HADRONIC STRUCTURE OF THE NUCLEON

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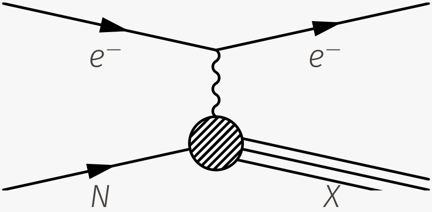
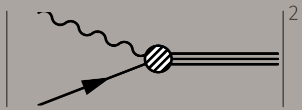
Collaborators: Jacob Bickerton, Alex Chambers, Roger Horsley, Yoshifumi Nakamura, Holger Perlt, Paul Rakow, Gerrit Schierholz, Arwed Schiller, Hinnurk Stüben, Ross Young & James Zanotti
(QCDSF Collaboration)

DEEP INELASTIC SCATTERING



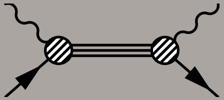
$$\omega = \frac{2p \cdot Q}{Q^2} = \frac{m_X^2 - m_N^2}{Q^2} + 1$$

Hadron Tensor



$\rightarrow$ Hadron Tensor has structure functions F_1, F_2

Compton Amplitude



$\rightarrow$ Compton Amplitude has Lorenzt-scalar functions T_1, T_2

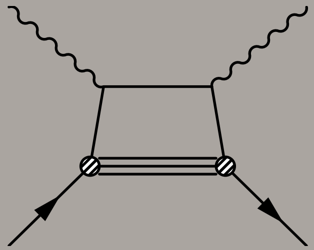
$$F_i = \frac{1}{2\pi} Im T_i$$

TWIST AND OPE

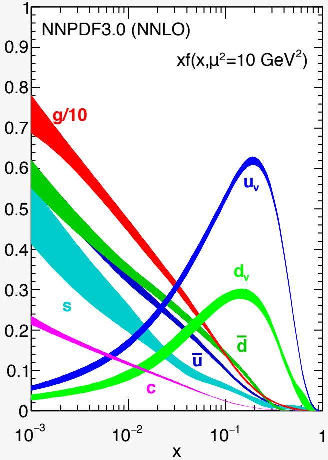
→ Non local operator in terms of a series of local operators

→ $\mathcal{N} = c_1\mathcal{O}_1 + \frac{c_2}{Q^2}\mathcal{O}_2 + \frac{c_3}{Q^4}\mathcal{O}_3 + \cdots$

"Handbag" Diagram



Twist 2 or Leading Twist



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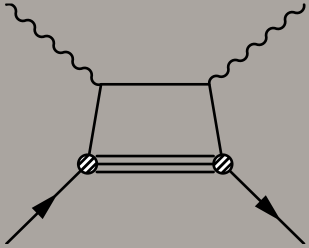
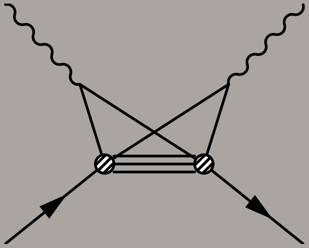


Twist 2 or Leading Twist

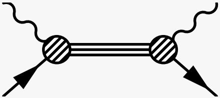
"Cat's Ears" Diagram



Higher Twist

COMPTON AMPLITUDE IN EXPERIMENT

$T_{\mu\nu} = \rho_{ss'} \int d^4\xi e^{iq\cdot\xi} \langle p,s' \,|\, T J_\mu(\xi) J_\nu(0) \,|\, p,s \rangle$



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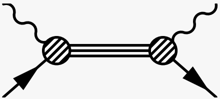

$\rightarrow$ Two photon exchange part of Hydrogen spectroscopic transition's contribution to proton charge radius uncertainty

COMPTON AMPLITUDE IN EXPERIMENT

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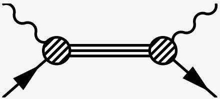

- → Two photon exchange part of Hydrogen spectroscopic transition's contribution to proton charge radius uncertainty
- → Unpolarised Compton amplitude proton neutron difference contribution to P-N mass splitting

COMPTON AMPLITUDE IN EXPERIMENT

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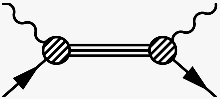

- $\rightarrow$ Two photon exchange part of Hydrogen spectroscopic transition's contribution to proton charge radius uncertainty
- $\rightarrow$ Unpolarised Compton amplitude proton neutron difference contribution to P-N mass splitting
- $\rightarrow$ Subtraction term $T(\omega = 0, Q^2)$

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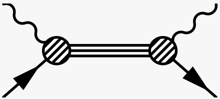

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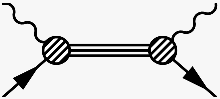

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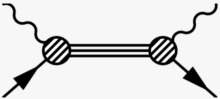

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- $\rightarrow$ Unpolarised Compton amplitude proton neutron difference contribution to P-N mass splitting
- $\rightarrow$ Subtraction term $T(\omega = 0, Q^2)$
  - Muonic hydrogen lamb shift uncertainty
  - P-N self energy uncertainty
  - Reggeon dominance hypothesis

COMPTON AMPLITUDE

$$
\begin{aligned}
T_{\mu\nu} &= \rho_{ss'} \int d^4\xi e^{iq\cdot\xi} \langle p, s' \,|\, TJ_\mu(\xi) J_\nu(0) \,| p, s\rangle \\
&= \left(-g_{\mu\nu} + \frac{q_\mu q_\nu}{q^2}\right) \boldsymbol{T_1} + \left(p_\mu - \frac{1}{2}\omega q_\mu\right)\left(p_\nu - \frac{1}{2}\omega q_\nu\right) \frac{\boldsymbol{T_2}}{\nu} \\
&+ \epsilon_{\mu\nu\alpha\beta} q^\alpha \left[\frac{s^\beta}{\nu}\boldsymbol{G_1} + \frac{\nu M s^\beta - s\cdot q p^\beta}{\nu^2}\boldsymbol{G_2}\right]
\end{aligned}
$$

where

$$
\begin{aligned}
\nu &= p \cdot Q \\
\omega &= \frac{2p\cdot Q}{Q^2}
\end{aligned}
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where

$$\nu = p \cdot Q$$

$$\omega = \frac{2p \cdot Q}{Q^2}$$

restrict to a subset

$$p_3 = q_3 = 0$$

$$\mu = \nu = 3$$

$$\rho = \frac{1}{2}\mathbb{I}$$

COMPTON AMPLITUDE

$$
\begin{aligned}
T_{33} &= \int d^4\xi e^{iq\cdot\xi} \langle p, s' \,|\, TJ_3(\xi) J_3(0) \,|p, s\rangle \\
&= T_1\left(\omega, Q^2\right)
\end{aligned}
$$

where

$$
\begin{aligned}
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\end{aligned}
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Via dispersion relation one can relate T_1 at $|\omega| < 1$ to physical region $1 < |\omega| < \infty$

$$T_1\left(\omega, Q^2\right) - T_1\left(0, Q^2\right) = 4\omega^2 \int_0^1 dx \frac{xF_1\left(x, Q^2\right)}{1 - (\omega x)^2}$$

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but we know

$$F_2\left(x, Q^2\right) = 2xF_1\left(x, Q^2\right) \qquad \text{(Callan-Gross)}$$

$$F_2\left(x, Q^2\right) = e_q^2 x\left(q(x) + \overline{q}(x)\right) \qquad \text{(Parton Model)}$$

MOMENTS

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so

$$= \sum_n 2\omega^{2n} \left\langle x^{2n-1} \right\rangle$$

LATTICE SITUATION

$\rightarrow$ »Just« calculate the four point function
$\int d^3x d^4y d^4z e^{i\mathbf{p}\cdot\mathbf{x}} e^{iq\cdot y} e^{-iq\cdot z} \left\langle \chi(x)\, T\left\{J_\mu(y)\, J_\nu(z)\right\} \chi^\dagger(0) \right\rangle$

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→ Other techniques to get at PDFs

- Heavy quark currents
- Current-current correlators
- Quasi-PDFs
- Pseudo-PDFs

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- $\rightarrow$ Other techniques to get at PDFs
  - Heavy quark currents
  - Current-current correlators
  - Quasi-PDFs
  - Pseudo-PDFs
- $\rightarrow$ Complementary new method using Feynman-Hellmann theorem

- → Calculate matrix elements using two point methods, ie. energy eigenstates

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Modify Action

$$S \to S + \lambda \int d^4x \;\cos\left(\mathbf{q}\cdot\mathbf{x}\right) J\left(x\right)$$

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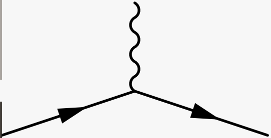
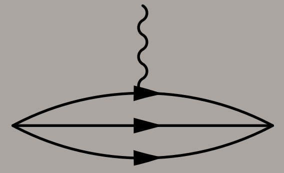
Feynman-Hellmann Theorem

$$\left.\frac{dE_{X,\mathbf{p}}}{d\lambda}\right|_{\lambda=0} = \frac{1}{2E_{X,\mathbf{p}}} \langle X, \mathbf{p} \,|\, J(0) \,|X, \mathbf{p} \pm \mathbf{q}\rangle$$



LATTICE CAVEAT

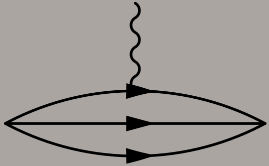
Connected



- $\rightarrow$ Modify quark propagator
- $\rightarrow$ High correlation for different λ

LATTICE CAVEAT

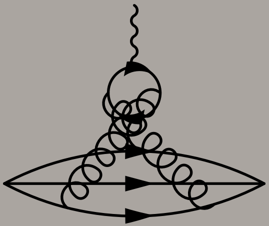
Connected



→ Modify quark propagator

→ High correlation for different λ

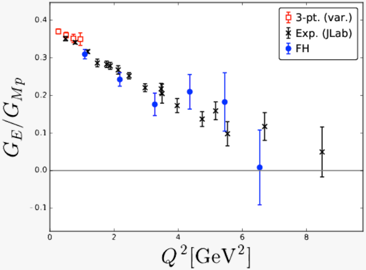
Disconnected

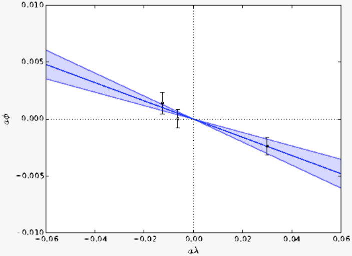


→ Modify weighting

→ No correlation for different λ

THINGS FEYNMAN-HELLMANN CAN DO





→ Large momentum EM form factors

→ Disconnected spin contribution to nucleon spin

Modify Action

$$S \to S + \lambda \int d^4x \;\cos\left(\mathbf{q} \cdot \mathbf{x}\right) J\left(x\right)$$

Modify Action

$$S \to S + \lambda \int d^4x \;\cos(\mathbf{q}\cdot\mathbf{x})\, J(x)$$

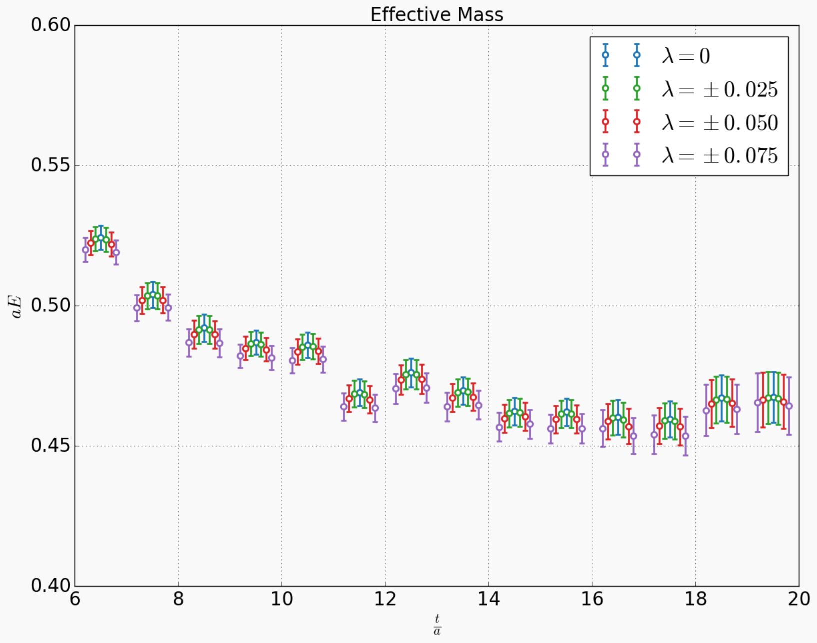
Second Order FHT

$$\left.\frac{d^2E}{d\lambda^2}\right|_{\lambda=0} = -\frac{\left\langle p \;\middle|\; \int d^4\xi\, 2\cos(\mathbf{q}\cdot\xi) T J(\xi) J(0) \;\middle|\; p \right\rangle}{2E}$$

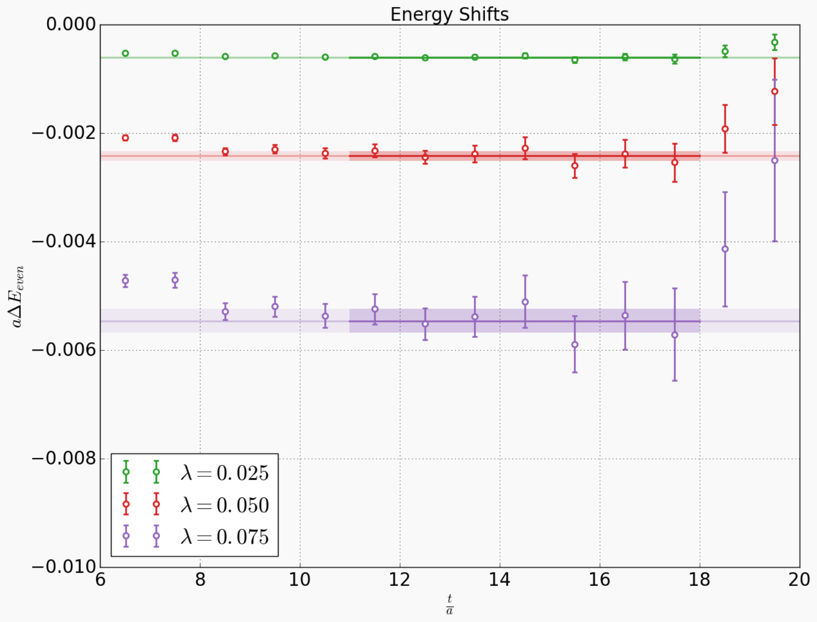
LATTICE DETAILS

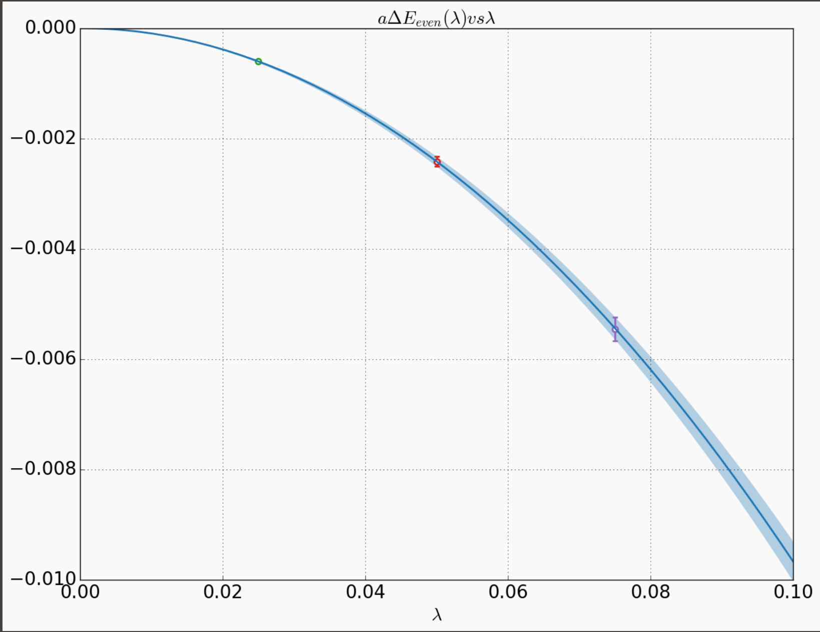
$L^3 \times T$	β	κ	$m_\pi(MeV)$	N_{cfg}	$N_{src/cfg}$
$32^3 \times 64$	5.5	0.1209	470	$\mathcal{O}(2000)$	$1-3$
$32^3 \times 64$	5.4	0.11993	470	$\mathcal{O}(1000)$	1
$32^3 \times 64$	5.65	0.122005	470	$\mathcal{O}(500)$	1

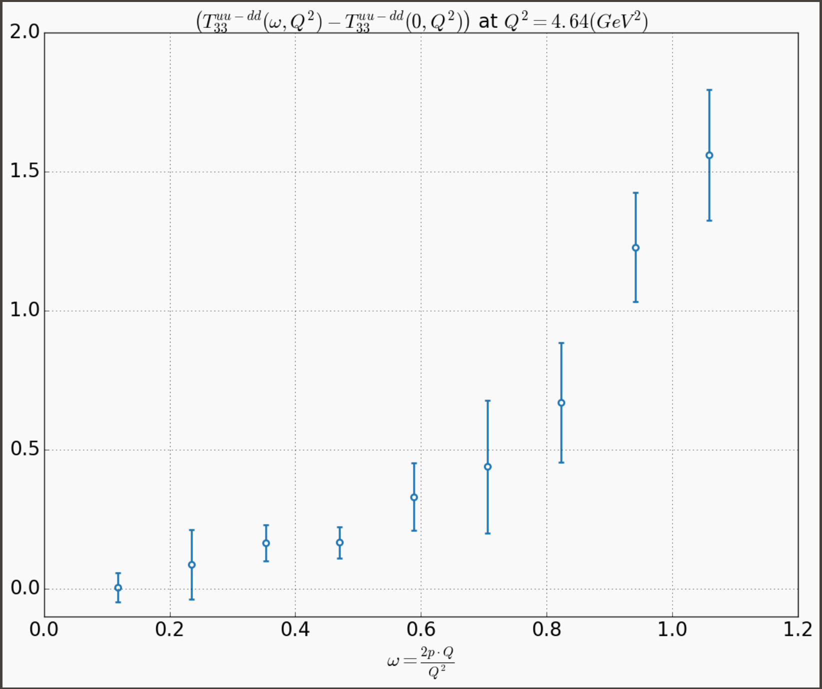
- $\rightarrow$ Q^2 from $1-16$ GeV^2, via various q in between $[2,1,0]$ and $[7,4,0]$
- $\rightarrow$ Different $\omega = \frac{2p\cdot Q}{Q^2}$ by different sink momenta
- $\rightarrow$ Renormalisation constant Z_V for lattices known
- $\rightarrow$ QCDSF $\mathcal{O}(a)$ improved action

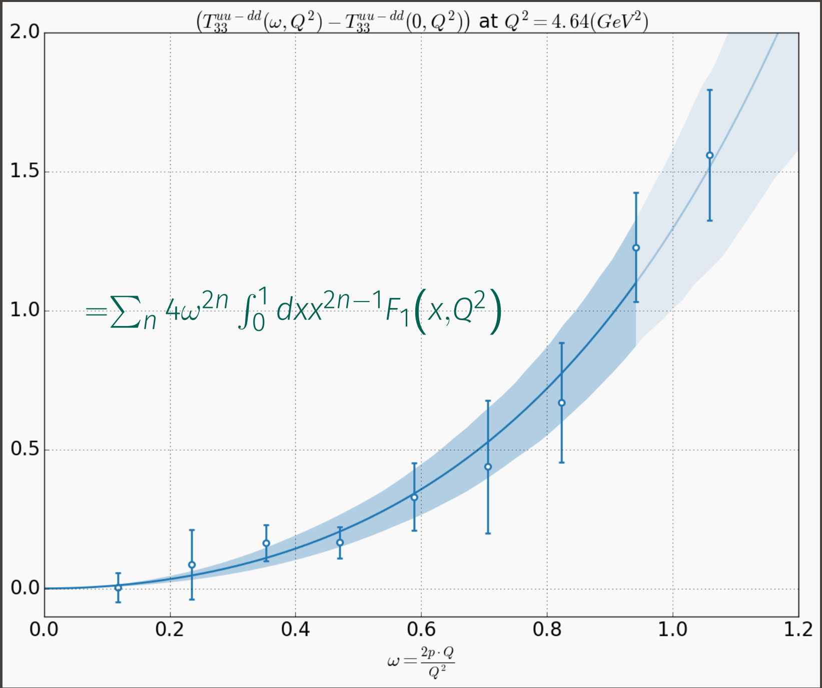


Effective Mass
0.60
0.55
0.50
0.45
0.40
6
8
10
12
14
16
18
20
aE
$\frac{t}{a}$
$\lambda = 0$
$\lambda = \pm 0.025$
$\lambda = \pm 0.050$
$\lambda = \pm 0.075$

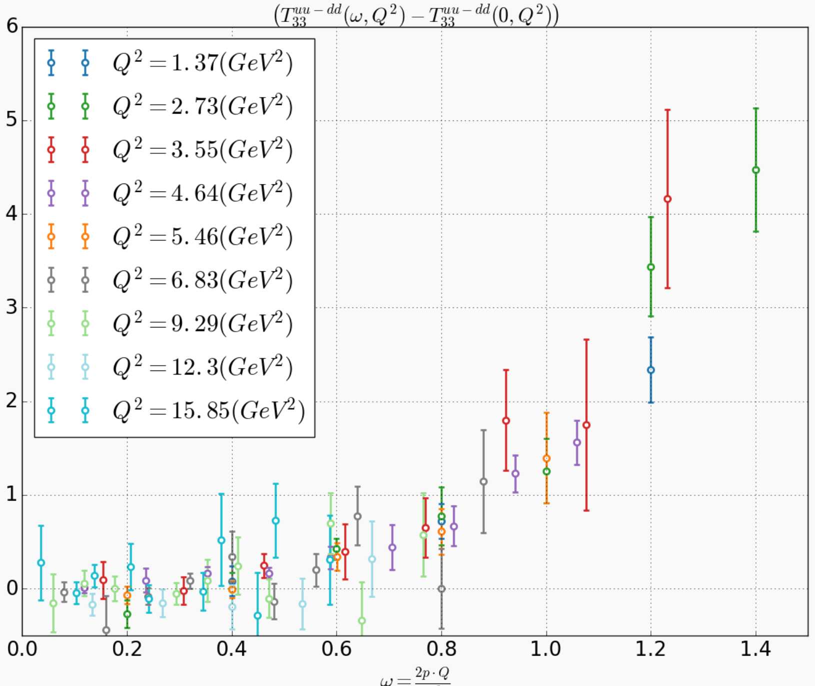

Energy Shifts
0.000
−0.002
−0.004
−0.006
−0.008
−0.010
6
8
10
12
14
16
18
20
$a\Delta E_{even}$
$\frac{t}{a}$
$\lambda = 0.025$
$\lambda = 0.050$
$\lambda = 0.075$


$a\Delta E_{even}(\lambda) vs \lambda$
0.000
−0.002
−0.004
−0.006
−0.008
−0.010
0.00
0.02
0.04
0.06
0.08
0.10
λ

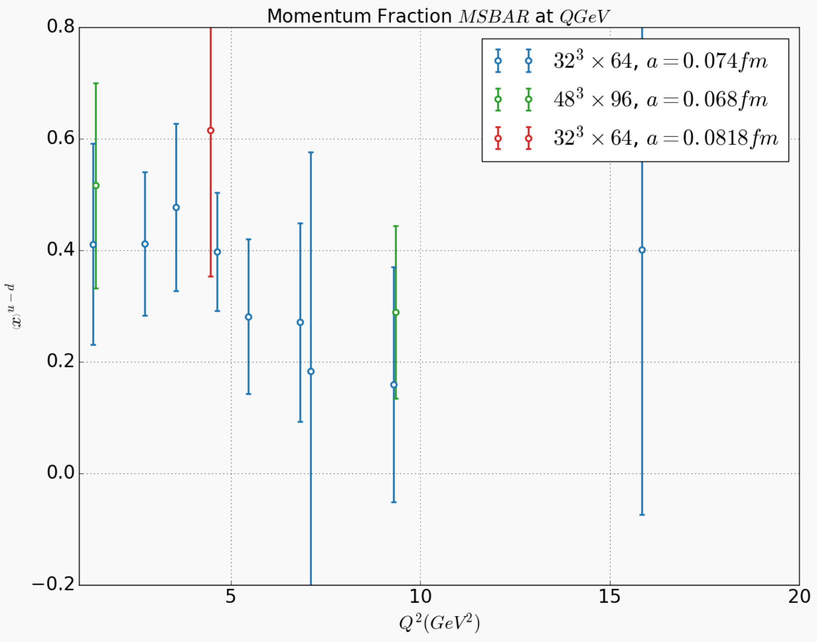

$\left(T_{33}^{uu-dd}(\omega,Q^2) - T_{33}^{uu-dd}(0,Q^2)\right)$ at $Q^2 = 4.64(GeV^2)$
2.0
1.5
1.0
0.5
0.0
0.0
0.2
0.4
0.6
0.8
1.0
1.2
$\omega = \frac{2p\cdot Q}{Q^2}$



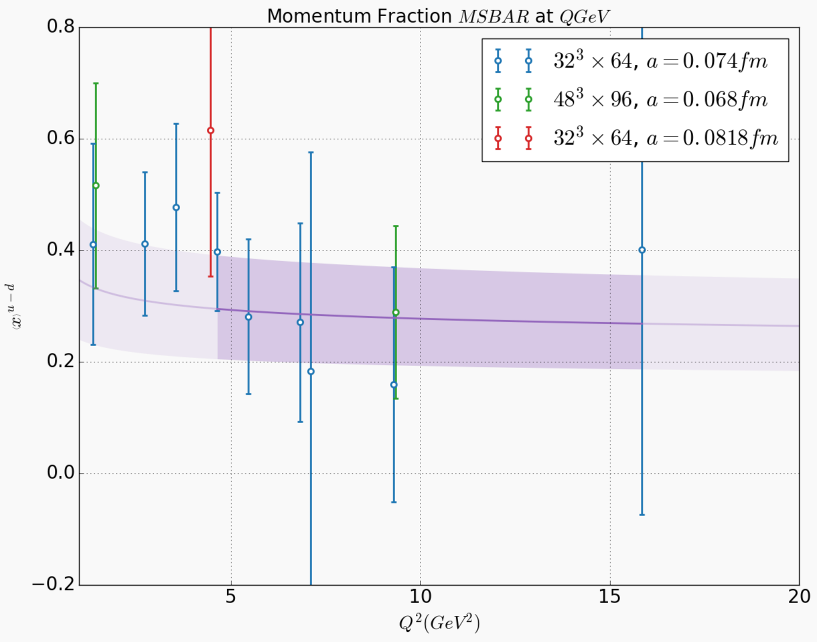
$\left(T_{33}^{uu-dd}(\omega,Q^2) - T_{33}^{uu-dd}(0,Q^2)\right)$ at $Q^2 = 4.64(GeV^2)$
$= \sum_n 4\omega^{2n} \int_0^1 dx x^{2n-1} F_1(x,Q^2)$
2.0
1.5
1.0
0.5
0.0
0.0
0.2
0.4
0.6
0.8
1.0
1.2
$\omega = \frac{2p\cdot Q}{Q^2}$



$\left(T_{33}^{uu-dd}(\omega,Q^2) - T_{33}^{uu-dd}(0,Q^2)\right)$
$Q^2 = 1.37(GeV^2)$
$Q^2 = 2.73(GeV^2)$
$Q^2 = 3.55(GeV^2)$
$Q^2 = 4.64(GeV^2)$
$Q^2 = 5.46(GeV^2)$
$Q^2 = 6.83(GeV^2)$
$Q^2 = 9.29(GeV^2)$
$Q^2 = 12.3(GeV^2)$
$Q^2 = 15.85(GeV^2)$
0
1
2
3
4
5
6
0.0
0.2
0.4
0.6
0.8
1.0
1.2
1.4



Momentum Fraction $MSBAR$ at $QGeV$
$32^3 \times 64,\ a = 0.074fm$
$48^3 \times 96,\ a = 0.068fm$
$32^3 \times 64,\ a = 0.0818fm$
$\langle x \rangle^{u-d}$
$Q^2(GeV^2)$
0.8
0.6
0.4
0.2
0.0
−0.2
5
10
15
20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$\langle x \rangle_{u-d}$
$Q^2(GeV^2)$
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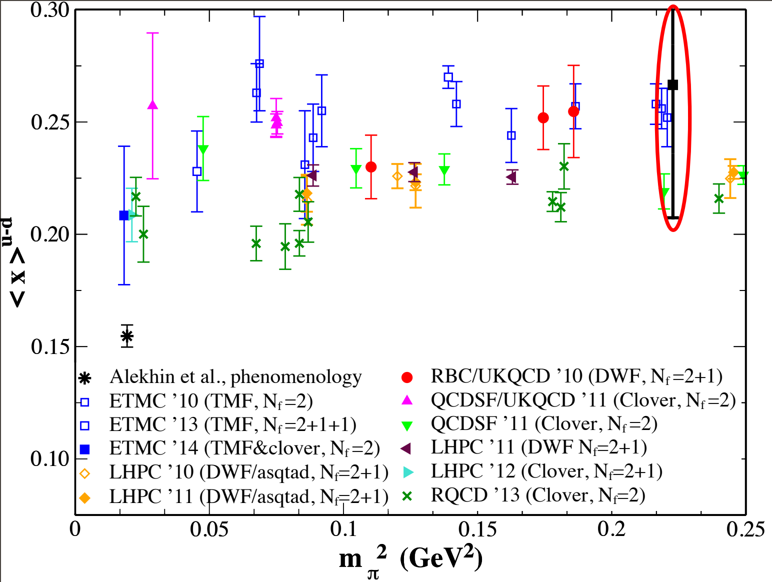
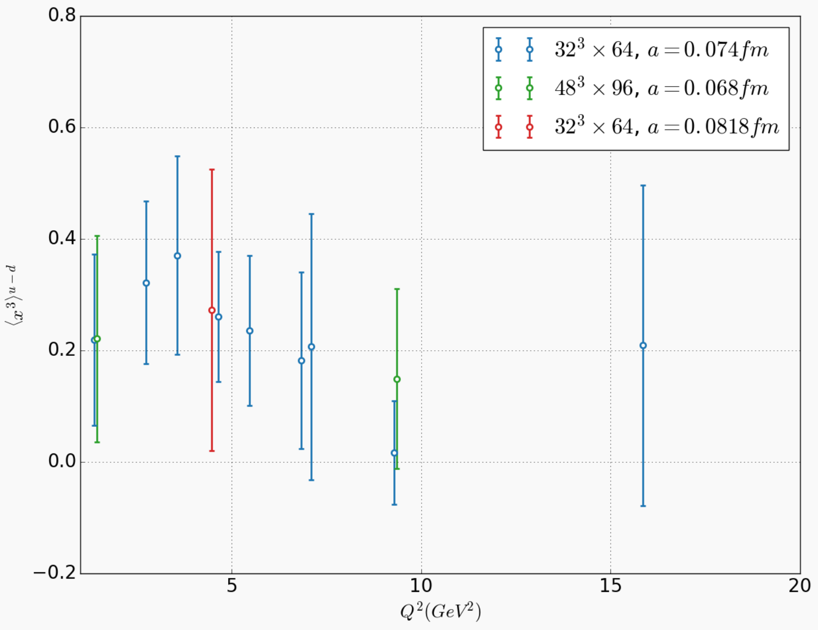
0.6
0.4
0.2
0.0
−0.2
5
10
15
20



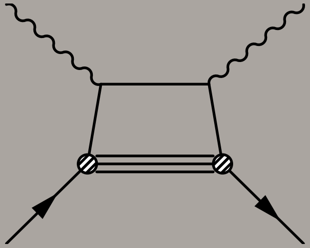
Constantinou [arXiv: 1511.00214]



$32^3 \times 64,\ a = 0.074fm$
$48^3 \times 96,\ a = 0.068fm$
$32^3 \times 64,\ a = 0.0818fm$
0.8
0.6
0.4
0.2
0.0
−0.2
5
10
15
20
$\langle x^3 \rangle_{u-d}$
$Q^2 (GeV^2)$

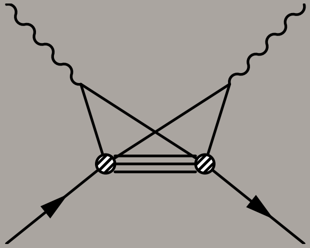
REMEMBER CAT'S EARS

"Handbag" Diagram



Twist 2 or Leading Twist

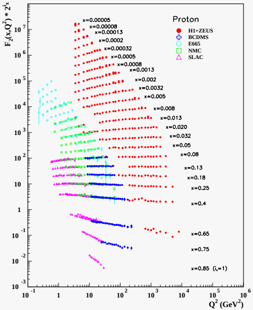
"Cat's Ears" Diagram



Higher Twist

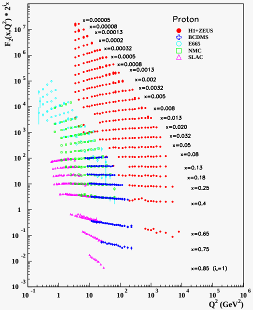
→ Experiment limited in quark content of targets (Ignoring heavier quark terms)

$$T^{proton} = \frac{4}{9}T^{uu} + \frac{1}{9}T^{dd} - \frac{2}{9}T^{ud+du}$$
$$T^{neutron} = \frac{1}{9}T^{uu} + \frac{4}{9}T^{dd} - \frac{2}{9}T^{ud+du}$$

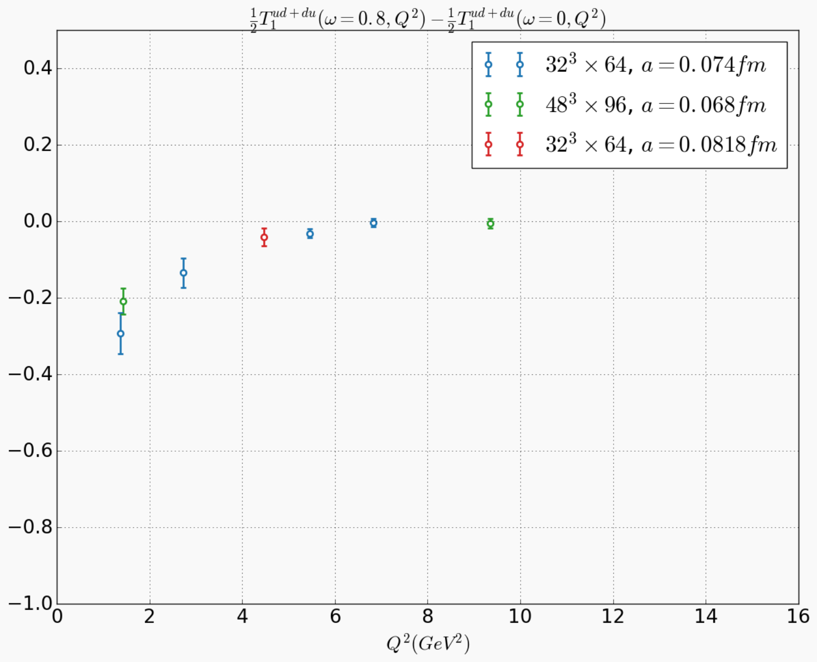


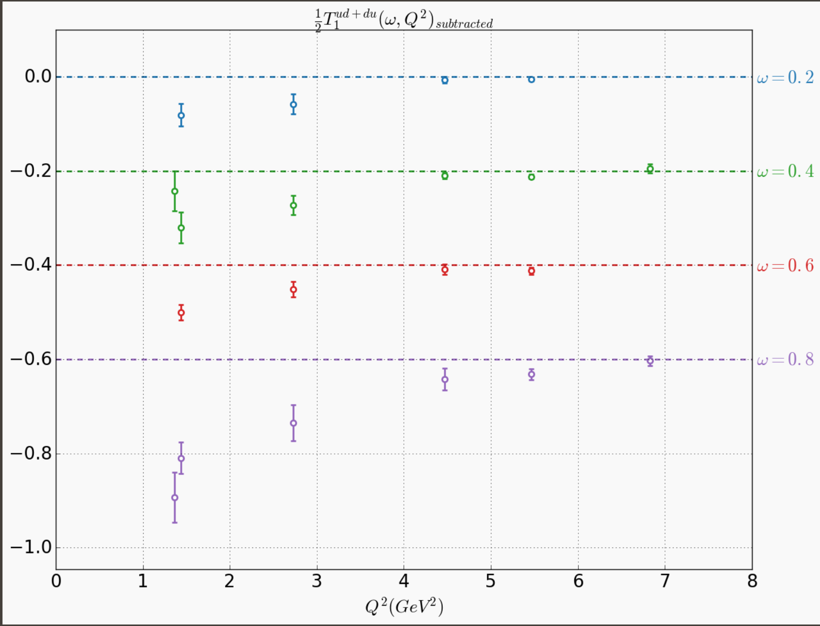
→ Experiment limited in quark content of targets (Ignoring heavier quark terms)

$$T^{proton} = \frac{4}{9}T^{uu} + \frac{1}{9}T^{dd} - \frac{2}{9}T^{ud+du}$$
$$T^{neutron} = \frac{1}{9}T^{uu} + \frac{4}{9}T^{dd} - \frac{2}{9}T^{ud+du}$$

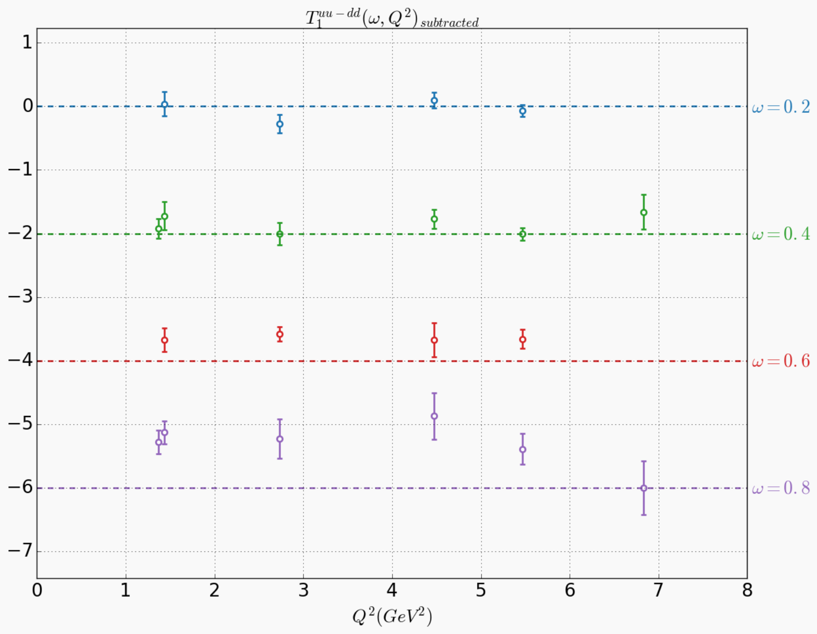


→ Lattice extraction of more combinations T^{uu}, T^{dd}, $T^{uu} + T^{dd} + T^{ud+du}$, $T^{uu} + T^{dd} - T^{ud+du}$


$\frac{1}{2}T_1^{ud+du}(\omega=0.8,Q^2)-\frac{1}{2}T_1^{ud+du}(\omega=0,Q^2)$
$32^3\times64$, $a=0.074fm$
$48^3\times96$, $a=0.068fm$
$32^3\times64$, $a=0.0818fm$
0.4
0.2
0.0
−0.2
−0.4
−0.6
−0.8
−1.0
0
2
4
6
8
10
12
14
16
$Q^2(GeV^2)$

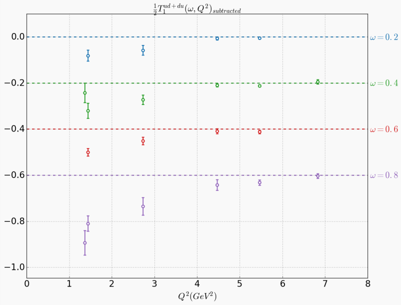


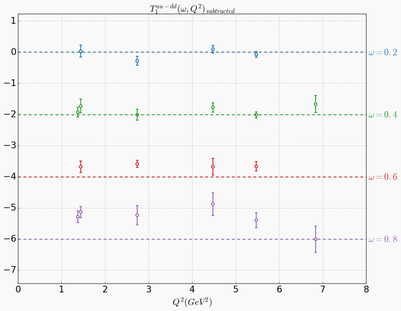
$\frac{1}{2}T_1^{ud+du}(\omega, Q^2)_{subtracted}$
0.0
−0.2
−0.4
−0.6
−0.8
−1.0
0
1
2
3
4
5
6
7
8
$Q^2(GeV^2)$
$\omega = 0.2$
$\omega = 0.4$
$\omega = 0.6$
$\omega = 0.8$



$T_1^{uu-dd}(\omega, Q^2)_{subtracted}$
1
0
−1
−2
−3
−4
−5
−6
−7
0
1
2
3
4
5
6
7
8
$Q^2(GeV^2)$
$\omega = 0.2$
$\omega = 0.4$
$\omega = 0.6$
$\omega = 0.8$

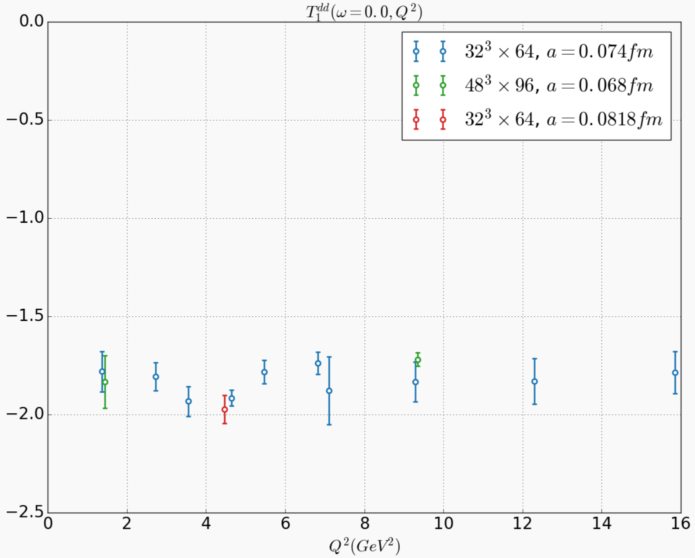
CONCLUSION

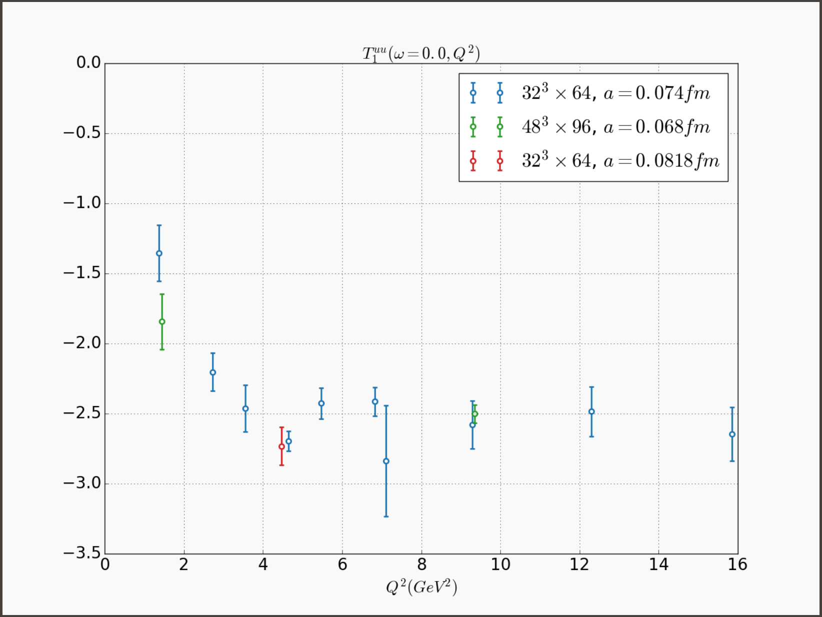




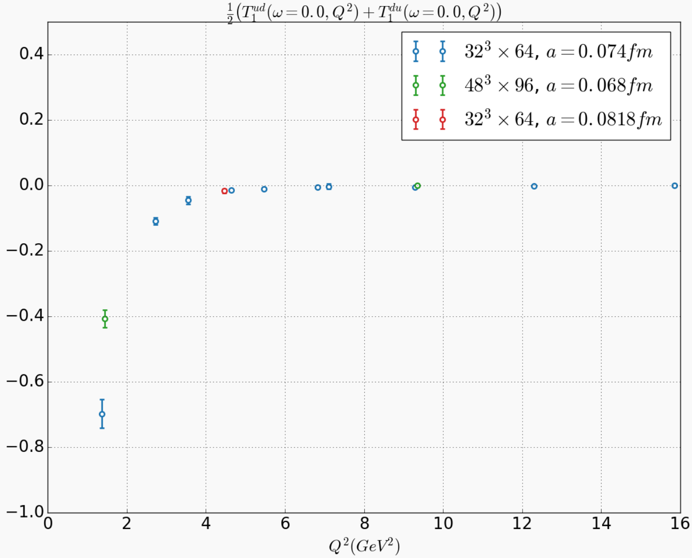
- → Looked at higher twist contributions, and quark flavour decomposition
- → Want to determine F_2, g_1 and g_2

BACKUP


$T_1^{dd}(\omega = 0.0, Q^2)$
$32^3 \times 64,\ a = 0.074fm$
$48^3 \times 96,\ a = 0.068fm$
$32^3 \times 64,\ a = 0.0818fm$
0.0
−0.5
−1.0
−1.5
−2.0
−2.5
0
2
4
6
8
10
12
14
16
$Q^2(GeV^2)$



$T_1^{uu}(\omega = 0.0, Q^2)$
0.0
−0.5
−1.0
−1.5
−2.0
−2.5
−3.0
−3.5
0
2
4
6
8
10
12
14
16
$Q^2(GeV^2)$
$32^3 \times 64,\ a = 0.074fm$
$48^3 \times 96,\ a = 0.068fm$
$32^3 \times 64,\ a = 0.0818fm$



$\frac{1}{2}\left(T_1^{ud}(\omega=0.0, Q^2) + T_1^{du}(\omega=0.0, Q^2)\right)$
$32^3 \times 64,\ a = 0.074fm$
$48^3 \times 96,\ a = 0.068fm$
$32^3 \times 64,\ a = 0.0818fm$
0.4
0.2
0.0
−0.2
−0.4
−0.6
−0.8
−1.0
0
2
4
6
8
10
12
14
16
$Q^2(GeV^2)$