Confinement of quarks in higher representations in view of dual superconductivity

Akihiro Shibata (KEK)

In collaboration with

Seikou Kato (Oyama college)
Kei-Ichi Kondo (Chiba University)
Ryutaro Matsudo (Chiba Univers)

Introduction:: dual superconductivity

- Dual superconductivity is a promising mechanism for quark confinement.
  [Y.Nambu (1974). G.'t Hooft, (1975). S.Mandelstam, (1976) A.M. Polyakov (1975)]
- In this scenario, QCD vacuum is considered as a dual super conductor.

superconductor

- Condensation of electric charges (Cooper pairs)
- Meissner effect: Abrikosov string (magnetic flux tube) connecting monopole and anti-monopole
- Linear potential between monopoles

dual superconductor

- Condensation of magnetic monopoles
- Dual Meissner effect: formation of a hadron string (chromo-electric flux tube) connecting quark and antiquark
- Linear potential between quarks

Evidence for the dual superconductivity :: fundamental rep. (I)

Abelian projection in Maximal Abelian gauge

Extracted the relevant mode for quark confinement as a diagonal part in some gauge

■ SU(2) case : Abelian projection $SU(2) \to U(1)$

✓ Abelian Dominance in the string tension by Suzuki-Yotsuyanagi (1990) , by Stack-Tucker-Wensley (2002)

✓ Monopole dominance in the string tension (DeGrant-Toussaint) by Stack-Tucker-Wensley (2002)

■ SU(3) case; Abelian projection SU(2)→U(1)×U(1)

✓ Abelian Dominance by Shiba-Suzuki (1994)

✓ perfect dominance by Sakumichi-Suganuma (2016)

✓ Monopole dominance by Stack-Tucker-Wensley (2002)

Problem:

Color (global) symmetry and gauge symmetry is broken.

Evidence for the dual superconductivity :: fundamental rep. (II)

Gauge decomposition method (our new formulation)

- Extracting the relevant mode *V* for quark confinement by solving the defining equation in the gauge independent way (gauge-invariant way)
- SU(2) case: a lattice compact representation of the Cho-Duan-Ge-Faddeev-Niemi-Shabanov (CDGFNS) decomposition.

we have showed that

- ➢ almost perfect V-field dominance, magnetic monopole dominance in string tension
- ➢ chromo-electric flux tube and dual Meissner effect.
- ➢ The vacuum of dual superconductor is of Type I

[Phys.Rev. D91 (2015) 3, 034506]

Evidence for the dual superconductivity :: fundamental rep. (III)

Gauge decomposition method SU(3) case

✓Extension of SU(2) case and two options

- **Maximal option** (Cho-Faddev-Niemi decomposition also N Cundy, Y.M. Cho et.al])
- **Minimal option** (our proposed non-Abelian dual superconductivity)

➔**for minimal option that we have showed in the series works**

➢ V-field dominance, non-Abalian magnetic monopole dominance in string tension,

➢ chromo-flux tube and dual Meissner effect.

➢The first observation on quark confinement/deconfinement phase transition in terms of dual Meissner effect at finite temperature

➔**for minimal option**

➢The same with the minimal option , [ours][N Cundy, Y.M. Cho et.al]

To establish dual superconductivity

- We must show that monopole plays a dominant role for the Wilson loops in higher representations as well as in the fundamental representation.
- In the previous studies, these sometimes made naïve replacement of Wilson loop operator between the Yan-Mills field and Abelian projected filed
  - E.g., recentry in order to test the mechanism of quark confinement, J.Greensite and R.Hollwieser compares the double winding Wilson loop in SU(2) Yang-Mills theory made of Yang-Mills field, Abelian projected field in the MAG, and the center in the maximal center gauge. [PRD91 054509 (2015)]
- In this talk, we investigate the Wilson loop by using our presented new formulation of the Lattice Yang-Mills theory based on the non-Abelian Stokes theorem.

Non-Abelian Stokes theorem

Kondo and Matsudo RRD92 125083 (2015)

Non-Abelian theorem in the presentation R can be given by

$$W_C[A] = \int [d\mu(g)]_C \exp\left(ig \oint \langle \Lambda | A^U | \Lambda \rangle \right) = \int [d\mu(g)]_\Sigma \exp\left(ig \int_{\Sigma:\partial\Sigma = C} d(\langle \Lambda | A^U | \Lambda \rangle) \right)$$

where $[d\mu(g)]_C$ and $[d\mu(g)]_\Sigma$ are the product of the Haar measure over the loop and a surface, respectively. $A^{U^\dagger} := UAU^\dagger + ig^{-1}UdU$, and $|\Lambda\rangle$ the highest weight state of the representation R.

OR

$$W_C[\mathscr{A}] = \int [d\mu(g)]_\Sigma \exp\left[-ig_{\mathrm{YM}} \int_{\Sigma:\partial\Sigma = C} F^g \right] \qquad F^g := \frac{1}{2} f^g_{\mu\nu}(x) dx^\mu \wedge dx^\nu$$

$$F^g_{\mu\nu}(x) = \Lambda_j \{ \partial_\mu [n^A_j(x) \mathscr{A}^A_\nu(x)] - \partial_\nu [n^A_j(x) \mathscr{A}^A_\mu(x)] - g^{-1}_{\mathrm{YM}} f^{ABC} n^A_j(x) \partial_\mu n^B_k(x) \partial_\nu n^C_k(x) \},$$

$$\boldsymbol{n}_j(x) = g(x) H_j g^\dagger(x) = n^A_j(x) T_A (j = 1, \ldots, r).$$

Lattice study of the Wilson loop in the representation R

- Wilson loops in the representation R can be calculated by using the multi-winding Wilson loop in the fundamental representation. .For example,

$$SU(2)\ \text{case}:\ 2\otimes 2 = 2\otimes 2^* = 1\oplus 3$$

$$\langle W(C\times C)\rangle = -\frac{1}{2} + \frac{3}{2}\langle W_{\mathrm{adj}}(C)\rangle$$

$$W_{Ad} = \frac{1}{2}\int D\mu[\xi]\mathrm{tr}(V^2) \qquad V := \prod V_{x,\mu}$$

$$SU(3)\ \text{case}:\ 3\otimes 3 = 3^*\oplus 6$$

$$\langle W(C\times C)\rangle = -\langle W(C)_{[0,1]}\rangle + 2\langle W(C)_{[2,0]}\rangle$$

$$W_{(0,1)} = \frac{1}{3}\int D\mu[\xi]\mathrm{tr}(V)^* \quad V := \prod V_{x,\mu}$$

$$W_{(2,0)} = \frac{1}{3}\int D\mu[\xi]\mathrm{tr}(V^2)$$

Dμ[ξ] is Invariant integration measure from non-Abelain Stokes theorem.
We carry out integral by using the **reduction condition, i.e.,** the integral is replaced by V-field which is decomposed by using the color field determined from the reduction condition. (see follwing)

A new formulation of Yang-Mills theory (on a lattice) for **fundamental representation**

Decomposition of SU(N) gauge links Phys.Rept. 579 (2015) 1-226

- For SU(N) YM gauge link, there are several possible options of decomposition *discriminated by its stability groups*:

□SU(2) Yang-Mills link variables: unique $U(1) \subset SU(2)$

□SU(3) Yang-Mills link variables: **Two options**

minimal option : $U(2) \cong SU(2) \times U(1) \subset SU(3)$

✓Minimal case is derived for the Wilson loop, defined for quark in the fundamental representation, which follows from the non-Abelian Stokes' theorem

maximal option : $U(1) \times U(1) \subset SU(3)$

✓Maximal case is a gauge invariant version of Abelian projection in the maximal Abelian (MA) gauge. (the maximal torus group)

The decomposition of SU(3) link variable: minimal option

$$W_C[U] := \mathrm{Tr}\left[P \prod_{\langle x, x+\mu\rangle \in C} U_{x,\mu} \right] / \mathrm{Tr}(\mathbf{1})$$

$$U_{x,\mu} = X_{x,\mu} V_{x,\mu}$$

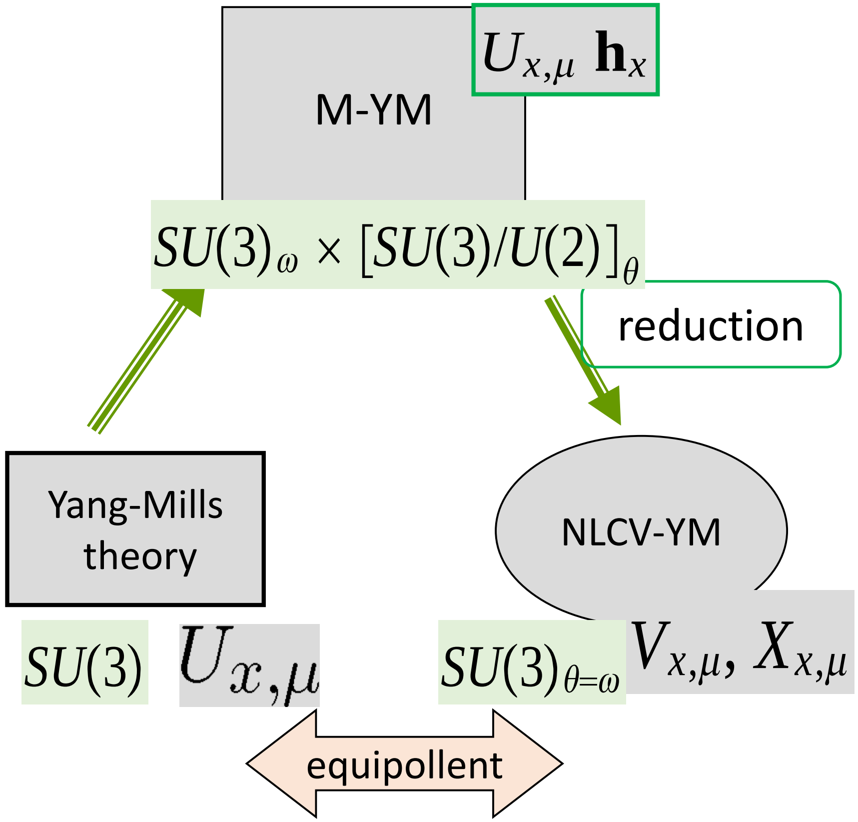
$$U_{x,\mu} \to U'_{x,\mu} = \Omega_x U_{x,\mu} \Omega^\dagger_{x+\mu}$$

$$V_{x,\mu} \to V'_{x,\mu} = \Omega_x V_{x,\mu} \Omega^\dagger_{x+\mu}$$

$$X_{x,\mu} \to X'_{x,\mu} = \boxed{\Omega_x X_{x,\mu} \Omega^\dagger_x}$$

$$\Omega_x \in G = SU(N)$$

$$W_C[V] := \mathrm{Tr}\left[P \prod_{\langle x, x+\mu\rangle \in C} V_{x,\mu} \right] / \mathrm{Tr}(\mathbf{1})$$



$$W_C[U] = \mathrm{const.} W_C[V] \quad !!$$

Defining equation for the decomposition : minimal option

Introducing a color field $\mathbf{h}_x = \xi(\lambda^8/2)\xi^\dagger \in SU(3)/U(2)$ with $\xi \in SU(3)$, a set of the defining equation of decomposition $U_{x,\mu} = X_{x,\mu}V_{x,\mu}$ is given by

$$D_\mu^\epsilon[V]\mathbf{h}_x = \frac{1}{\epsilon}(V_{x,\mu}\mathbf{h}_{x+\mu} - \mathbf{h}_x V_{x,\mu}) = 0,$$

$$g_x = e^{-2\pi q_x/N}\exp(-a_x^{(0)}\mathbf{h}_x - i\sum_{i=1}^{3} a_x^{(l)}u_x^{(i)}) = 1,$$

which correspond to the continuum version of the decomposition, $\mathcal{A}_\mu(x) = \mathcal{V}_\mu(x) + \mathcal{X}_\mu(x)$,

$$D_\mu[\mathcal{V}_\mu(x)]\mathbf{h}(x) = 0, \qquad \mathrm{tr}(\mathcal{X}_\mu(x)\mathbf{h}(x)) = 0.$$

Note that the field strength of V-field appears in the Non-Abelian Stokes theorem for fundamental representation.

Exact solution (N=3)

$$X_{x,\mu} = \hat{L}_{x,\mu}^\dagger(\det\hat{L}_{x,\mu})^{1/N} g_x^{-1} \quad V_{x,\mu} = X_{x,\mu}^\dagger U_{x,} = g_x\hat{L}_{x,\mu}U_{x,}(\det\hat{L}_{x,\mu})^{-1/N}$$

$$\hat{L}_{x,\mu} = \left(\sqrt{L_{x,\mu}L_{x,\mu}^\dagger}\right)^{-1} L_{x,\mu}$$

$$L_{x,\mu} = \frac{N^2-2N+2}{N}\mathbf{1} + (N-2)\sqrt{\frac{2(N-2)}{N}}(\mathbf{h}_x + U_{x,\mu}\mathbf{h}_{x+\mu}U_{x,\mu}^{-1}) + 4(N-1)\mathbf{h}_x U_{x,\mu}\mathbf{h}_{x+\mu}U_{x,\mu}^{-1}$$

continuum limit

$$\mathbf{V}_\mu(x) = \mathbf{A}_\mu(x) - \frac{2(N-1)}{N}[\mathbf{h}(x),[\mathbf{h}(x),\mathbf{A}_\mu(x)]] - ig^{-1}\frac{2(N-1)}{N}[\partial_\mu\mathbf{h}(x),\mathbf{h}(x)],$$

$$\mathbf{X}_\mu(x) = \frac{2(N-1)}{N}[\mathbf{h}(x),[\mathbf{h}(x),\mathbf{A}_\mu(x)]] + ig^{-1}\frac{2(N-1)}{N}[\partial_\mu\mathbf{h}(x),\mathbf{h}(x)].$$

The decomposition of SU(3) link variable: maximal option

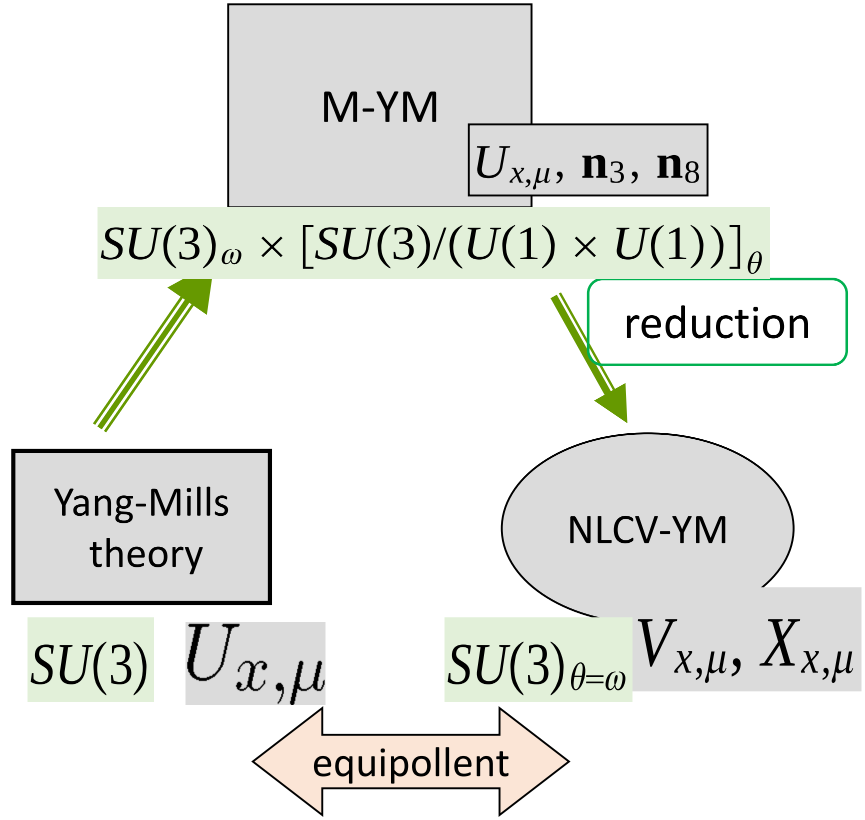
$$U_{x,\mu} = X_{x,\mu} V_{x,\mu}$$

$$U_{x,\mu} \to U'_{x,\mu} = \Omega_x U_{x,\mu} \Omega^\dagger_{x+\mu}$$

$$V_{x,\mu} \to V'_{x,\mu} = \Omega_x V_{x,\mu} \Omega^\dagger_{x+\mu}$$

$$X_{x,\mu} \to X'_{x,\mu} = \boxed{\Omega_x X_{x,\mu} \Omega^\dagger_x}$$

$$\Omega_x \in G = SU(N)$$



Gauge invariant construction of the Abelian projection to maximal torus group U(1) x U(1) in MA gauge.

Defining equation for the decomposition: maximal option

By introducing color fields $\mathbf{n}_x^{(3)} = \Theta_x(\lambda^3/2)\Theta^\dagger$, $\mathbf{n}_x^{(8)} = \Theta_x(\lambda^8/2)\Theta^\dagger$ $\in SU(3)_\omega \times [SU(3)/(U(1)\times U(1))]_\theta$, a set of the defining equation for the decomposition $U_{x,\mu} = X_{x,\mu}V_{x,\mu}$ is given by

$$D_\mu^\varepsilon[V]n_x^{(k)} = \frac{1}{\varepsilon}(V_{x,\mu}n_{x+\mu}^{(k)} - n_x^{(k)}V_{x,\mu}) = 0, \quad (k = 3,8)$$

$$g_x = \exp(2\pi i n/N)\exp(i\sum\nolimits_{j=3,8} a^{(j)}n_x^{(j)}) = 1$$

Coressponding to the continuum version of the dexomposition $\mathcal{A}_\mu(x) = \mathcal{V}_\mu(x) + \mathcal{X}_\mu(x)$

$$D_\mu[\mathcal{V}_\mu]\mathbf{n}^{(k)}(x) = 0, \quad tr(\mathbf{n}^{(k)}(x)\mathcal{X}_\mu(x)) = 0, \quad (k = 3,8)$$

$$X_{x,\mu} = \hat{K}_{x,\mu}^\dagger \det(K_{x,\mu})^{1/3} g_x^{-1}, \quad V_{x,\mu} = g_x \hat{K}_{x,\mu} \det(K_{x,\mu})^{-1/3}$$

where

$$\hat{K}_{x,\mu} := \left(\sqrt{K_{x,\mu}K_{x,\mu}^\dagger}\right)^{-1} K_{x,\mu}, \quad \hat{K}_{x,\mu}^\dagger = K_{x,\mu}^\dagger\left(\sqrt{K_{x,\mu}K_{x,\mu}^\dagger}\right)^{-1}$$

$$K_{x,\mu} = 1 + 6\mathbf{n}_x^{(3)}U_{x,\mu}\mathbf{n}_{x+\mu}^{(3)}U_{x,\mu}^\dagger + 6\mathbf{n}_x^{(8)}U_{x,\mu}\mathbf{n}_{x+\mu}^{(8)}U_{x,\mu}^\dagger$$

Reduction condition

- The decomposition is uniquely determined for a given set of link variables $U_{x,\mu}$ and color fields which is given by minimizing the reduction condition.
- The reduction condition is introduced such that the theory in terms of new variables is equipollent to the original Yang-Mills theory, i.e., defining an effective gauge-Higgs model whose kinetic term is given by the reduction condition

for given $U_{x,\mu}$

$$F[\Theta;U] = \begin{cases} \sum_{x,\mu}\mathrm{tr}\left[\sum_{j=3,8}(D_\mu^\epsilon[U]\mathbf{n}^{(j)})^\dagger(D_\mu^\epsilon[U]\mathbf{n}^{(j)})\right] & \text{represented MA} \\ \sum_{x,\mu}\mathrm{tr}\left[\sum_{j}(D_\mu^\epsilon[U]\mathbf{n}^{(8)})^\dagger(D_\mu^\epsilon[U]\mathbf{n}^{(8)})\right] & \text{representied n8} \\ \sum_{x,\mu}\mathrm{tr}\left[\sum_{j}(D_\mu^\epsilon[U]\mathbf{n}^{(3)})^\dagger(D_\mu^\epsilon[U]\mathbf{n}^{(3)})\right] & \text{represented n3} \end{cases}$$

$\mathbf{n}^{(3)}$where $\mathbf{n}_j := \Theta^\dagger H_j\Theta$, H_j Cartan generators, and $D_\mu^\epsilon[U]\mathbf{n}^{(j)} := U_{x,\mu}\mathbf{n}_{x+\mu}^{(j)} - \mathbf{n}_x^{(j)}U_{x,\mu}$

Note that the n8-reduction determines only n8 color field, n3 is arbitrary but $(\mathbf{n}^{(3)})^2 = \frac{1}{6}\mathbf{1} + \frac{1}{2\sqrt{3}}\mathbf{n}^{(8)}$

Maximal option with the MA reduction as gauge invariant version of Abelian projection in MA gauge

MA reduction condition is rewritten into the gauge fixing of maximal Abelian gauge.

$$F_{MA}[\Theta;U] = \sum_{x,\mu} tr\left[\sum_{j=3,8}(D^{\epsilon}_{\mu}[U]\mathbf{n}^{(j)})^{\dagger}(D^{\epsilon}_{\mu}[U]\mathbf{n}^{(j)})\right] = \sum_{x,\mu}\left[2-2\sum_{j=3,8}\mathrm{tr}(U^{\dagger}_{x,\mu}\mathbf{n}^{(j)}_{x}U_{x,\mu}\mathbf{n}^{(j)}_{x+\mu})\right]$$

$$= \sum_{x,\mu}\left[2-2\sum_{j=3,8}\mathrm{tr}([\Theta_x U^{\dagger}_{x,\mu}\Theta^{\dagger}_{x}]H_j[\Theta_{x+\mu}U_{x,\mu}\Theta^{\dagger}_{x+\mu}]H_j)\right] = \sum_{x,\mu} 2 - F_{MAG}[\Theta;U]$$

Decomposition for maximal option id given by

$$K_{x,\mu} = U_{x,\mu} + 6\mathbf{n}^{(3)}_{x}U_{x,\mu}\mathbf{n}^{(3)}_{x+\mu} + 6\mathbf{n}^{(8)}_{x}U_{x,\mu}\mathbf{n}^{(8)}_{x+\mu} = \Theta_x\left[{}^{\Theta^{\dagger}}U_{x,\mu} + 6\frac{\lambda_3}{2}{}^{\Theta^{\dagger}}U_{x,\mu}\frac{\lambda_3}{2} + 6\frac{\lambda_8}{2}{}^{\Theta^{\dagger}}U_{x,\mu}\frac{\lambda_8}{2}\right]\Theta^{\dagger}_{x+\mu}$$

$$= \Theta_x[\mathrm{diag}({}^{\Theta^{\dagger}}u^{11}_{x,\mu}, {}^{\Theta^{\dagger}}u^{22}_{x,\mu}, {}^{\Theta^{\dagger}}u^{33}_{x,\mu},)]\Theta^{\dagger}_{x+\mu}$$

$$V_{x,\mu} = (K_{x,\mu}K^{\dagger}_{x,\mu})^{-1/2}K_{x,\mu}(\det K_{x,\mu})^{-1/3} = \mathrm{diag}\left(\frac{{}^{\Theta^{\dagger}}u^{11}_{x,\mu}}{\left|{}^{\Theta^{\dagger}}u^{11}_{x,\mu}\right|}, \frac{{}^{\Theta^{\dagger}}u^{22}_{x,\mu}}{\left|{}^{\Theta^{\dagger}}u^{22}_{x,\mu}\right|}, \frac{{}^{\Theta^{\dagger}}u^{33}_{x,\mu}}{\left|{}^{\Theta^{\dagger}}u^{33}_{x,\mu}\right|}\right)(\det({}^{\Theta^{\dagger}}u^{11}_{x,\mu}\,{}^{\Theta^{\dagger}}u^{22}_{x,\mu}\,{}^{\Theta^{\dagger}}u^{33}_{x,\mu}))^{-1/3}$$

Lattice data

SU(2) case ::

standard Wilson action 24^4 lattice β=2.5

hyper-blocking smearing

SU(3) case ::

standard Wilson action 24^4 lattice β=6.2, β=6.0

APE smearing

various reduction condition (MA, n3, n8)

SU(2) case: preceding study

- Naively extended Abelian projection does not reproduce the correct behavior of Wilson loops in higher representations.
- For example, in the adjoint rep. in SU(2) gauge theory, the Abelian projected Wilson loop,

$$W^{\mathrm{Abel}} = \frac{1}{3}(\mathrm{tr}V^2 + 1) \qquad V = \prod_{<x,\mu>\in C} V_{x,\mu}$$

approaches 1/3 other than 0 [G.I.Poulis, PRD 54 , 6974 (1996)]

■After testing several possible operators, the correct expression has found. [Piulis (1996)]

■M.N.Chernodub et.al performed numerical simulation according to Poulis. [PR**D70**,14506 (2004)]

SU(2) :: General formula

General form of the higher dimensional Wilson loop by using the fundamental representation.

$$W^{(j)}[U] = \frac{1}{2j+1} \sum_{n=0}^{\lfloor j \rfloor} \mathrm{tr}(U^{2(j-n)}) \qquad U = \prod_{<x,\mu>\in C} U_{x,\mu}$$

$$W^{(j)}[V] = \frac{1}{2} \int d\mu[\xi] \mathrm{tr}(V^{2j}) \qquad V = \prod_{<x,\mu>\in C} V_{x,\mu}$$

In case of j =1

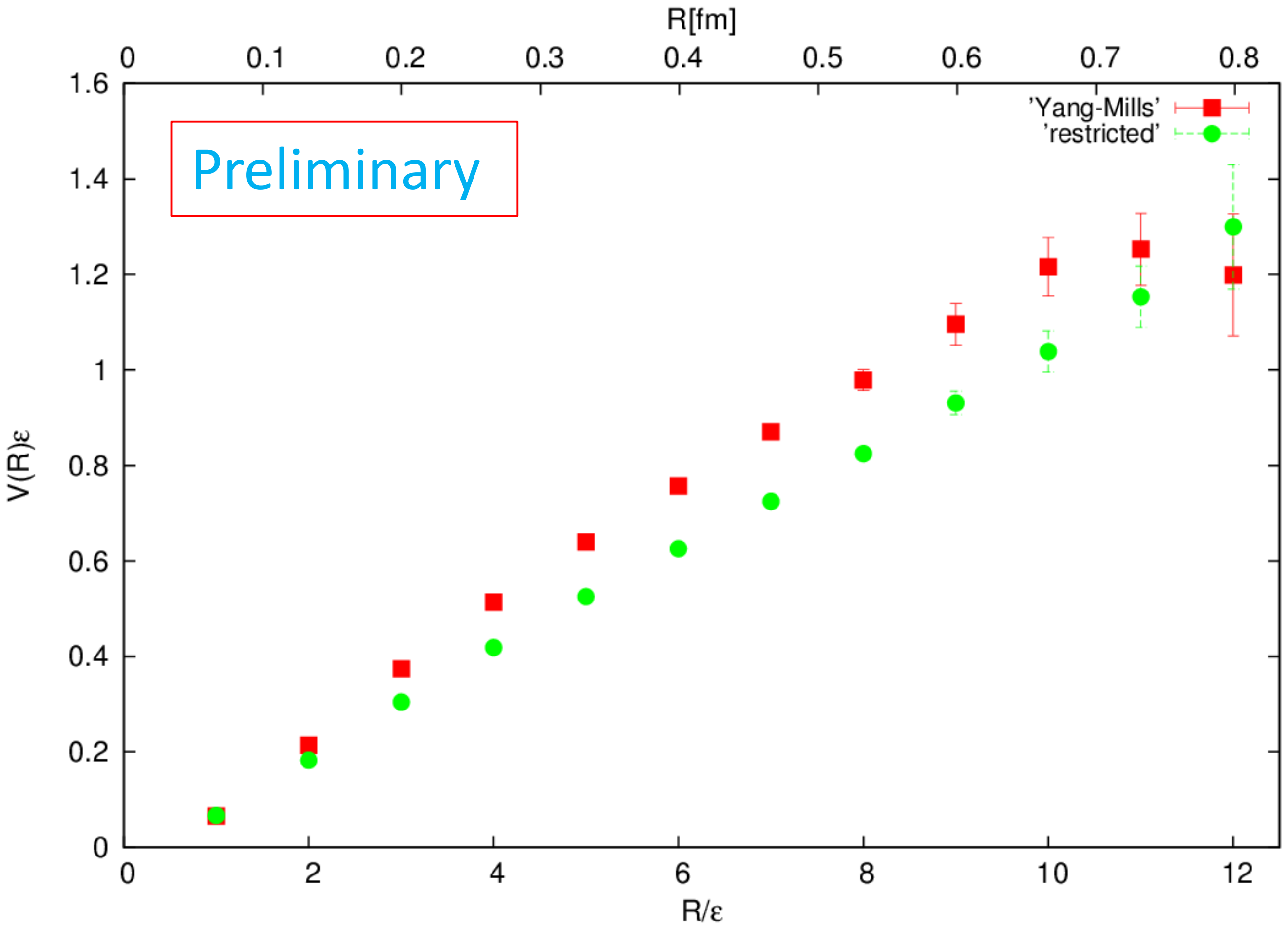
$$\boxed{\begin{aligned} W^{(j=1)}[U](C) &= \frac{1}{3}[\mathrm{tr}(U^2) + 1] \\ W^{(j=1)}[V](C) &= \frac{1}{2} \int D\mu[\xi] \mathrm{tr}(V^2) \approx \frac{1}{2} \mathrm{tr}(V(\mathbf{n})^2) \end{aligned}}$$

SU(2) adj.

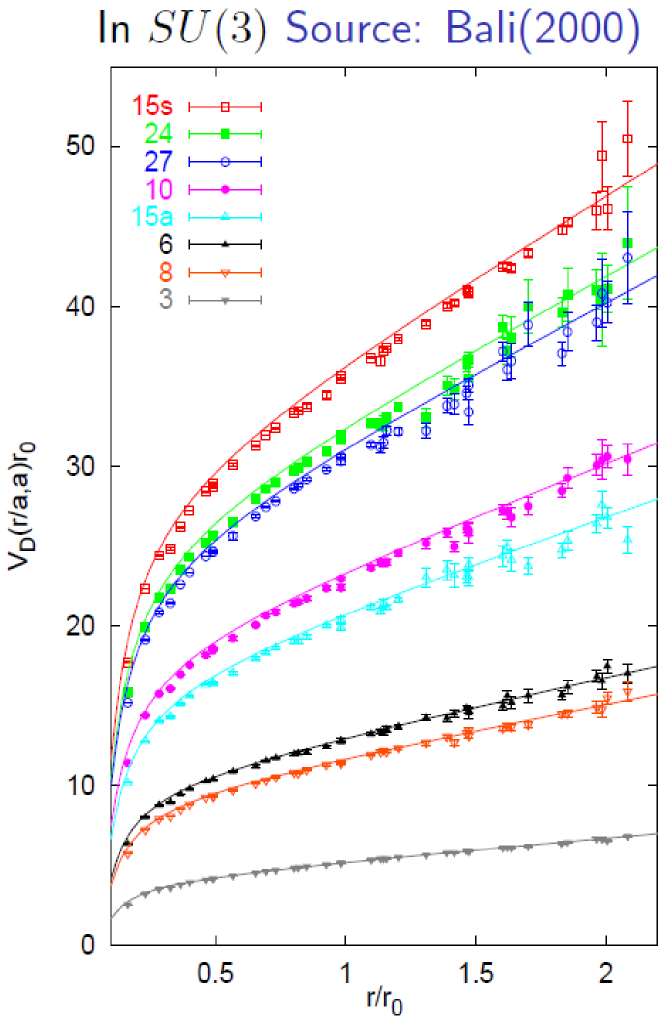
$$V^{adj.}(R) = -log\left[\frac{W(R,T)}{W(R,T-1)}\right]$$

T=6

- V dominance for the static potential is found.
- It seems that the string breaking for YM field starts at R = 0.7fm, but not for V (Abelian) field.
- This is consistent with Chernodub et.al (2004)



SU(3) case : Yang-Mills field



$$U = \prod_{<x,\mu>\in C} U_{x,\mu}$$

$$3 = [1,0]$$
$$8 = [1,1]$$
$$6 = [2,0]$$
$$15a = [2,1]$$
$$10 = [3,0]$$
$$\cdots\cdot$$

$$\mathrm{tr}(U_{[1,0]}) = \mathrm{tr}(U)$$
$$\mathrm{tr}(U_{[1,1]}) = |\mathrm{tr}(U)|^2 - 1$$
$$\mathrm{tr}(U_{[2,0]}) = \frac{1}{2}(\mathrm{tr}(U)^2 + \mathrm{tr}(U^2))$$
$$\mathrm{tr}(U_{[2,1]}) = \mathrm{tr}(U^*)\mathrm{tr}(U_{[2,0]}) - \mathrm{tr}(U)$$
$$\mathrm{tr}(U_{[3,0]}) = \frac{1}{6}(\mathrm{tr}(U)^3 + 3\mathrm{tr}(U)\mathrm{tr}(U^2) + 2\mathrm{tr}(U^3))$$

$$\cdots\cdots$$

SU(3) :: general formula for the restricted field

$$W_{(m_1,m_2)} = \frac{1}{6}\int D\mu[\xi][\mathrm{tr}(V^{m_1})\mathrm{tr}((V^{m_2})^*) - \mathrm{tr}(V^{m_1-m_2})] \quad (m_1 \neq 0, m_2 \neq 0)$$

$$W_{(m_1,0)} = \frac{1}{3}\int D\mu[\xi]\mathrm{tr}(V^{m_1}) \quad (m_2 = 0)$$

$$W_{(0,m_2)} = \frac{1}{3}\int D\mu[\xi]\mathrm{tr}((V^{m_2})^*) \quad (m_1 = 0)$$

- Where V-field must be decomposed by using the maximal option.
- Invariant integration measure Dμ[ξ] is dropped by using the reduction condition, **i.e.,** V-field is obtained by using the color field determined from the reduction condition.
- Note that we have arbitrariness in choice of the reduction condition

➔ In this talk, we use 3 types of the reduction conditions: MA, n3, n8

SU(3) [0,1] (fundamental) representation

$$V(R,T) = -\log\langle W_{[R,T+1]}[*]\rangle / \langle W_{[R,T+1]}[*]\rangle$$

APE smearing N = xx, $\alpha = 0.2$

$\beta = 6.2$, fit range [3,12]

full (N=28) :

σ = 0.0293314 +/- 0.00061

n3 (N=28) :

σ = 0.0293018 +/-0.000395

MA (N=8) :

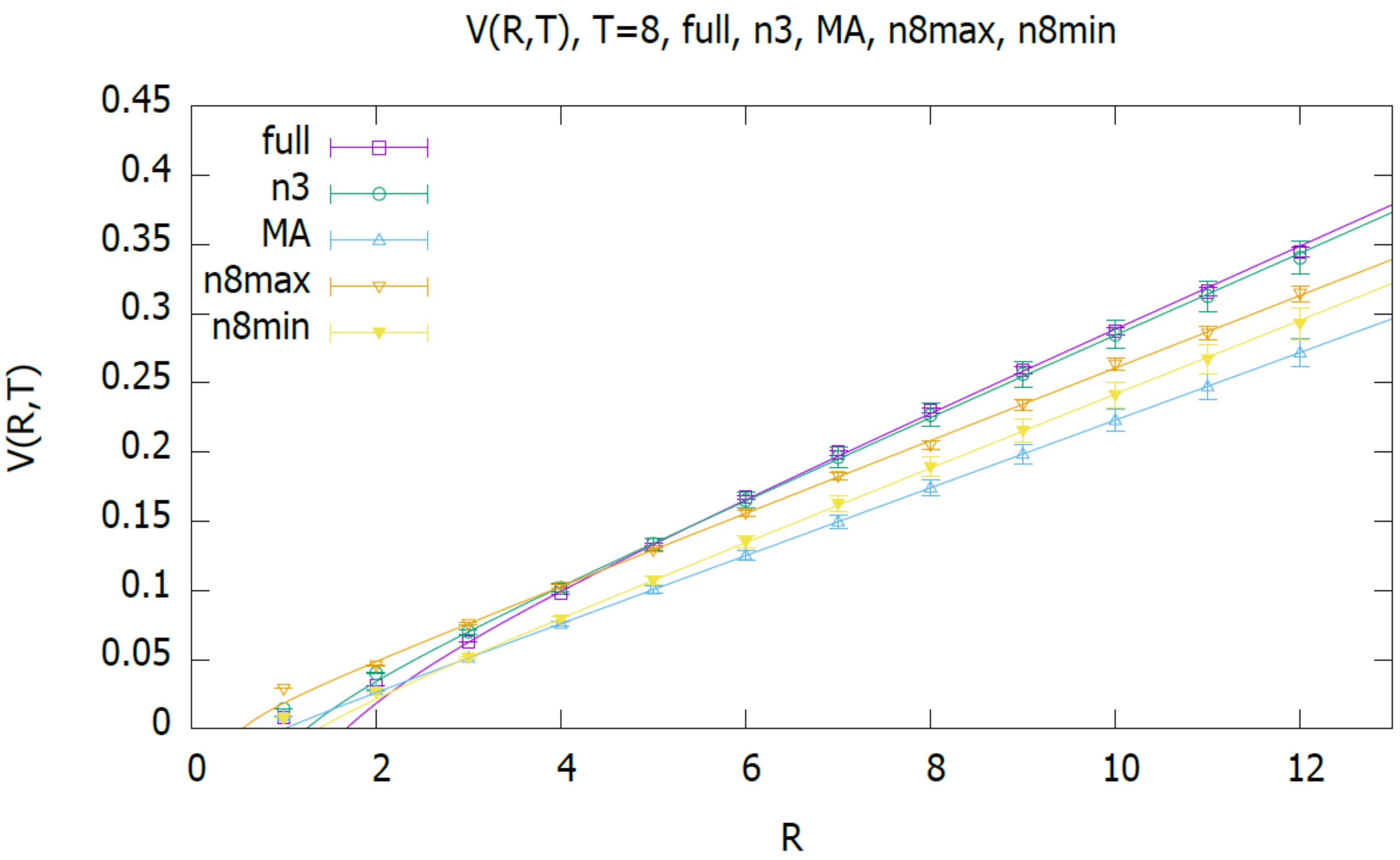
σ = 0.0244112 +/- 0.0000338

n8 (decomsed by minmal op, N=28):

σ = 0.026503 +/- 0.0002966

n8max (N=4) :

σ = 0.0261281 +/- 0.000332



SU(3) [1,1] (Adjoint) representation

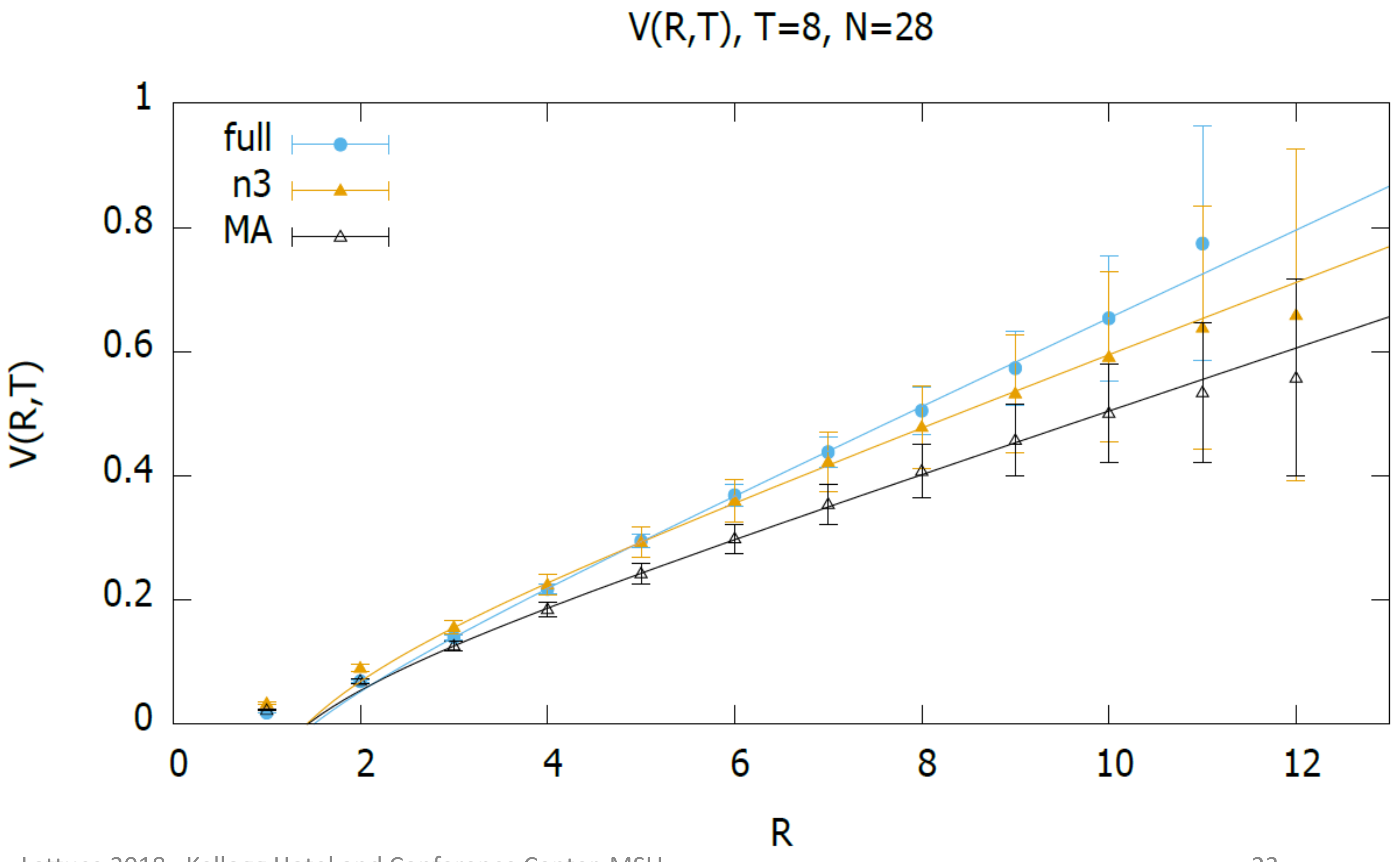
$$V(R,T) \ = -\log\langle W_{[R,T+1]}[*]\rangle/\langle W_{[R,T+1]}[*]\rangle$$

APE smearing N = xx, $\alpha = 0.2$
β=6.2 , fit range [3,12]

full (N=28) :
σ = 0.0699755 +/- 0.002781

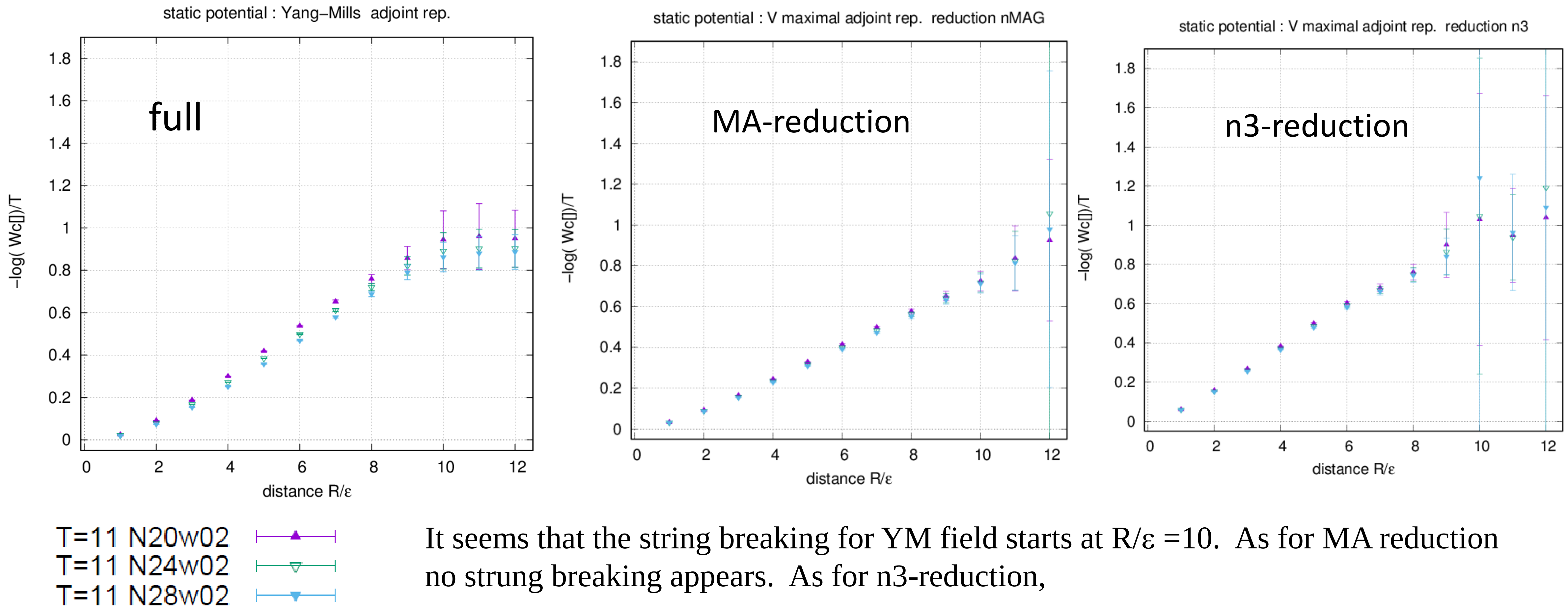
n3 (N=28) :
σ = 0.056761 +/- 0.002224

MA (N=8) :
σ = 0.0495824 +/- 0.002005



Adjoint representation : β=6.0

$$V(R,T) = -\frac{1}{T}\log(\langle W_{[R,T]}[*]\rangle)$$



It seems that the string breaking for YM field starts at R/ε =10. As for MA reduction no strung breaking appears. As for n3-reduction,

SU(3) [0,2] (6-dimension) representation

$$V(R,T) \;=\; -\tfrac{1}{T}\log(\langle W_{[R,T]}[*]\rangle)$$

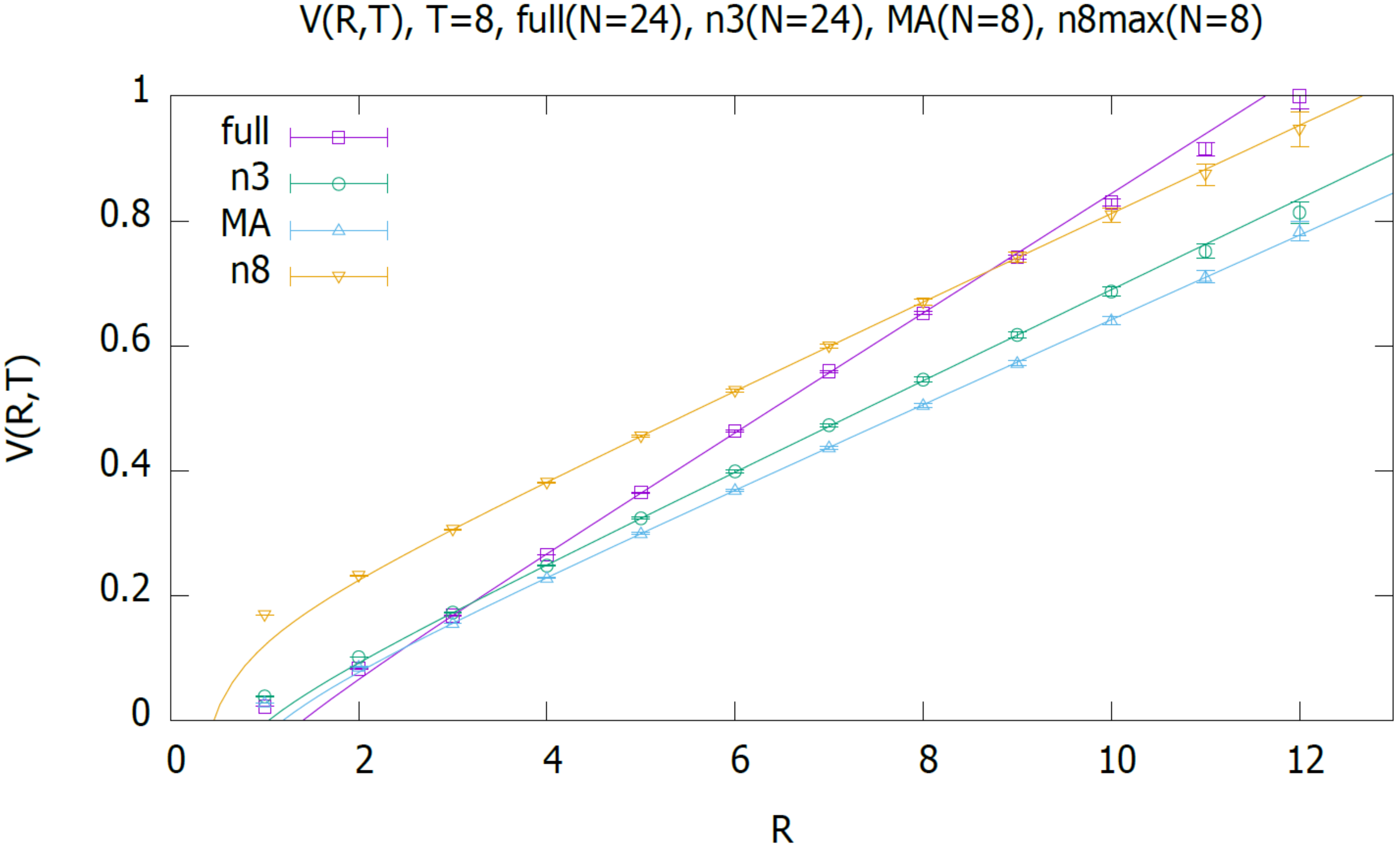
APE smearing N = xx, α=0.2
β=6.2 , fit range [3,12]

full (N=24) :
σ = 0.0923932 +/- 0.001724

n3 (N=24) :
σ = 0.0702499 +/- 0.0008097

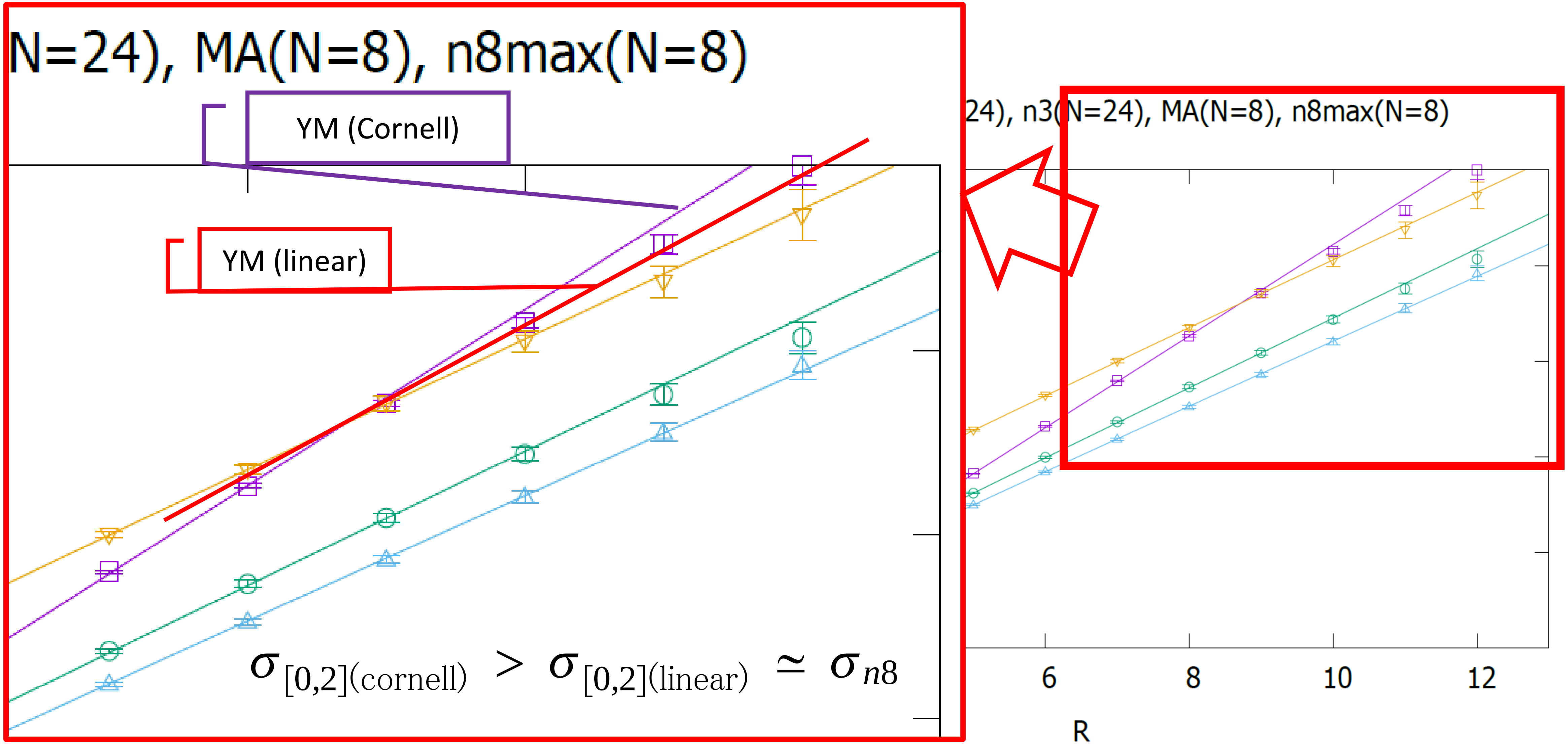
MA (N=8) :
σ = 0.063367 +/- 0.0008151

n8 (max, N=8) :
σ = 0.0567384 +/- 0.00288



SU(3) [0,2] representation

$$V(R,T) = -\frac{1}{T}\log(\langle W_{[R,T]}[*]\rangle)$$



$$\sigma_{[0,2](\mathrm{cornell})} > \sigma_{[0,2](\mathrm{linear})} \simeq \sigma_{n8}$$

Summary

- The Abelian projected Wilson loop for a higher representation, which made of naïve replacement of gauge link variable, does not reproduce the correct behavior of the original Wilson loop.
- Through the non-Abelian Stokes theorem (NAST), we have obtained the another Wilson loop, which is essentially same as the Abelian projected Wilson loop in the fundamental representation, but is different from that in higher representation.
- We have investigated Wilson loop average in the higher representation by using lattice simulation, and obtained correct behavior, i.e.,   restricted field (V) dominance in the string tension for the higher representation:
    - ➢ in the adjoint representation for SU(2) Yang-Mills theory
    - ➢ in the adjoint and 6-dimansiona representation for SU(3) YM theory.

Outlook

- Need higher statistics and tuning of the smearing parameters
- Check in other representations
- String breaking, N-arity.
- Monopole dominance in the string tension for higher representation.
- Casimir scaling is achieved or not in the intermediate scale and on in the string tension.
- ………..