Lattice Quantum Gravity and Asymptotic Safety

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July, 2018

Asymptotic Safety

Weinberg proposed idea that gravity might be Asymptotically Safe in 1976 [Erice Subnucl. Phys. 1976:1]. This scenario would entail:

- Gravity is effectively renormalizable when formulated non-perturbatively. Problem lies with perturbation theory, not general relativity.

- In a Euclidean lattice formulation the fixed point would show up as a continuous phase transition point, the approach to which would define a continuum limit.

Lattice gravity

- Euclidean dynamical triangulations (EDT) is a lattice formulation that was introduced in the '90's. [Ambjorn, Carfora, and Marzuoli, The geometry of dynamical triangulations, Springer, Berlin, 1997] Lattice geometries are approximated by triangles with fixed edge lengths. The dynamics is contained in the connectivity of the triangles, which can be added or deleted.
- In lattice gravity, the lattice itself is a dynamical entity, which evolves in Monte Carlo time. The dimension of the building blocks can be fixed, but the effective fractal dimension must be calculated from simulations.
- EDT works in 2d, where it reproduces the results of non-critical bosonic string theory.
- The EDT formulation in 4d was shown to have two phases, a “collapsed" phase with infinite Hausdorff dimension and a branched polymer phase, with Hausdorff dimension 2. The critical point separating them was shown to be first order, so that new continuum physics is not expected. [Bialas et al, Nucl. Phys. B472, 293 (1996), hep-lat/9601024; de Bakker, Phys. Lett. B389, 238 (1996), hep-lat/9603024]

Einstein Hilbert Action

Continuum Euclidean path-integral:

$$Z = \int \mathscr{D}g \; e^{-S[g]}, \tag{1}$$

$$S[g_{\mu\nu}] = -\frac{k}{2} \int d^d x \sqrt{\det g} (R - 2\Lambda), \tag{2}$$

where $k = 1/(8\pi G_N)$.

Discrete action

Discrete Euclidean (Regge) action is

$$S_E = k \sum 2 V_2 \delta - \lambda \sum V_4, \tag{3}$$

where $\delta = 2\pi - \sum \theta$ is the deficit angle around a triangular face, V_i is the volume of an i-simplex, and $\lambda = k\Lambda$. Can show that

$$S_E = -\frac{\sqrt{3}}{2}\pi k N_2 + N_4 \left(\frac{5\sqrt{3}}{2} k \arccos \frac{1}{4} + \frac{\sqrt{5}}{96} \lambda \right) \tag{4}$$

where N_i is the total number of i-simplices in the lattice. Conveniently written as

$$S_E = -\kappa_2 N_2 + \kappa_4 N_4. \tag{5}$$

Measure term

Continuum calculations suggest a form for the measure

$$Z = \int \mathscr{D}g \prod_x \sqrt{\det g}^{\beta} \; e^{-S[g]}, \tag{6}$$

Going to the discretized theory, we have

$$\prod_x \sqrt{\det g}^{\beta} \rightarrow \prod_{j=1}^{N_2} \mathscr{O}(t_j)^{\beta}, \tag{7}$$

where $\mathscr{O}(t_j)$ is the order of triangle t_j, i.e. the number of 4-simplices to which a triangle belongs. Can incorporate this term in the action by taking exponential of the log. β is a free parameter in simulations. Can interpret as an ultra-local measure term, since it looks like a product over local 4-volumes.

New Idea

Revisiting the EDT approach because other formulations (renormalization group and other lattice approaches) suggest that gravity is asymptotically safe.

New work done in collaboration with students (past and present) and postdoc: JL, S. Bassler, D. Coumbe, Daping Du, J. Neelakanta, (arXiv:1604.02745).

- Key new idea that inspired this study is that a fine-tuning of bare parameters in EDT is necessary to recover the correct continuum limit. This is in analogy to using Wilson fermions in lattice gauge theory to study quantum chromodynamics (QCD) with light or massless quarks. Striking similarities are seen. Coincidence?
- Previous work did not implement this fine-tuning, leading to negative results.

Main problems to overcome

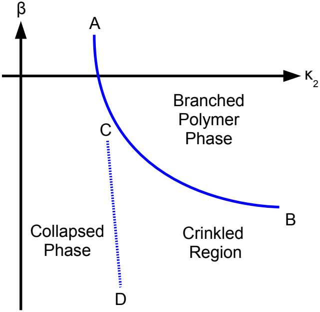
- Must show recovery of semiclassical physics in 4 dimensions.
- Must show existence of continuum limit at continuous phase transition.

Simulations

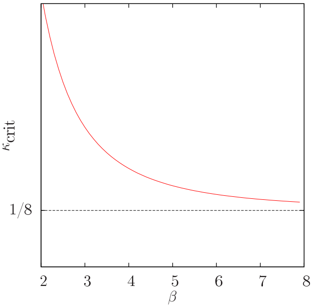
Methods for doing these simulations were introduced in the 90's. We wrote new code from scratch.

- The Metropolis Algorithm is implemented using a set of local update moves.
- We introduce a new algorithm for parallelizing the code, which we call parallel rejection. Exploits the low acceptance of the model, and partially compensates for it. Checked that it reproduces the scalar code configuration-by-configuration. Buys us a factor of ~ 10.

Phase diagram EDT vs. QCD with Wilson fermions

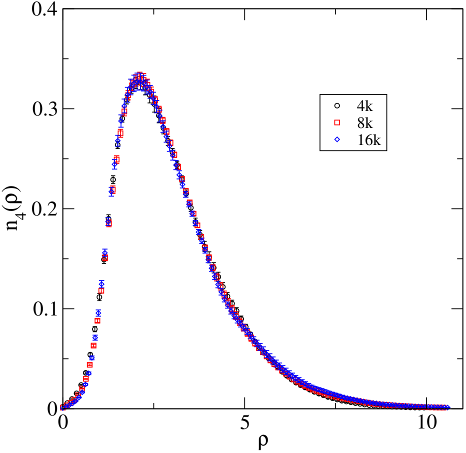


EDT

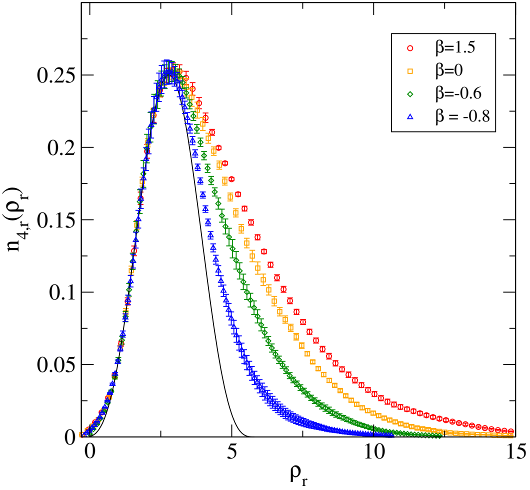


QCD

Three volume distribution



Three volume distribution



What does it mean?

Interesting results that suggest that the correct classical result might be restored in the continuum, large-volume limit. Analogy with Wilson fermions that inspired this study may tell us more.

We have to perform a fine-tuning, and long distance physics gets messed up by discretization effects. These things happen when the regulator breaks a symmetry of the quantum theory. In this case, natural to identify the symmetry as continuum diffeomorphism invariance.

If true, then β would not be a relevant parameter in a continuum formulation with diffeo invariance unbroken. (Would still need a measure term if the regulator broke scale invariance, but β would be fixed.)

Interesting because if true, number of relevant couplings in continuum theory could be less than 3.

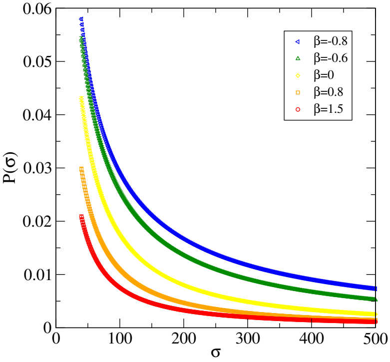
Diffusion process and the spectral dimension

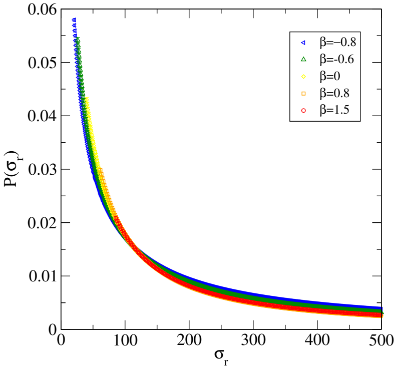
Spectral dimension is defined by a diffusion process

$$D_S(\sigma) = -2\frac{d \log P(\sigma)}{d \log \sigma}, \tag{8}$$

where σ is the diffusion time step on the lattice, and $P(\sigma)$ is the return probability, i.e. the probability of being back where you started in a random walk after σ steps.

Relative lattice spacing

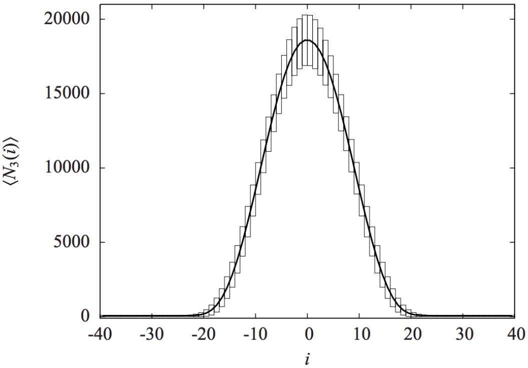




Return probability left and rescaled return probability right.

Causal dynamical triangulations

Euclidean de Sitter space solution from arXiv:1604.02745 (Ambjorn et al.)



Semiclassical fluctuations

Looking at quantum fluctuations about de Sitter space allow one to fix M_{Planck}. A simple minisuperspace model fits the CDT data well. Ambjorn et al. (arXiv:1604.02745) look at the correlator

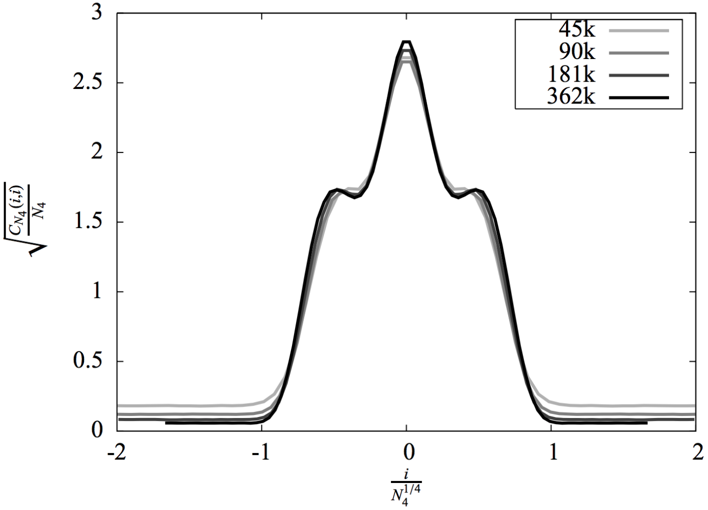
$$C(i,j) = \frac{1}{K} \sum_k (N_3^{(k)}(i) - \overline{N}_3(i))(N_3^{(k)}(j) - \overline{N}_3(j)), \tag{9}$$

where one can show that $C(i,j) \propto G_N$.

The size of these quantum fluctuations compared to the width of the de Sitter universe can be used to fix the lattice spacing.

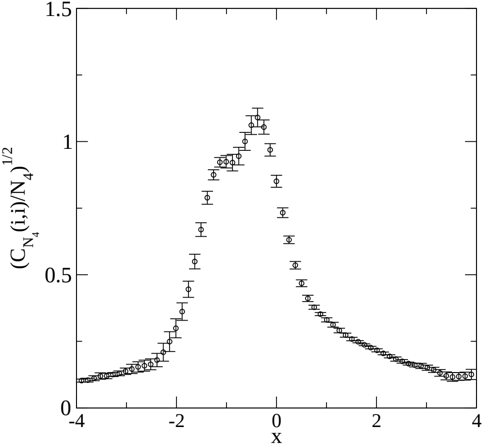
Causal dynamical triangulations

Semiclassical fluctuations about de Sitter, arXiv:1604.02745 (Ambjorn et al.)



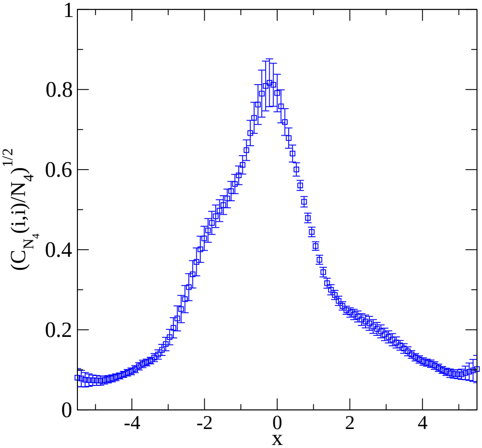
Semiclassical fluctuations from EDT

4k, $\beta = 0$

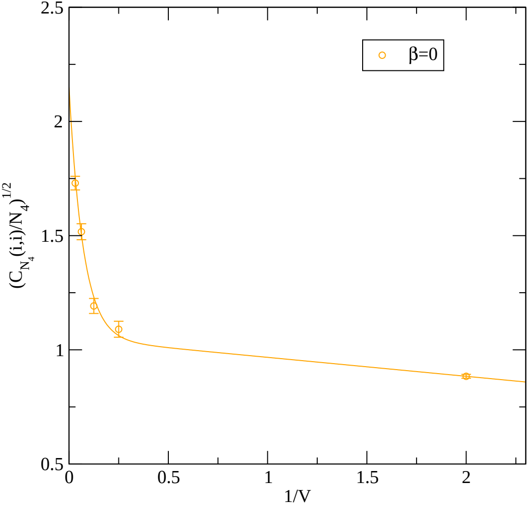


Semiclassical fluctuations from EDT

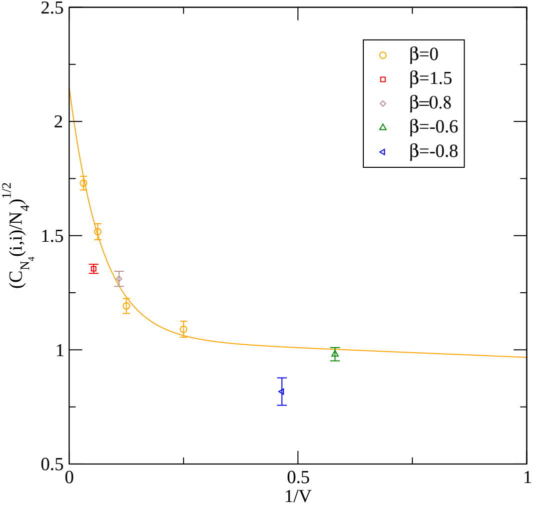
8k, $\beta = -0.8$



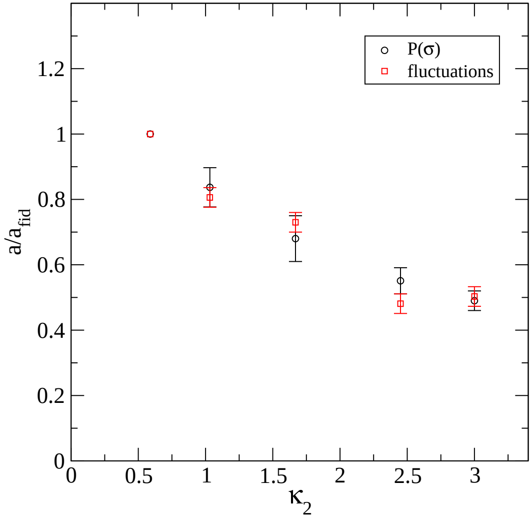
Finite volume effects



Finite volume effects



Relative lattice spacing



Relative lattice spacing

The agreement between these two approaches for obtaining the relative lattice spacing bolsters the interpretation of the theory as quantum gravity, and strengthens the link between EDT and CDT.

Could they be in the same universality class?

Conclusions

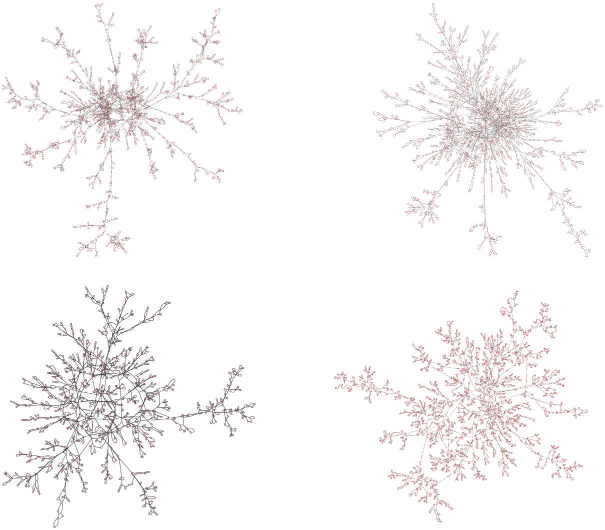
Important to test the picture presented here against other approaches, renormalization group and other lattice formulations.

If this holds, lattice provides a nonperturbative definition of a renormalizable quantum field theory of gravity.

Beginning to probe the matter sector (in the quenched approximation) by looking at effects of the geometry on scalar and fermion fields.

Back-up Slides

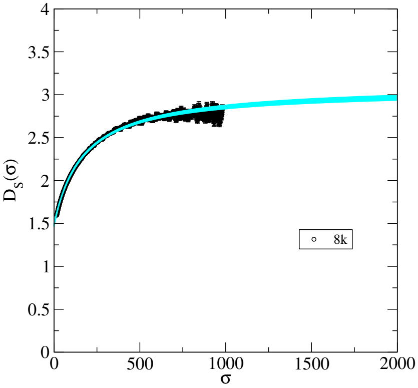
Visualization of geometries



Coarser to finer, left to right, top to bottom.

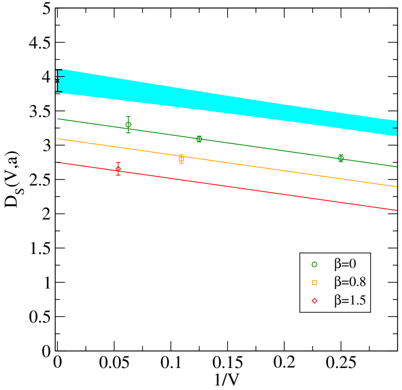
Spectral Dimension

χ^2/dof=1.25, p-value=17%
$D_S(\infty) = 3.090 \pm 0.041$, $D_S(0) = 1.484 \pm 0.021$



Infinite volume, continuum extrapolation

χ^2/dof=0.52, p-value=59%
$D_S(\infty) = 3.94 \pm 0.16$



Infinite volume, continuum extrapolation

χ^2/dof=0.17, p-value=84%
$D_S(0) = 1.44 \pm 0.19$