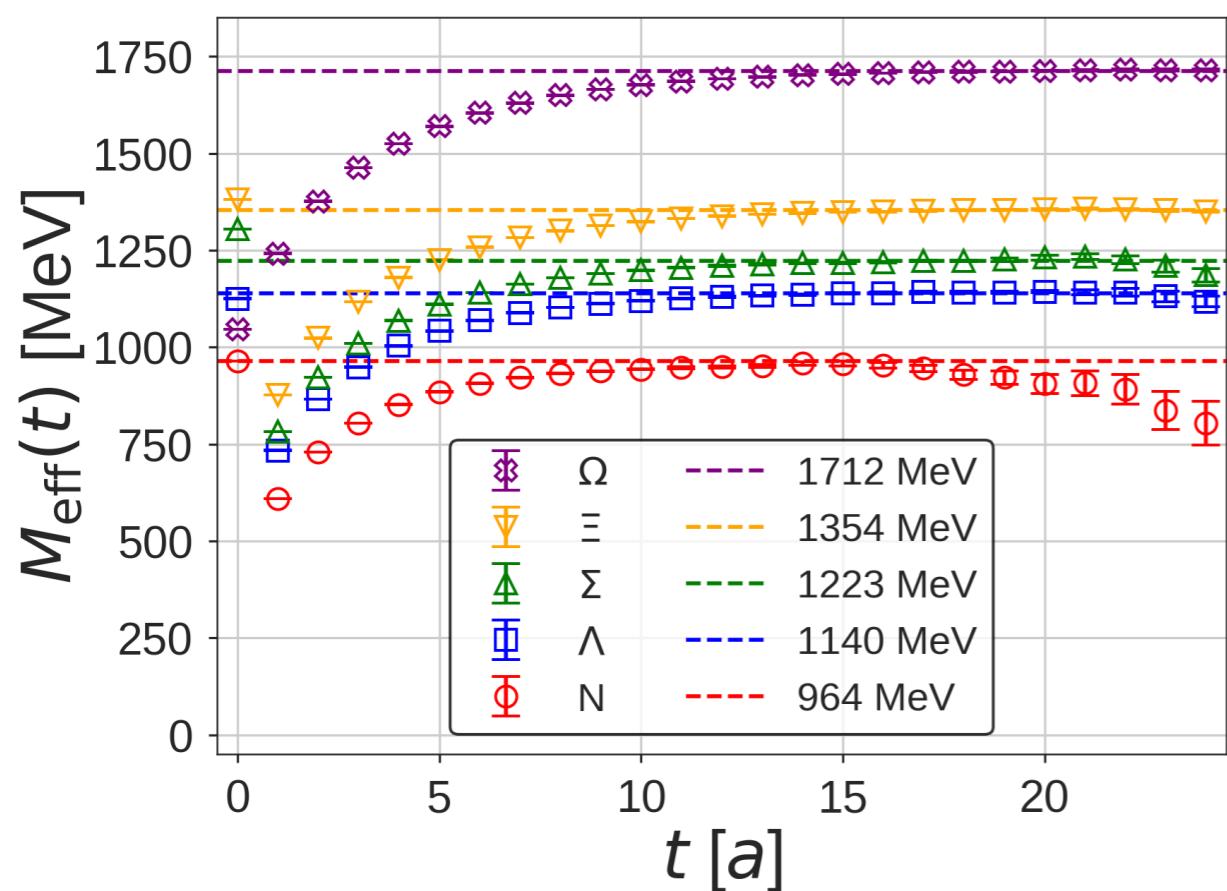
**single saturation is later  
than smeared src.**



**potential & observable are stable even at early time**



# Effective Masses at almost Physical Point



# Supplemental Material

*two nucleons in a finite volume*

# NN Systems from Lattice QCD

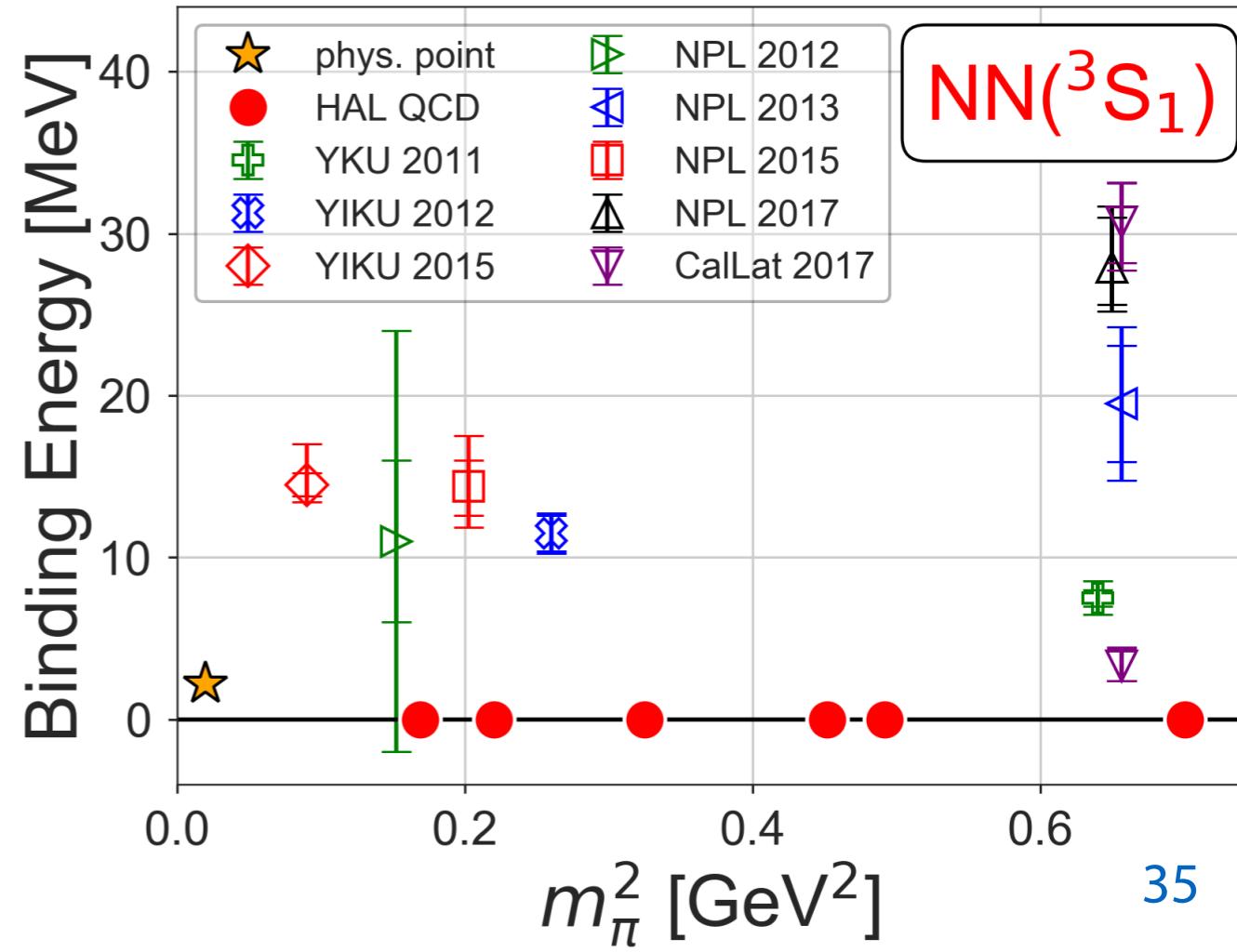
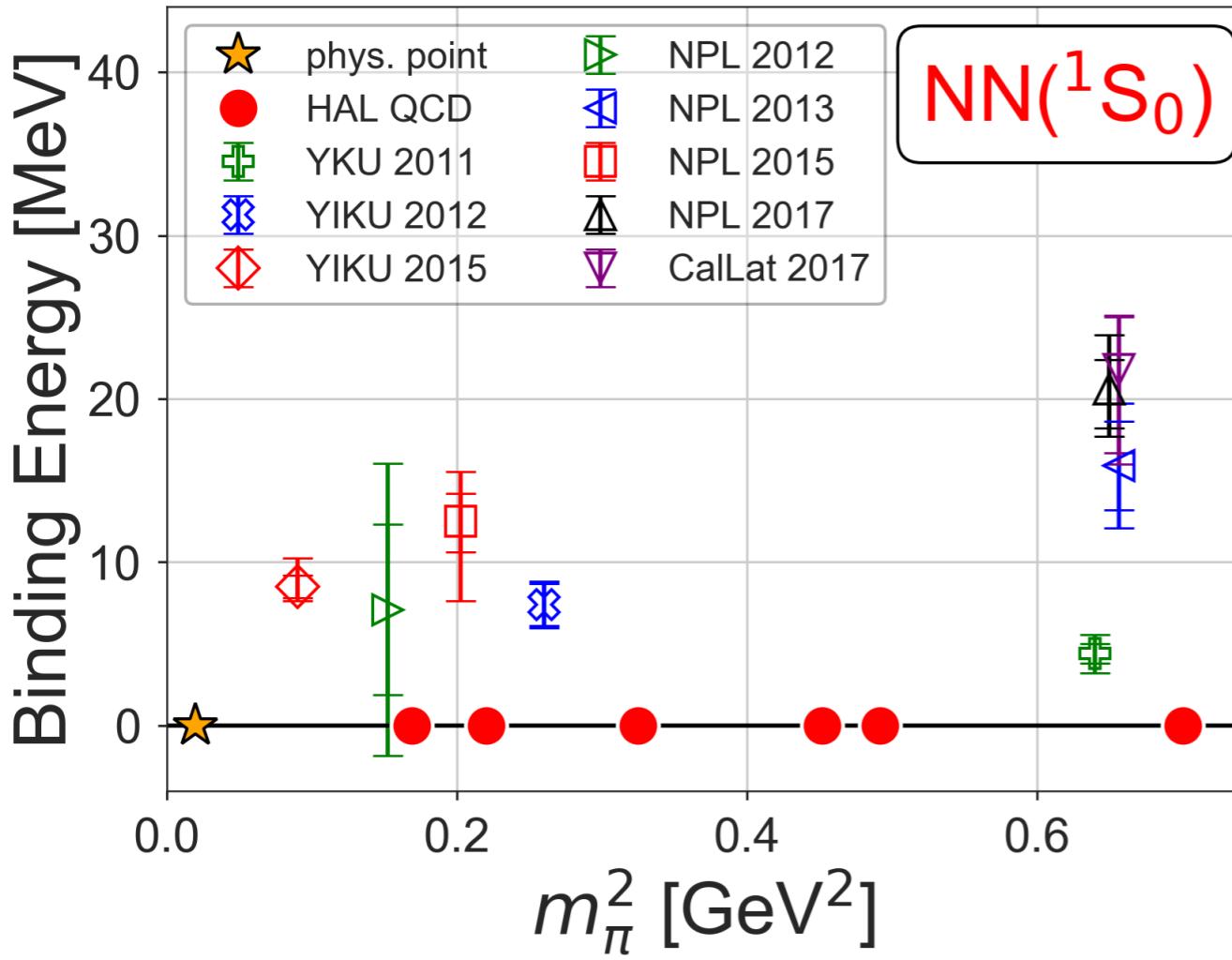
	“Direct”	HAL QCD	phys. point
Dineutron ( ${}^1S_0$ )	bound	unbound	unbound
Deuteron ( ${}^3S_1$ )	bound	unbound	bound

Inconsistent conclusions → which is correct?

Due to **the fake plateaux problem**,  
all results from **the direct method** show **anomalous behaviors**.

Refs. **TI** for HAL QCD Coll., [PRD96.034521\(2017\)](#)

S. Aoki, T. Doi, **TI**, LATTICE2017 Proc., [arXiv:1707.08800](#).

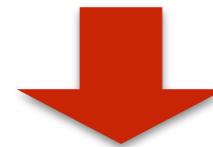


# Summary of Consistency/Normality Checks

Refs. **TI** for HAL QCD Coll., [PRD96.034521\(2017\)](#)  
 S. Aoki, T. Doi, **TI**, LATTICE2017 Proc., [arXiv:1707.08800](#).

These *Effective Range Expansion* analyses  
 are also **problematic & unreliable**.

These exhibit unreasonable behaviors  
 due to the fake plateaux.



	source independence	consistency of $k^2 < 0$ and $k^2 > 0$	non-singular ERE	physical residue	ERE fitting with constraints	ERE fitting for 2-pole system	Reasonable ERE for bound states
Yamazaki et al. 2011		No	No				
NPLQCD 2012			No				
Yamazaki et al. 2012	No		No				
NPLQCD 2013	No			No	No		No
NPLQCD 2015		No		No	No		No
Yamazaki et al. 2015			No	No			
CalLat 2017	No			No	No	No	No
NPLQCD 2017				No	No	No	No

# Lüscher's Finite Volume Method & Bound State

1. Lüscher's formula

$$k \cot \delta_0(k) = \frac{1}{\pi L} \sum_{\mathbf{n} \in \mathbb{Z}^3} \frac{1}{|\mathbf{n}|^2 - (kL/2\pi)^2}$$

2. Interpolation by **Effective Range Expansion (ERE)**

$$k \cot \delta_0(k) \simeq -\frac{1}{a_0} + \frac{1}{2} r_{\text{eff}} k^2 + \dots$$

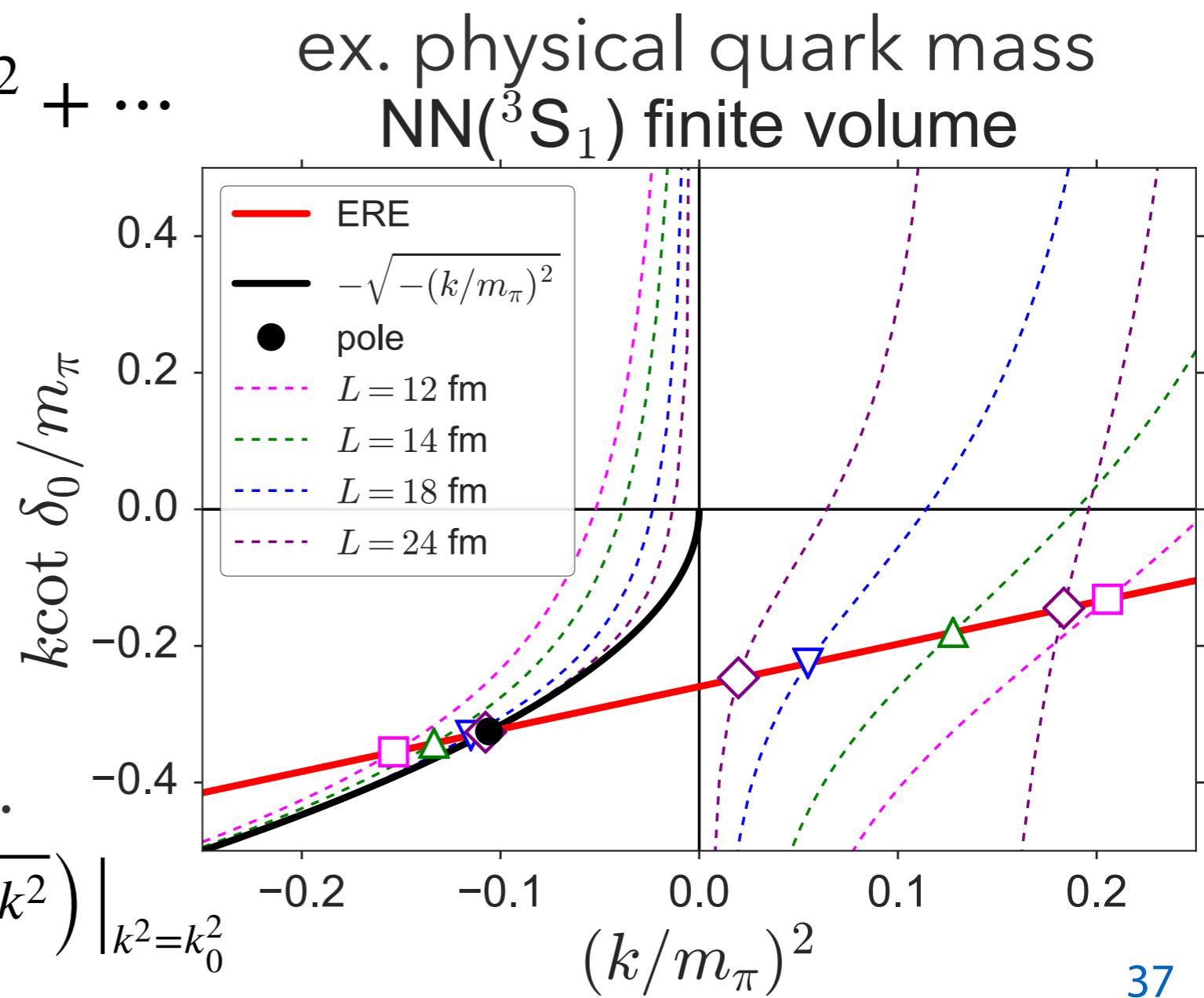
3. Search **bound state**

@ S-matrix's pole

$$k_0 \cot \delta_0(k_0) = -\sqrt{-k_0^2}$$

with physical pole cond.

$$\frac{d}{dk^2}(k \cot \delta_0(k)) \Big|_{k^2=k_0^2} < \frac{d}{dk^2} \left( -\sqrt{-k^2} \right) \Big|_{k^2=k_0^2}$$



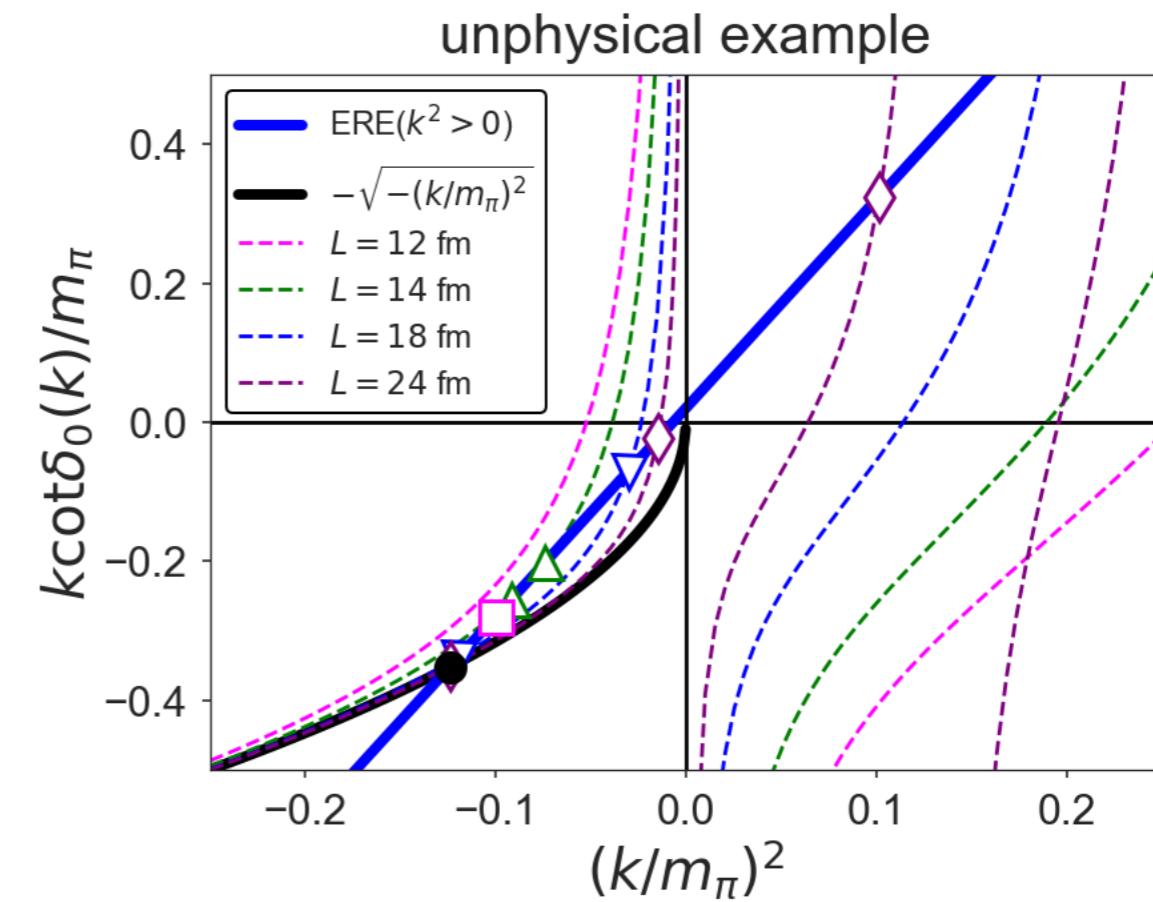
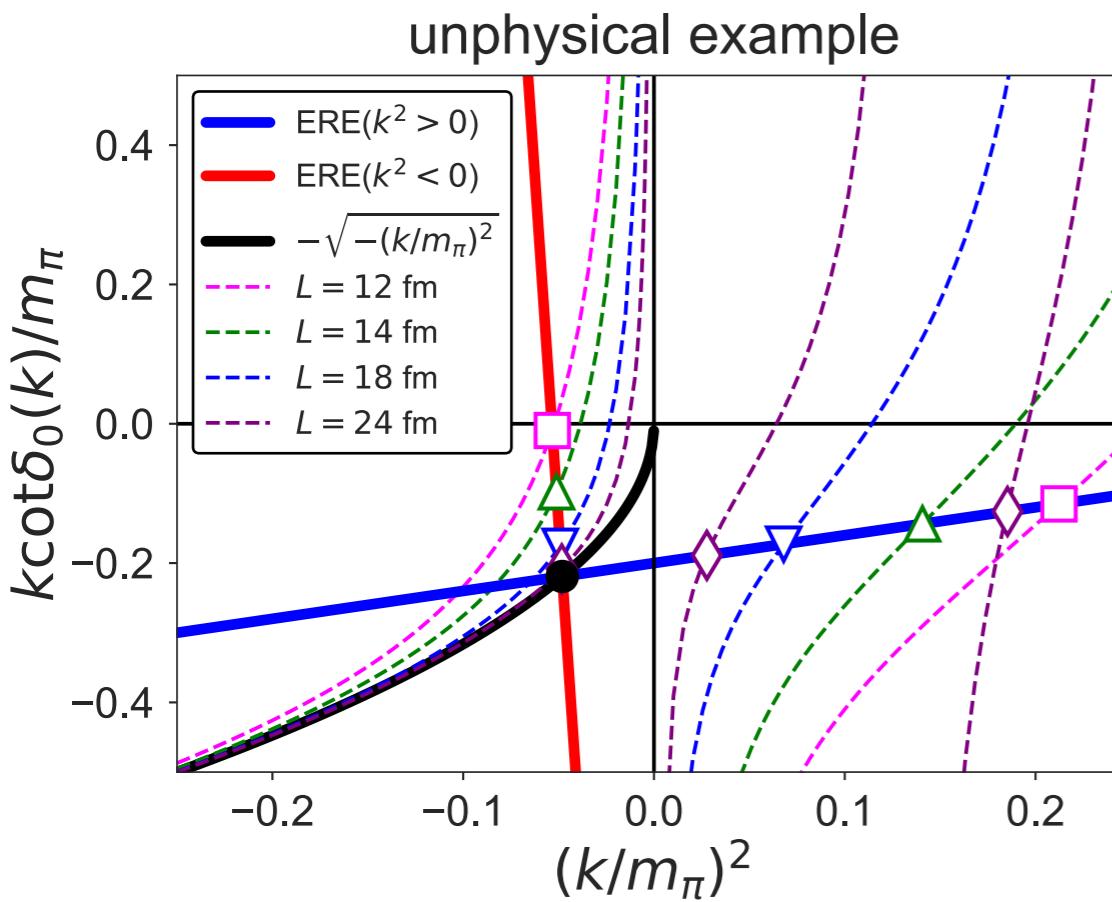
# Consistency/Normality Check

These are **the necessary conditions**.

Passing these tests cannot guarantee the correctness.

1. Quark Src. and Op. independence of the plateau
2. Effective Range Expansion (ERE)
  1. **Consistency** between ERE( $k^2 > 0$ ) and ERE( $k^2 < 0$ ).
  2. ERE parameters are **non-singular**.
  3. ERE satisfies **the physical pole condition**.

## Example of **the incorrect EREs**



# Frequently Asked Questions

- *Consistency checks are useful to test the validity of the result?*
  - No, **these are necessary conditions.** Passing these test is mandatory, **but it cannot guarantee the correctness of the plateaux.**
- Volume independence is the evidence of the bound state?
  - No, a deeply bound state is almost volume independent, but volume independence does not mean a deeply bound state.  
For example, the results in Yamazaki et al. 2012 are roughly volume independent, but it shows singular phase shifts.
- *ERE passes the physical pole condition within error bars.*
  - **All ERE parameters must satisfy the condition.**  
In the first place, if some parameters violate this condition, chi-square cannot be defined. Therefore such **error bars are incorrect.**

# ERE Fitting with the Finite Volume Constraint (1)

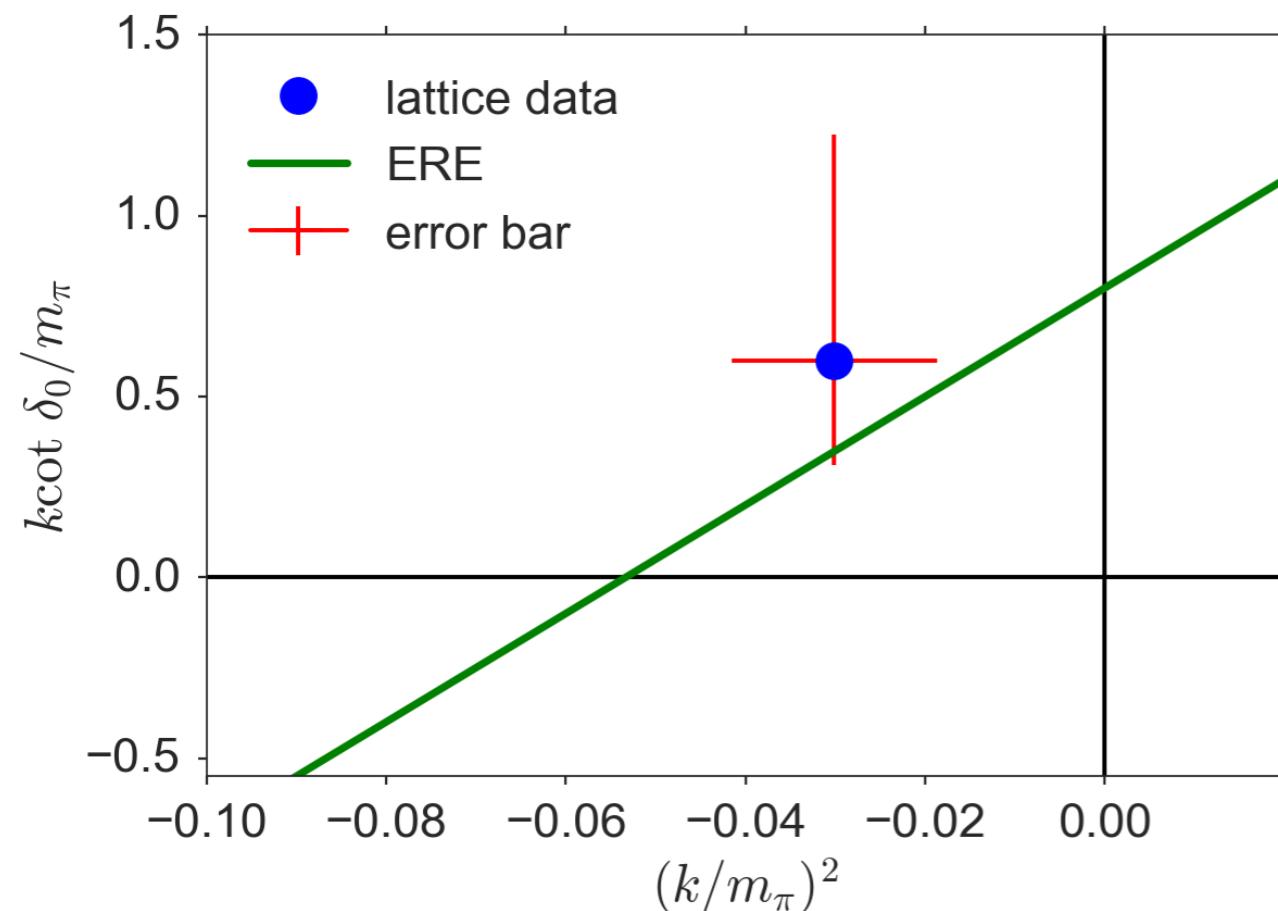
- $k^2$  and  $k \cot \delta_0$  are correlated.

$$k \cot \delta_0(k) = \frac{1}{\pi L} \sum_{\mathbf{n} \in \mathbb{Z}^3} \frac{1}{|\mathbf{n}|^2 - (kL/2\pi)^2}$$

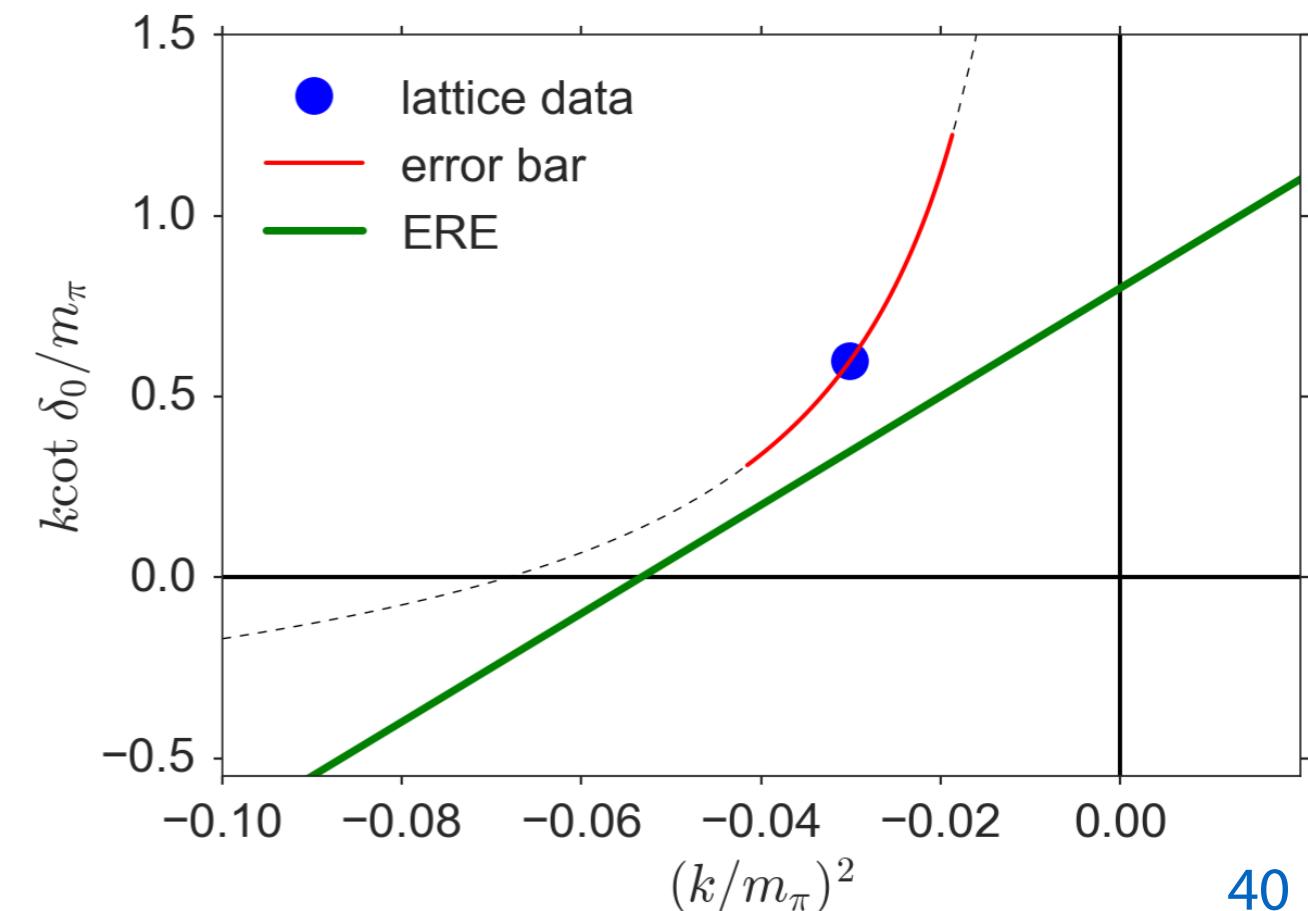
- It is incorrect to fit without using this constraint.

- **However, this fact is ignored in the previous studies.**

incorrect error & fitting

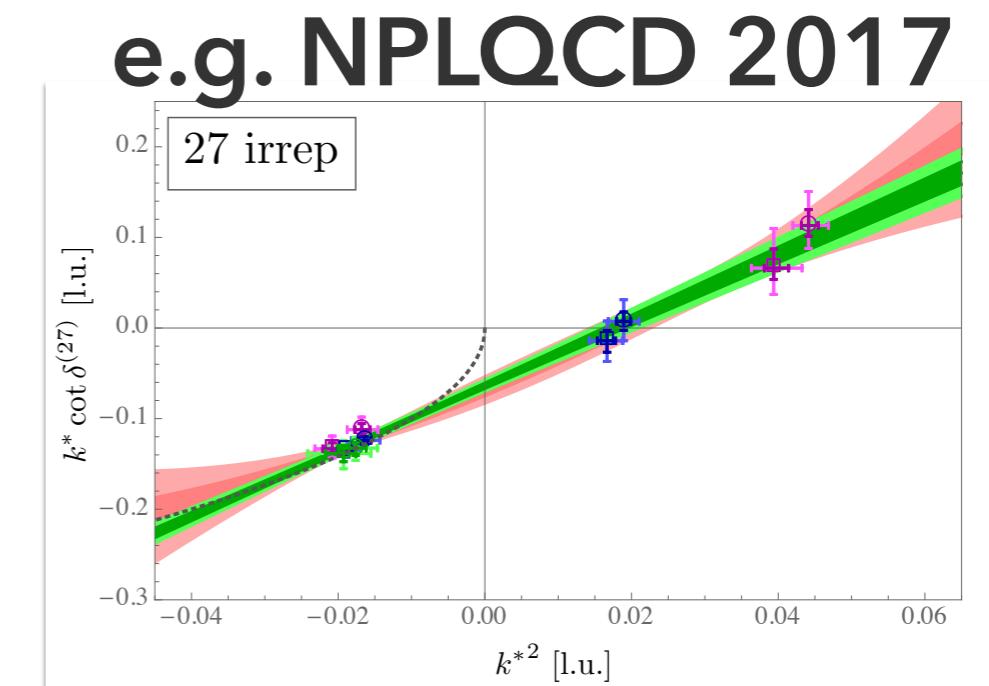
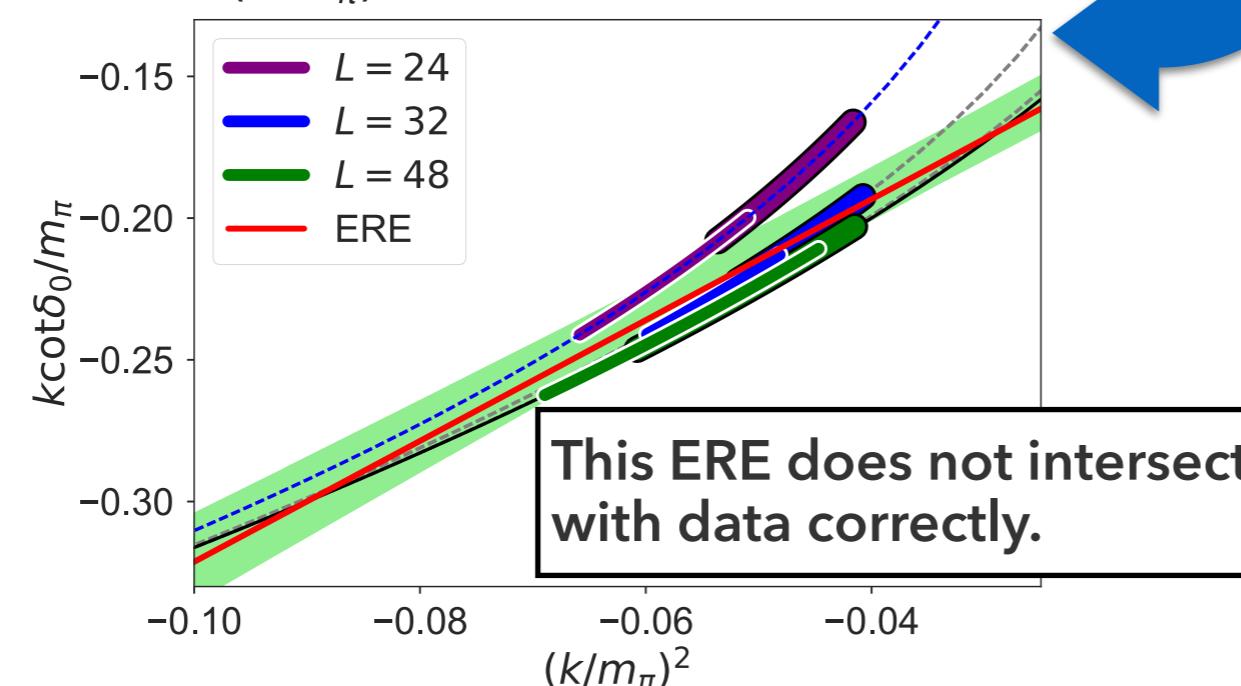
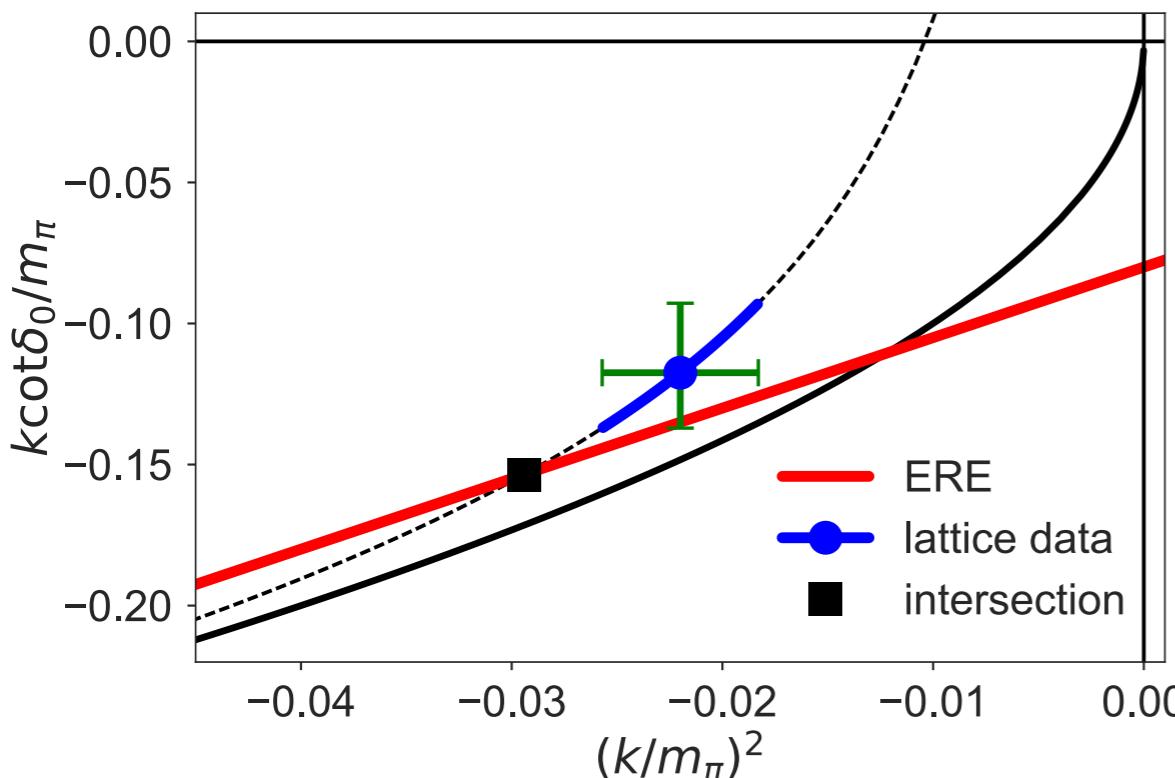


chi-square is ill-defined

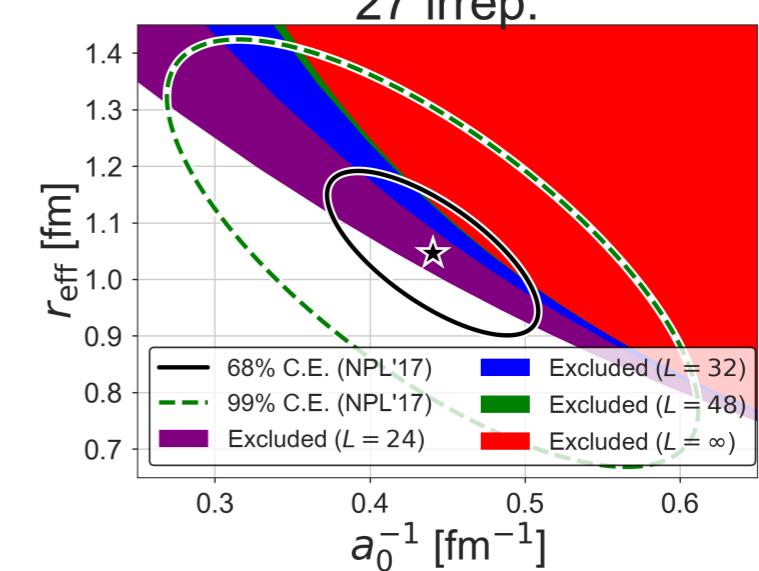


# ERE Fitting with the Finite Volume Constraint (2)

- $k^2$  and  $k \cot \delta_0$  are correlated.
- For ERE fitting, one has to consider this constraint.



wrong ERE params.  
constraint is ignored



# ERE Fitting with the Bound State Pole

considering a tangent at  $k^2 = -\kappa_b^2 > 0$

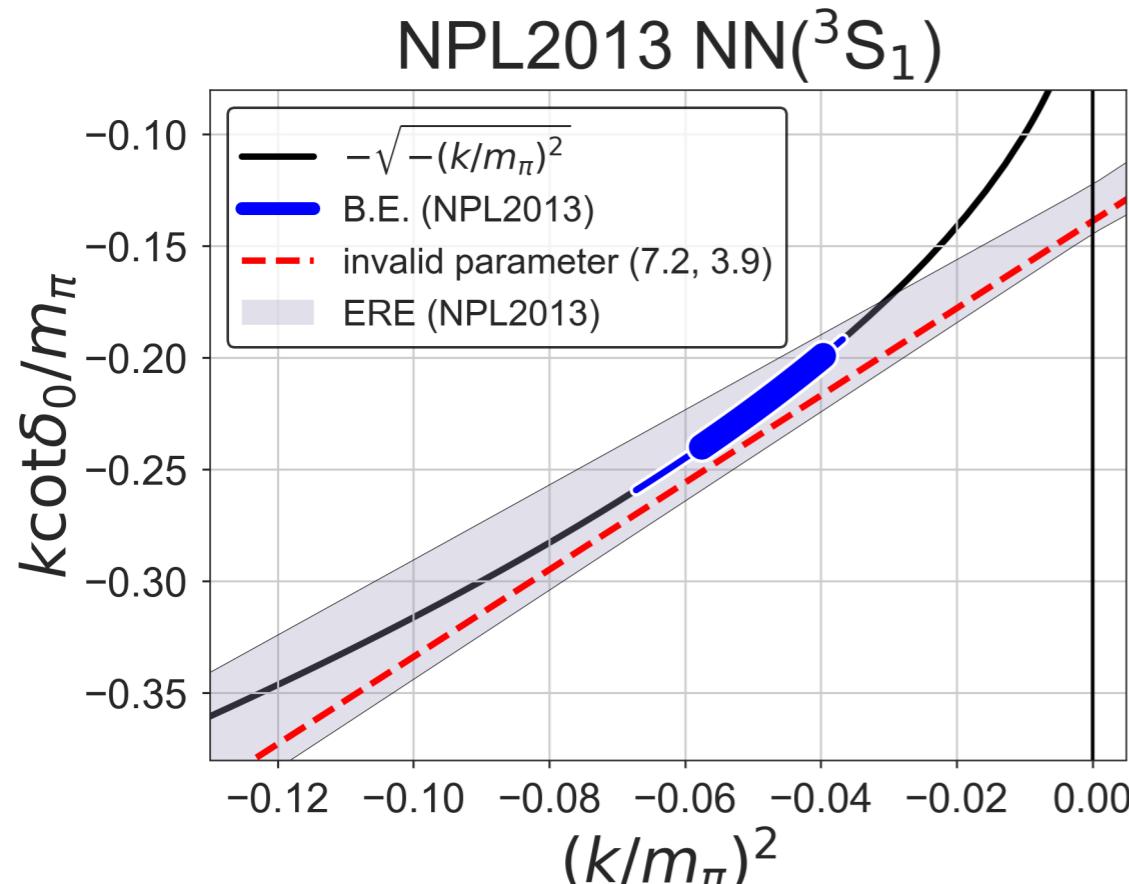
$$-\frac{1}{2}\sqrt{-\kappa_b^2} + \frac{1}{2}\frac{1}{\sqrt{-\kappa_b^2}}k^2$$

**allowed ERE parameter (in NLO)**

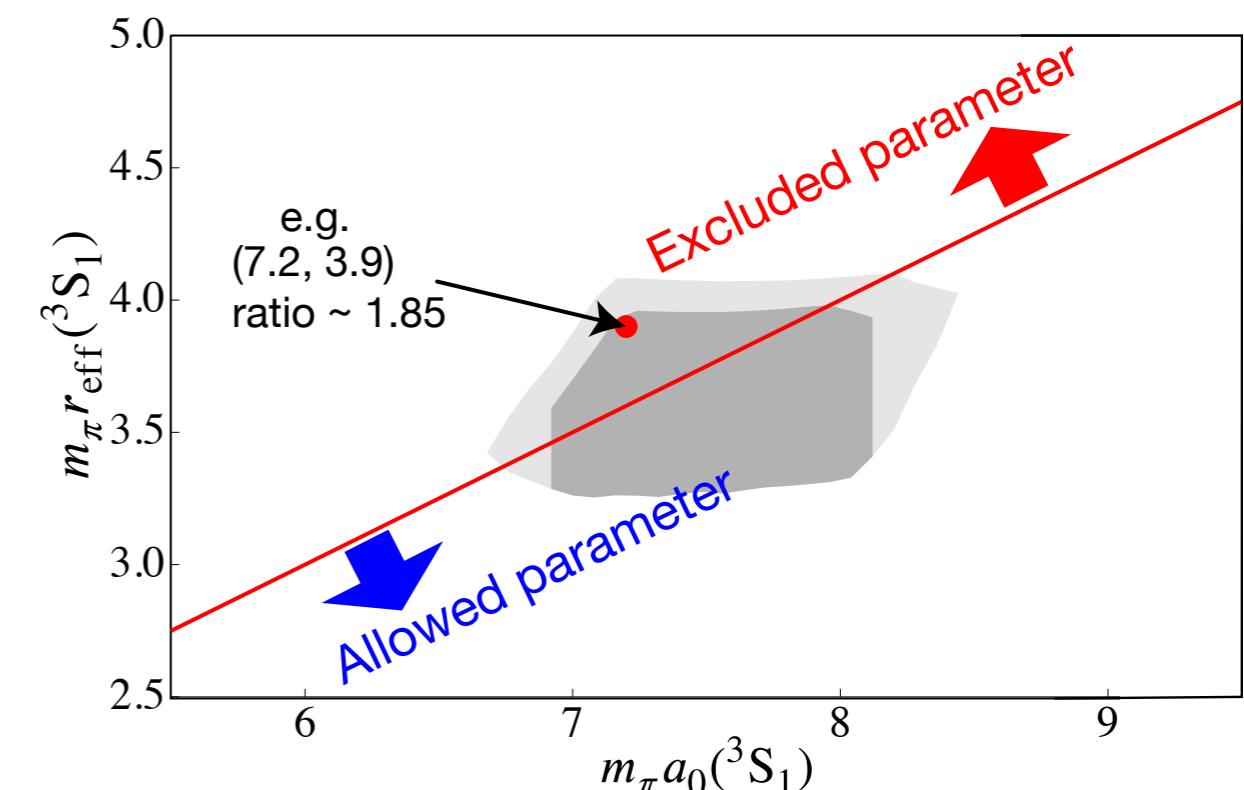
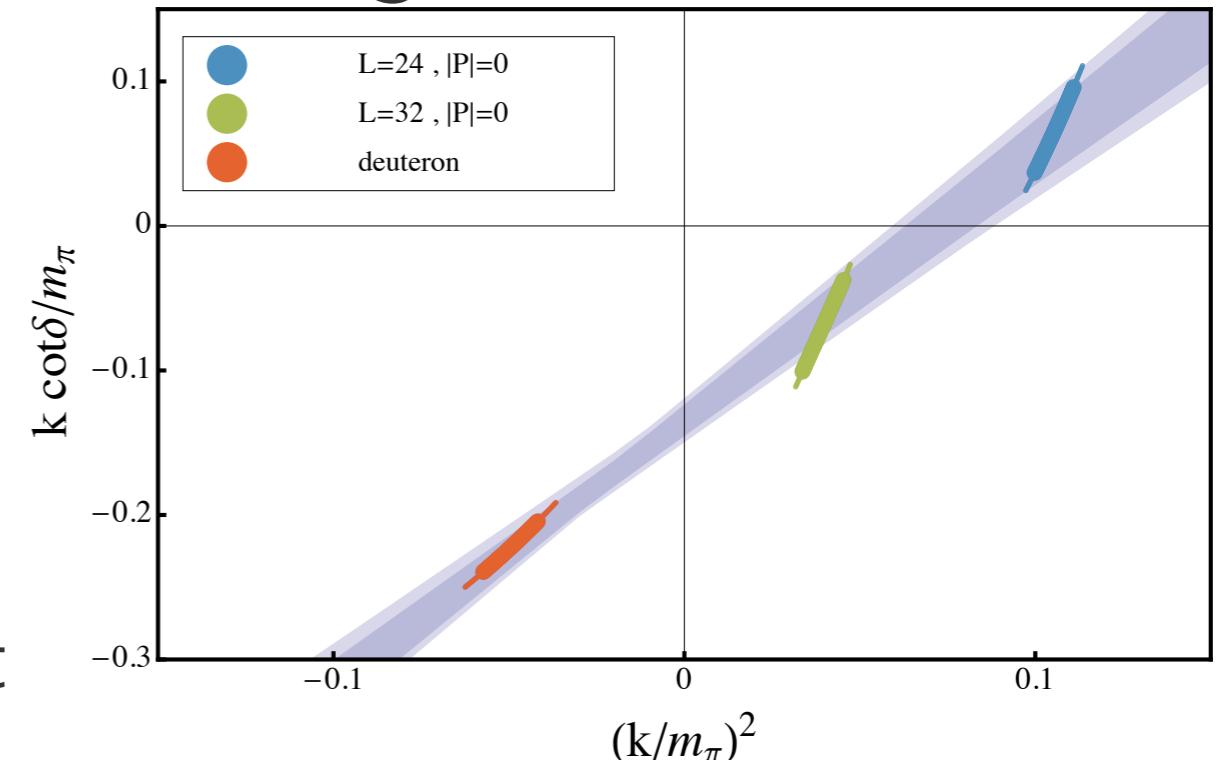
$$a_0/r_{\text{eff}} > 2$$

otherwise, ERE does not cross the pole condition

this ERE fitting ignores the constraint



e.g. NPLQCD 2013

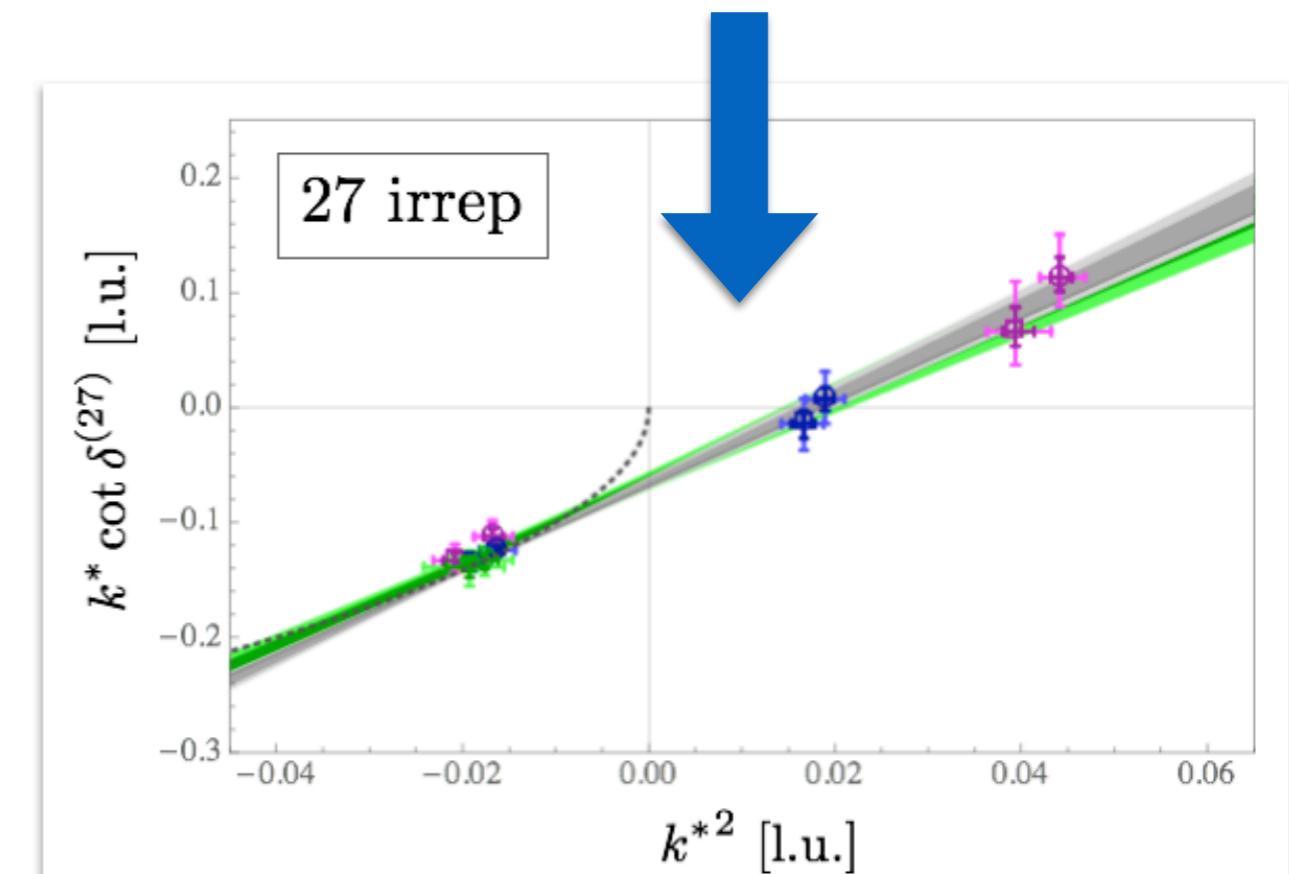
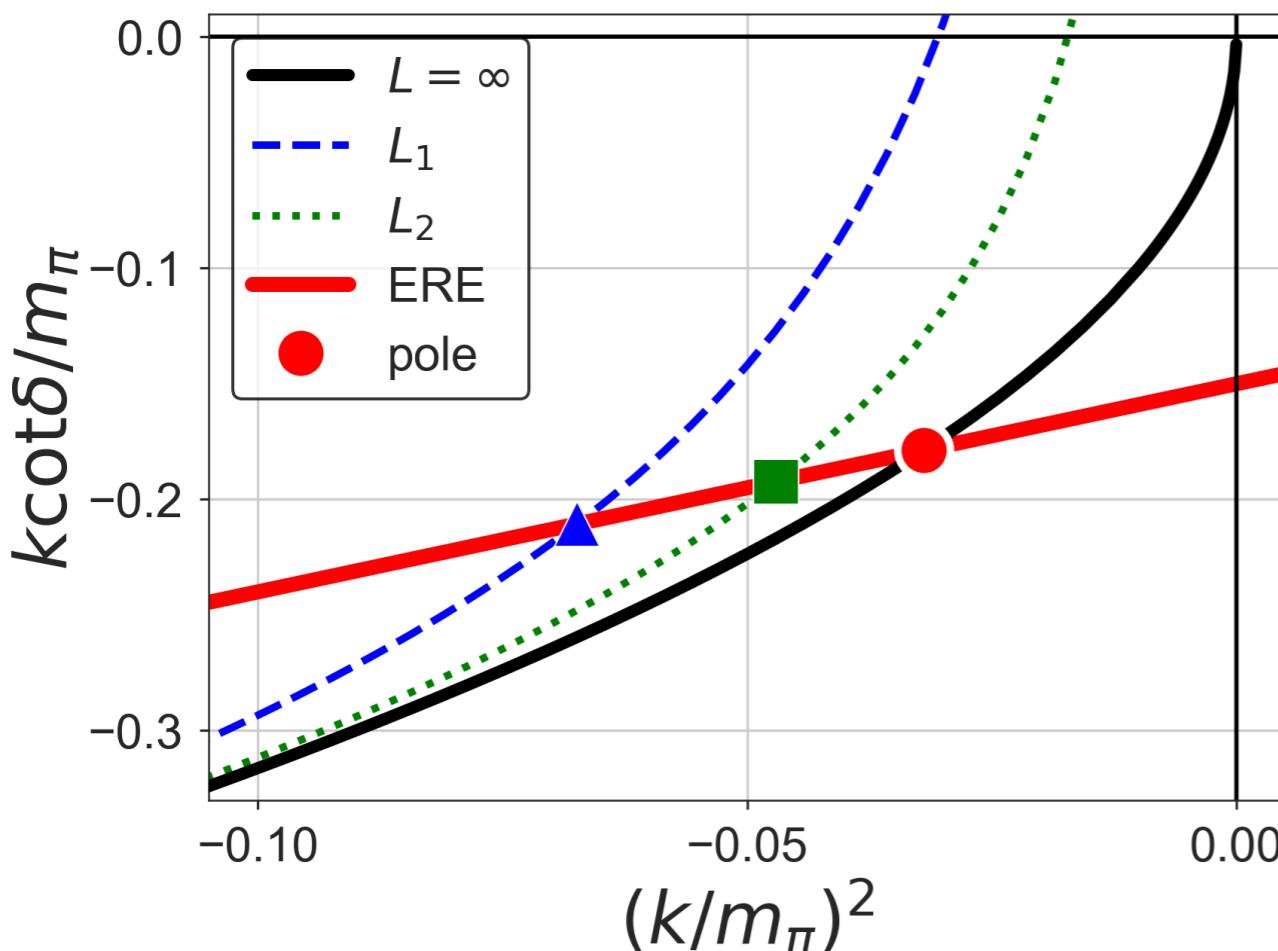


this confidence region is **incorrect** 42

# Physical Residue of the S-matrix

$$\frac{d}{dk^2} \left[ k \cot \delta_0(k) - \left( -\sqrt{-k^2} \right) \right] \Big|_{k^2 = -\kappa_b^2} = -\frac{1}{\beta_b^2} < 0$$

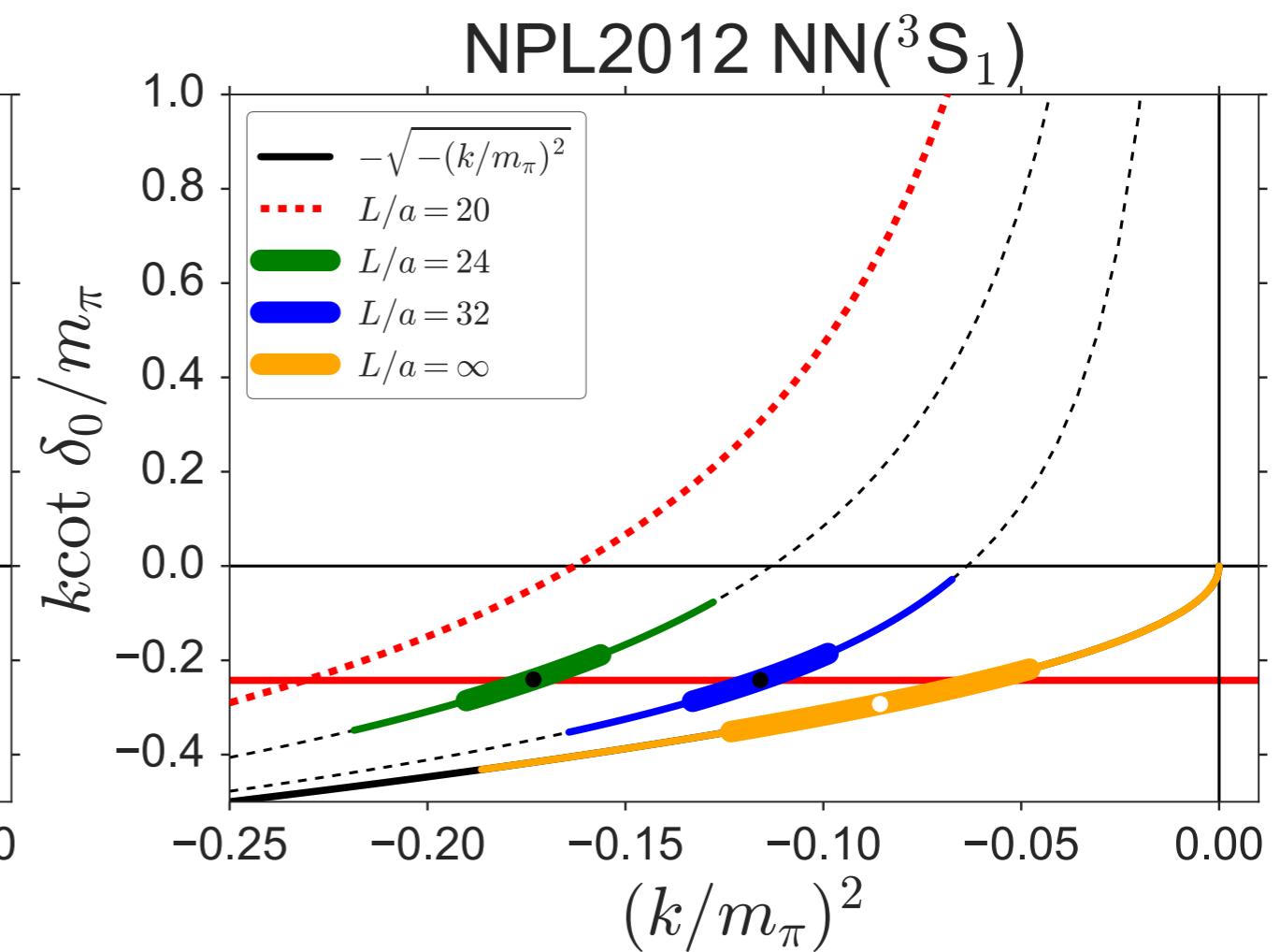
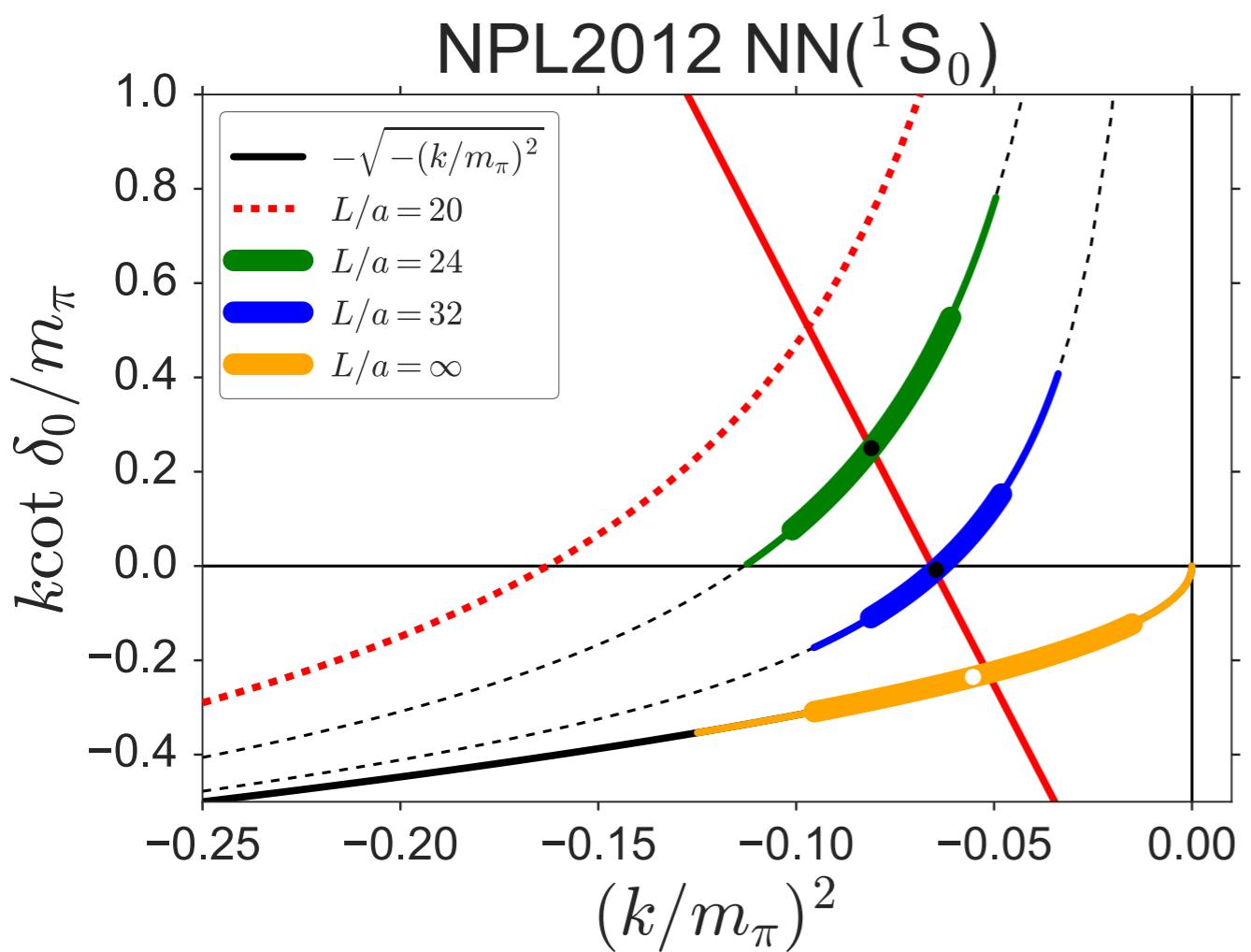
- This condition should be always satisfied around the pole.
- **A bound state pole** is an outcome of **the ERE fitting**, it is inappropriate to compare the ERE and its tangent.



e.g. NPLQCD 2017

# NPLQCD 2011 – Singular & Inconsistent ERE

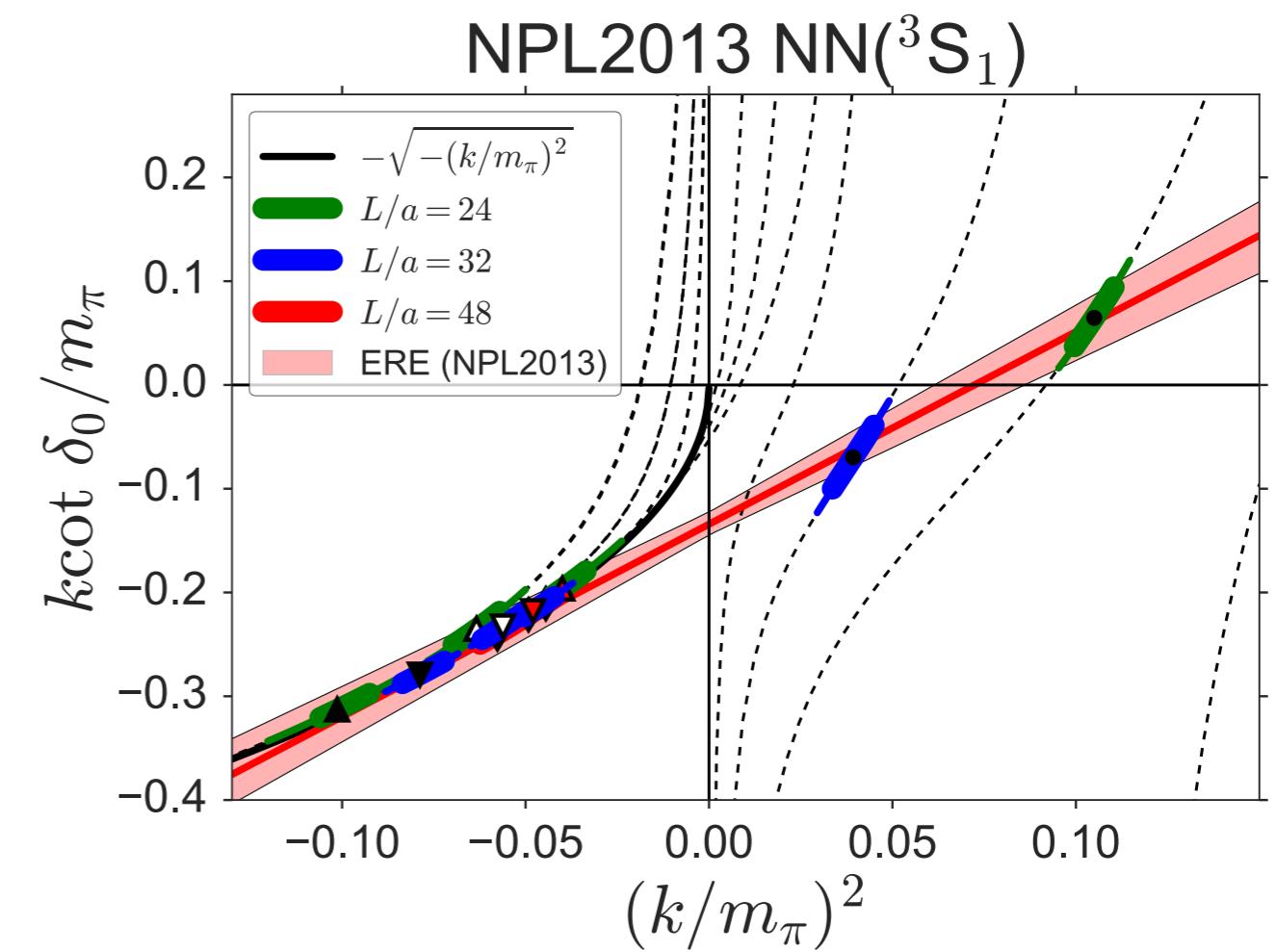
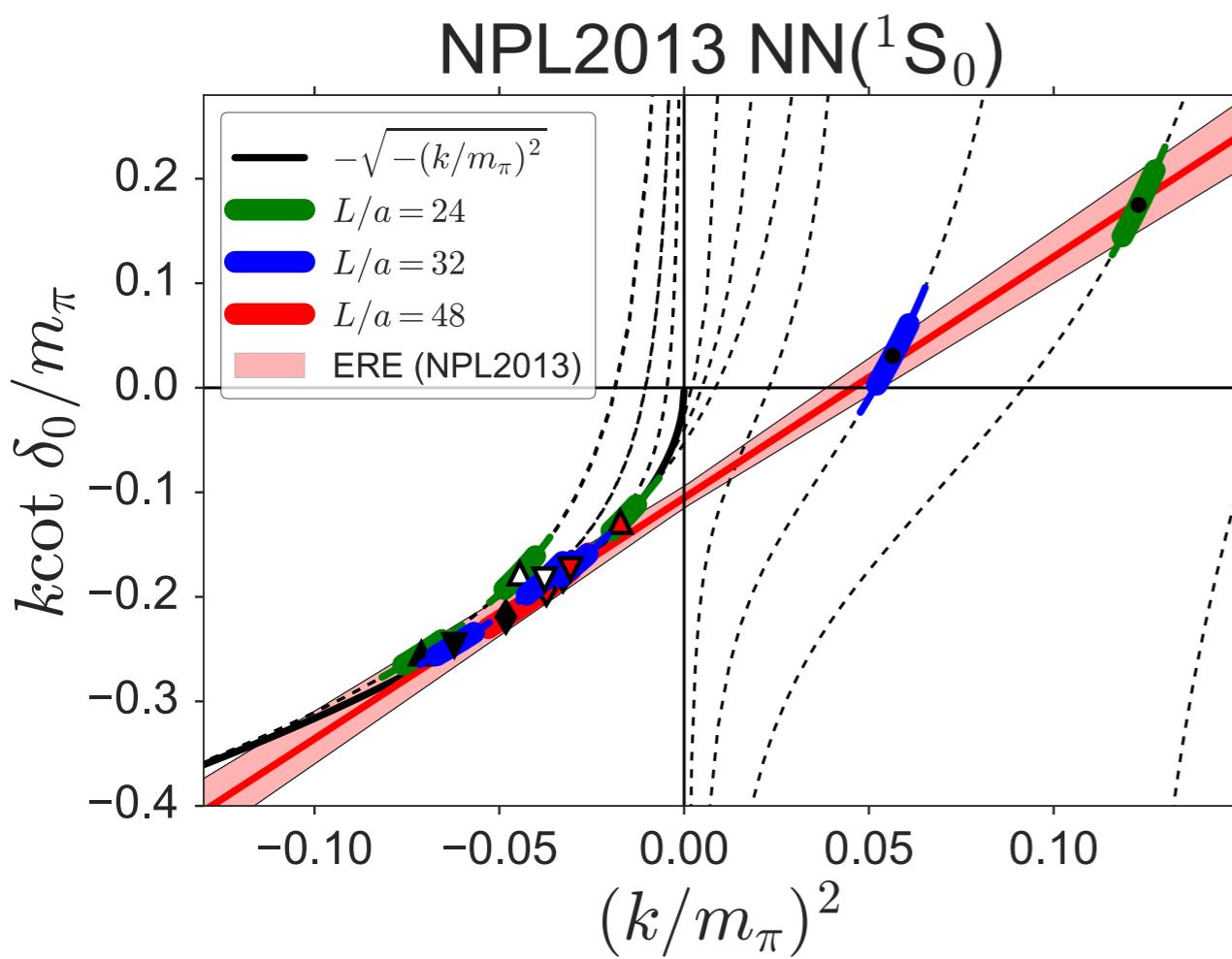
- Singular behavior in NN( ${}^1S_0$ )
- Positive energy shift  $(k/m_\pi)^2 > 0$  at  $L/a = 20$  is incompatible with  $L = 24$  &  $32$ . NPLQCD Coll. PRD81,054505(2010).



# NPLQCD 2013 – Unphysical Pole

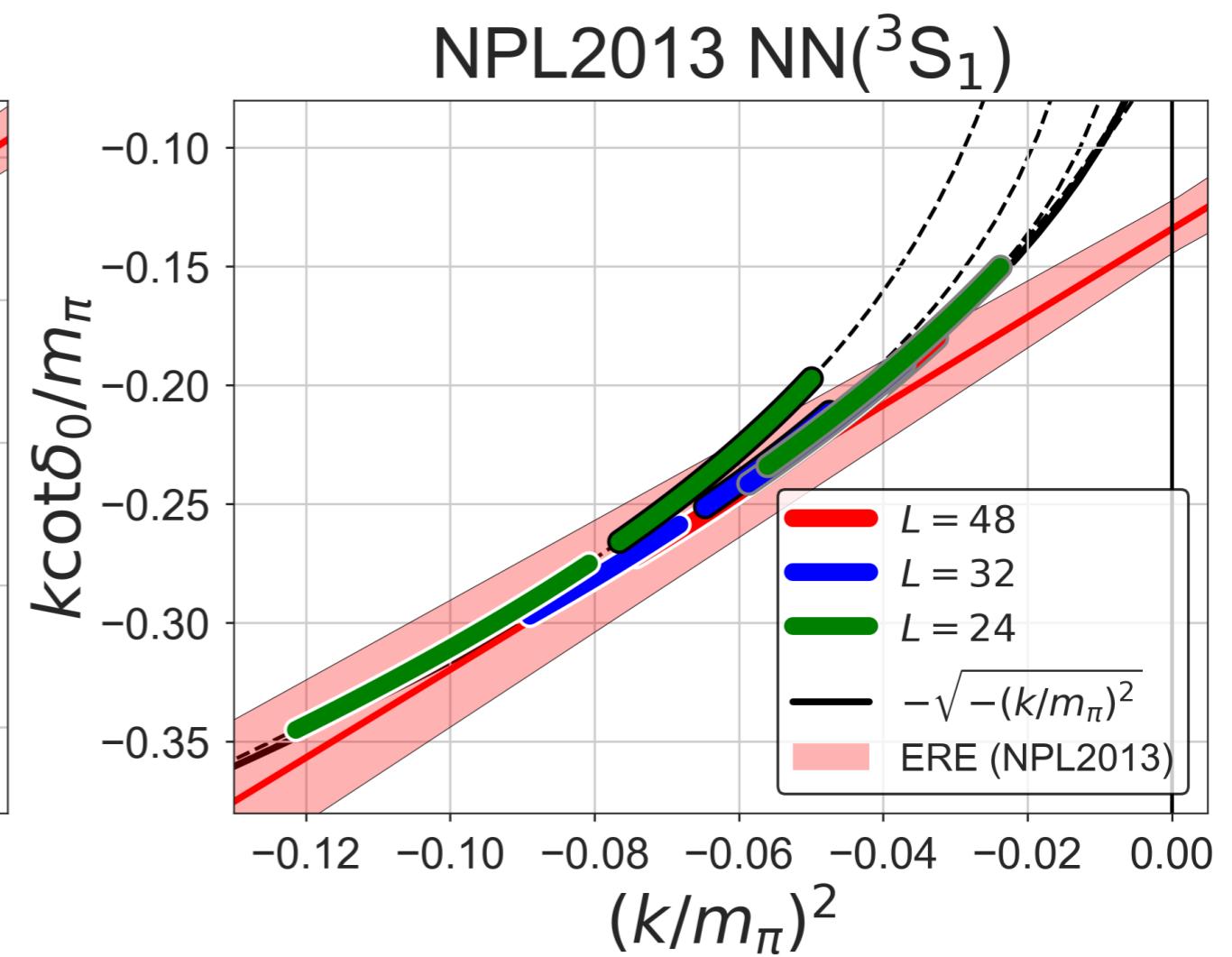
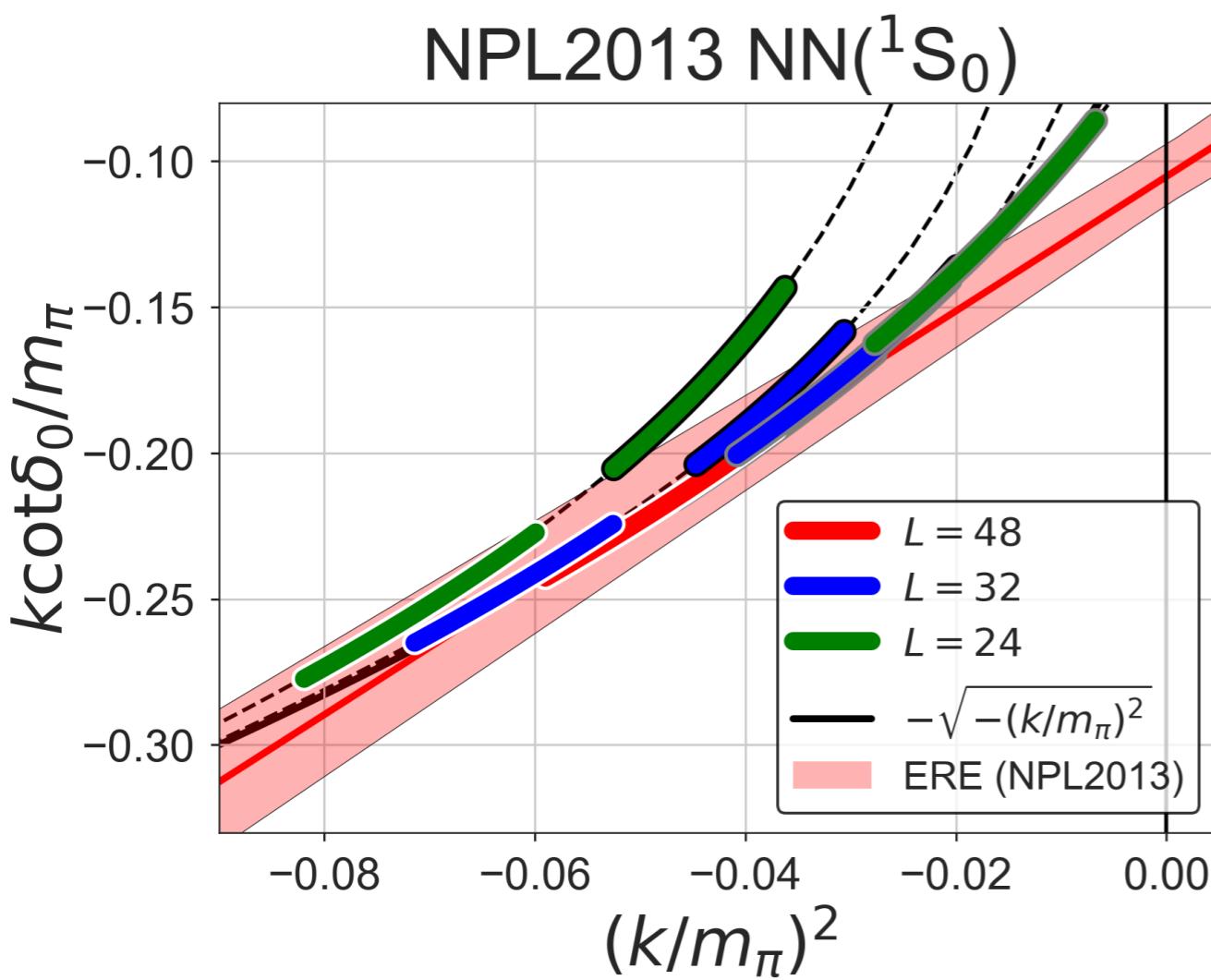
- physical pole condition is violated

$$\frac{d}{dk^2}(k \cot \delta_0(k)) \Big|_{k^2=k_0^2} < \frac{d}{dk^2} \left( -\sqrt{-k^2} \right) \Big|_{k^2=k_0^2}$$



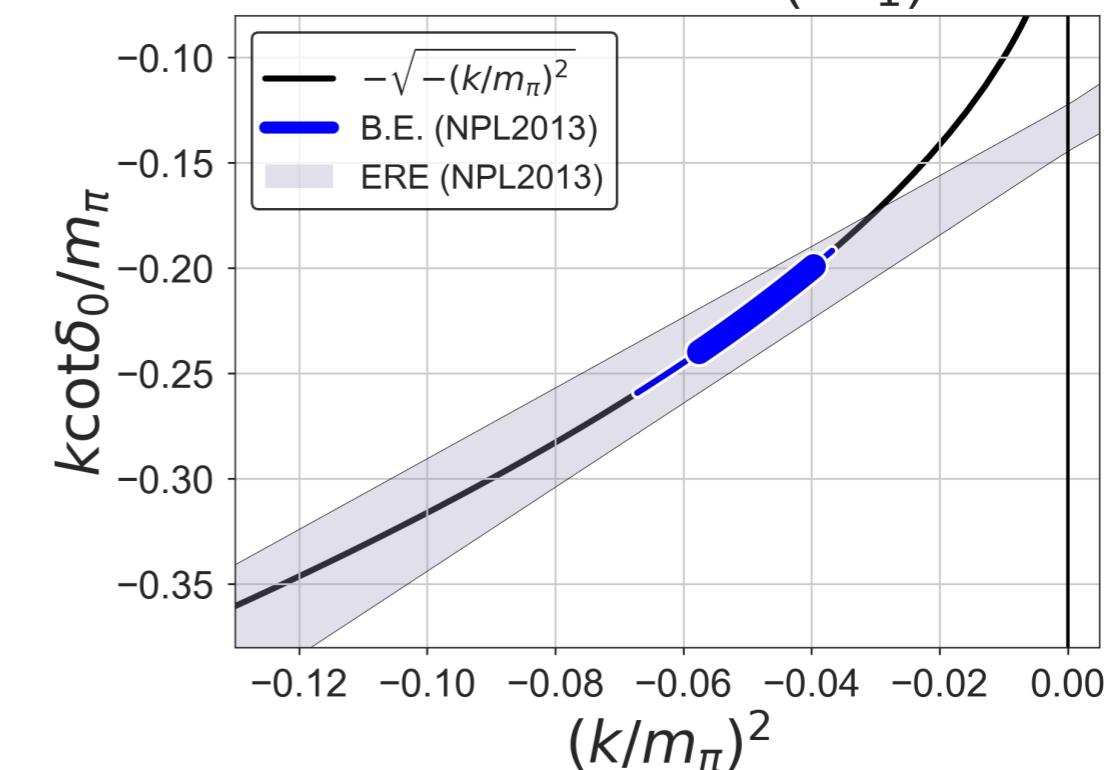
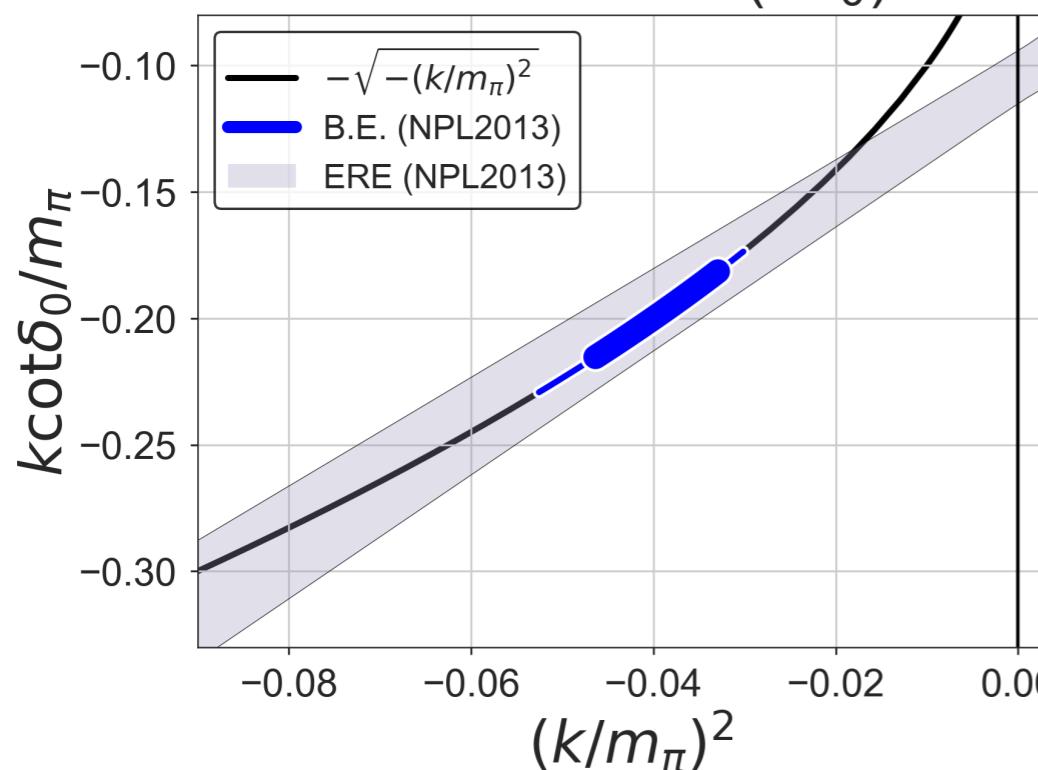
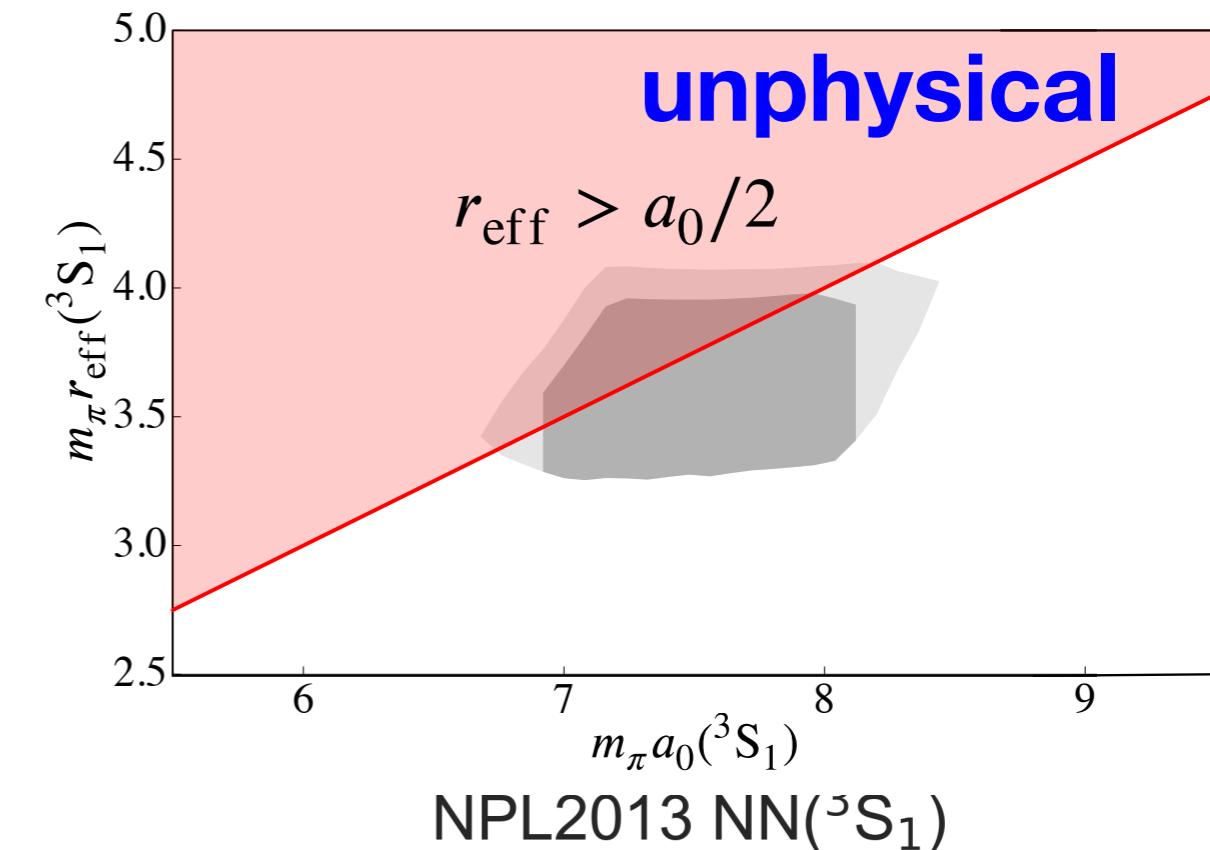
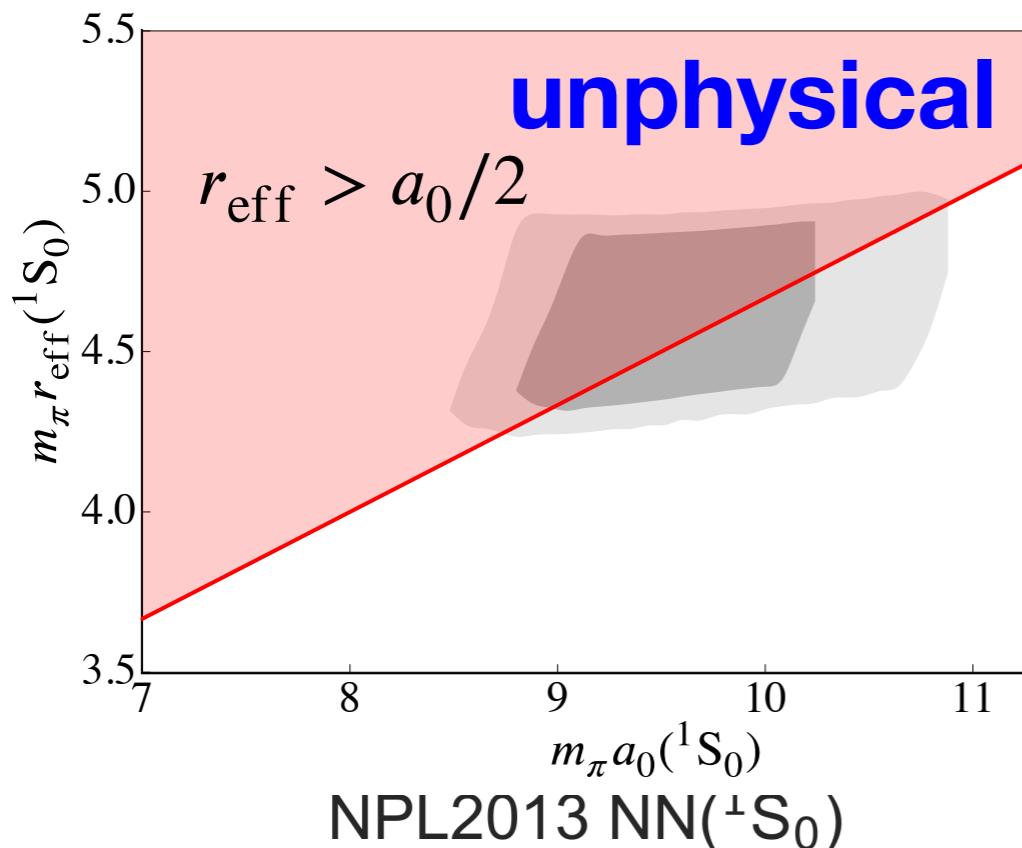
# NPLQCD 2013 – Incorrect ERE Fitting & Unphysical Pole

- ERE does not intersect with the Lüscher's constraint & pole condition correctly.
- These error bars are incorrect.



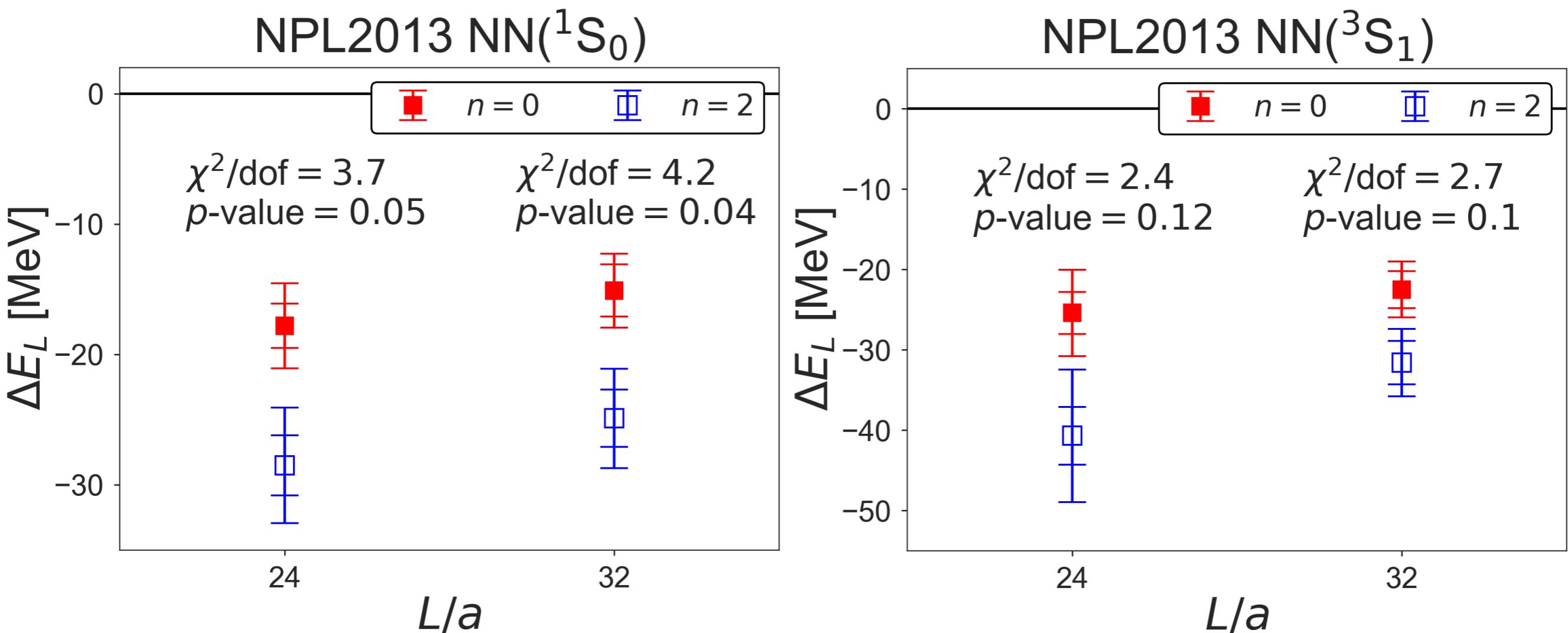
# NPLQCD 2013 – Incorrect ERE parameter

These confidence region are **incompatible with the pole & unreliable**.



# NPLQCD 2013 – Operator Dependence

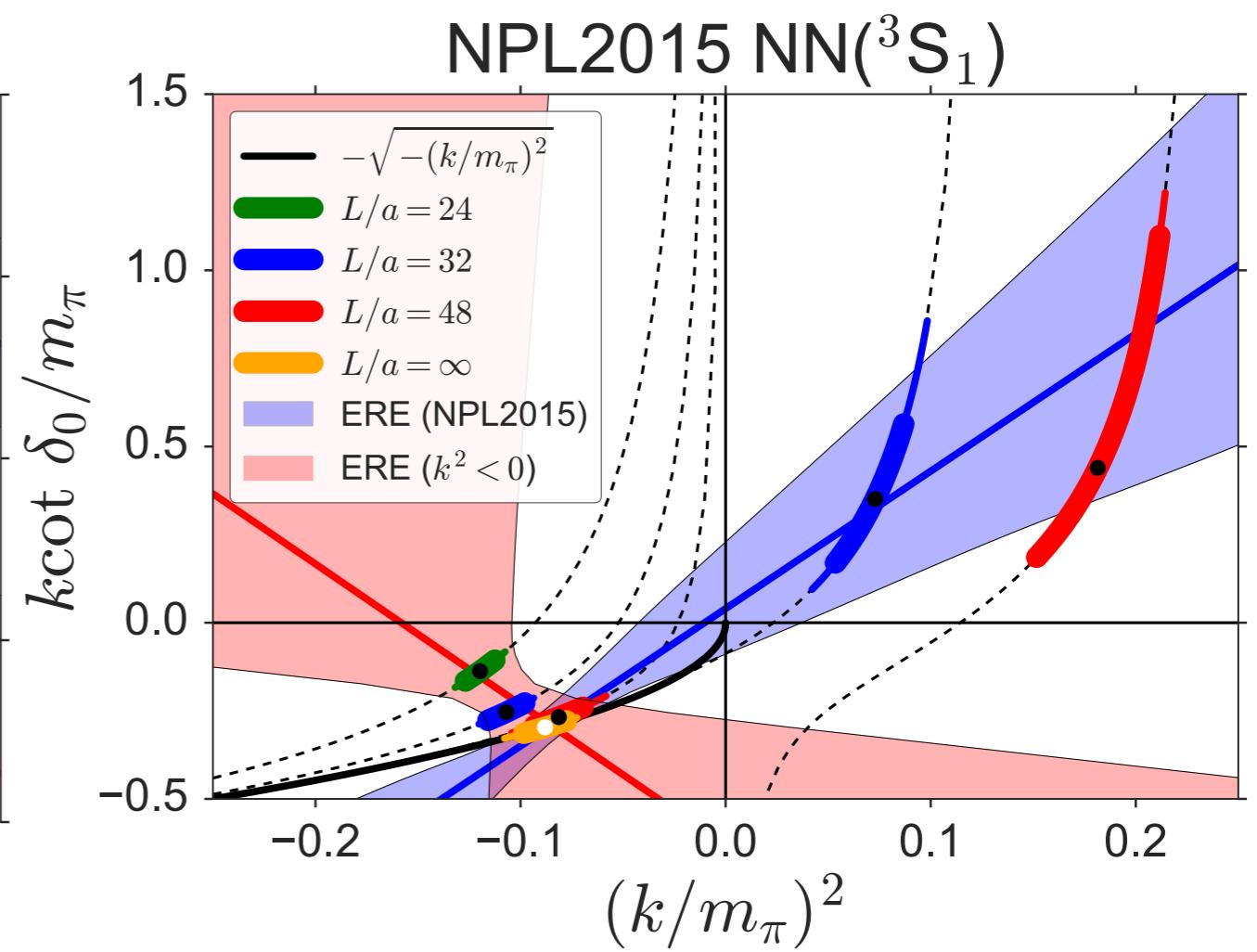
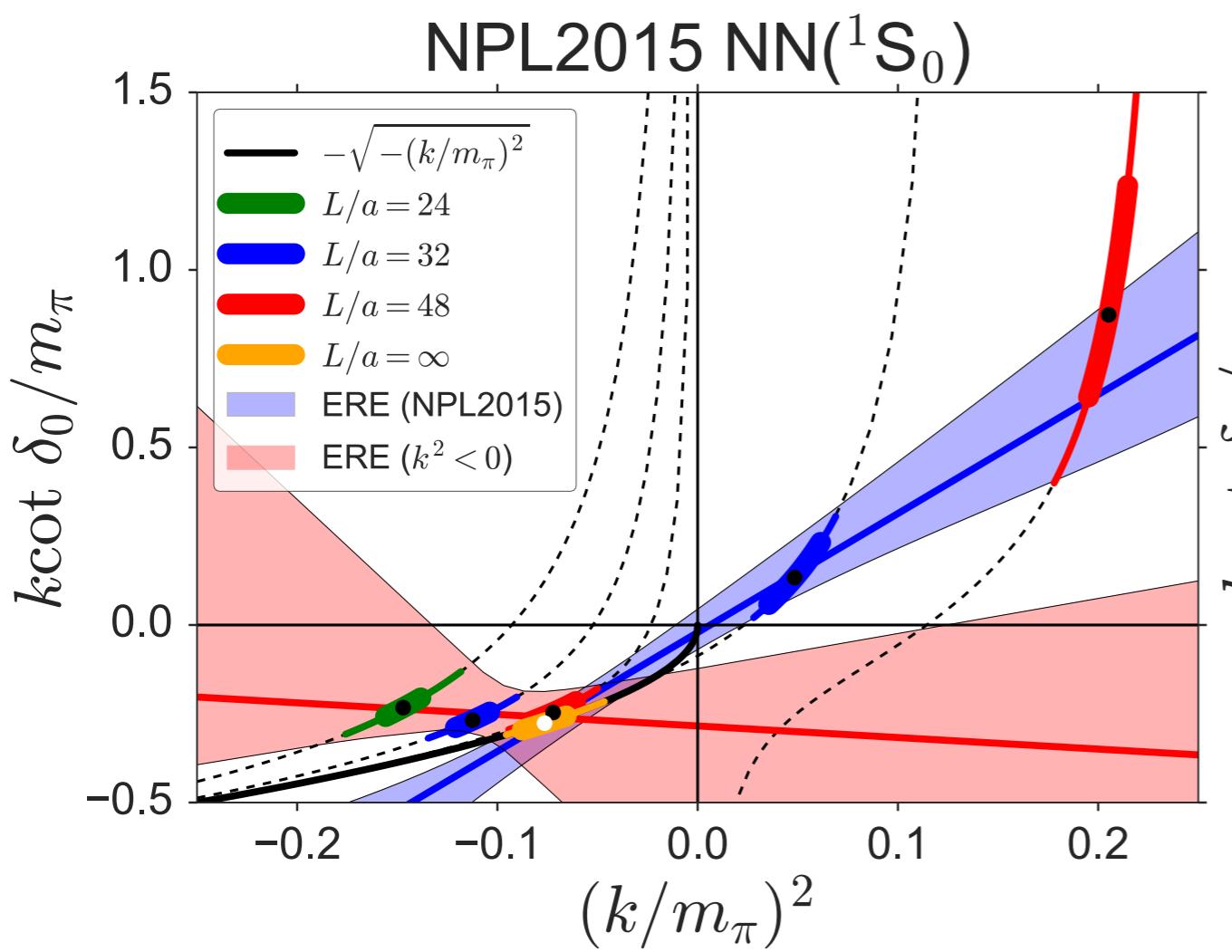
- Operator dependence is statistically significant.
  - n: boosted momentum
  - n = 0 and n = 2 should be almost the same value



# NPLQCD 2015 – Unphysical Pole & Inconsistent ERE

- Inconsistency between  $\text{ERE}(k^2 < 0)$  and  $\text{ERE}(k^2 > 0)$

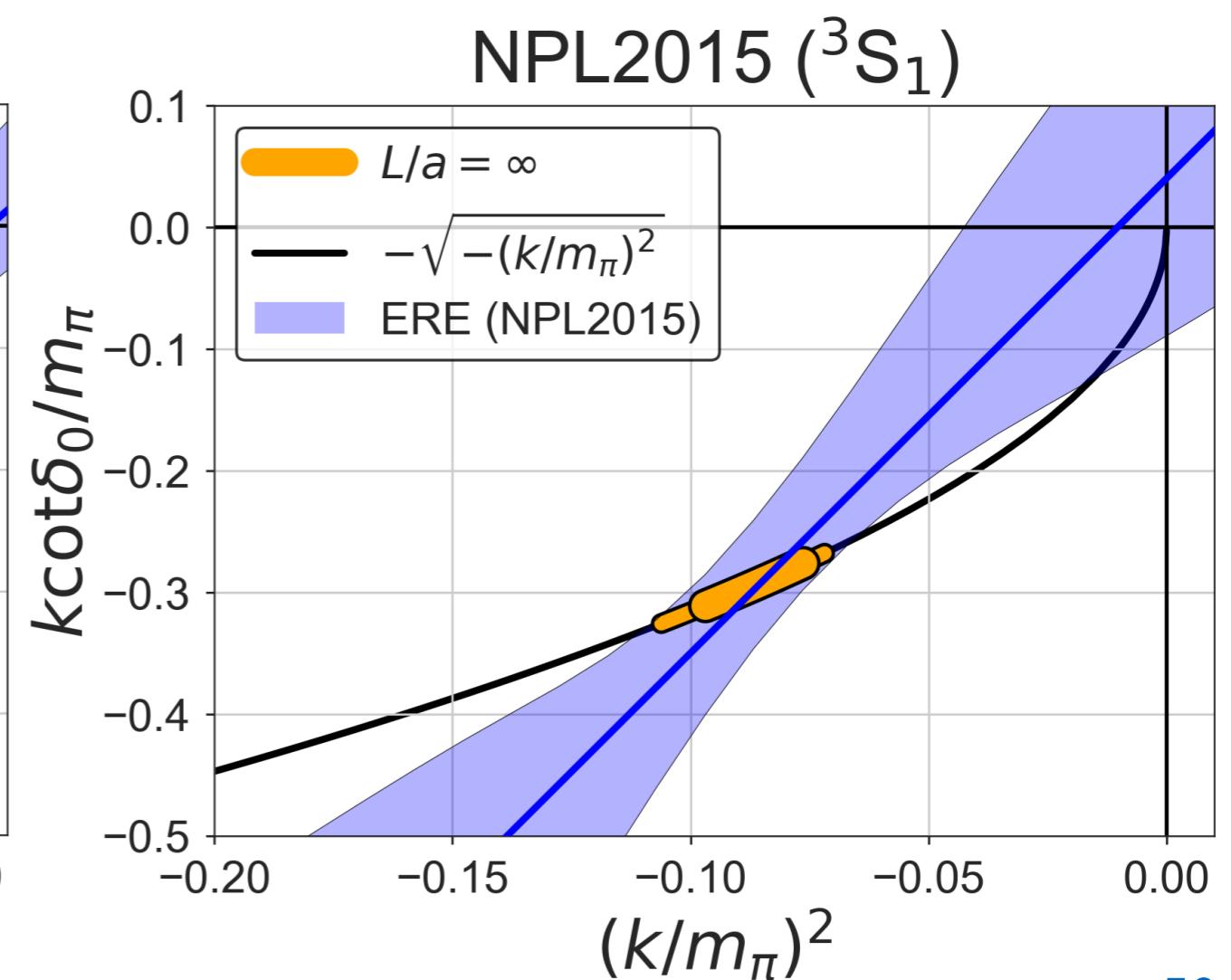
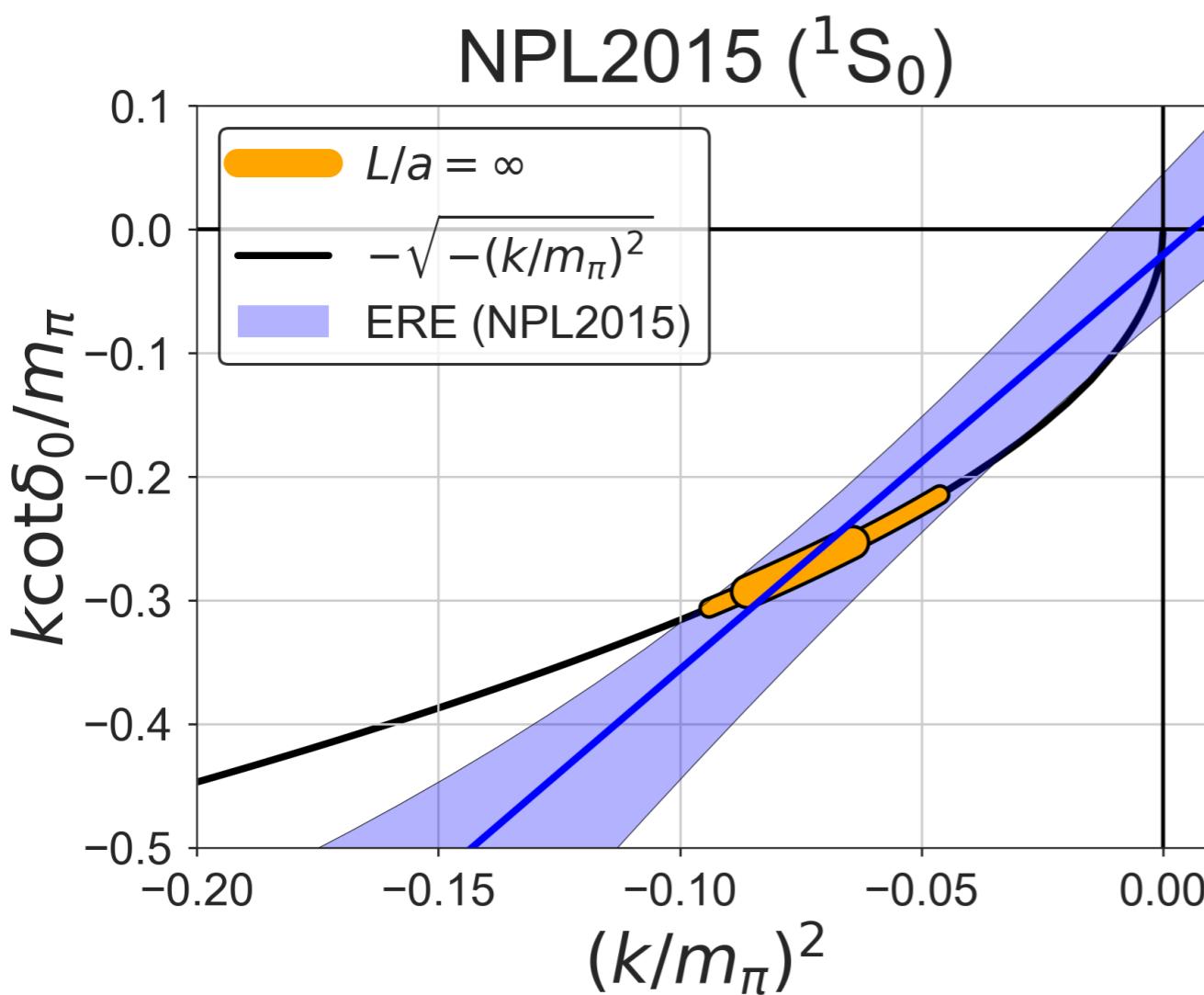
- Physical pole condition  $\frac{d}{dk^2}(k \cot \delta_0(k)) \Big|_{k^2=k_0^2} < \frac{d}{dk^2}(-\sqrt{-k^2}) \Big|_{k^2=k_0^2}$   
is violated ( $\text{ERE}(k^2 > 0)$ )



# NPLQCD 2015 – Unphysical Pole

- Physical pole condition is violated (ERE( $k^2 > 0$ ))

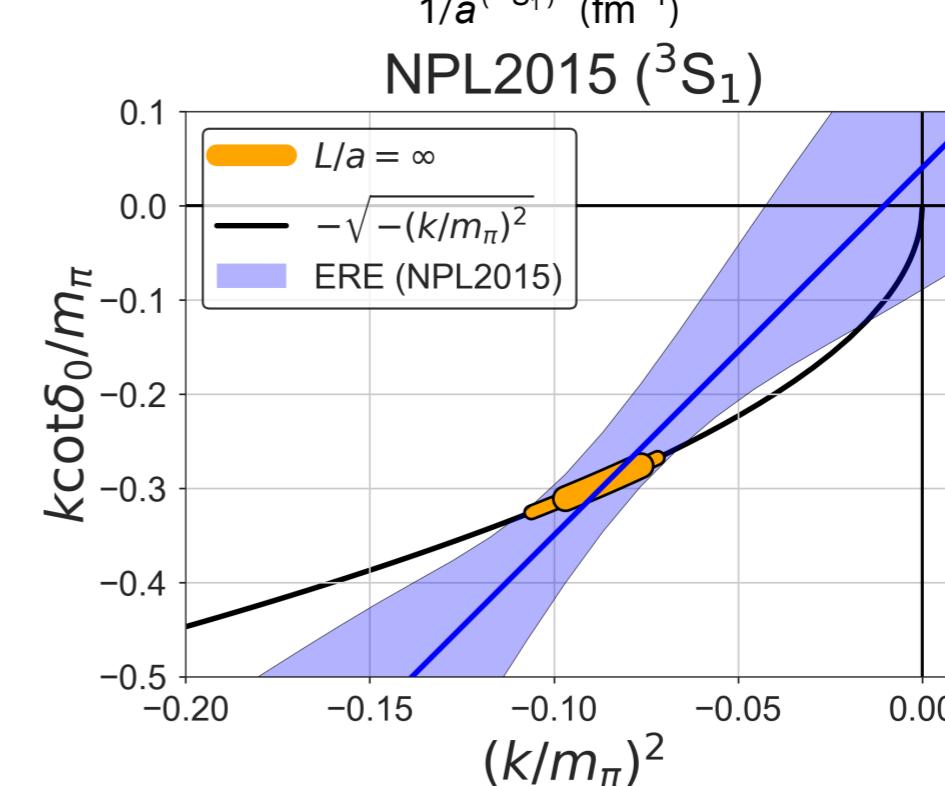
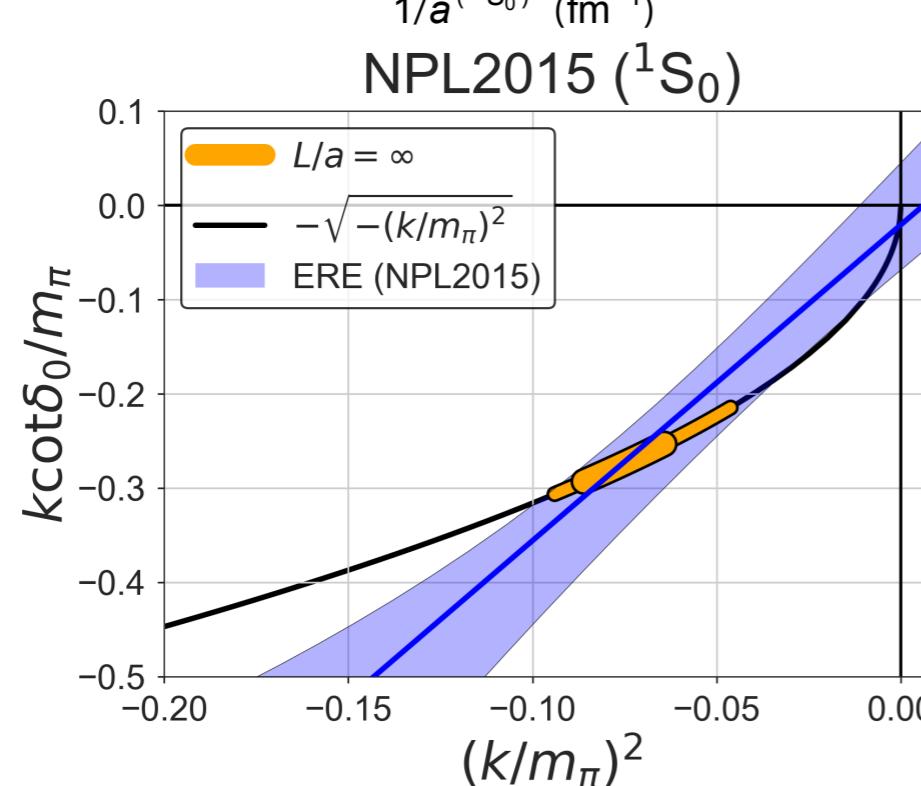
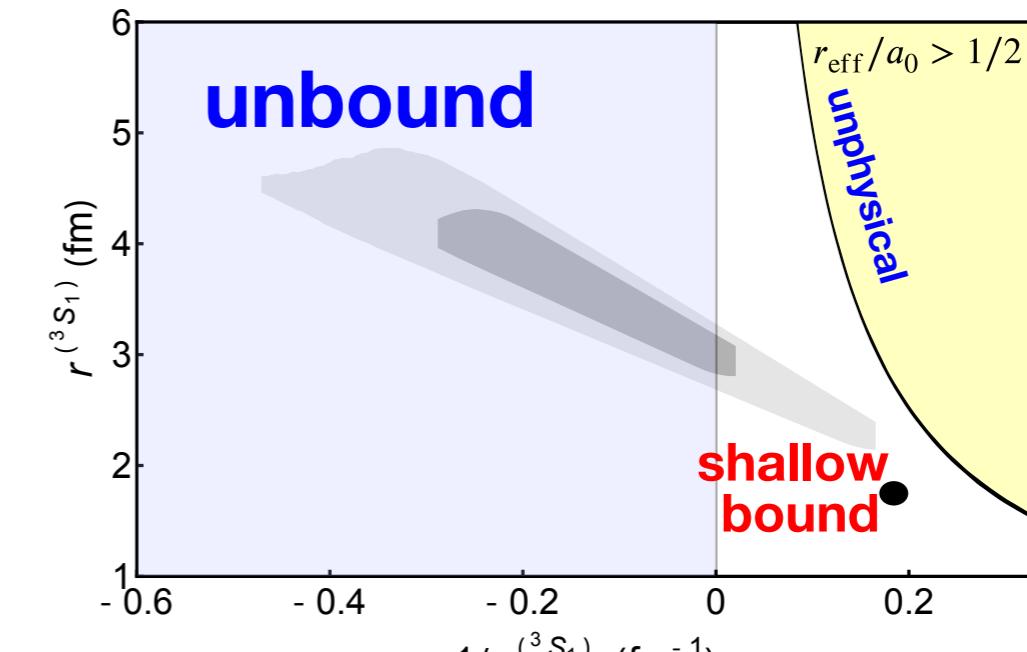
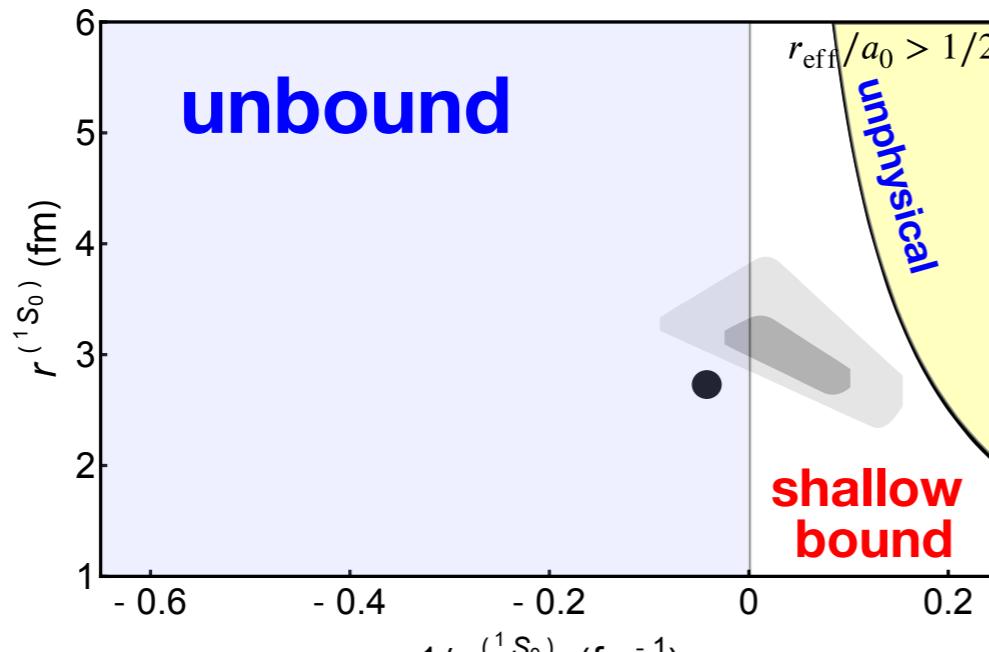
$$\frac{d}{dk^2}(k \cot \delta_0(k)) \Big|_{k^2=k_0^2} < \frac{d}{dk^2} \left( -\sqrt{-k^2} \right) \Big|_{k^2=k_0^2}$$



# NPLQCD 2015 – Incorrect ERE parameter

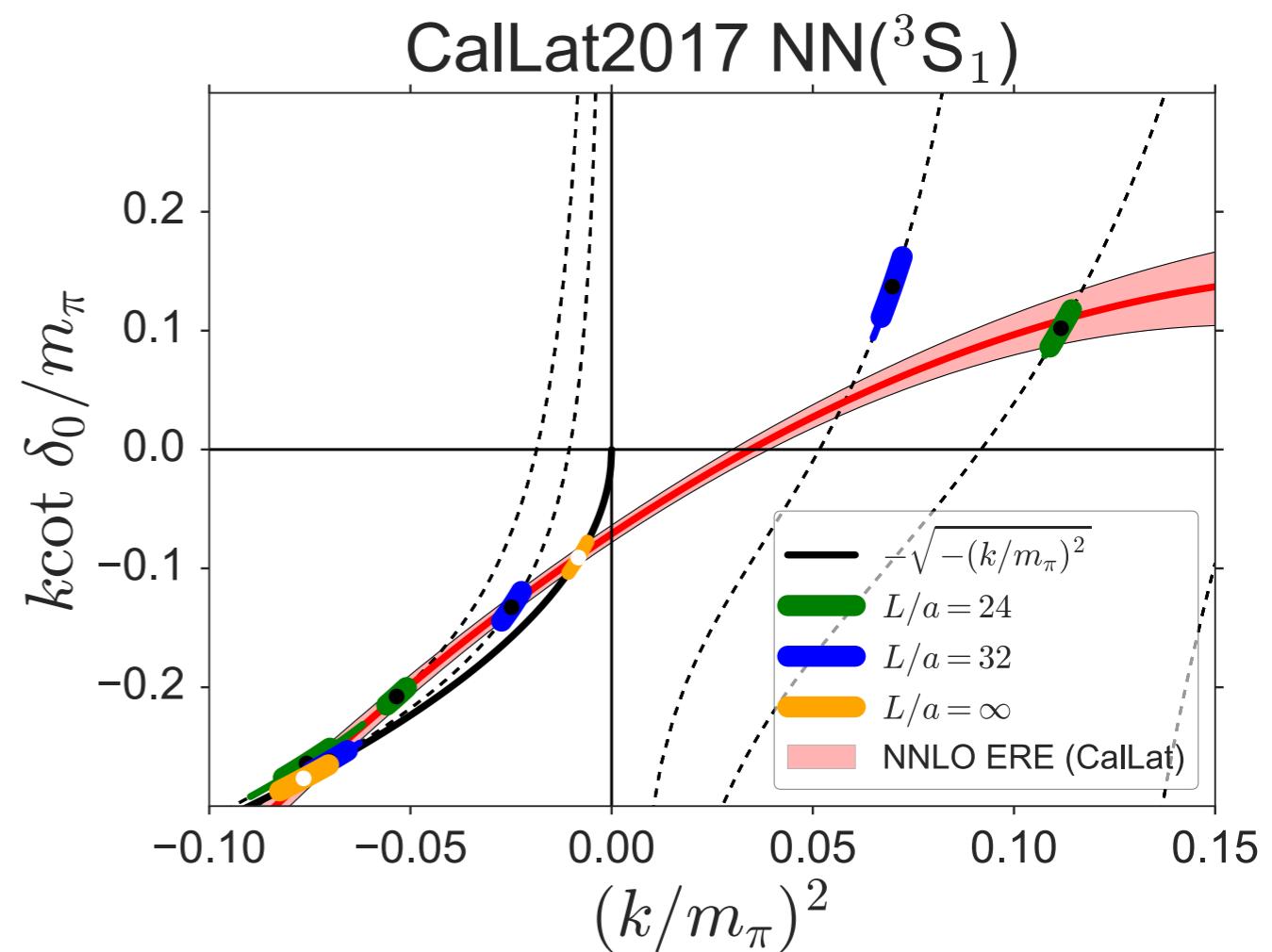
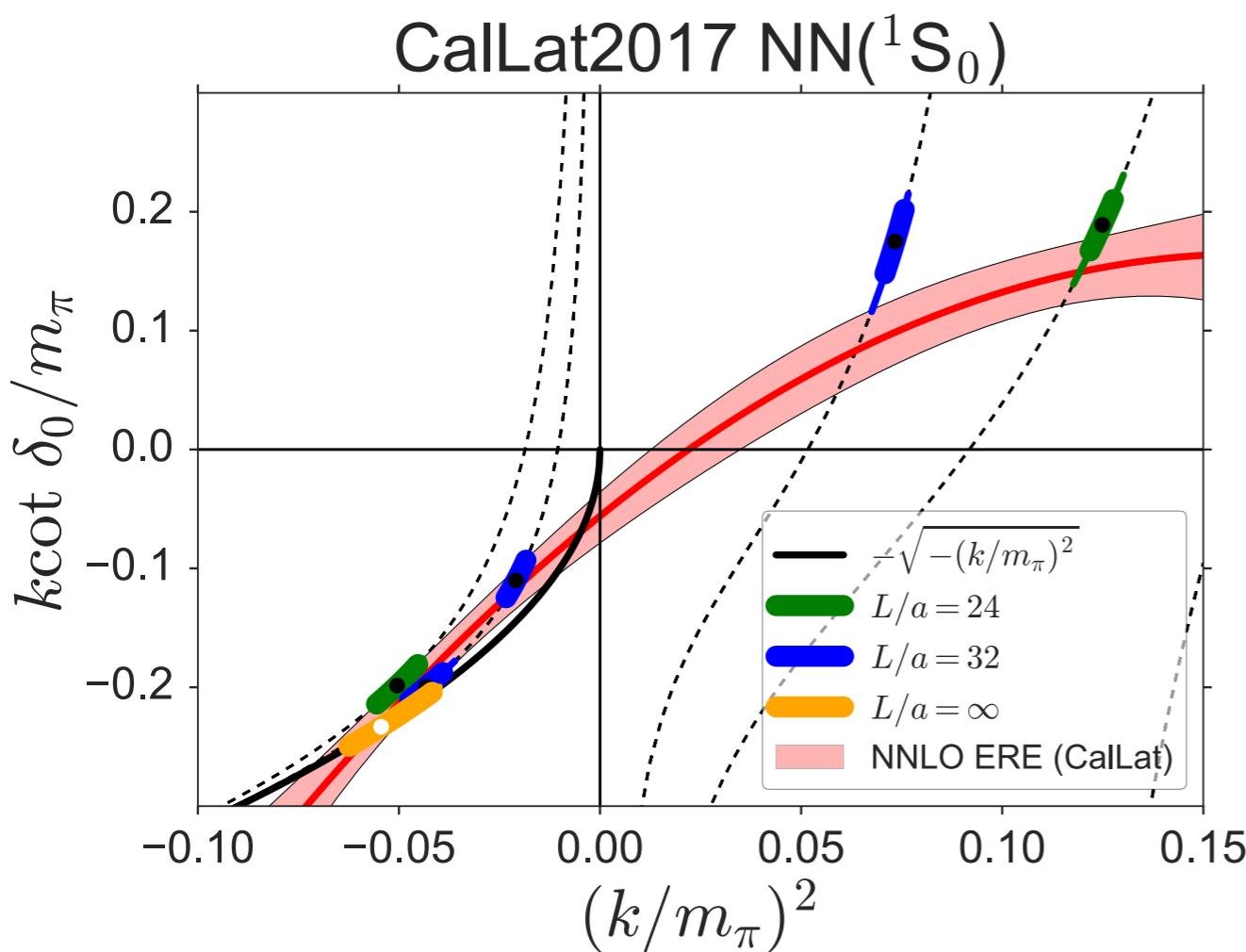
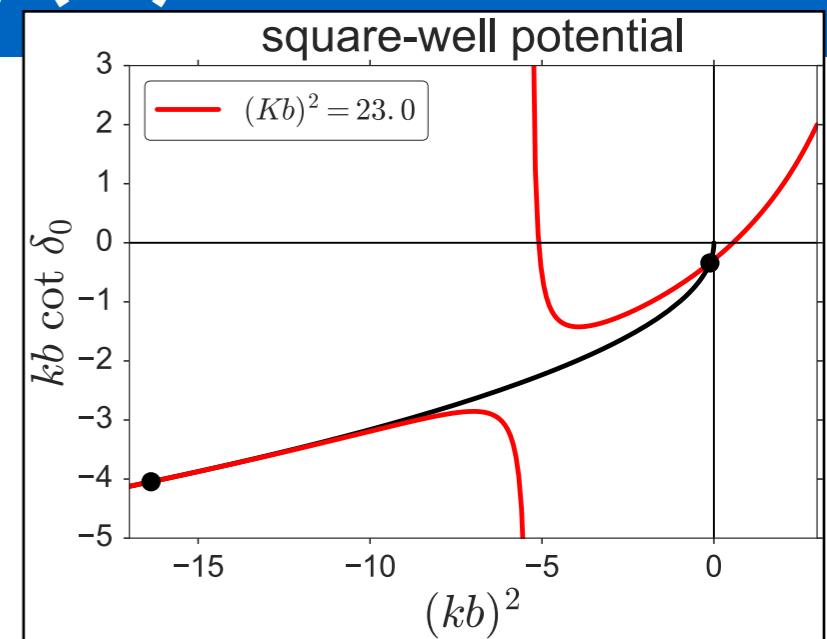
These confidence region of ERE parameters are **unreliable**.

These parameter might allow a shallow bound state  $(k/m_{\pi})^2 \sim -0.01$ , however, which is inconsistent with a deeply bound state around  $(k/m_{\pi})^2 \sim -0.1$



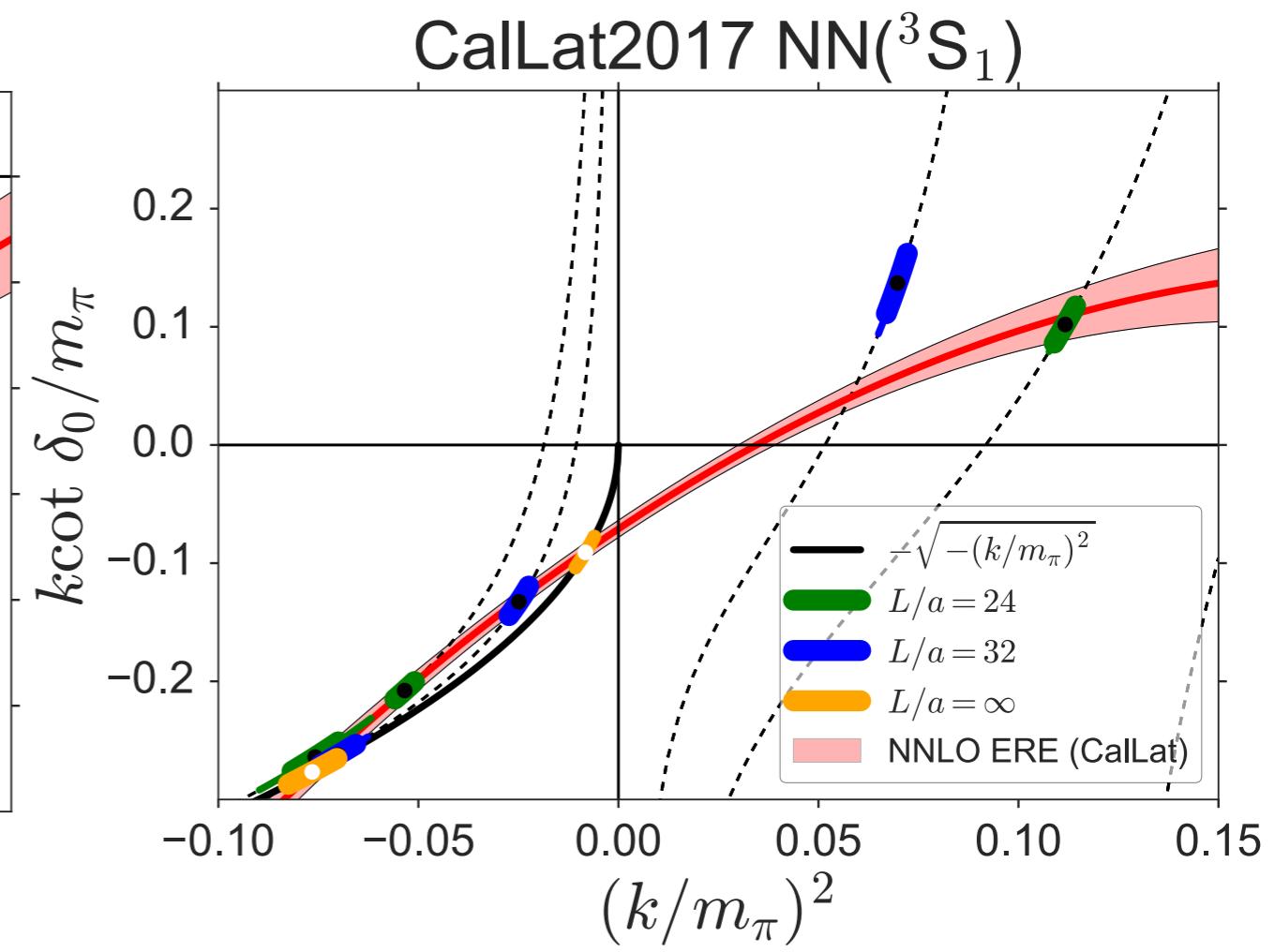
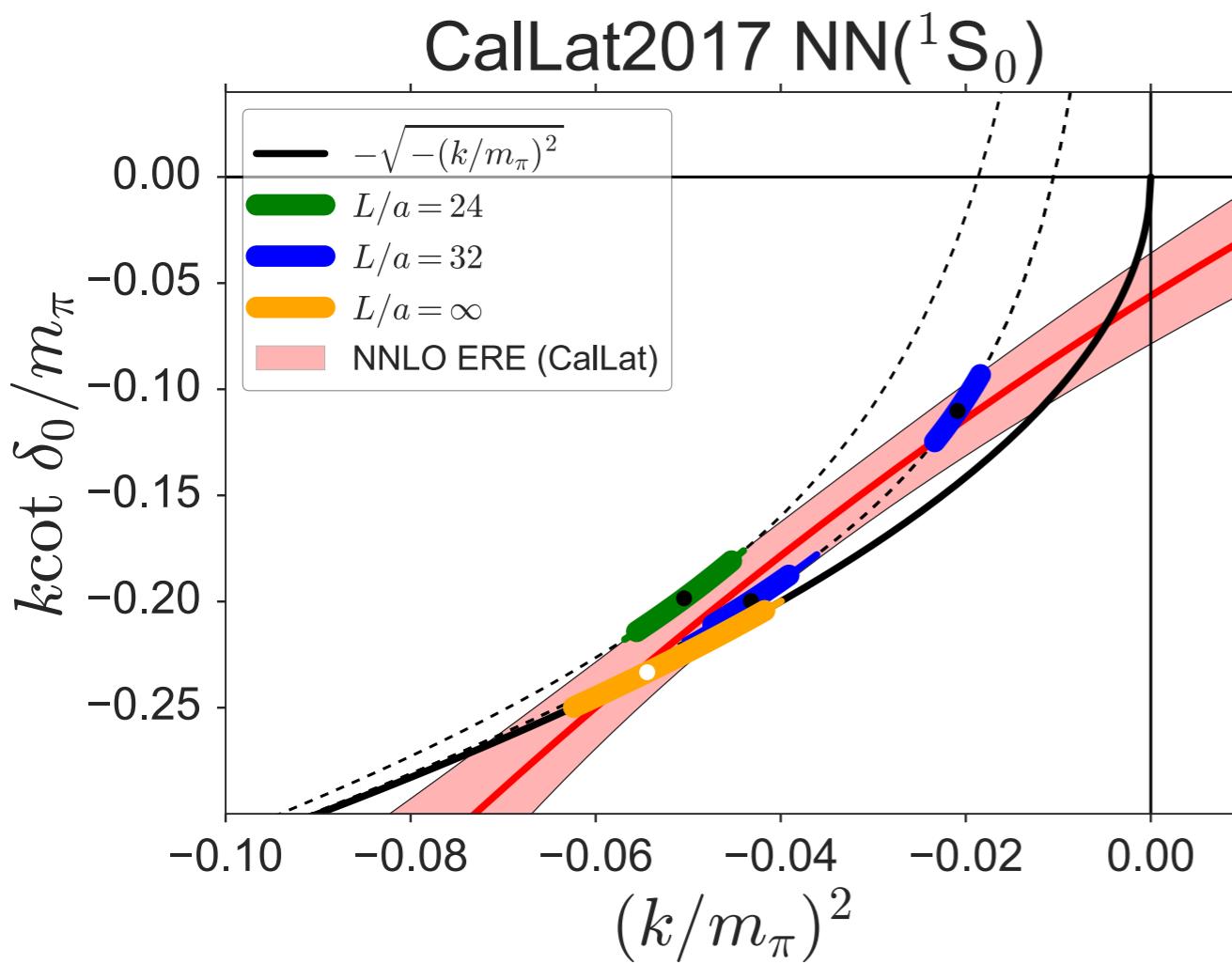
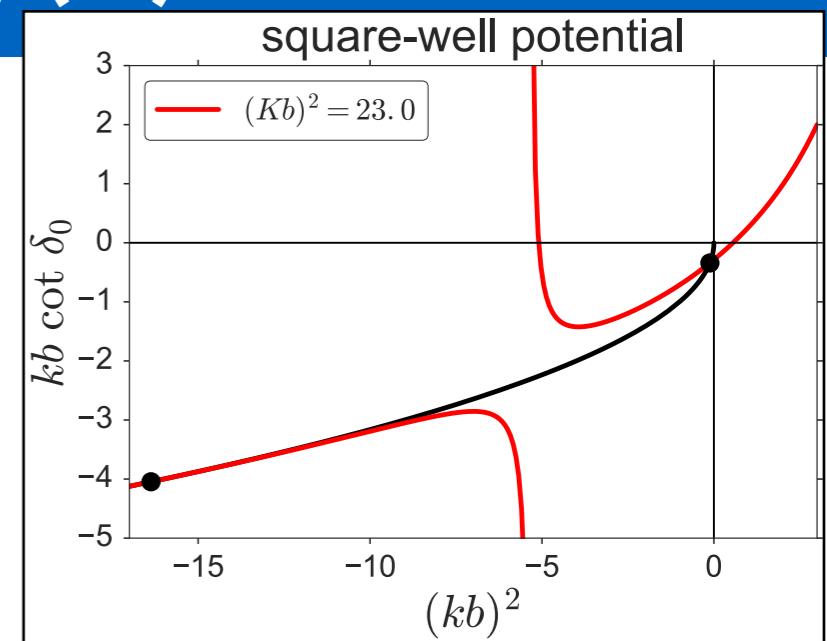
# CalLat2017 – Incorrect ERE Fitting (1)

- Physical pole condition is violated.
- Incorrect single ERE fitting for two-pole system.

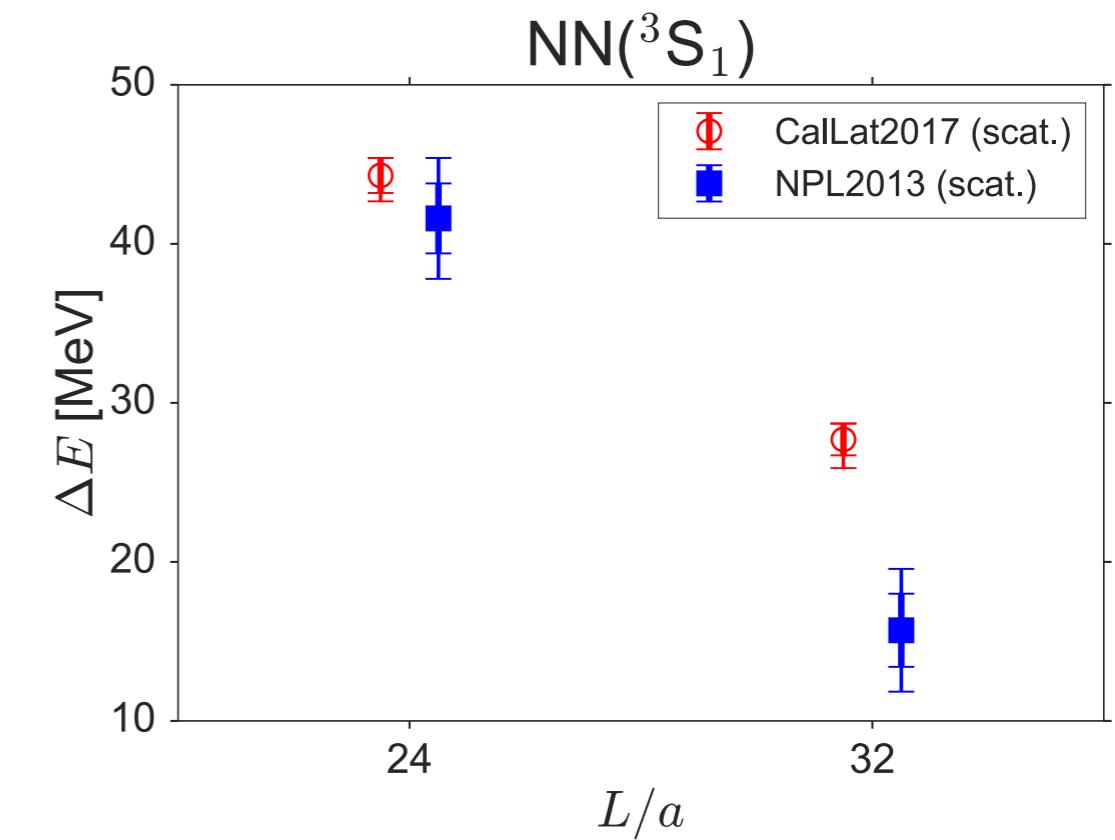
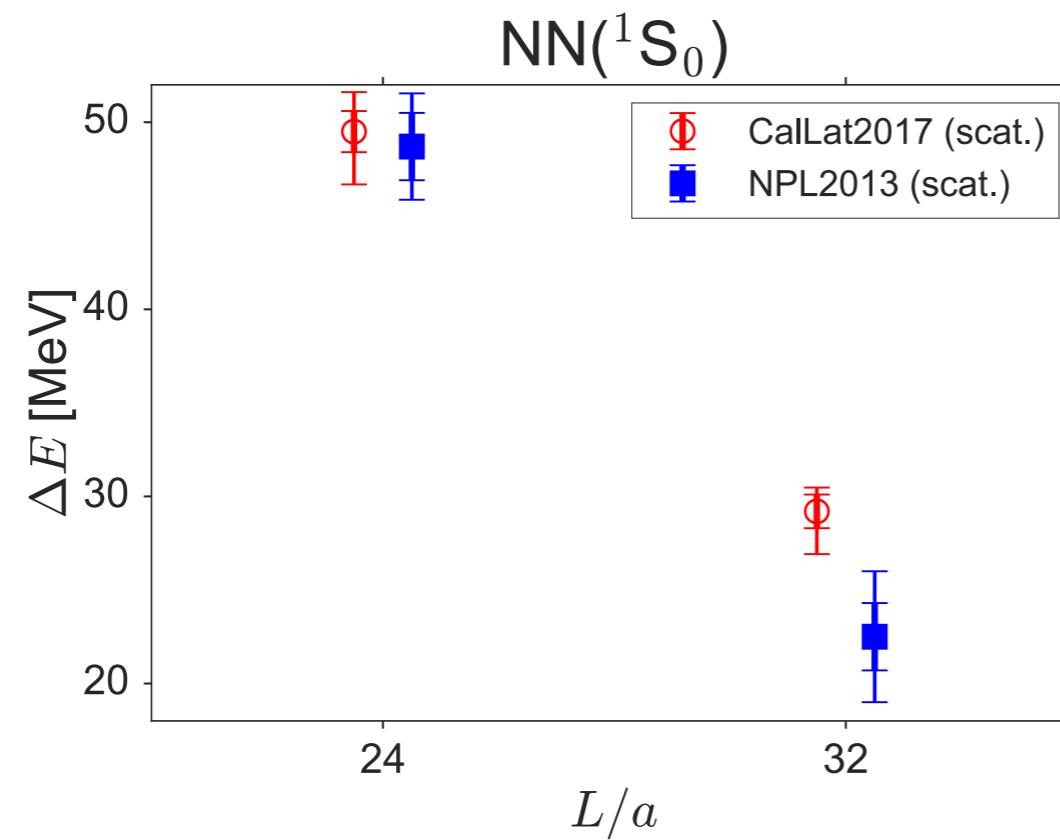
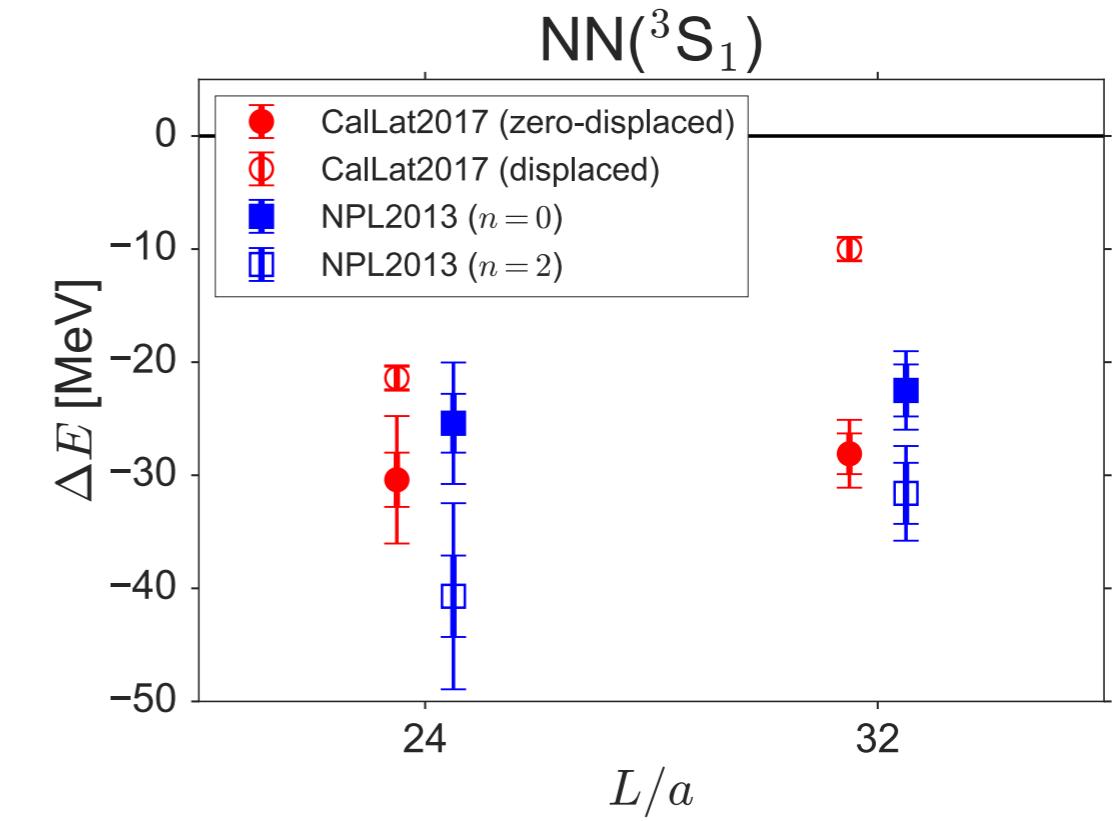
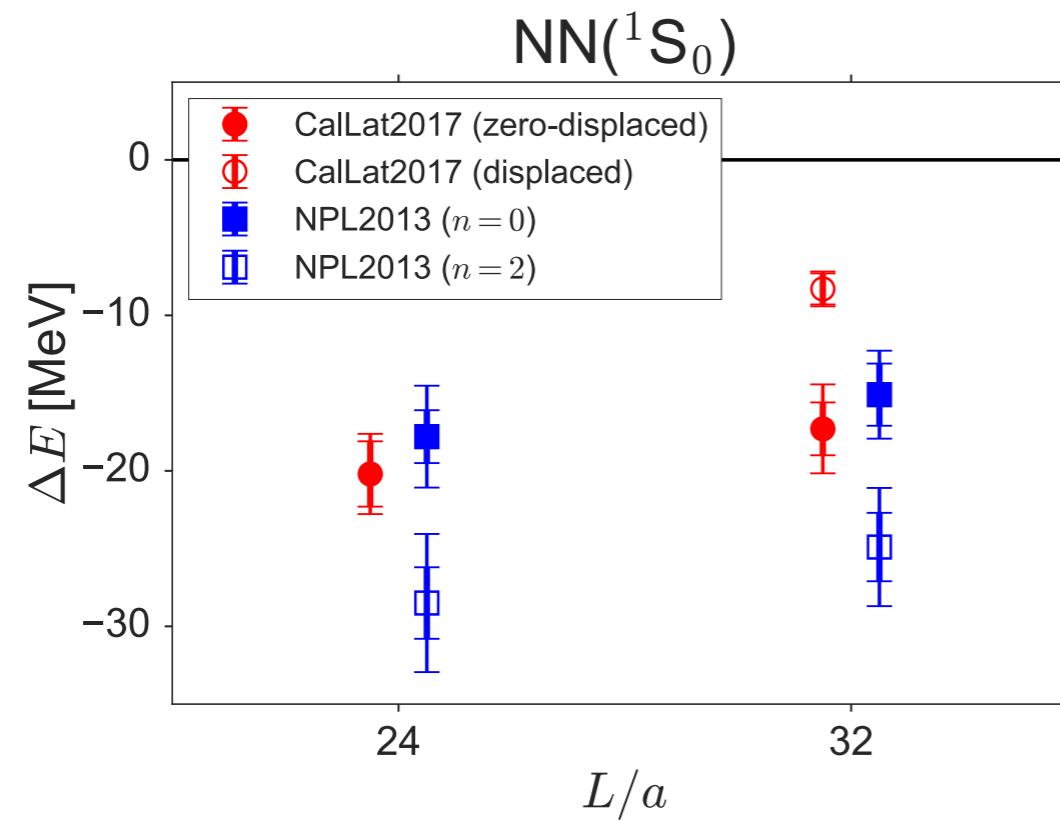


# CalLat2017 – Incorrect ERE Fitting (2)

- Physical pole condition is violated.
- ERE does not intersect with  $L = 24$  correctly.



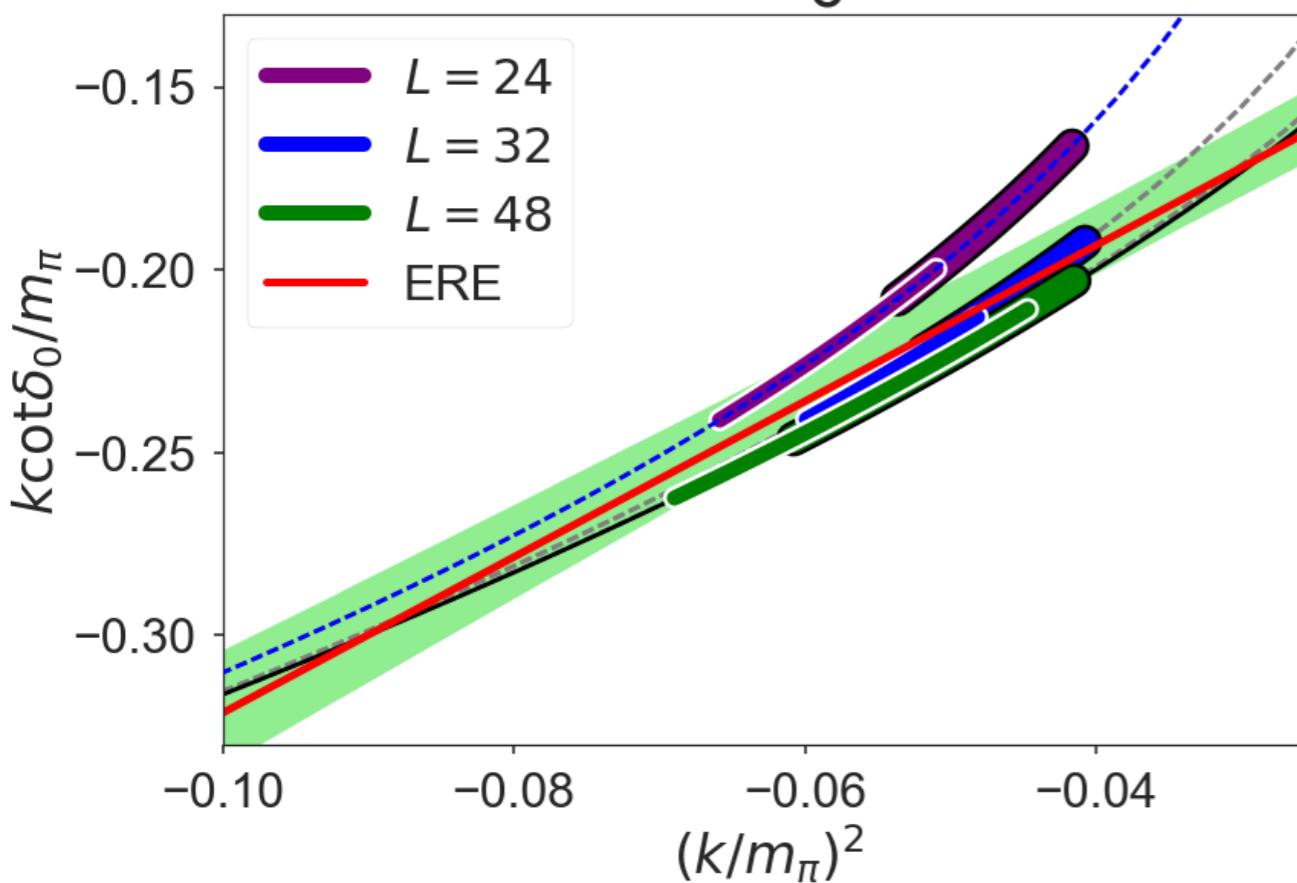
# CalLat2017 – Quark Source Dependence



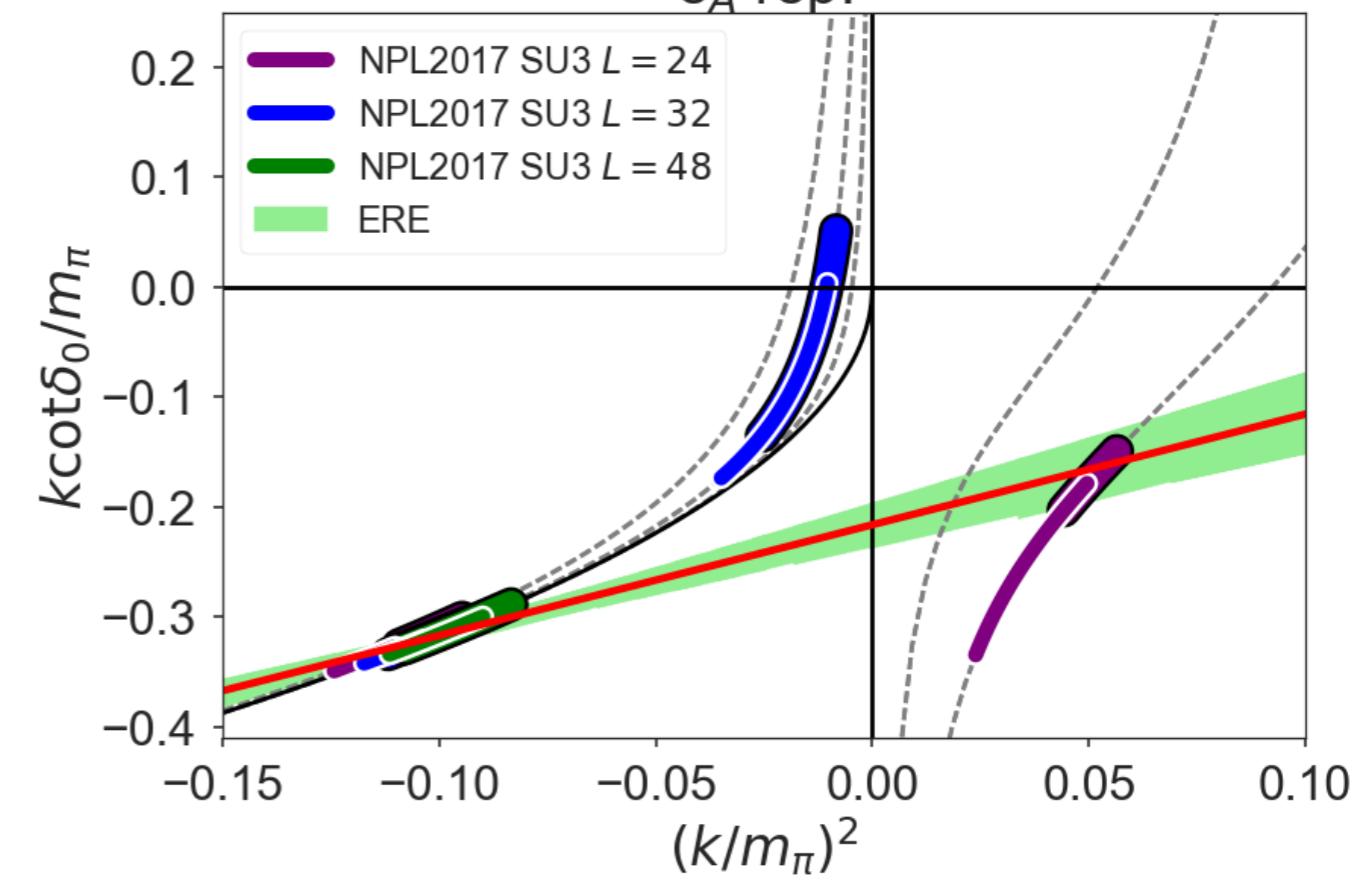
# NPLQCD 2017 – Incorrect ERE Fitting

ERE fitting w/o constraint

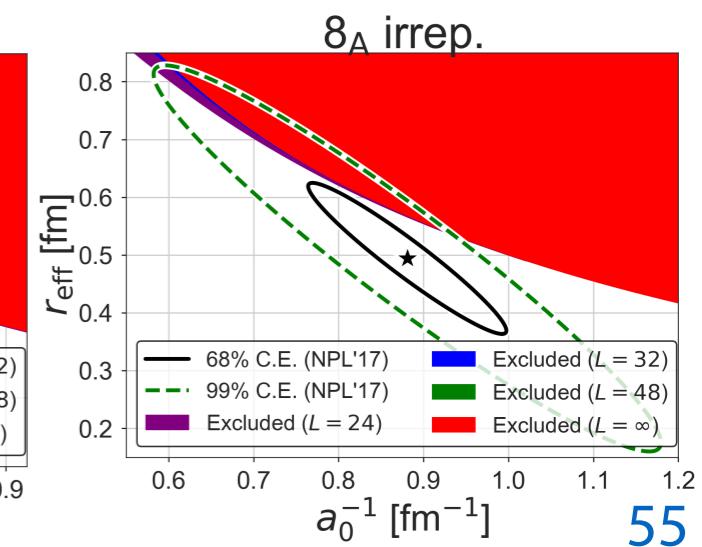
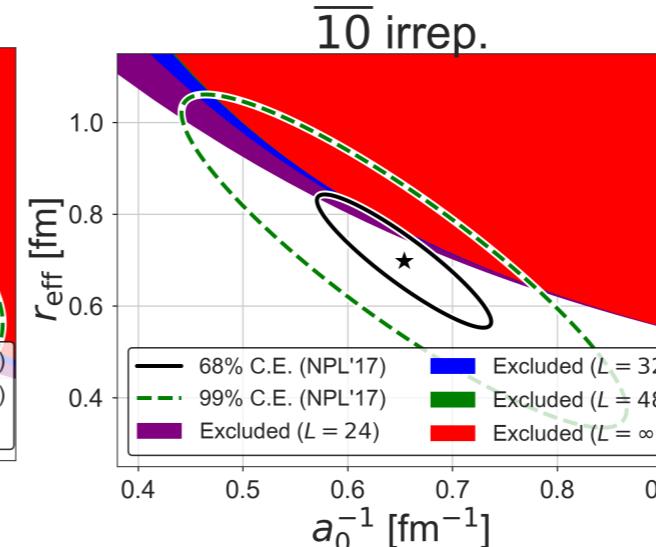
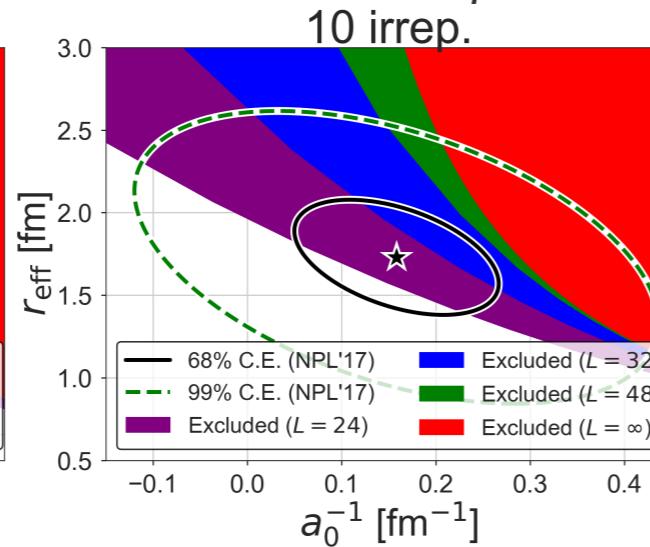
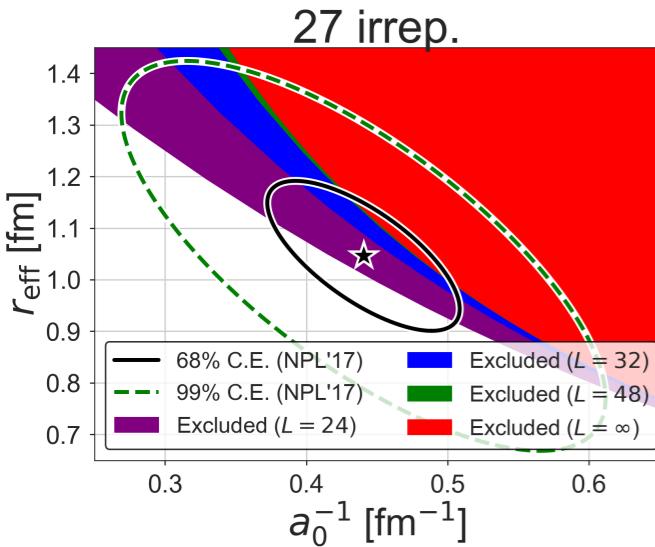
$^1S_0$



incorrect single ERE analysis  
for two-pole system  
 $8_A$  rep.

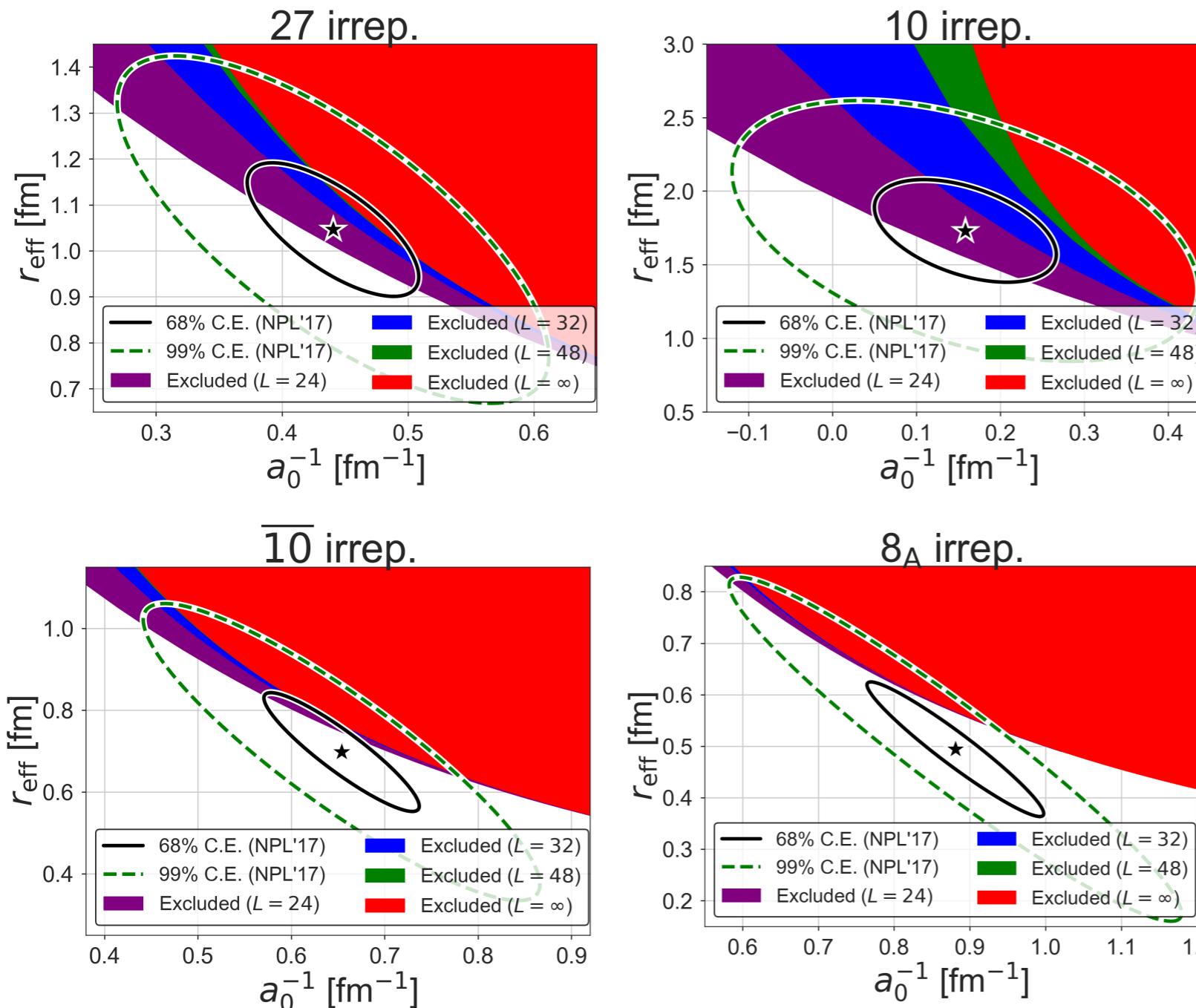


These confidence ellipses of ERE parameter are **unreliable**.

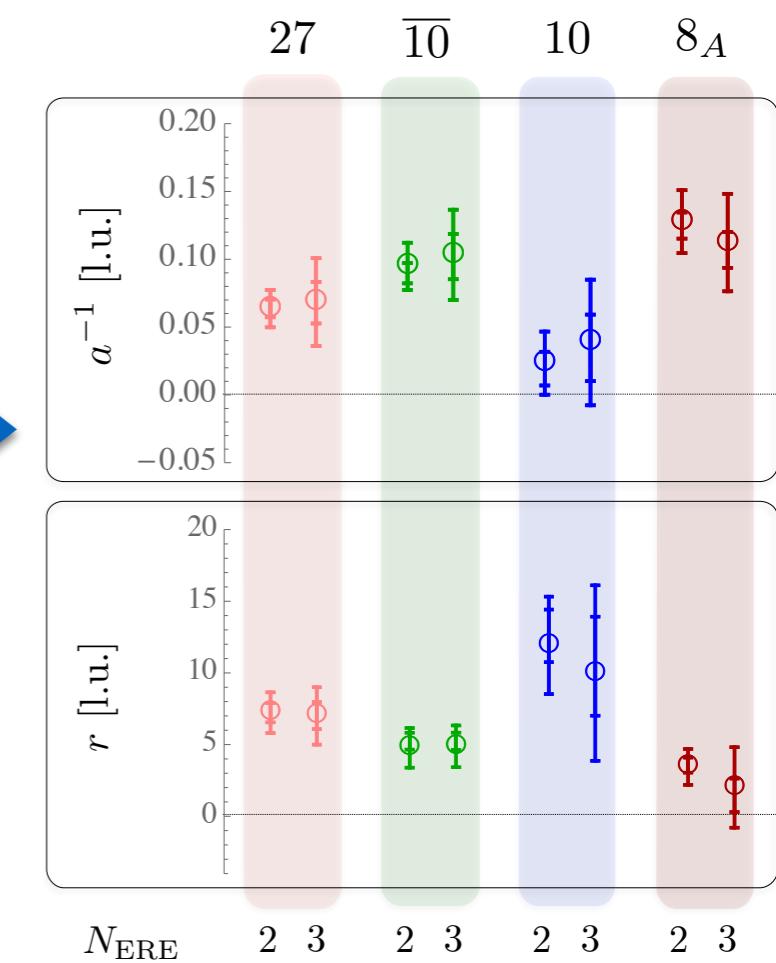


# NPLQCD 2017 – Incorrect ERE Parameter

These confidence ellipses of ERE parameter ignore the finite volume constraints.

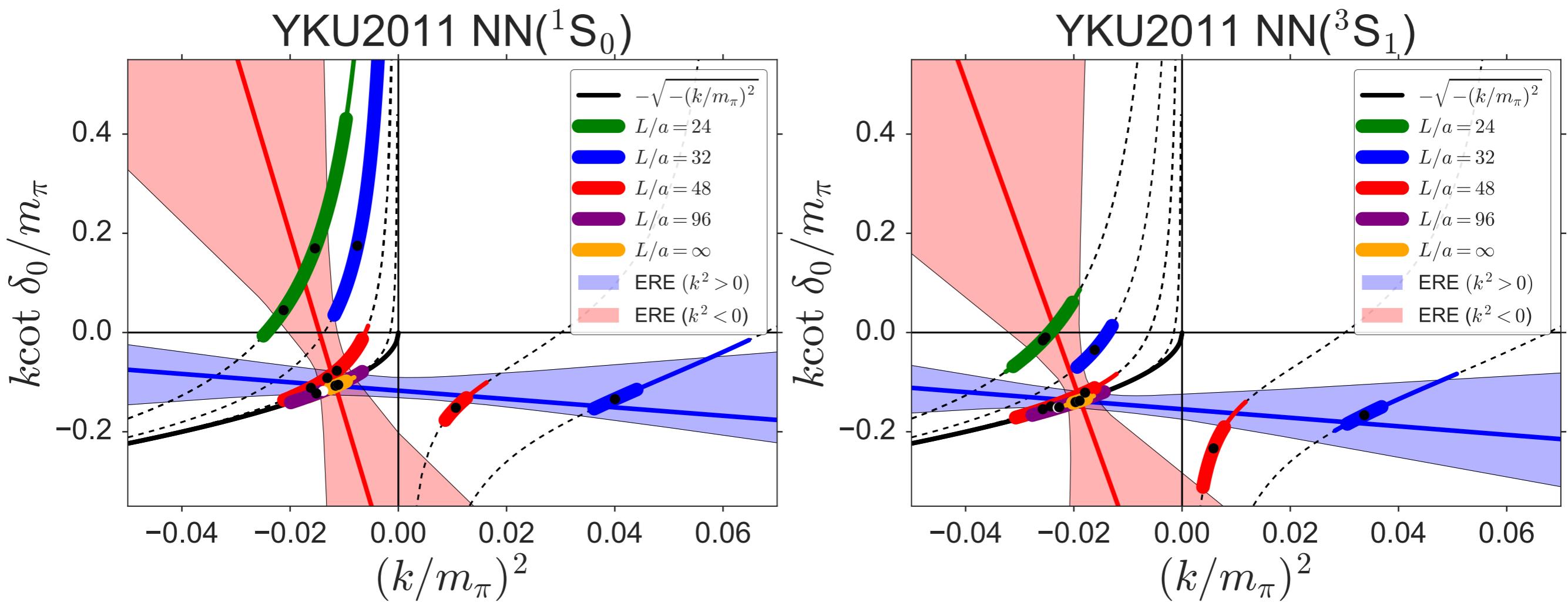


These parameters are  
**incompatible with**  
the Lüscher's formula.



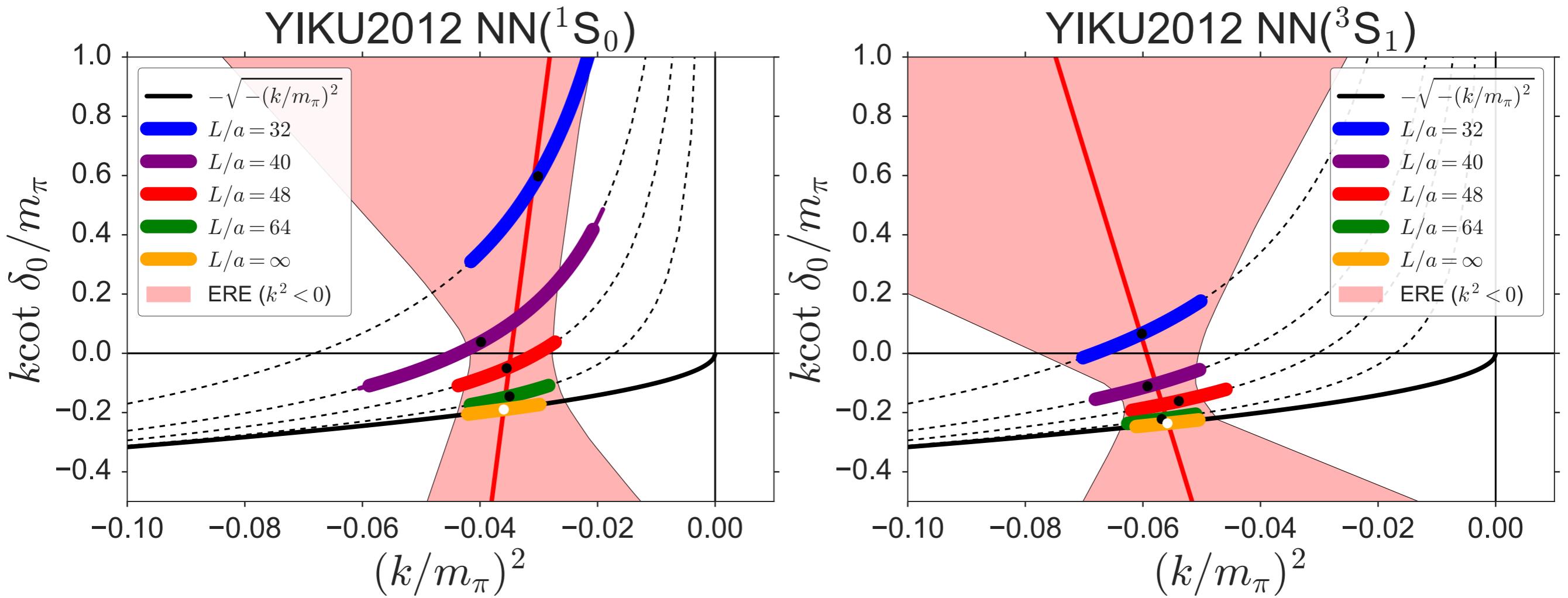
# Yamazaki et al. 2011 – Anomalous ERE

- Inconsistency between  $\text{ERE}(k^2 < 0)$  and  $\text{ERE}(k^2 > 0)$
- $\text{ERE}(k^2 < 0)$  is singular (*Volume independent plateaux*)



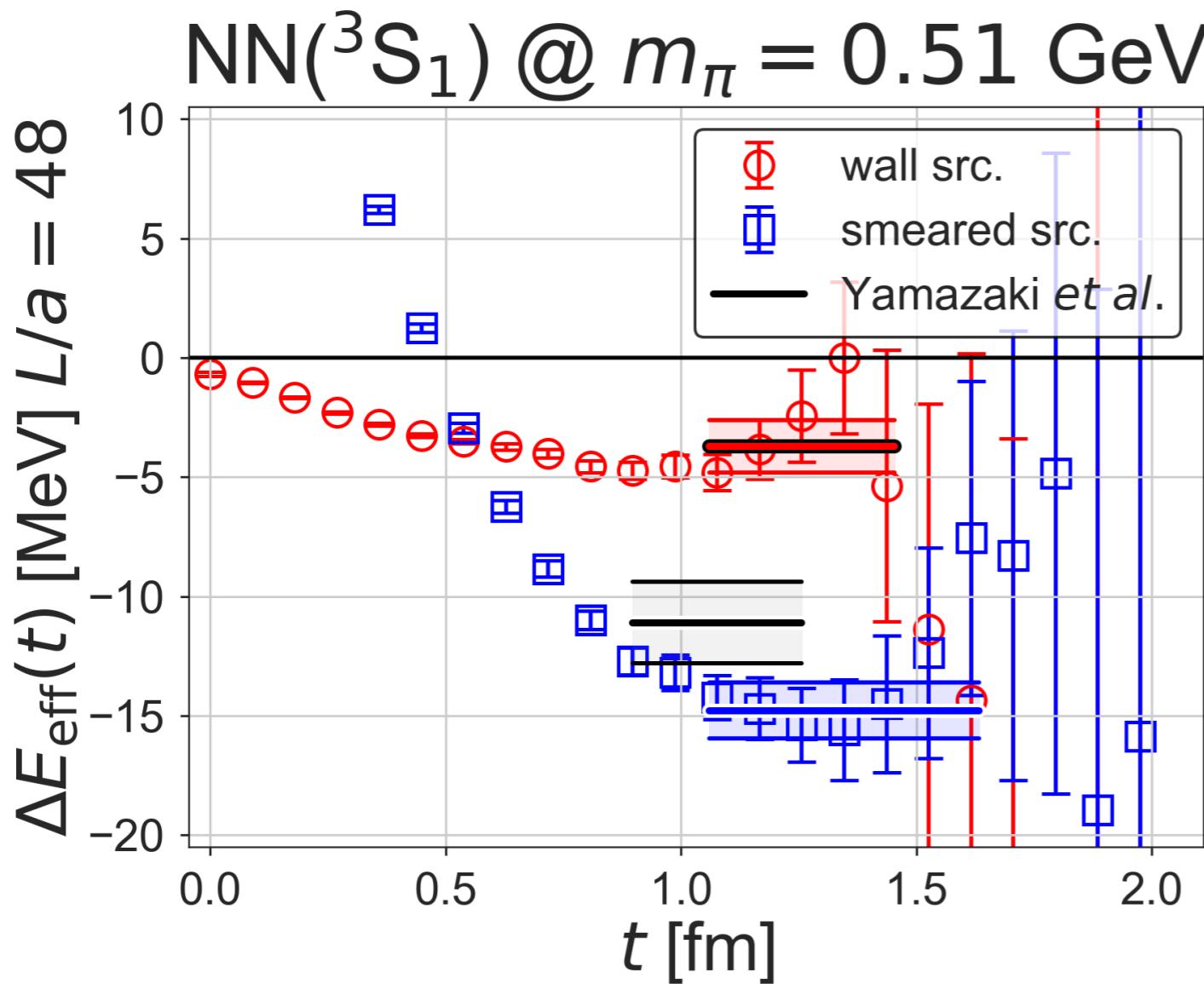
# Yamazaki et al. 2012 – Singular ERE

- Singular ERE (*Volume independent plateaux*)



# Yamazaki et al. 2012 – Quark Source Dependence

Plateau depends on the quark source & statistics

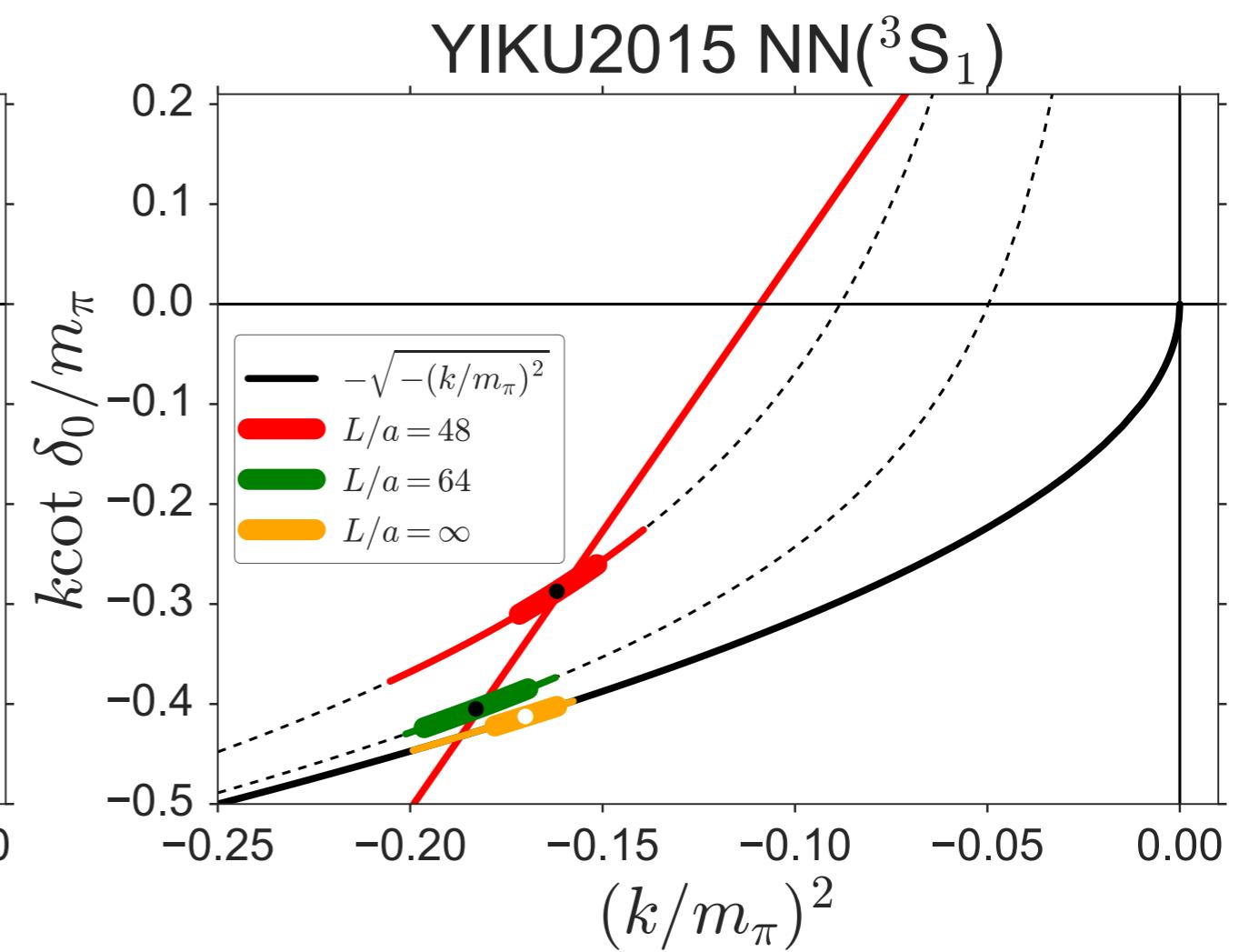
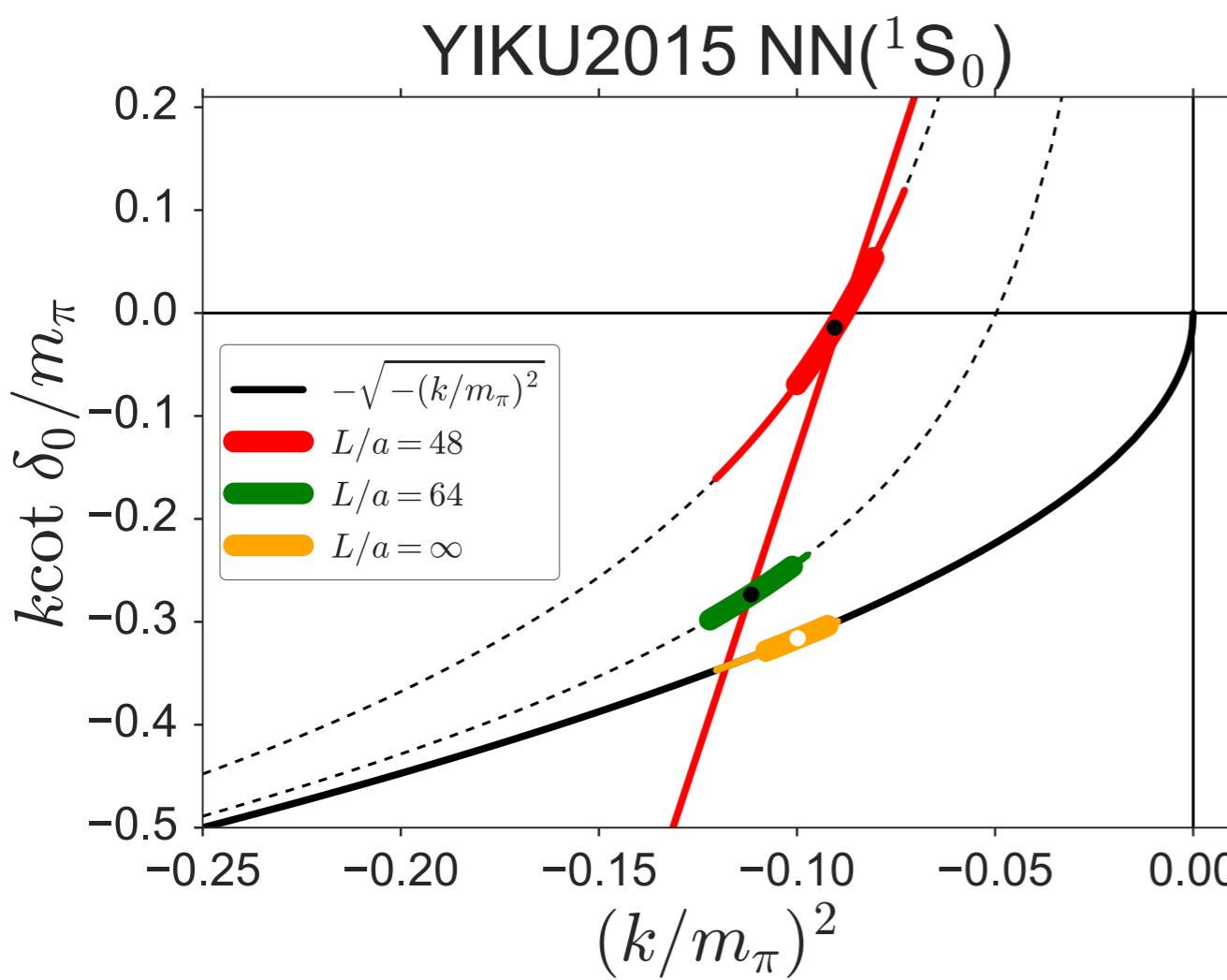


# of our smeared src.  
8 times larger than the original

Ref. TI for HAL QCD Coll.  
JHEP1610(2016)101, arXiv:1607.06371

# Yamazaki et al. 2015 – Singular ERE

- Singular ERE (*Volume independent plateaux*)
- Physical pole condition  $\frac{d}{dk^2}(k \cot \delta_0(k)) \Big|_{k^2=k_0^2} < \frac{d}{dk^2}(-\sqrt{-k^2}) \Big|_{k^2=k_0^2}$   
is violated

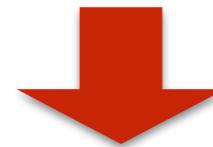


# Summary of Consistency/Normality Checks

Refs. **TI** for HAL QCD Coll., [PRD96.034521\(2017\)](#)  
 S. Aoki, T. Doi, **TI**, LATTICE2017 Proc., [arXiv:1707.08800](#).

These *Effective Range Expansion* analyses  
 are also **problematic & unreliable**.

These exhibit unreasonable behaviors  
 due to the fake plateaux.



	source independence	consistency of $k^2 < 0$ and $k^2 > 0$	non-singular ERE	physical residue	ERE fitting with constraints	ERE fitting for 2-pole system	Reasonable ERE for bound states
Yamazaki et al. 2011		No	No				
NPLQCD 2012			No				
Yamazaki et al. 2012		No	No				
NPLQCD 2013		No		No	No		No
NPLQCD 2015		No		No	No		No
Yamazaki et al. 2015			No	No			
CalLat 2017		No		No	No	No	No
NPLQCD 2017				No	No	No	No