Topological structures in finite temperature QCD

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Motivation and Method

- Motivation
  - Understand the vacuum at finite temperature
  - Find the distribution of Topological objects
- Method
  - Finite Temperature lattice
  - Zero and almost-zero modes of Fermions
  - Chiral symmetry + Index theorem → Overlap Operator
- Configurations
  - Physical mass, $N_t = 8$ $N_s = 32$, around T_c
  - Generated with dynamical domain wall fermions

Approach

- 1) Find Eigenvectors of Overlap Operator
- 2) Analyze Zero-modes and almost zero modes
  - 2.1) Vectors of lowest eigenvalues are believed to be made up of topological objects
- 3) Compare to Caloron solutions
  - 3.1) SU(3) Caloron with non-trivial holonomy is made up of 3 Instanton-dyons
  - 3.2) Dyons for one Caloron can sit at different positions in space
- 4) Observe how eigenvectors change for different boundary conditions and temperature
  - 4.1) If a fermionic eigenvector changes position at different boundary conditions, it indicates that it moved from one dyon to another

Configurations

- We use configurations generated with dynamical domain wall fermions, used before in [Phys. Rev. Lett. 113, 082001]

T/T_c	N_s	N_t	Configurations
1.00	32	8	3
1.08	32	8	5

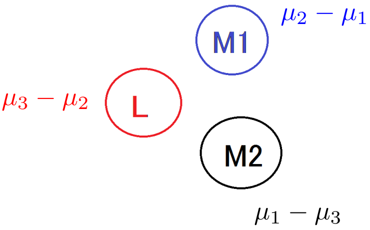
- We look in configurations with $|Q_{top}| = 1$
- We use zero-mode and almost zero-mode solutions to the overlap operator, to probe for topological objects
- Will look at density and chiral density, since they are gauge invariant

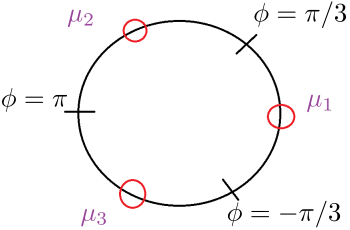
$$\rho(x) = \psi^{\dagger}_{a,i}\psi_{a,i} \quad (1)$$

$$\rho_5(x) = \psi^{\dagger}_{a,i}\gamma_{5(i,j)}\psi_{a,j} \quad (2)$$

Calorons and Instanton-Dyons

- Caloron made up of 3 dyons
- Polyakov loop $P = \frac{1}{3} Tr[\exp[iDiag(\mu_1, \mu_2, \mu_3)]]$
- Fermionic zero mode exist on dyon with $\mu_m < \phi < \mu_{m+1}$
- $\exp i\phi$ is the boundary condition of the Dirac operator in temporal direction





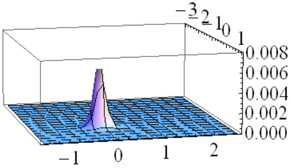
- Zero mode falloff depends on distance from ϕ to closest μ
- For $\phi = \pi$ falloff much stronger when $\mu_1 = \mu_2 = \mu_3 = 0$.
- Effective mass $= \log(\rho(x)/\rho(x+a))/a$

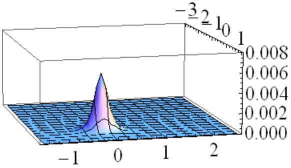
Lattice vs Analytic at $T = 1.08T_c$

- Comparison between zero-modes from overlap operator and analytic formula for Caloron with confining holonomy (Polyakov loop $= 0$)

Lattice | Caloron

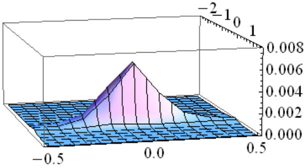
$\rho(x, y)$

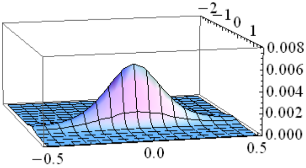




- Anti periodic fermions. Other Dyons at $(0.17fm, 0, 0)$ and $(-0.17fm, 0, 0)$.

$\rho(t, x)$



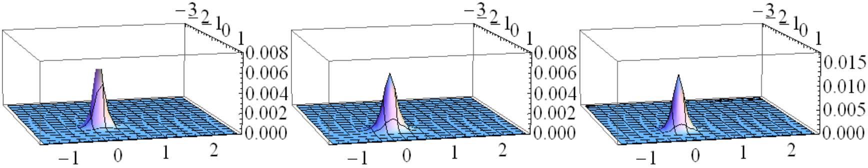


- Anti periodic fermions. Effective mass is 0.78 ± 0.09 (Lattice), 0.84 (Caloron).

Lattice vs Analytic at $T = 1.08T_c$

- Comparison between zero-modes from overlap operator and analytic formula for Caloron with confining holonomy (Polyakov loop $= 0$)

Lattice | Caloron | Trivial Holonomy



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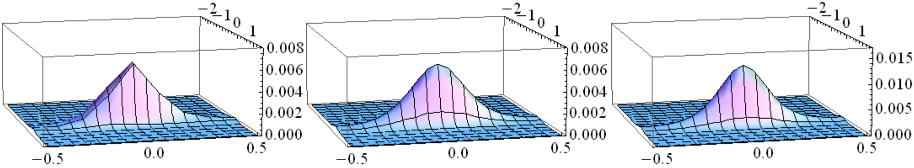

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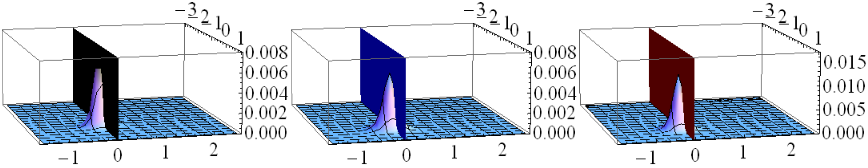

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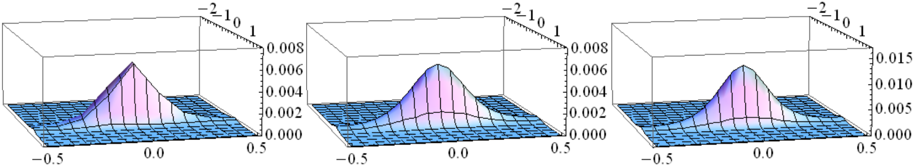

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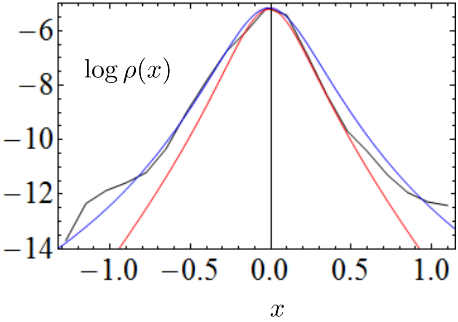
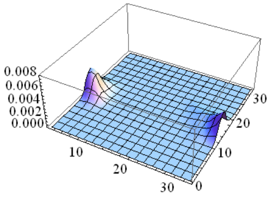
Falloff Comparison

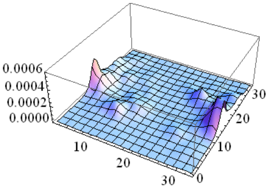
- 3d plots can be hard to compare
- Slice through the maximum
- Lattice (black), Confining Holonomy (blue), Trivial holonomy (red)
- Trivial holonomy scaled to same height as lattice

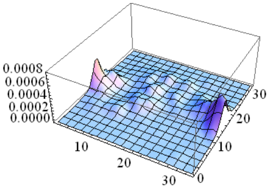


Different Boundary locations at $T = 1.08T_c$

- Peaks change location for different boundary conditions, unlike Instantons
- $\rho(x, y)$ sliced though third position coordinate z and summed over t







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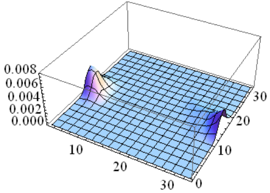
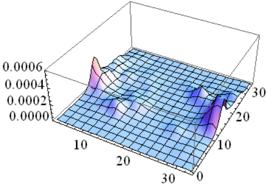
$\phi = \pi,\ z = 28$



$\phi = -\pi/3,\ z = 28$



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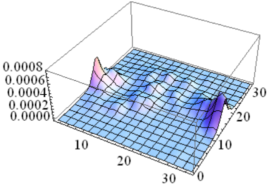

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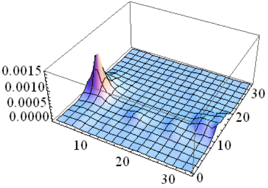

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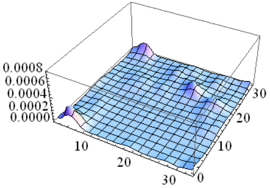

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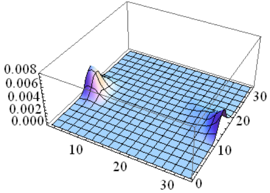

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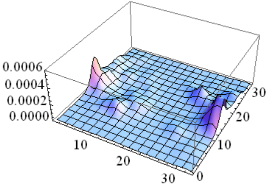

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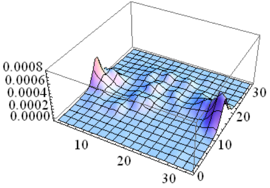

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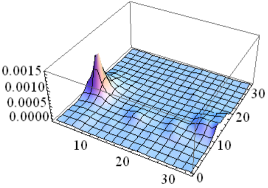

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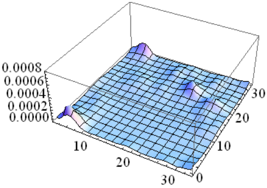
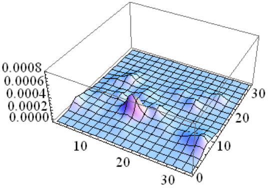
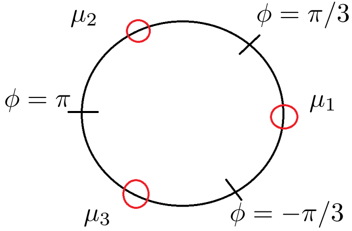


$\phi = \pi/3,\ z = 7$



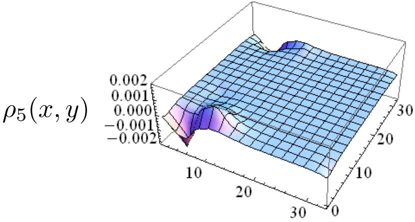
Why do we think it is Dyons

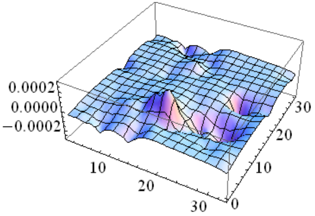
- The shape in the density ρ plots are well reproduced by different combinations of dyons
- Solutions move around for different boundary conditions
- The fall off (effective mass) is too fast for trivial holonomy
- Boundary conditions different from $\phi = \pi$ have many peaks, which indicate that they have not been affected by fermionic interactions.



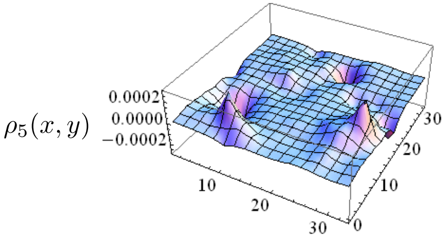
Almost Zero modes $\rho_5(x, y)$

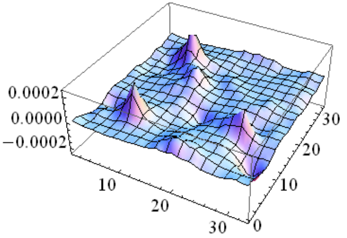
- Above T_c at $\phi = \pi$ and $\phi = \pi/3$





- At T_c at $\phi = \pi$ and $\phi = \pi/3$

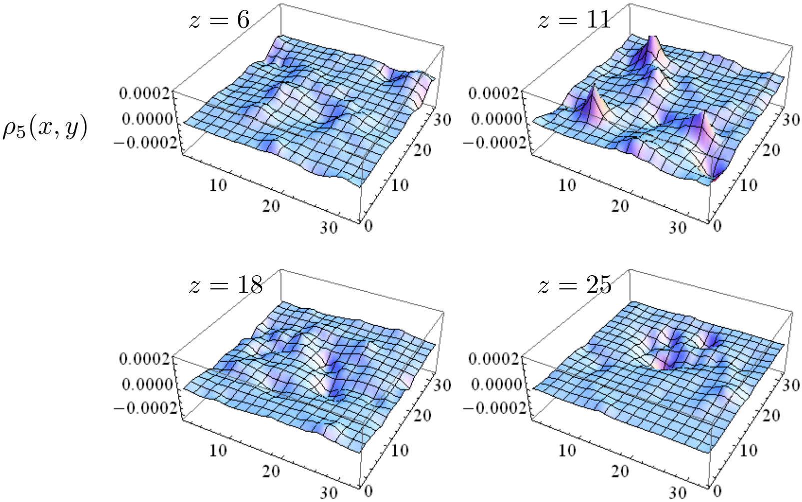




- Indicate picture of collectivization of interactions

Almost zero-mode compared to Caloron at $\phi = \pi/3$

- Almost zero modes also resembles calorons with non-trivial holonomy
- $\rho_5(x, y)$ sliced though third position coordinate z and summed over t
- $\phi = \pi/3$, $T = T_c$



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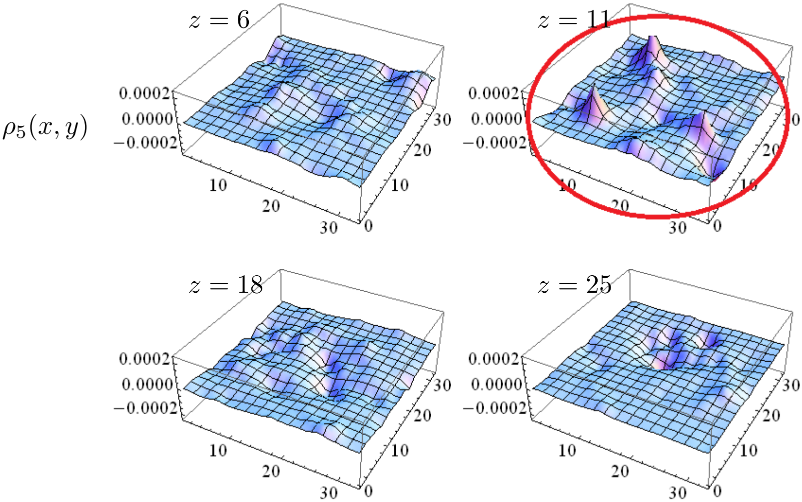
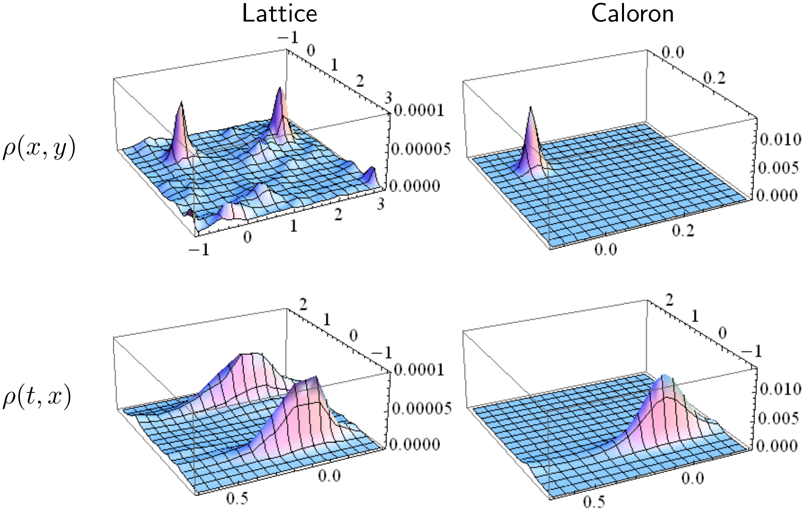


Almost zero-mode compared to Caloron at $\phi = \pi/3$

- Chosen at time such that peak of interest is maximum
- Two peaks appear along x direction, slightly different y position



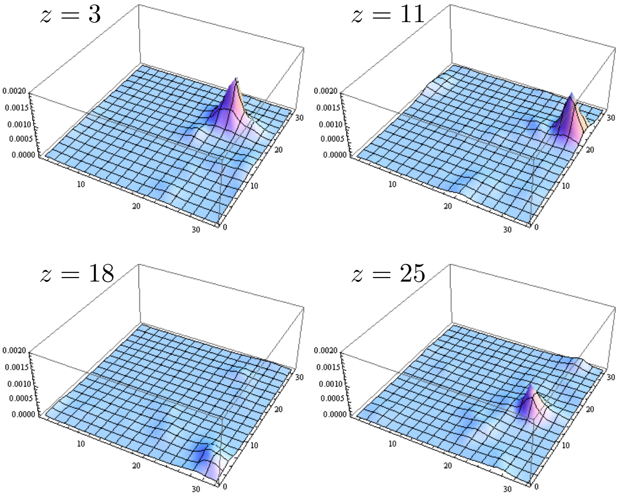
Conclusion

- We have seen that the density of the zero modes fit well with that of calorons with confining holonomy
- Solutions move around for different boundary conditions
- This indicates that Instanton-dyons make up the topoloical objects in the finite temperature vacuum
- We found that only $T > T_c$ has isolated pairs of almost zero modes
- Typical distance to closest dyons are $0.1fm - 0.5fm$
- Disclaimer
  - Low amount of data

Comparison at $\phi = -\pi/3$ Boundary Condition

- At ϕ not in the anti periodic region like $\phi = -\pi/3$ several peaks contribute
- $\rho(x, y)$ sliced though third position coordinate z and summed over t

$\rho(x, y)$



- ρ_5 positive everywhere (negative for $Q_{top} = -1$)

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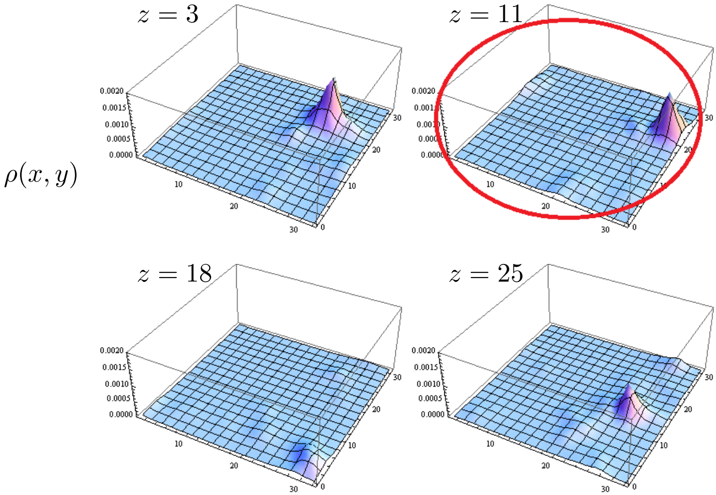

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Lattice | Caloron

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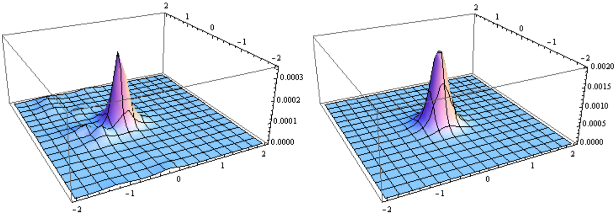
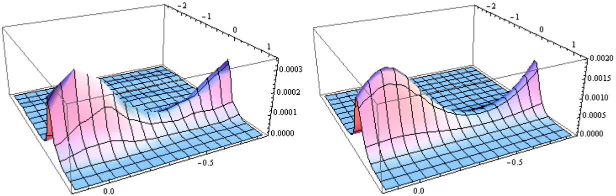


- The two other dyons sit at $(-0.2, 0, 0)$ and $(0, 0.3, 0)$.

$\rho(t, x)$



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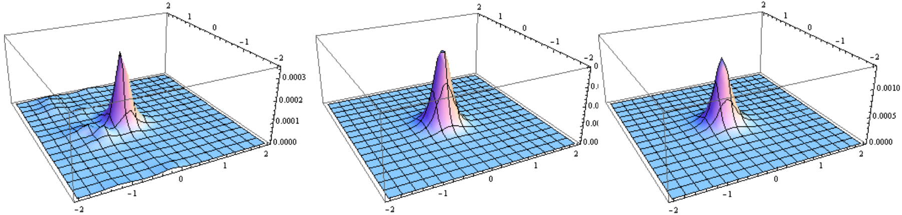

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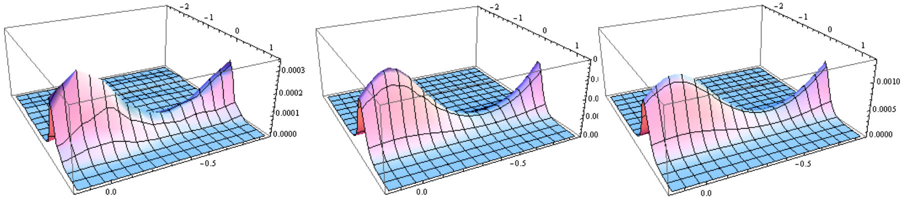

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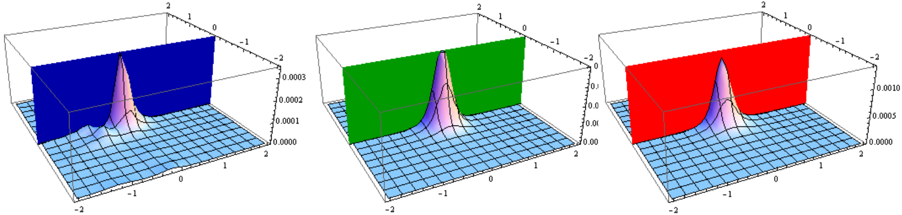

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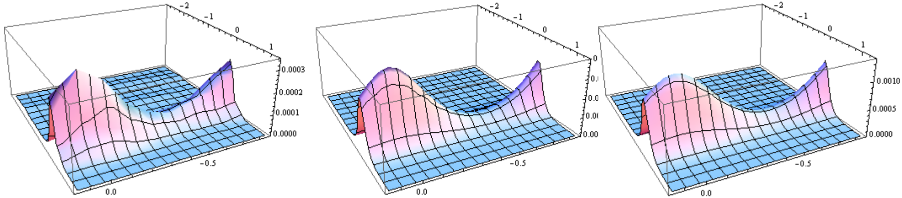

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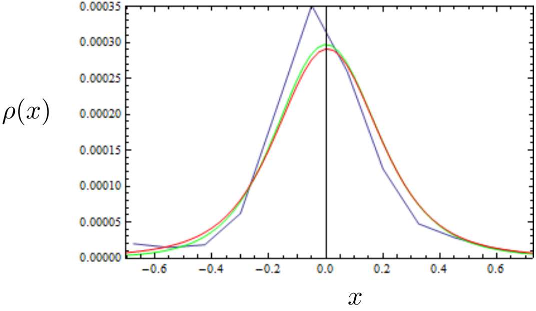
Falloff at other Boundary Condition

- Slice through the maximum
- Lattice (blue), Confining Holonomy (green), Trivial holonomy (red)
- Scaled to around the same height as lattice

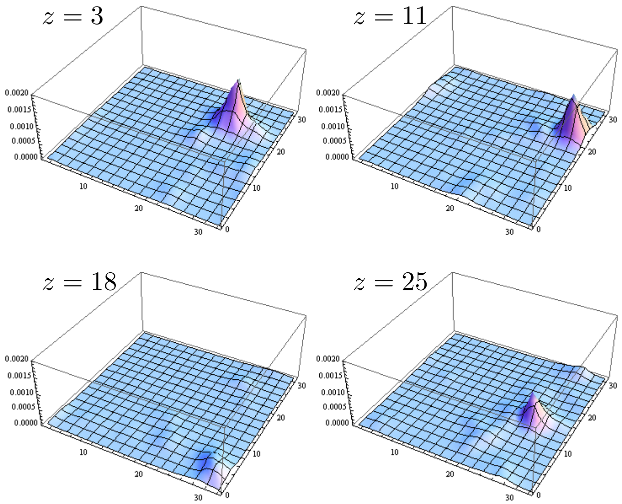


- At $\phi = -\pi/3$ falloff is the same for confining and trivial holonomy

Comparison at $\phi = -\pi/3$ Boundary Condition 2

- At ϕ not in the anti periodic region like $\phi = -\pi/3$ several peaks contribute
- $\rho(x, y)$ sliced though third position coordinate z and summed over t

$\rho(x, y)$



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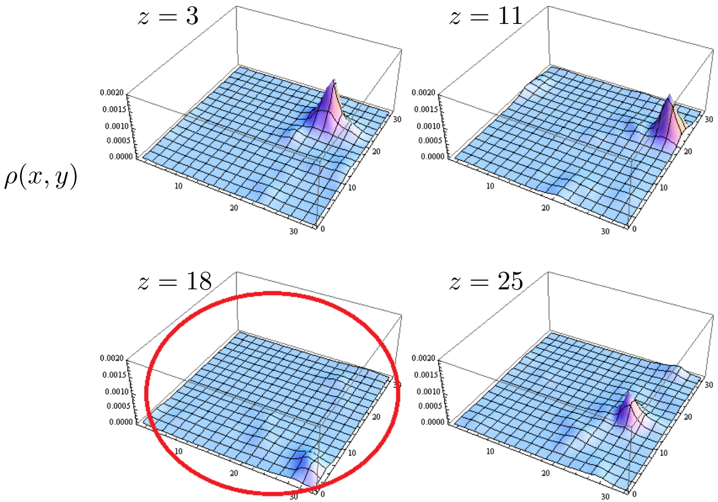

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Lattice | Caloron

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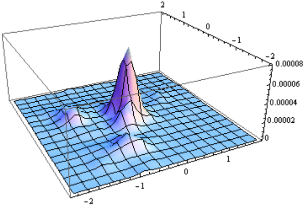

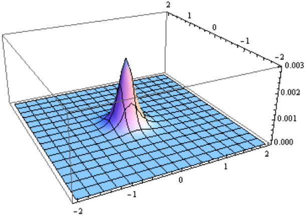

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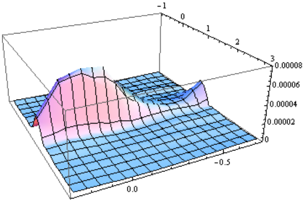
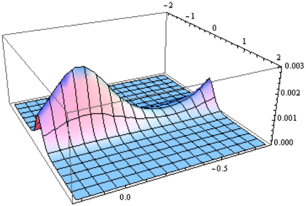
$\rho(t, x)$

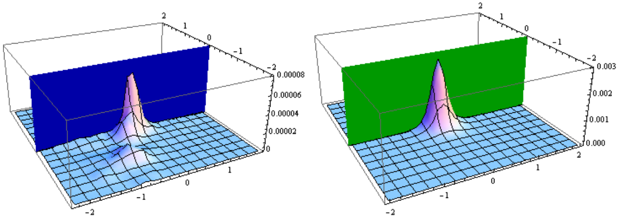




Comparison at $\phi = -\pi/3$ Boundary Condition 2

Lattice Caloron

$\rho(x,y)$



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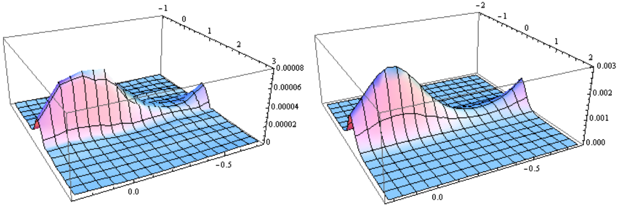

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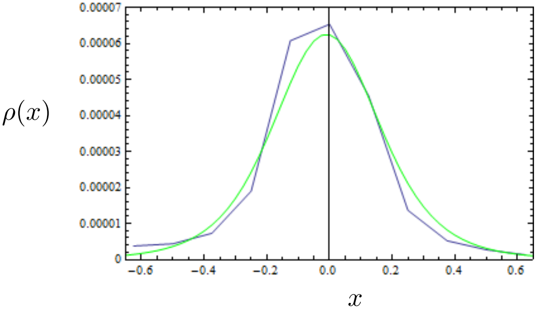

Some Comments

- Distance to other dyons between $0.1/T$ and $0.5/T$
- Might be biased due to $|Q_{top}| = 1$ requirement

- ρ_5 is completely positive(negative) for (anti)zero modes
- Zero modes can be made up of several calorons
- Sum of ρ_5 is 1(-1), but each peak contribute less than 1(-1)

Overlapping Peaks

- Each peak is consistent with Calorons made up of dyons