Calculation of Pion Valence Distribution from Hadronic Lattice Cross Sections

Raza Sabbir Sufian

in Collaboration with

J. Karpie, C. Egerer, D. Richards, J.W. Qiu, B. Chakraborty, R. Edwards, K. Orginos

Jefferson Lab
Thomas Jefferson National Accelerator Facility

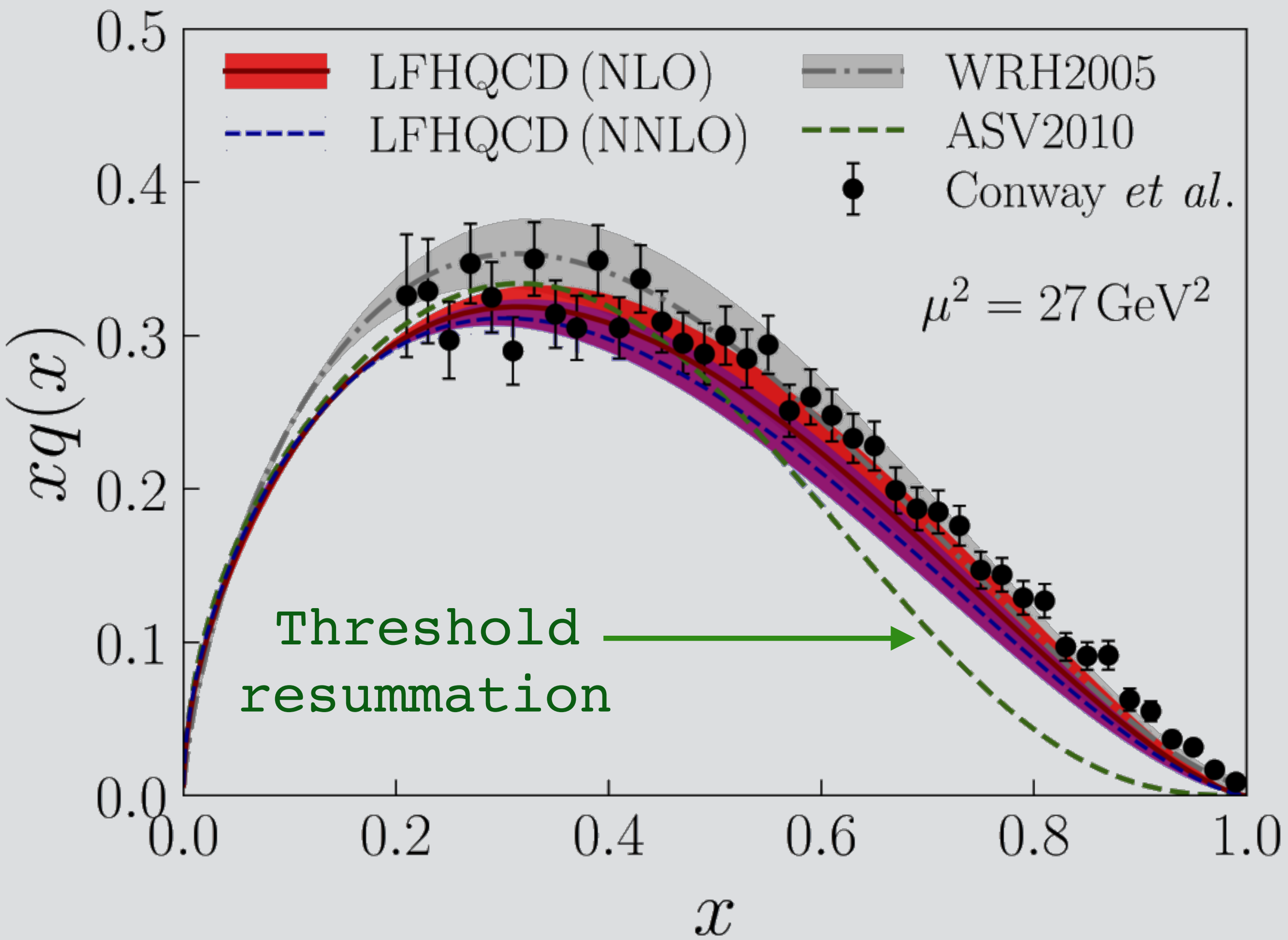
Why Pion Valence Distribution

★ Pion valence distribution large-x behavior an unresolved problem

★ From pQCD and different models : $(1-x)^2$ or $(1-x)^1$?

★ Large-x region: small configuration constrained by confinement dynamics

de Téramond, Liu, **RSS**, Dosch, Brodsky, Deur
PRL (2018)
Light-Front Holographic QCD



Lattice QCD can help understanding large-x behavior and test different models

C12-15-006 experiment at JLab to explore large-x behavior

Calculations of Parton Distributions on the Lattice

- ★ Hadronic tensor (K. F. Liu, PRL 1994, PRD 200)
- ★ Position-space correlators (V. M. Braun & D. Müller, EPJ 2008)
- ★ Inversion Method (A. Chambers, et al PRL 2017)
- ★ Quasi PDFs (X. Ji, PRL 2013)
- ★ Pseudo-PDFs (A. Radyushkin, PLB 2017)

Extensive efforts and significant achievements in recent years

- ★ Hadronic Lattice Cross Sections (LCSs) (Y. Q. Ma, J.-W. Qiu, PRL 2018)

Altogether, a community approach complementary to global fits of PDFs

What are Good Lattice "Cross Sections" (LCSs)

Single hadron matrix elements:

Ma & Qiu
PRL (2018)

1. Calculable using lattice QCD with Euclidean time

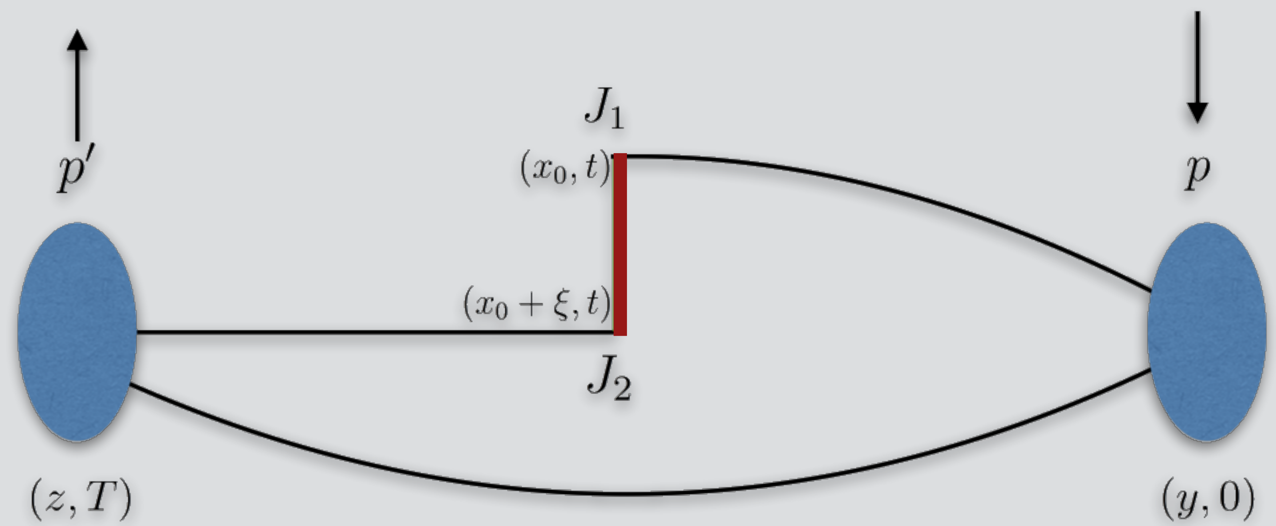
2. Well defined continuum limit ($a \to 0$), UV finite
i.e. no power law divergence from Wilson line in non-local operator

3. Share the same perturbative collinear divergences with PDFs

4. Factorizable to PDFs with IR-safe hard coefficients
with controllable power corrections

A good theory can identify its limitations - no free lunch

★ 4-point correlation function is numerically expensive



★ Equal time current insertion : sum over all energy modes can saturate phase space

Use heavy-light flavor changing current to suppress noise from spectator propagator in a systematic way

Simple and controllable approximations to problems

Good Lattice Cross Sections (LCSs)

★ Hadron matrix elements: $\sigma_n(\omega, \xi^2, P^2) = \langle P|T\{\mathcal{O}_n(\xi)\}|P\rangle$

$$\omega \equiv P \cdot \xi$$

★ Current-current correlators

$$\mathcal{O}_{j_1 j_2}(\xi) \equiv \xi^{d_{j_1}+d_{j_2}-2}\, Z_{j_1}^{-1}\, Z_{j_2}^{-1} j_1(\xi)\, j_2(0)$$

d_j : Dimension of the current

Z_j : Renormalization constant of the current

Z_j already known for the lattice ensembles being used

★ Different choices of currents

$$j_S(\xi) = \xi^2 Z_S^{-1}[\overline{\psi}_q \psi_q](\xi), \qquad j_V(\xi) = \xi Z_V^{-1}[\overline{\psi}_q \gamma \cdot \xi \psi_q](\xi),$$

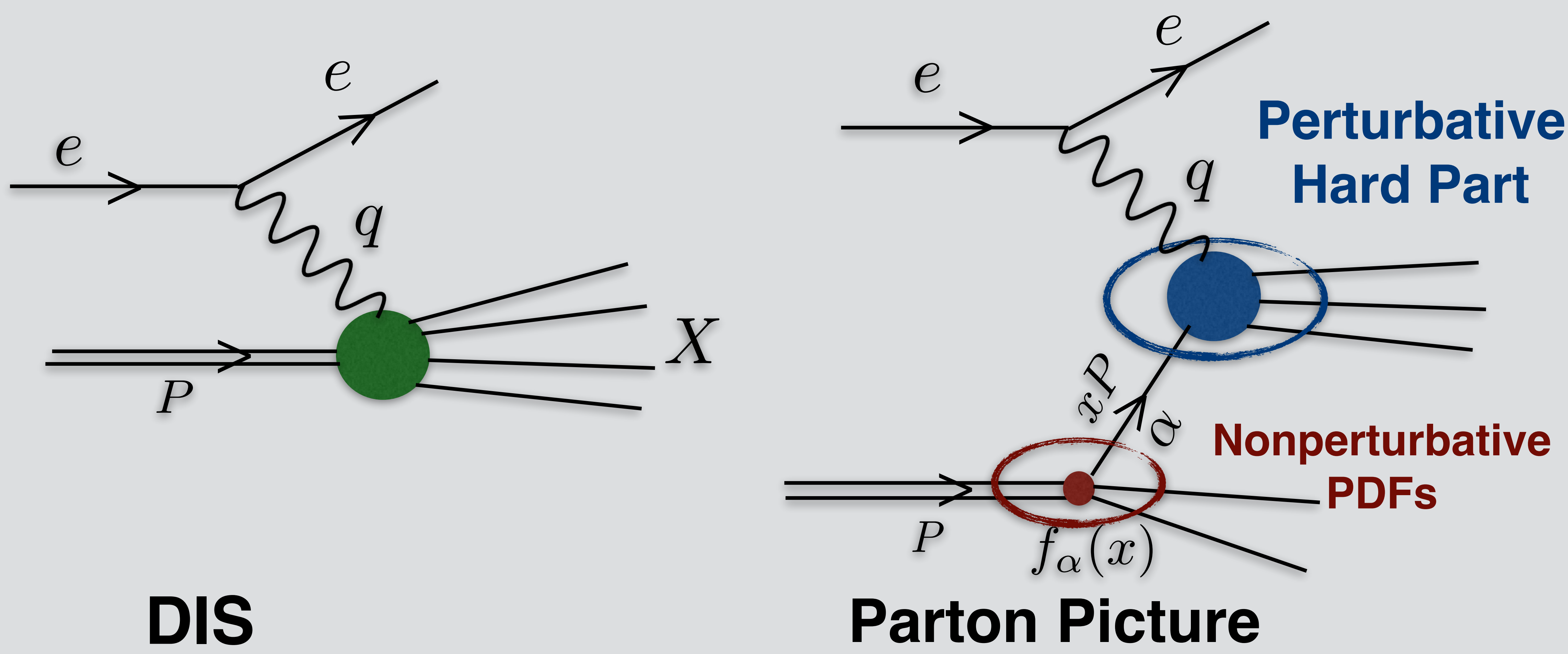
$$j_{V'}(\xi) = \xi Z_{V'}^{-1}[\overline{\psi}_q \gamma \cdot \xi \psi_{q'}](\xi), \qquad j_G(\xi) = \xi^3 Z_G^{-1}[-\frac{1}{4} F^c_{\mu\nu} F^c_{\mu\nu}](\xi), \ldots$$

flavor changing current

gluon distribution

Parton Distribution Functions (PDFs) & Factorization

$$\sigma^{DIS}(x, Q^2, \sqrt{s}) = \sum_{\alpha=q,\bar{q},g} C_\alpha\left(x, \frac{Q^2}{\mu^2}, \sqrt{s}\right) \otimes f_\alpha(x,\mu^2) + \mathcal{O}\left(\frac{\Lambda^2_{QCD}}{Q^2}\right)$$



Factorization scale μ describes which fluctuations should be included in the PDFs and which can be included in the hard scattering part

LCSs: Lattice Calculable + Renormalizable + Factorizable

$$\sigma_n(\omega,\xi^2,P^2) = \sum_a \int_{-1}^{1} \frac{dx}{x} f_a(x,\mu^2) \times K_n^a(x\omega,\xi^2,x^2P^2,\mu^2) + \mathcal{O}(\xi^2\Lambda_{QCD}^2)$$

Nonperturbative PDFs of flavor $a = q, g$

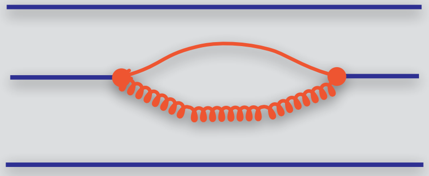
Perturbatively calculable hard hoefficients

$$f_{\bar{a}}(x,\mu^2) = -f_a(-x,\mu^2)$$

P and ξ → Collision Kinematics

$P \to \sqrt{s}$ Collision energy

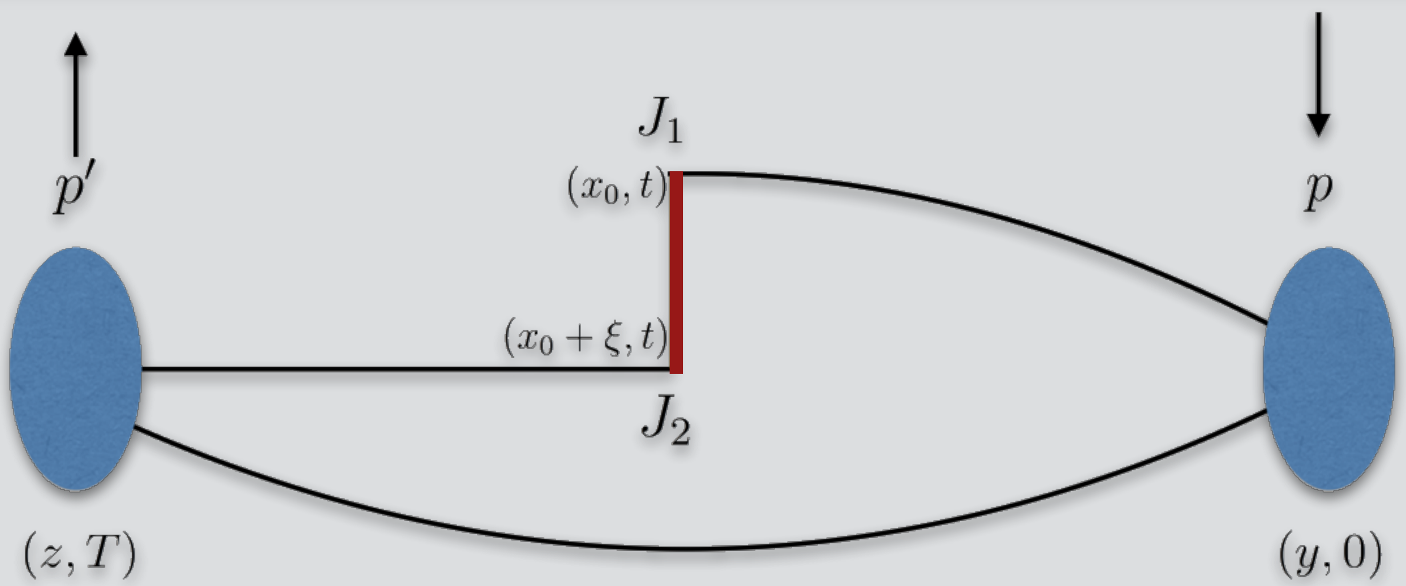
$\xi \to \frac{1}{Q}$ Hard Probe



$\mathcal{O}_n$ → Dynamical Features of LCSs

LCSs : Factorization holds for any finite ω and $P^2\xi^2$ if ξ is short distance

Lattice Calculation Setup



possible ξ on/off axis

$$
\begin{aligned}
&\langle \Pi(-p')|\mathcal{O}_{J_1}(x_0)\mathcal{O}_{J_2}(\xi)|\Pi(-p')\rangle = \\
&\quad = \sum_{y,z} e^{i(p'.z-p.y)} \langle \bar{q}\,\Gamma_\Pi\, q(z,T)\, \bar{q}\, J_2\, q(x_0+\xi,t)\, \bar{q}\, J_1\, q(x_0,t)\, \bar{q}\,\Gamma_\Pi\, q(y,0)\rangle \\
&\quad = \sum_{y,z} e^{i(p'.z-p.y)} \mathrm{tr}[J_2\, D^{-1}(x_0+\xi,t;x_0,t)\, J_1\, D^{-1}(x_0,t;y,0)\,\Gamma_\Pi \\
&\qquad \times D^{-1}(y,0;z,T)\Gamma_\Pi\, D^{-1}(z,T;x_0+\xi,t)],
\end{aligned}
$$

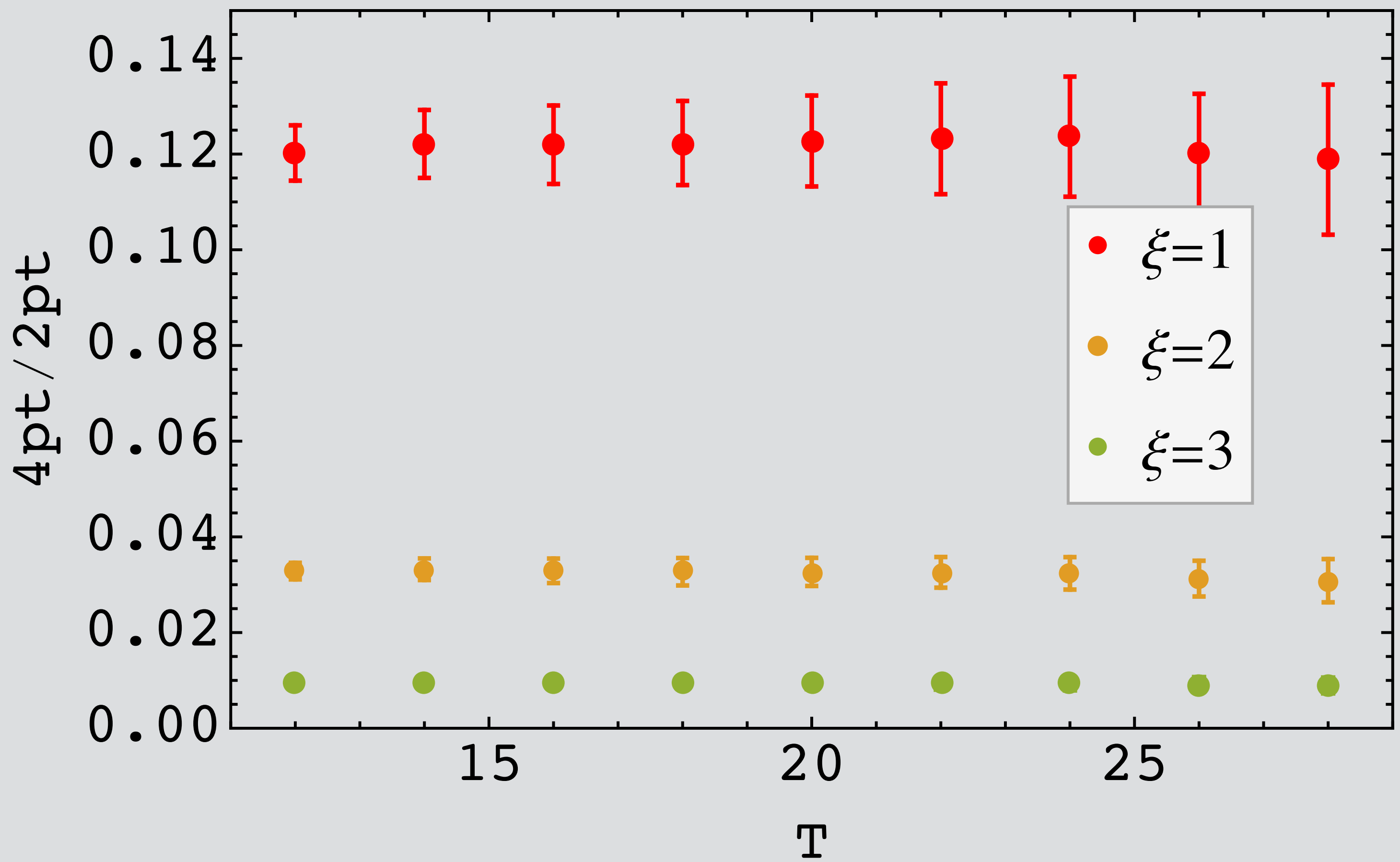
- ★ Analysis shown here on isoClover with 450 Configurations
- ★ Lattice spacing a~ 0.127 fm, $m_\pi \approx 430$ MeV $(32^3 \times 96)$
- ★ Projected calculations with $m_\pi \approx 380$ MeV, $a \approx 0.09$ fm $(32^3 \times 64)$; $m_\pi \approx 170$ MeV $(64^3 \times 128)$

Example Lattice Matrix Elements

- About 10 different current-current correlations are being analyzed

- Momentum smearing used for higher momentum

Gunnar S. Bali, et al (PRD 2016)



source-sink separation

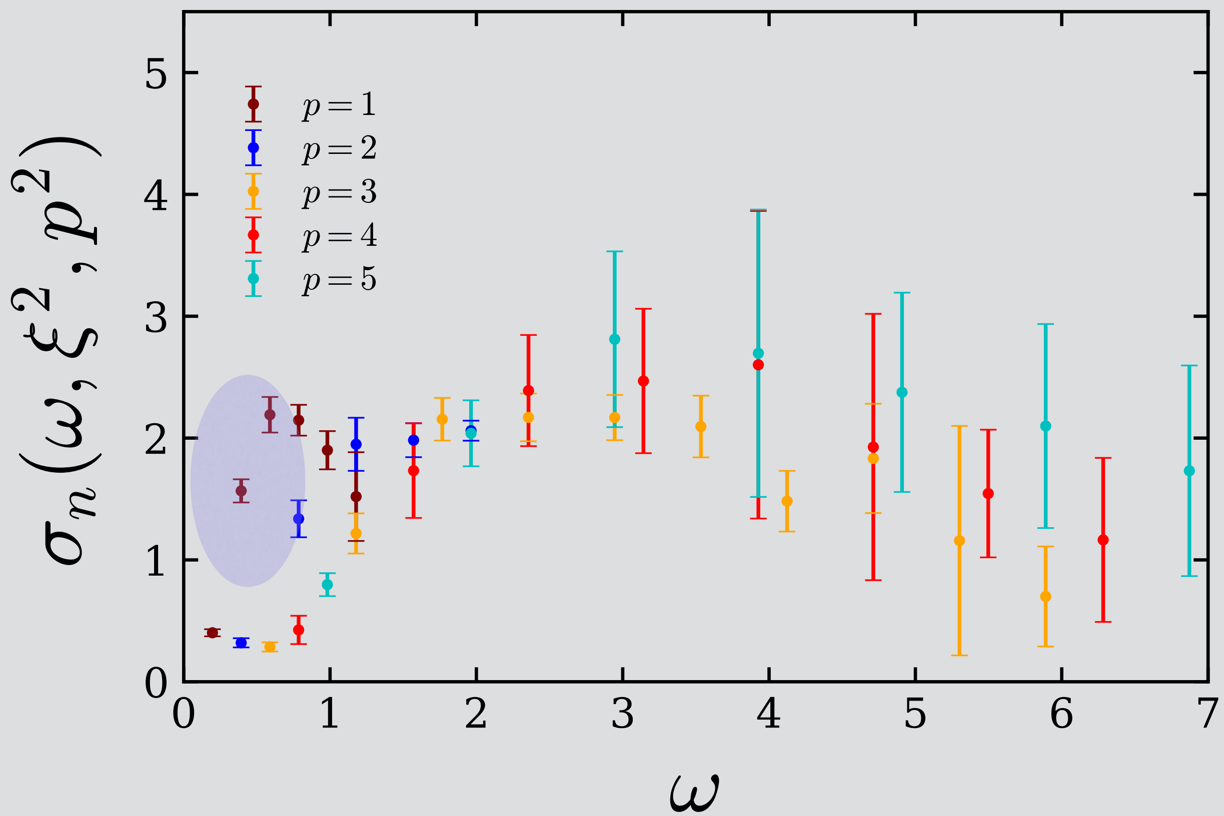
V-A matrix element

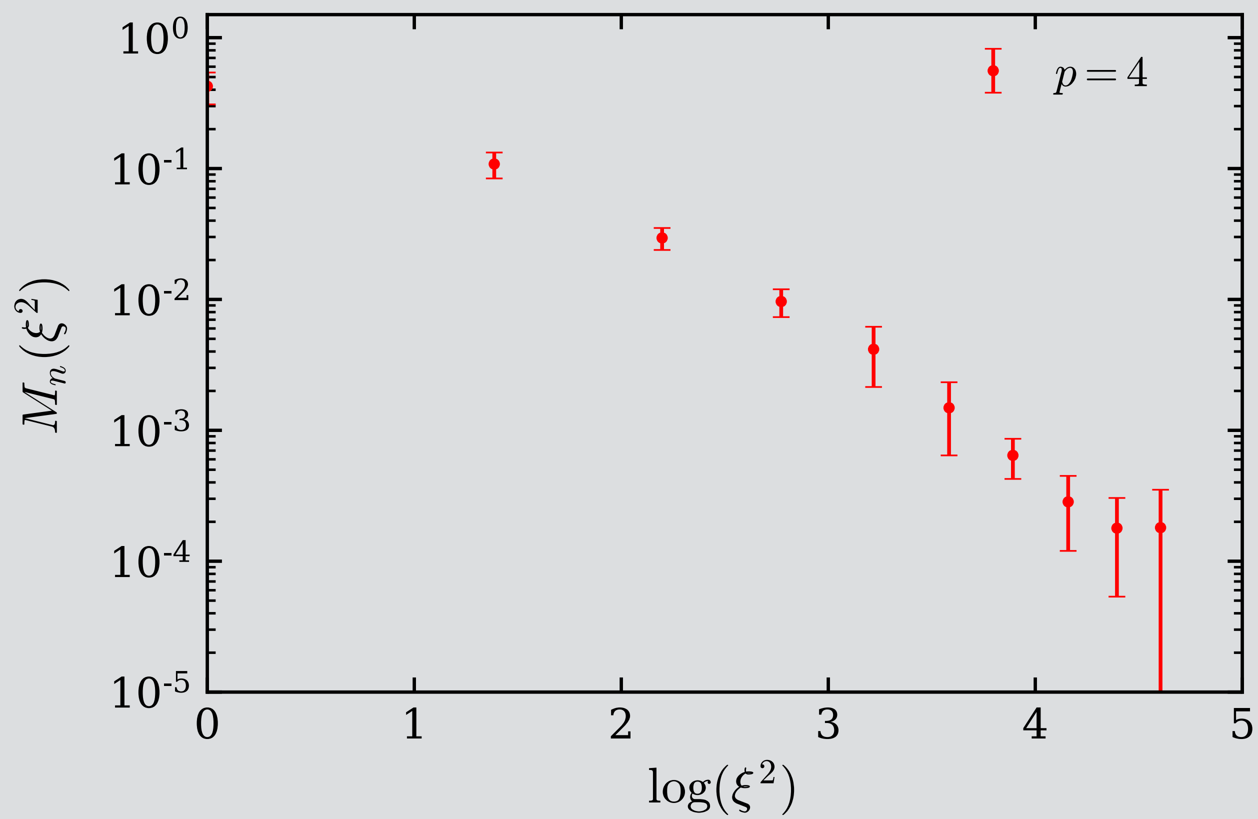
Idea by **D. Richards** for reliable extraction of matrix elements

Preliminary Lattice Results

★ Only about 1/3 statistics of p=3,4,5 data analyzed

V-V current correlation





★ p=1 (0.3 GeV) data deviates

Does the calculated correlation matrix lead to consistent description of pion PDF ?

$$f(x) \approx A x^{\alpha}(1-x)^{\beta}(1+\gamma\sqrt{x}+\delta x)$$

Preliminary Lattice Results

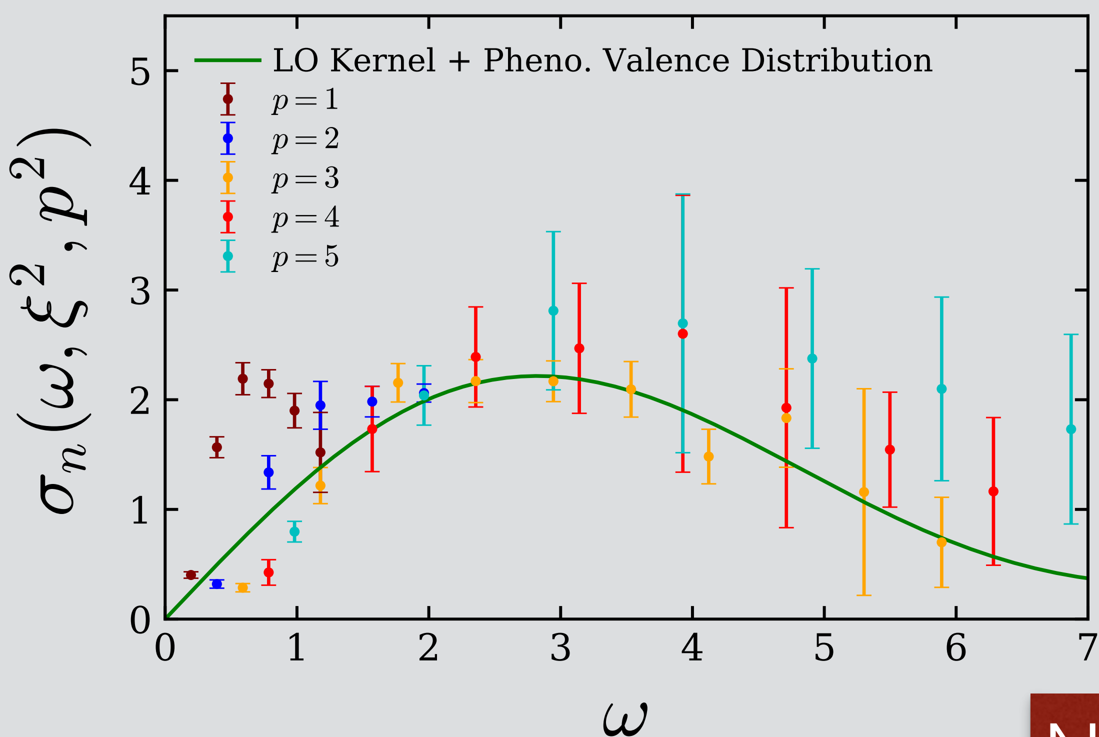
$$\sigma_n(\omega,\xi^2,P^2) = \sum_a \int_{-1}^{1} \frac{dx}{x} f_a(x,\mu^2) \times K_n^a(x\omega,\xi^2,x^2P^2,\mu^2) + \mathcal{O}(\xi^2\Lambda_{QCD}^2)$$

calculate on lattice

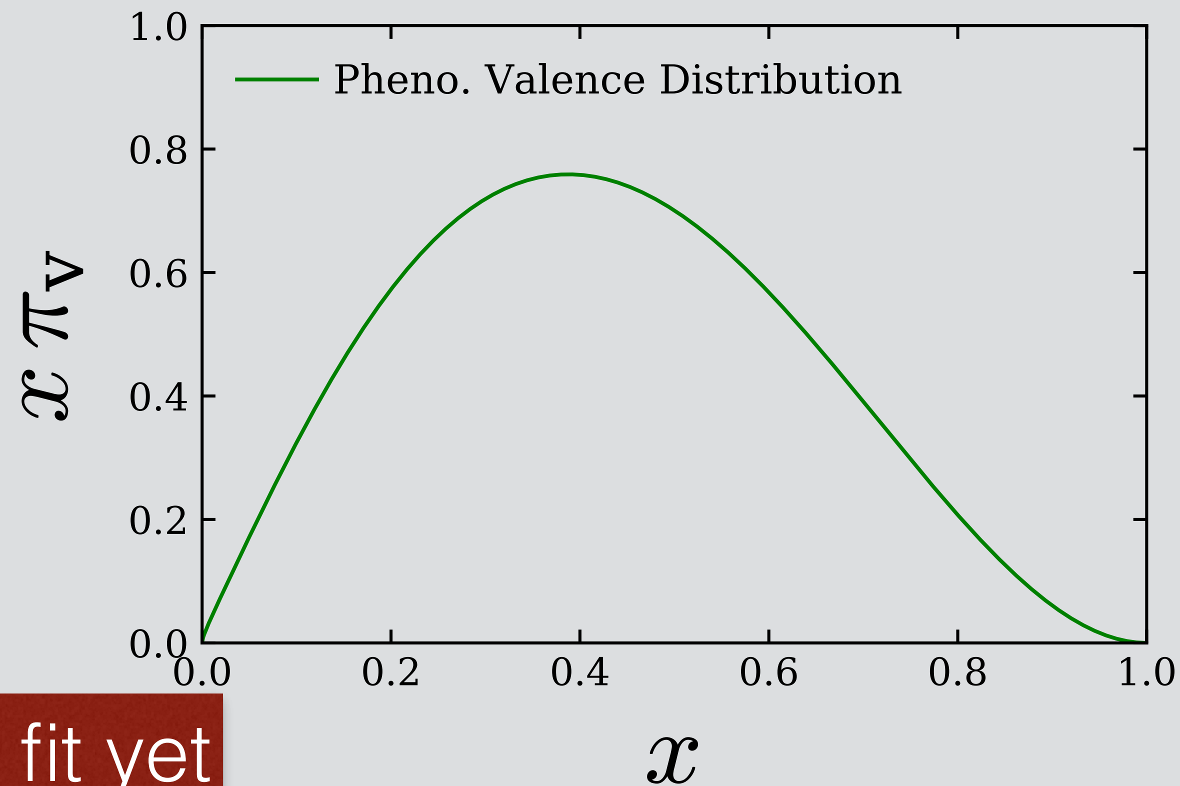
extract PDF

PQCD

K_n^a being calculated at LO and NLO for different currents



NOT a fit yet



★ A combined fit to many LCSs on an ensemble will lead to precise determination of PDFs

e.g. like global fits to data from different experiments !

With these encouraging results, we are very excited !!!

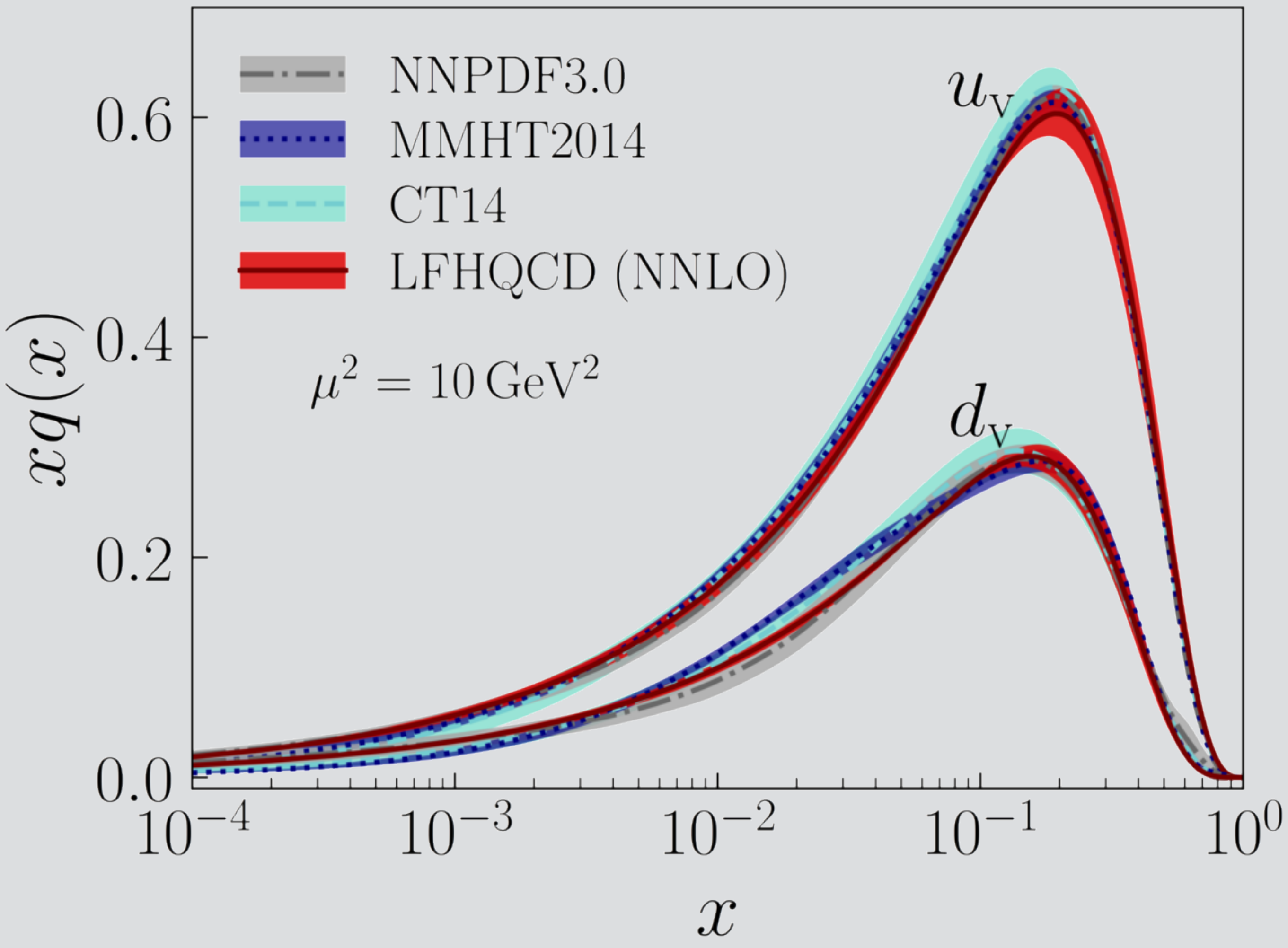
Collaboration between lattice QCD and perturbative QCD

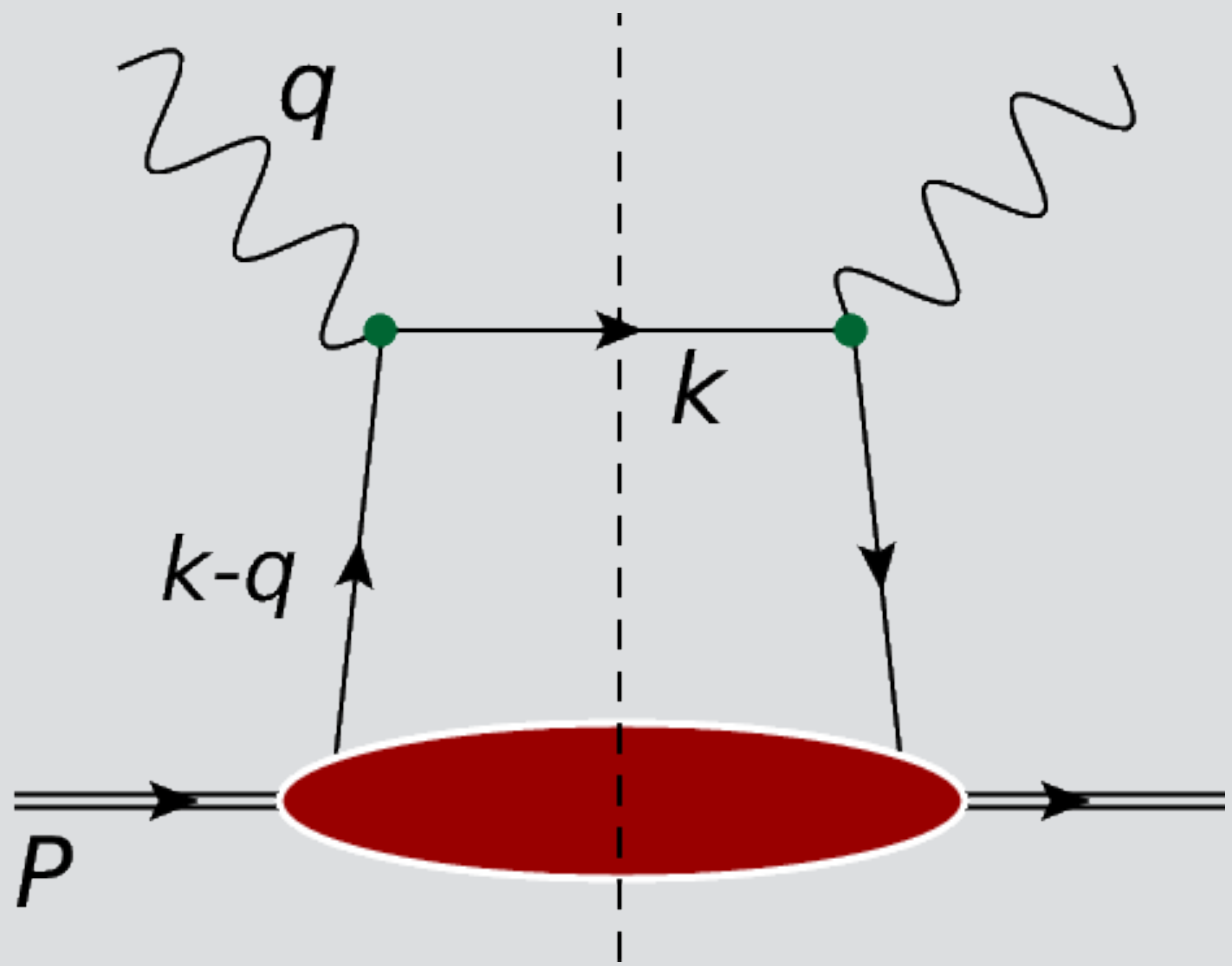
LCSs can be a tool to test different model calculations

Extensions such as kaon, nucleon PDFs on their way....

Thank You

Backup

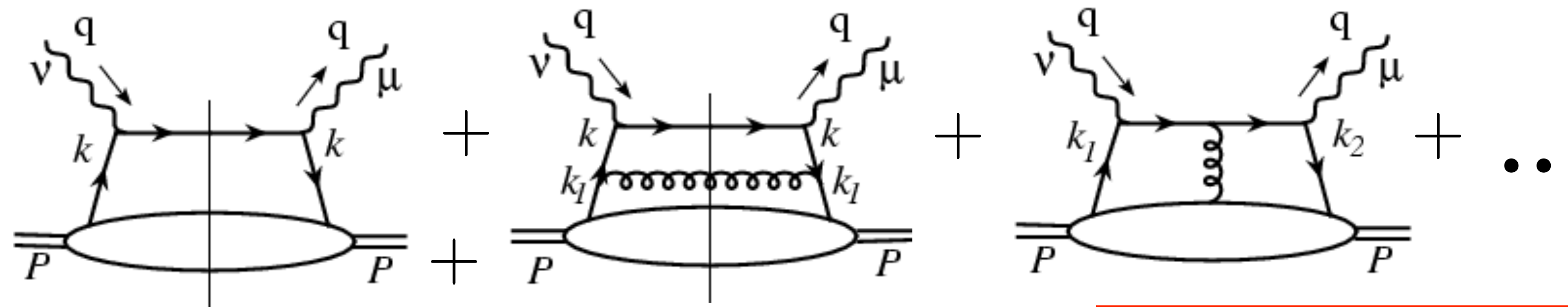

NNPDF3.0
MMHT2014
CT14
LFHQCD (NNLO)
$\mu^2 = 10\,\mathrm{GeV}^2$
u_V
d_V
0.6
0.4
0.2
0.0
$xq(x)$
10^{-4}
10^{-3}
10^{-2}
10^{-1}
10^{0}
x



q
k
k-q
P

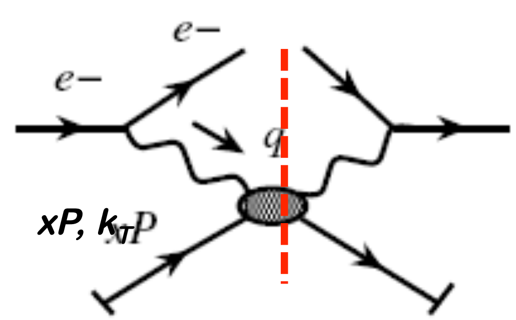
❑ DIS cross section is infrared divergent, and nonperturbative!

$$\sigma^{\rm DIS}_{\ell p\to\ell' X(\text{everything})} \propto$$

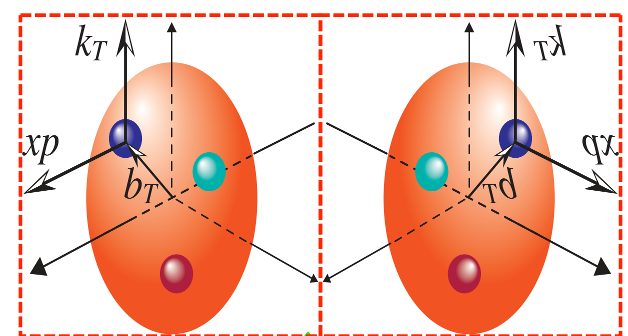


❑ QCD factorization (approximation!)

$$\sigma^{\rm DIS}_{\ell p\to\ell' X(\text{everything})} =$$



$\otimes$



$$+\ O\left(\frac{1}{QR}\right)$$

Color entanglement
Approximation

Physical
Observable

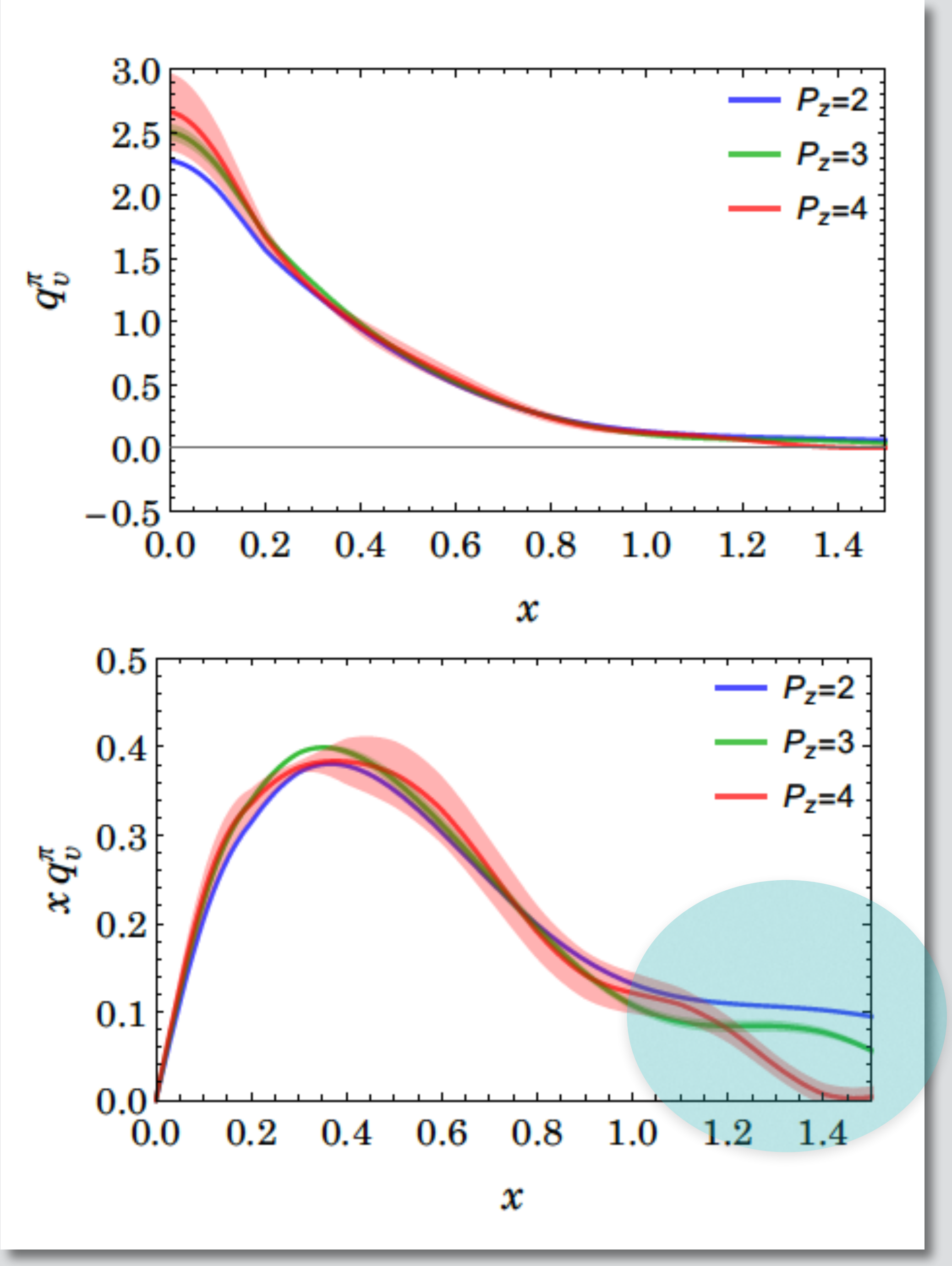
Controllable
Probe

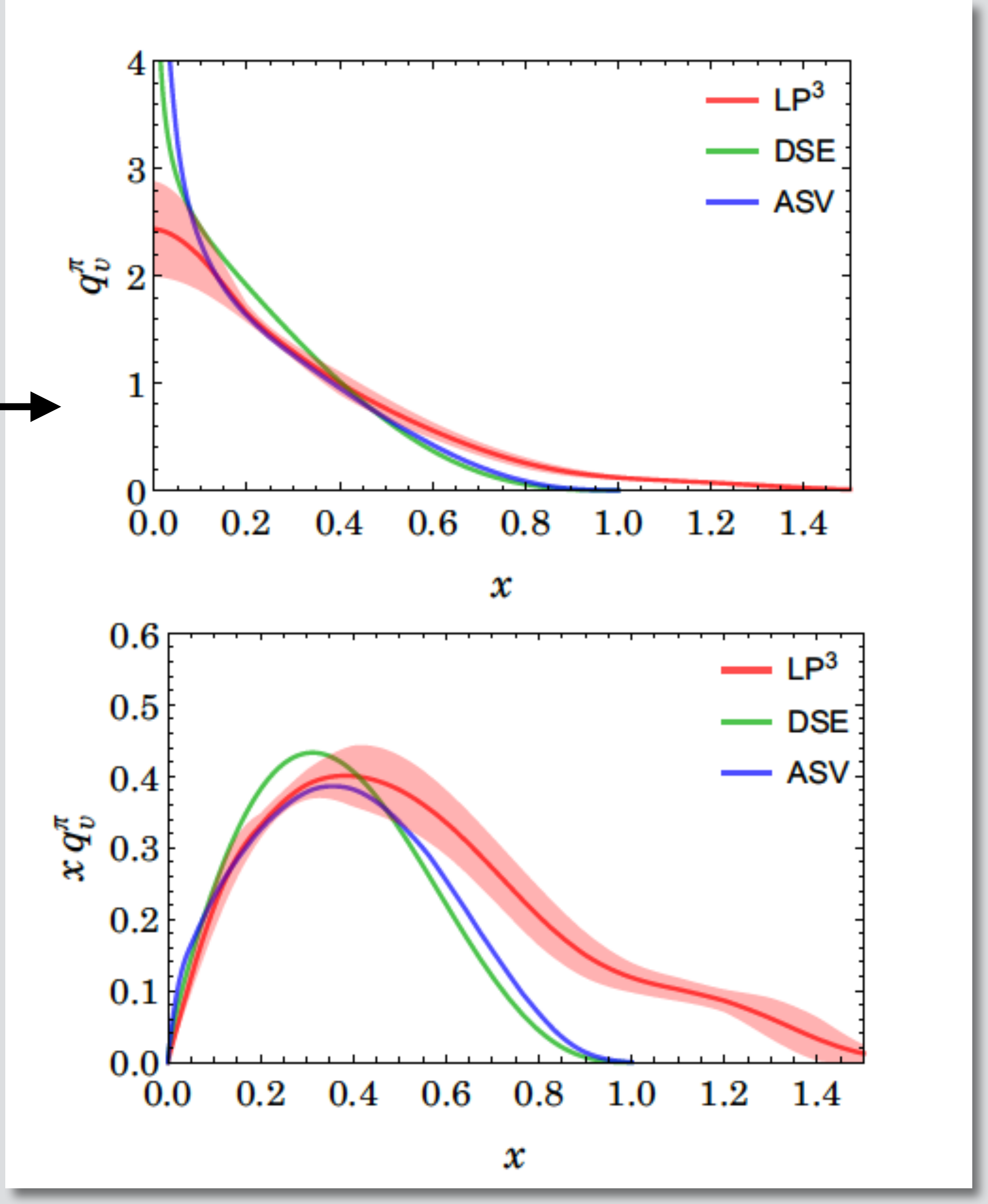
Quantum Probabilities
Structure

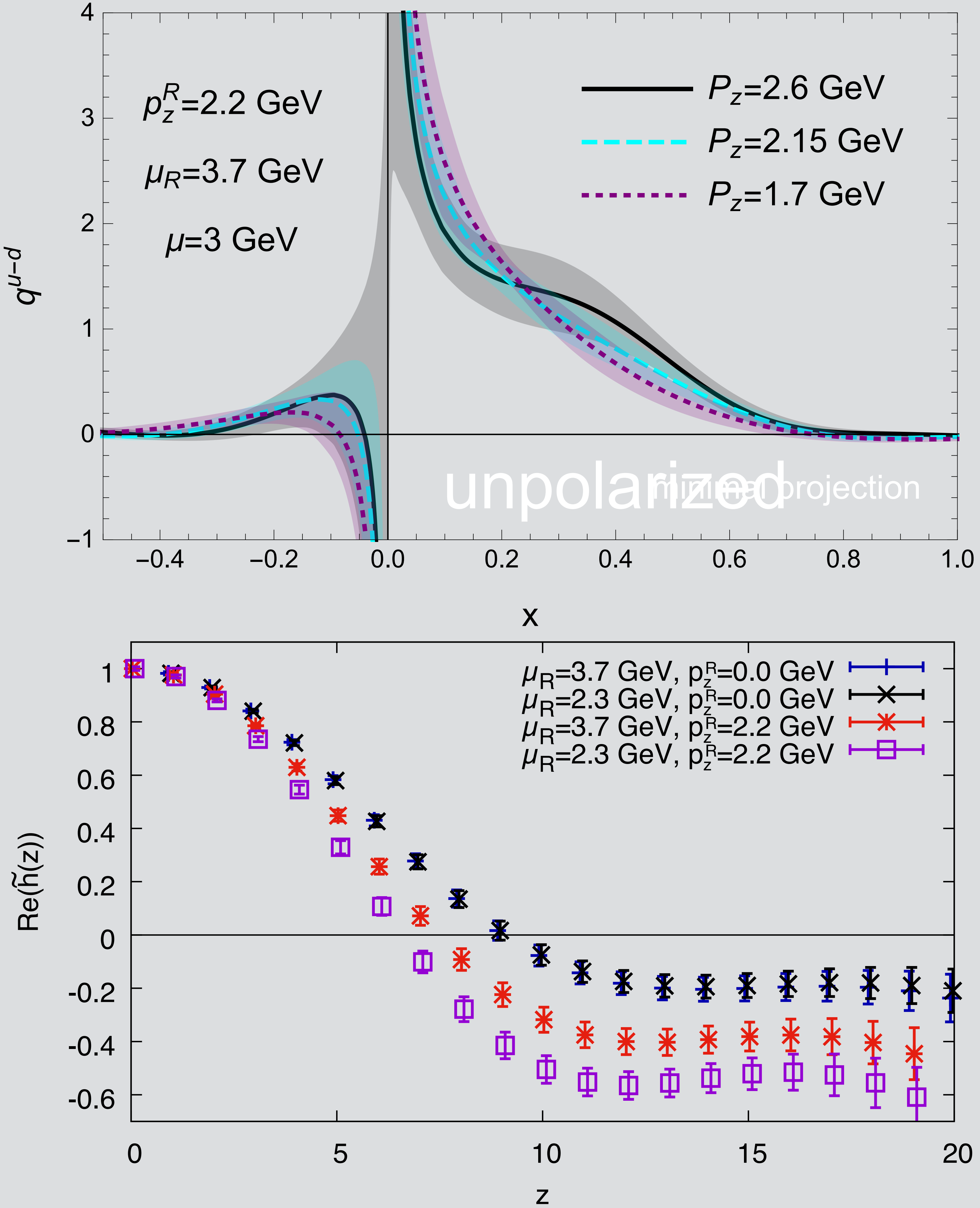
Quasi-Distribution of Pion

$m_\pi \simeq 300\,\mathrm{MeV}$

LP3, arXiv:1804.01483







p_z^R=2.2 GeV
μ_R=3.7 GeV
μ=3 GeV
P_z=2.6 GeV
P_z=2.15 GeV
P_z=1.7 GeV
q^{u-d}
x
unpolarized
minimal projection
μ_R=3.7 GeV, p_z^R=0.0 GeV
μ_R=2.3 GeV, p_z^R=0.0 GeV
μ_R=3.7 GeV, p_z^R=2.2 GeV
μ_R=2.3 GeV, p_z^R=2.2 GeV
Re($\tilde{h}$(z))
z

where

$$
\begin{aligned}
\tilde{f}_\alpha(x,\rho) = {} & \frac{\alpha_s C_F}{2\pi}\begin{cases} \frac{x-\rho}{(1-x)(1-\rho)} + \frac{2x(2-x)-\rho(1+x)}{2(1-x)(1-\rho)^{3/2}} \ln\frac{2x-1+\sqrt{1-\rho}}{2x-1-\sqrt{1-\rho}} & x>1 \\ \frac{-3x+2x^2+\rho}{(1-x)(1-\rho)} + \frac{2x(2-x)-\rho(1+x)}{2(1-x)(1-\rho)^{3/2}} \ln\frac{1+\sqrt{1-\rho}}{1-\sqrt{1-\rho}} & 0<x<1 \\ -\frac{x-\rho}{(1-x)(1-\rho)} - \frac{2x(2-x)-\rho(1+x)}{2(1-x)(1-\rho)^{3/2}} \ln\frac{2x-1+\sqrt{1-\rho}}{2x-1-\sqrt{1-\rho}} & x<0 \end{cases} \\
& + \frac{\alpha_s C_F}{2\pi}(1-\tau)\begin{cases} \frac{\rho(-3x+2x^2+\rho)}{2(1-x)(1-\rho)(4x-4x^2-\rho)} + \frac{-\rho}{4(1-\rho)^{3/2}} \ln\frac{2x-1+\sqrt{1-\rho}}{2x-1-\sqrt{1-\rho}} & x>1 \\ \frac{-x+\rho}{2(1-x)(1-\rho)} + \frac{-\rho}{4(1-\rho)^{3/2}} \ln\frac{1+\sqrt{1-\rho}}{1-\sqrt{1-\rho}} & 0<x<1 \\ -\frac{\rho(-3x+2x^2+\rho)}{2(1-x)(1-\rho)(4x-4x^2-\rho)} - \frac{-\rho}{4(1-\rho)^{3/2}} \ln\frac{2x-1+\sqrt{1-\rho}}{2x-1-\sqrt{1-\rho}} & x<0 \end{cases},
\end{aligned}
\tag{44}
$$

$$
\begin{aligned}
\tilde{f}_z(x,\rho) = {} & \frac{\alpha_s C_F}{2\pi}\begin{cases} \frac{-2\rho(1-7x+6x^2)-\rho^2(1+2x)}{(1-\rho)^2(4x-4x^2-\rho)} g_{z\alpha} + \frac{4x(1-3x+2x^2)-\rho(2-11x+12x^2-4x^3)-\rho^2}{(1-x)(1-\rho)^2(4x-4x^2-\rho)} \\ \quad + \left[\frac{\rho(4-6x-\rho)}{2(1-\rho)^{5/2}} g_{z\alpha} + \frac{2-4x+4x^2-5x\rho+2x^2\rho+\rho^2}{2(1-x)(1-\rho)^{5/2}}\right] \ln\frac{2x-1+\sqrt{1-\rho}}{2x-1-\sqrt{1-\rho}} & x>1 \\ \frac{-2+2x-\rho(1-4x)}{(1-\rho)^2} g_{z\alpha} + \frac{(-1+2x)(2-3x+\rho)}{(1-x)(1-\rho)^2} \\ \quad + \left[\frac{\rho(4-6x-\rho)}{2(1-\rho)^{5/2}} g_{z\alpha} + \frac{2-4x+4x^2-5x\rho+2x^2\rho+\rho^2}{2(1-x)(1-\rho)^{5/2}}\right] \ln\frac{1+\sqrt{1-\rho}}{1-\sqrt{1-\rho}} & 0<x<1 \\ -\frac{-2\rho(1-7x+6x^2)-\rho^2(1+2x)}{(1-\rho)^2(4x-4x^2-\rho)} g_{z\alpha} - \frac{4x(1-3x+2x^2)-\rho(2-11x+12x^2-4x^3)-\rho^2}{(1-x)(1-\rho)^2(4x-4x^2-\rho)} \\ \quad - \left[\frac{\rho(4-6x-\rho)}{2(1-\rho)^{5/2}} g_{z\alpha} + \frac{2-4x+4x^2-5x\rho+2x^2\rho+\rho^2}{2(1-x)(1-\rho)^{5/2}}\right] \ln\frac{2x-1+\sqrt{1-\rho}}{2x-1-\sqrt{1-\rho}} & x<0 \end{cases} \\
& + \frac{\alpha_s C_F}{2\pi}(1-\tau)\begin{cases} \frac{\rho(1-2x)[-4x(1-x)(2+\rho)+3\rho^2]}{2(1-\rho)^2(4x-4x^2-\rho)^2} g_{z\alpha} + \frac{\rho[-4x(2-9x+6x^2)+\rho(1-10x+2\rho)]}{2(1-\rho)^2(4x-4x^2-\rho)^2} \\ \quad + \frac{\rho[(2+\rho)g_{z\alpha}+3)]}{4(1-\rho)^{5/2}} \ln\frac{2x-1+\sqrt{1-\rho}}{2x-1-\sqrt{1-\rho}} & x>1 \\ \frac{-3\rho g_{z\alpha}-1-2\rho}{2(1-\rho)^2} + \frac{\rho[(2+\rho)g_{z\alpha}+3)]}{4(1-\rho)^{5/2}} \ln\frac{1+\sqrt{1-\rho}}{1-\sqrt{1-\rho}} & 0<x<1 \\ -\frac{\rho(1-2x)[-4x(1-x)(2+\rho)+3\rho^2]}{2(1-\rho)^2(4x-4x^2-\rho)^2} g_{z\alpha} - \frac{\rho[-4x(2-9x+6x^2)+\rho(1-10x+2\rho)]}{2(1-\rho)^2(4x-4x^2-\rho)^2} \\ \quad - \frac{\rho[(2+\rho)g_{z\alpha}+3)]}{4(1-\rho)^{5/2}} \ln\frac{2x-1+\sqrt{1-\rho}}{2x-1-\sqrt{1-\rho}} & x<0 \end{cases},
\end{aligned}
\tag{45}
$$

$$
\begin{aligned}
\tilde{f}_p(x,\rho) = {} & \frac{\alpha_s C_F}{2\pi}\begin{cases} \frac{-4x\rho(3-5x+2x^2)+\rho^2(4-3x+4x^2-4x^3)-\rho^3}{(1-x)(1-\rho)^2(4x-4x^2-\rho)} g_{z\alpha} + \frac{-2x\rho(5-6x)+\rho^2(3-2x)}{(1-\rho)^2(4x-4x^2-\rho)} \\ \quad + \left[\frac{-2\rho(1-4x+2x^2)-\rho^2(2-x+2x^2)+\rho^3}{2(1-x)(1-\rho)^{5/2}} g_{z\alpha} + \frac{-\rho(2-6x+\rho)}{2(1-\rho)^{5/2}}\right] \ln\frac{2x-1+\sqrt{1-\rho}}{2x-1-\sqrt{1-\rho}} & x>1 \\ \frac{\rho(1-2x)(4-3x-\rho)}{(1-x)(1-\rho)^2} g_{z\alpha} + \frac{-2x+3\rho-4x\rho}{(1-\rho)^2} \\ \quad + \left[\frac{-\rho(2-8x+4x^2)-\rho^2(2-x+2x^2)+\rho^3}{2(1-x)(1-\rho)^{5/2}} g_{z\alpha} + \frac{-\rho(2-6x+\rho)}{2(1-\rho)^{5/2}}\right] \ln\frac{1+\sqrt{1-\rho}}{1-\sqrt{1-\rho}} & 0<x<1 \\ -\frac{-4x\rho(3-5x+2x^2)+\rho^2(4-3x+4x^2-4x^3)-\rho^3}{(1-x)(1-\rho)^2(4x-4x^2-\rho)} g_{z\alpha} - \frac{-2x\rho(5-6x)+\rho^2(3-2x)}{(1-\rho)^2(4x-4x^2-\rho)} \\ \quad - \left[\frac{-2\rho(1-4x+2x^2)-\rho^2(2-x+2x^2)+\rho^3}{2(1-x)(1-\rho)^{5/2}} g_{z\alpha} + \frac{-\rho(2-6x+\rho)}{2(1-\rho)^{5/2}}\right] \ln\frac{2x-1+\sqrt{1-\rho}}{2x-1-\sqrt{1-\rho}} & x<0 \end{cases} \\
& + \frac{\alpha_s C_F}{2\pi}(1-\tau)\begin{cases} \frac{16x\rho(1-3x+2x^2)+4x^2\rho^2(3-2x)-\rho^3(5-2x)+2\rho^4}{2(1-\rho)^2(4x-4x^2-\rho)^2} g_{z\alpha} \\ \quad + \frac{\rho(1-2x)[16x(1-x)-2\rho(1+2x-2x^2)-\rho^2]}{2(1-\rho)^2(4x-4x^2-\rho)^2} + \frac{-\rho(4-\rho)(g_{z\alpha}+1)}{4(1-\rho)^{5/2}} \ln\frac{2x-1+\sqrt{1-\rho}}{2x-1-\sqrt{1-\rho}} & x>1 \\ \frac{\rho(5-2\rho)g_{z\alpha}+2+\rho}{2(1-\rho)^2} + \frac{-\rho(4-\rho)(g_{z\alpha}+1)}{4(1-\rho)^{5/2}} \ln\frac{1+\sqrt{1-\rho}}{1-\sqrt{1-\rho}} & 0<x<1 \\ -\frac{16x\rho(1-3x+2x^2)+4x^2\rho^2(3-2x)-\rho^3(5-2x)+2\rho^4}{2(1-\rho)^2(4x-4x^2-\rho)^2} g_{z\alpha} \\ \quad - \frac{\rho(1-2x)[16x(1-x)-2\rho(1+2x-2x^2)-\rho^2]}{2(1-\rho)^2(4x-4x^2-\rho)^2} - \frac{-\rho(4-\rho)(g_{z\alpha}+1)}{4(1-\rho)^{5/2}} \ln\frac{2x-1+\sqrt{1-\rho}}{2x-1-\sqrt{1-\rho}} & x<0 \end{cases}.
\end{aligned}
\tag{46}
$$

ξ^2 be small but not vanishing

Apply OPE to non-local op $\mathcal{O}_n(\xi)$

$$\sigma_n(\omega, \xi^2, P^2) = \sum_{J=0} \sum_a W_n^{(J,a)}(\xi^2, \mu^2)\, \xi^{\nu_1} \cdots \xi^{\nu_J} \times \langle P | \mathcal{O}^{(J,a)}_{\nu_1 \cdots \nu_J}(\mu^2) | P \rangle \,,$$

$\mathcal{O}^{(J,a)}_{\nu_1 \cdots \nu_J}(\mu^2)$ Local, symmetric , traceless op transit of one parton from a hadron across the other hadron. The probability of undergoing a hard scattering event with a large momentum transfer Q is proportional to the probability for finding two partons, one from each proton, to be within a transverse separation of $1/Q$ of each other. Multi-parton hard scattering is suppressed because there is a small probability of finding more than two partons within a short distance of $1/Q$ when the two flat disks collide. Soft final-state interactions should not change the cross section, as long as we make

Drell-Yan process

$$\pi^- + p \to \mu^+ + \mu^- + X$$