East Lansing, MI, USA

Light-cone PDFs from lattice QCD

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Outline of the talk

1. PDFs and quasi-PDFs
2. Procedure
3. Renormalization
4. Matching
5. Systematic effects
   - excited states

Continued in next talk by Aurora Scapellato

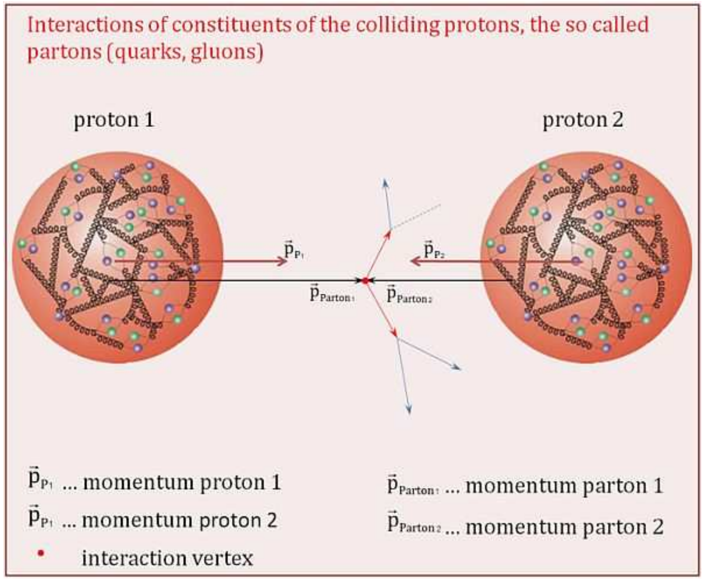
Based on:

- C. Alexandrou, K. Cichy, M. Constantinou, K. Jansen, A. Scapellato, F. Steffens, "Transversity parton distribution functions from lattice QCD ", arXiv: 1807.00232 [hep-lat]
- C. Alexandrou, K. Cichy, M. Constantinou, K. Jansen, A. Scapellato, F. Steffens, "Reconstruction of light-cone parton distribution functions from lattice QCD simulations at the physical point", arXiv: 1803.02685 [hep-lat]
- C. Alexandrou, K. Cichy, M. Constantinou, K. Hadjiyiannakou, K. Jansen, H. Panagopoulos, F. Steffens, "A complete non-perturbative renormalization prescription for quasi-PDFs", Nucl. Phys. B923 (2017) 394-415 (Frontier Article)
- M. Constantinou, H. Panagopoulos, "Perturbative Renormalization of quasi-PDFs", Phys. Rev. D96 (2017) 054506
- C. Alexandrou, K. Cichy, M. Constantinou, K. Hadjiyiannakou, K. Jansen, F. Steffens, C. Wiese, "Updated Lattice Results for Parton Distributions", Phys. Rev. D96 (2017) 014513

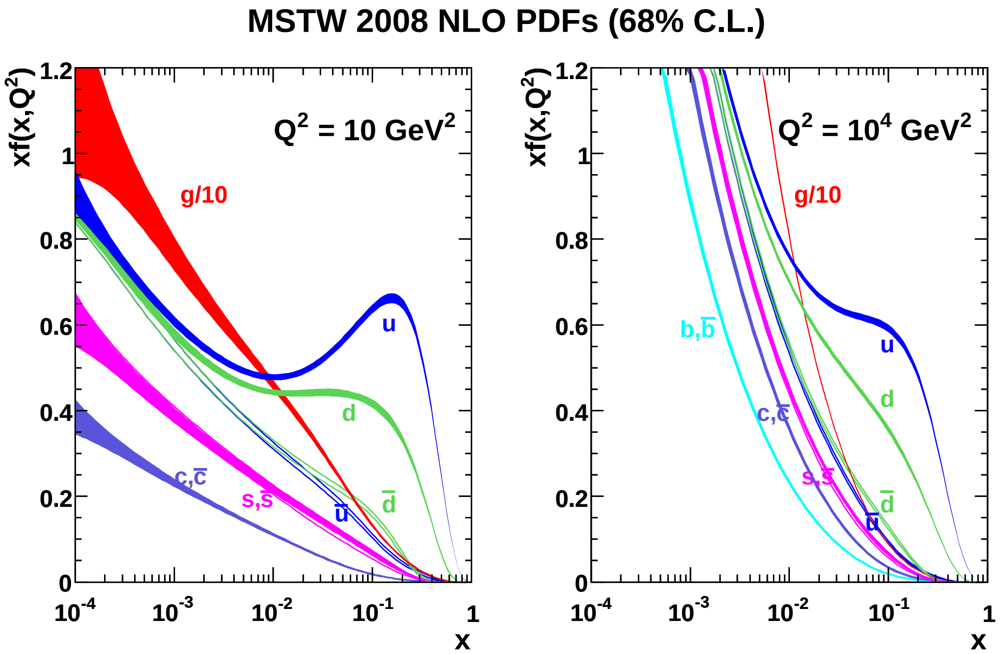
PDFs

- Hadrons are complicated systems with properties resulting from the strong dynamics of quarks and gluons inside them.
- This dynamics is characterized in terms of, among others, parton distribution functions (PDFs).
- PDFs are essential in making predictions for collider experiments.

$$\sigma_{AB} = \sum_{a,b=q,g} \sigma_{ab} \otimes f_{a|A}(x_1, Q^2) \otimes f_{b|B}(x_2, Q^2)$$



Source: LHC, CERN



MSTW2008, Eur. Phys. J. C63, 189

- PDFs have non-perturbative nature $\Rightarrow$ LATTICE?
- But: PDFs given in terms of non-local light-cone correlators – intrinsically Minkowskian – problem for the lattice!

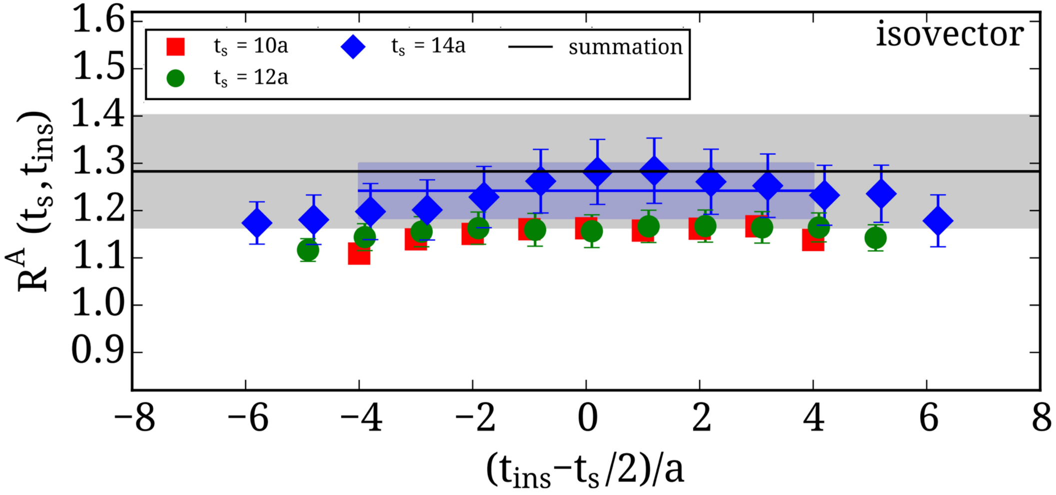
$$q(x) = \frac{1}{2\pi} \int d\xi^- e^{-ixp^+\xi^-} \langle N|\overline{\psi}(\xi^-)\Gamma\mathcal{A}(\xi^-,0)\psi(0)|N\rangle,$$

where: $\xi^- = \frac{\xi^0-\xi^3}{\sqrt{2}}$ and $\mathcal{A}(\xi^-,0)$ is the Wilson line from 0 to ξ^-.

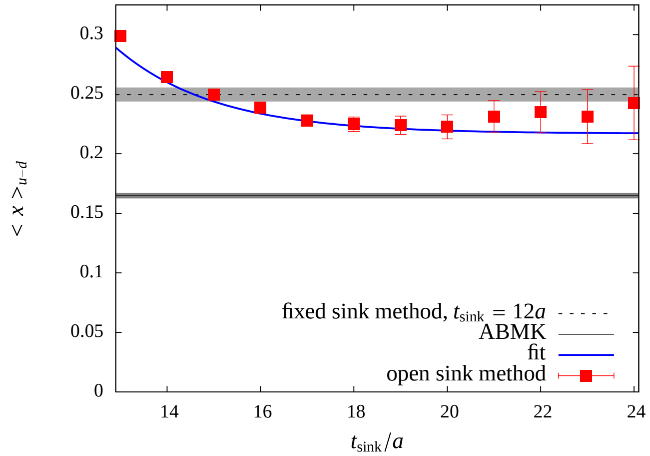
- This expression is light-cone dominated – needs $\xi^2 = \vec{x}^2 + t^2 \sim 0$ – very hard due to non-zero lattice spacing!
- Accessible on the lattice – moments of the distributions, but ...
  - ⋆ higher derivatives noisy,
  - ⋆ operator mixing.

Moments of PDFs on the lattice

There is, however, an important lesson to be learned from moments calculations:

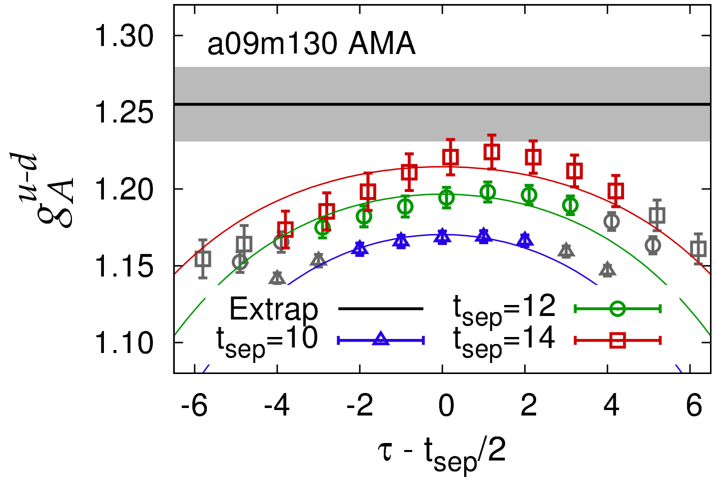


ETMC, C. Alexandrou et al.
Phys. Rev. D93, 039904 (2016)



ETMC, S. Dinter et al.
Phys. Lett. B704, 89 (2011)

- source-sink separation T_s has to be at least 1 fm!
- important to verify excited states contamination between different methods
- 2-state fits make sense **only** if one can get the safe=large T_s with good precision
- else, no comparison to the plateau method possible

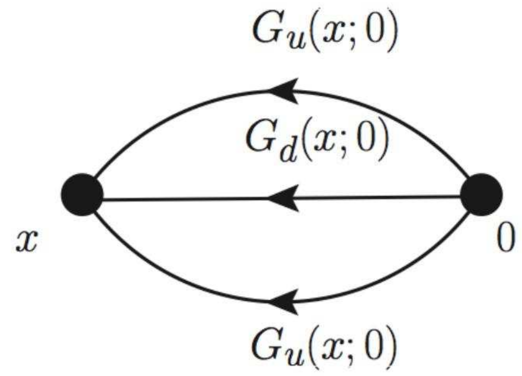


PNDME, T. Bhattacharya et al.
Phys. Rev. D94, 054508 (2016)

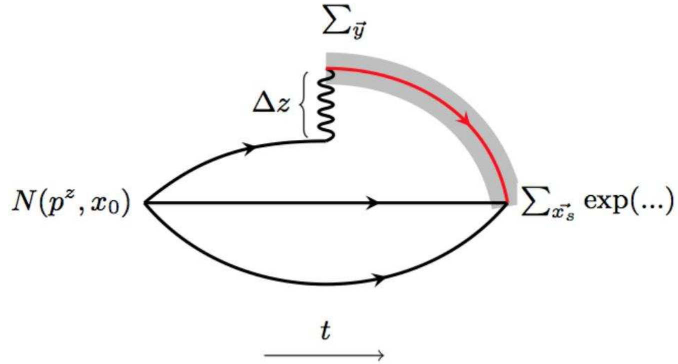
Quasi-PDFs

- Quasi-PDF approach:
  X. Ji, *Parton Physics on a Euclidean Lattice*, Phys. Rev. Lett. **110** (2013) 262002
- Compute a **quasi distribution** $\tilde{q}$, which is purely spatial and uses nucleons with finite momentum:

$$\tilde{q}(x,\mu^2,P_3) = \int \frac{dz}{4\pi} e^{ixP_3 z} \langle N|\overline{\psi}(z)\Gamma\mathcal{A}(z,0)\psi(0)|N\rangle.$$



  - z – distance in any *spatial* direction z,
  - P_3 – momentum boost in this direction.
  - $\Gamma = \gamma_0, \gamma_3$ – unpolarized, $\Gamma = \gamma_5\gamma_3$ – helicity, $\Gamma = \sigma_{31}, \sigma_{32}$ – transversity
  - Theoretically very appealing and intuitive!
  - Differs from light-front PDFs by $\mathcal{O}\left(\frac{\Lambda^2_{\rm QCD}}{P_3^2}, \frac{m_N^2}{P_3^2}\right)$.
  - The highly non-trivial aspect: how to relate $\tilde{q}(x,\mu^2,P_3)$ to the light-front PDF $q(x,\mu^2)$ (infinite momentum frame) $\Longrightarrow$ **LaMET**



Beautiful idea and solid theoretical framework!

BUT: lattice realization far from trivial!

- Signal for the relevant nucleon 2-pt and 3-pt function depends on:
  - ⋆ nucleon momentum P_3 – exponentially decaying with P_3!
  - ⋆ source-sink separation T_s – exponentially decaying with t_s!
  - ⋆ quark mass – worsens for smaller masses.
- Many systematics to control!

Spectrum becomes denser at larger nucleon momenta
$\Longrightarrow$ Careful analysis of excited states contamination required at least at the largest employed P_3.

2-state fits need to be checked against the plateau method with good precision of large T_s.

Summary of the procedure

The procedure to obtain light-cone PDFs from the lattice computation can be summarized as follows:

1. Compute bare matrix elements: $\langle N|\overline{\psi}(z)\Gamma\mathcal{A}(z,0)\psi(0)|N\rangle$
2. Compute vertex functions and the resulting renormalization functions in the intermediate RI'-MOM scheme $Z^{\mathrm{RI}'}(z,\mu)$.
3. Convert the renormalization functions to the $\overline{\mathrm{MS}}$ scheme and evolve to $\bar{\mu}=2$ GeV.
4. Apply the renormalization functions to the bare matrix elements, obtaining renormalized matrix elements in the $\overline{\mathrm{MS}}$ scheme.
5. Calculate the Fourier transform, obtaining quasi-PDFs:
$$\tilde{q}(x,\mu^2,P_3)=\int\frac{dz}{4\pi}\,e^{ixP_3z}\langle N|\overline{\psi}(z)\Gamma\mathcal{A}(z,0)\psi(0)|N\rangle.$$
6. Relate quasi-PDFs to light-cone PDFs via a matching procedure.
7. Apply target mass corrections to eliminate residual m_N/P_3 effects.

Renormalization

Bare matrix elements $\langle N|\overline{\psi}(z)\Gamma\mathcal{A}(z,0)\psi(0)|N\rangle$ contain divergences that need to be removed:

- standard logarithmic divergence w.r.t. the regulator, $\log(a\mu)$,
- power divergence related to the Wilson line; resums into a multiplicative exponential factor, $\exp\left(-\delta m|z|/a + c|z|\right)$
  δm – strength of the divergence, operator independent,
  c – arbitrary scale (fixed by the renormalization prescription).

Proposed renormalization programme described in:
C. Alexandrou, K. Cichy, M. Constantinou, K. Hadjiyiannakou, K. Jansen, H. Panagopoulos, F. Steffens, "A complete non-perturbative renormalization prescription for quasi-PDFs", Nucl. Phys. B923 (2017) 394-415 (Frontiers Article)

Important insights also from the lattice perturbative paper:
M. Constantinou, H. Panagopoulos, "Perturbative Renormalization of quasi-PDFs", Phys. Rev. D96 (2017) 054506
$\rightarrow$ mixing of $\Gamma = \gamma_3$ and $\Gamma = \mathbf{1}$, important guidance to non-pert. renormalization!

Non-perturbative renormalization scheme: RI'-MOM.

G. Martinelli et al., Nucl. Phys. B445 (1995) 81

RI'-MOM renormalization conditions (for cases without mixing):
for the operator:

$$Z_q^{-1}\, Z_{\mathcal{O}}(z)\, \frac{1}{12}\mathrm{Tr}\left[\mathcal{V}(p,z)\left(\mathcal{V}^{\mathrm{Born}}(p,z)\right)^{-1}\right]\Big|_{p^2=\bar{\mu}_0^2} = 1\,,$$

for the quark field:

$$Z_q = \frac{1}{12}\mathrm{Tr}\left[(S(p))^{-1}\, S^{\mathrm{Born}}(p)\right]\Big|_{p^2=\bar{\mu}_0^2}\,.$$

- momentum p in the vertex function is set to the RI′ renormalization scale $\bar{\mu}_0$
- $\mathcal{V}(p,z)$ – amputated vertex function of the operator,
- $\mathcal{V}^{\mathrm{Born}}$ – its tree-level value, $\mathcal{V}^{\mathrm{Born}}(p,z){=}i\gamma_3\gamma_5\, e^{ipz}$ for helicity,
- $S(p)$ – fermion propagator ($S^{\mathrm{Born}}(p)$ at tree-level).

This prescription handles all divergences that are present and applies the necessary finite renormalization related to the lattice regularization.

Matching to light-front PDFs (unpolarized, helicity)

The matching formula can be expressed as:

$$q(x,\mu) = \int_{-\infty}^{\infty} \frac{d\xi}{|\xi|}\, C\left(\xi, \frac{\mu}{xP_3}\right) \tilde{q}\left(\frac{x}{\xi}, \mu, P_3\right)$$

C – matching kernel: [C. Alexandrou et al., arXiv:1803.02685 [hep-lat]]

$$C\left(\xi, \frac{\xi\mu}{xP_3}\right) = \delta(1-\xi) + \frac{\alpha_s}{2\pi} C_F \begin{cases} \left[\frac{1+\xi^2}{1-\xi} \ln\frac{\xi}{\xi-1} + 1 + \frac{3}{2\xi}\right]_+ & \xi > 1, \\ \left[\frac{1+\xi^2}{1-\xi} \ln\frac{x^2 P_3^2}{\xi^2\mu^2}\,(4\xi(1-\xi)) - \frac{\xi(1+\xi)}{1-\xi} + 2\iota(1-\xi)\right]_+ & 0 < \xi < 1, \\ \left[-\frac{1+\xi^2}{1-\xi} \ln\frac{\xi}{\xi-1} - 1 + \frac{3}{2(1-\xi)}\right]_+ & \xi < 0, \end{cases}$$

$\iota=0$ for γ_0 and $\iota=1$ for $\gamma_3/\gamma_5\gamma_3$.

Plus prescription at $\xi=1$:

$$\int \frac{d\xi}{|\xi|} \left[C\left(\xi, \frac{\xi\mu}{xP_3}\right)\right]_+ \tilde{q}\left(\frac{x}{\xi}\right) = \int \frac{d\xi}{|\xi|}\, C\left(\xi, \frac{\xi\mu}{xP_3}\right) \tilde{q}\left(\frac{x}{\xi}\right) - \tilde{q}\,(x) \int d\xi\, C\left(\xi, \frac{\mu}{xP_3}\right).$$

Alternative matching: [T. Izubuchi et al., arXiv:1801.03917 [hep-ph]]

$$C\left(\xi, \frac{\xi\mu}{xP_3}\right) = \delta(1-\xi) \; + \; \frac{\alpha_s}{2\pi} C_F \begin{cases} \left[\frac{1+\xi^2}{1-\xi} \ln \frac{\xi}{\xi-1} + 1 + \frac{3}{2\xi}\right]_{+(1)}^{[1,\infty]} - \frac{3}{2\xi} & \xi > 1, \\ \left[\frac{1+\xi^2}{1-\xi} \ln \frac{x^2 P_3^2}{\xi^2\mu^2} \left(4\xi(1-\xi)\right) - \frac{\xi(1+\xi)}{1-\xi} + 2\iota(1-\xi)\right]_{+(1)}^{[0,1]} & 0 < \xi < 1 \\ \left[-\frac{1+\xi^2}{1-\xi} \ln \frac{\xi}{\xi-1} - 1 + \frac{3}{2(1-\xi)}\right]_{+(1)}^{[-\infty,0]} - \frac{3}{2(1-\xi)} & \xi < 0, \end{cases}$$

$$+ \; \frac{\alpha_s C_F}{2\pi} \delta(1-\xi) \left(\frac{3}{2} \ln \frac{\mu^2}{4y^2 P_3^2} + \frac{5}{2}\right)$$

violates particle number conservation:

$$\int_{-\infty}^{\infty} dx\, q(x,\mu) \neq \int_{-\infty}^{\infty} dx\, \tilde{q}(x,\mu,P_3) \qquad \text{and} \qquad \int_{-\infty}^{\infty} d\xi\, C(\xi, \xi\mu/xP_3) \neq 1,$$

which **increases** with growing P_3 (around 8% at $P_3 = 10\pi/48$).

In our procedure, particle number is **conserved**. This amounts to a modification of the $\overline{\mathrm{MS}}$ scheme; modification **decreases** with growing P_3.

Matching to light-front PDFs (transversity)

Recently we derived the matching formula for transversity PDFs ($\overline{\mathrm{MS}} \longrightarrow \overline{\mathrm{MS}}$):
[C. Alexandrou et al., arXiv:1807.00232 [hep-lat]]

$$\delta C\left(\xi, \frac{\xi\mu}{xP_3}\right) = \delta(1-\xi) + \frac{\alpha_s}{2\pi} C_F \begin{cases} \left[\frac{2\xi}{1-\xi}\ln\frac{\xi}{\xi-1} + \frac{2}{\xi}\right]_+ & \xi > 1, \\ \left[\frac{2\xi}{1-\xi}\left(\ln\frac{x^2 P_3^2}{\xi^2\mu^2}(4\xi(1-\xi))\right) - \frac{2\xi}{1-\xi}\right]_+ & 0 < \xi < 1, \\ \left[-\frac{2\xi}{1-\xi}\ln\frac{\xi}{\xi-1} + \frac{2}{1-\xi}\right]_+ & \xi < 0, \end{cases}$$

Formula for the transverse momentum cutoff scheme derived in: [X. Xiong et al., Phys. Rev. D 90, 014051]

Different systematic effects still need to be addressed:

- pion mass ✓
- cut-off effects ✓✗
- finite volume effects ✓✗
- contamination by excited states ✓✗
- higher-twist effects ✓✗
- truncation of conversion, evolution and matching ✗
- lattice artifacts in renormalization functions ✓✗
- ...

Biggest challenge:
Reach large momenta at large source-sink separations

Computation setup

- fermions: $N_f = 2$ twisted mass fermions + clover term
- gluons: Iwasaki gauge action, $\beta = 2.1$

β=2.10,	$c_{\rm SW}$=1.57751, a=0.0938(3)(2) fm
$48^3 \times 96$	$a\mu = 0.0009$ $\quad m_N = 0.932(4)$ GeV
$L = 4.5$ fm	$m_\pi = 0.1304(4)$ GeV $\quad m_\pi L = 2.98(1)$

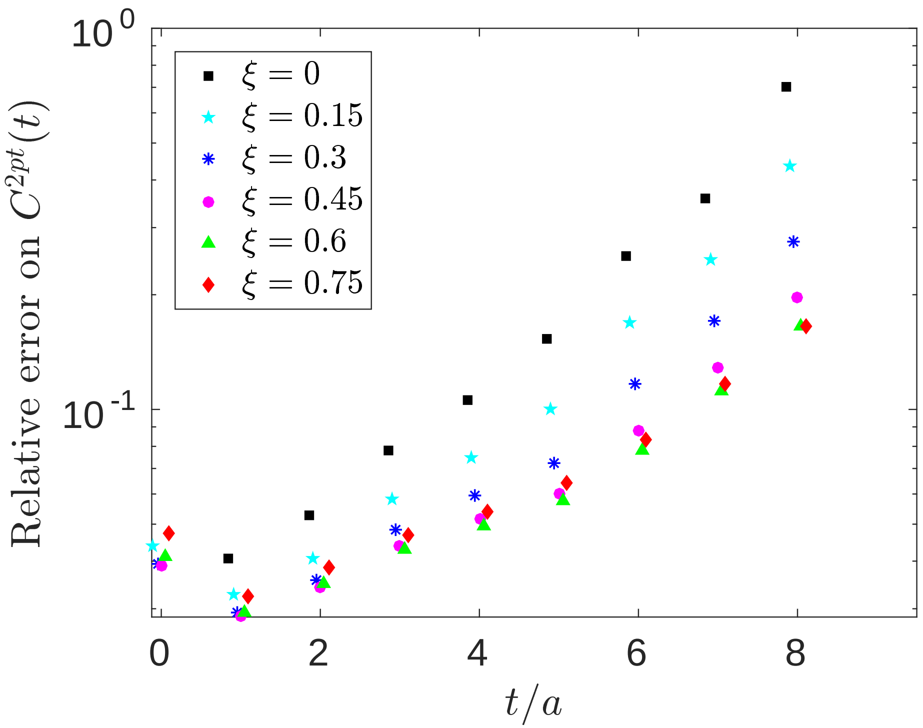
For each gauge field configuration, we use:

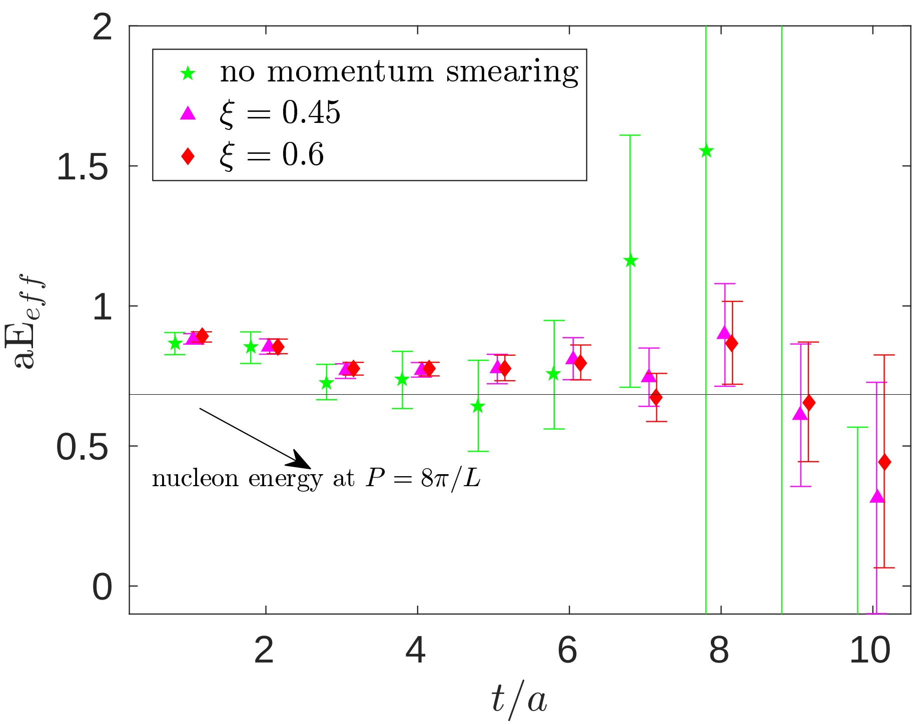
- 6 directions of Wilson line: $\pm x, \pm y, \pm z$
- 16 source positions: 1 HP inversion, 16 LP inversions
- Bias from the LP inversions corrected using the Covariant Approximation Averaging technique (CAA)
  [E. Shintani et al., Phys. Rev. D91, 114511 (2015)]

Momentum smearing

$$S_{\text{mom}}\psi(x) = \frac{1}{1+6\alpha}\left(\psi(x) + \alpha \sum_{j=\pm1}^{\pm3} U_j(x) e^{i\xi\hat{j}} \psi(x+\hat{j})\right)$$

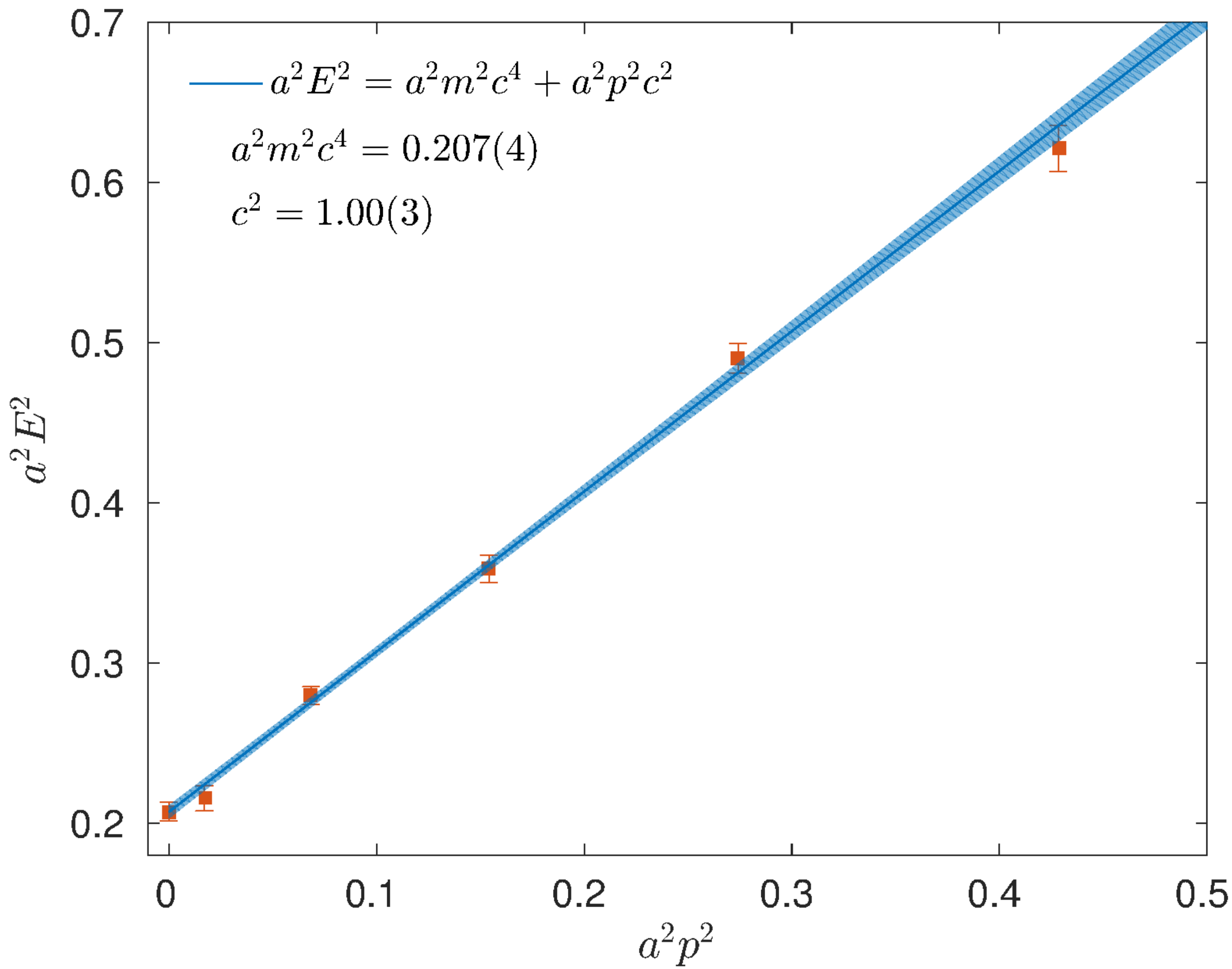
[G. Bali et al., Phys. Rev. D93, 094515 (2016)]



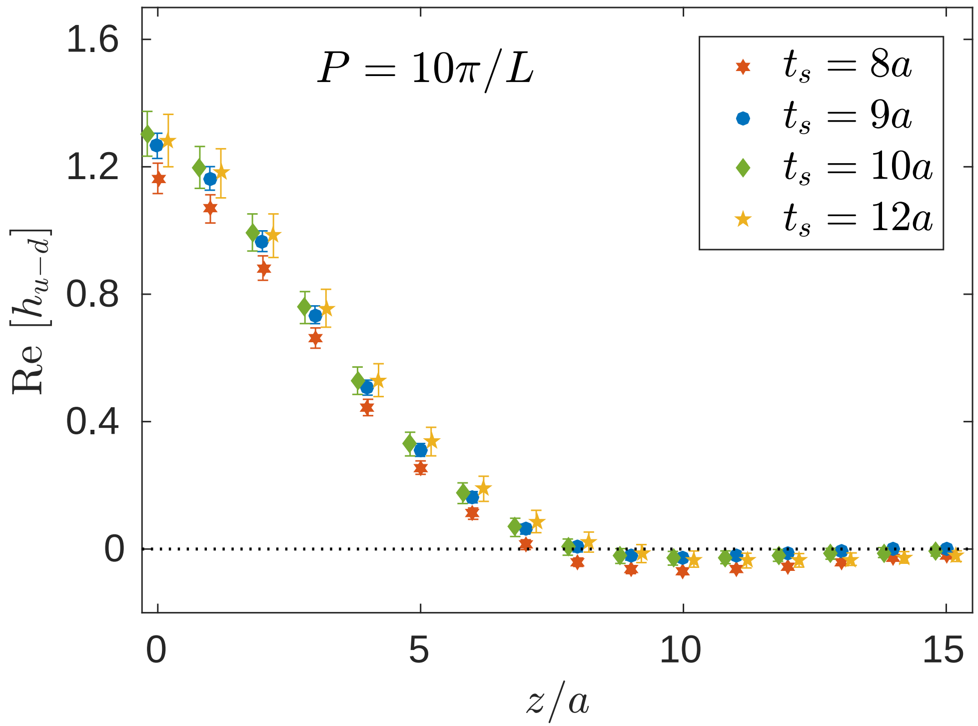


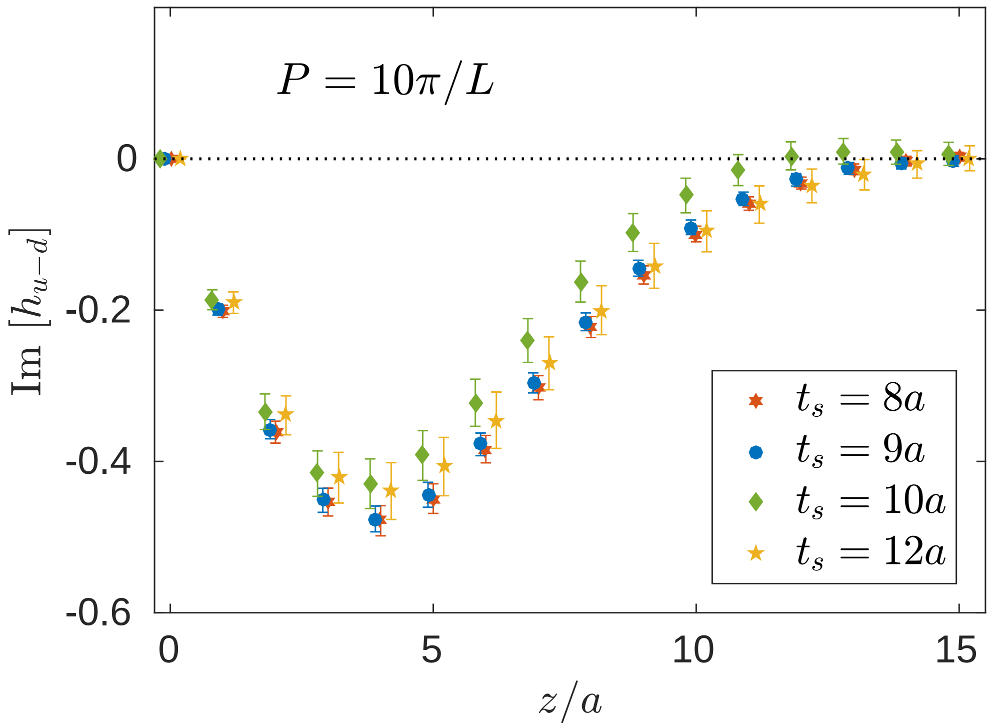
50 iterations of (Gaussian) momentum smearing, $\alpha = 4$

Dispersion relation



Excited states – plateau method





Statistics:

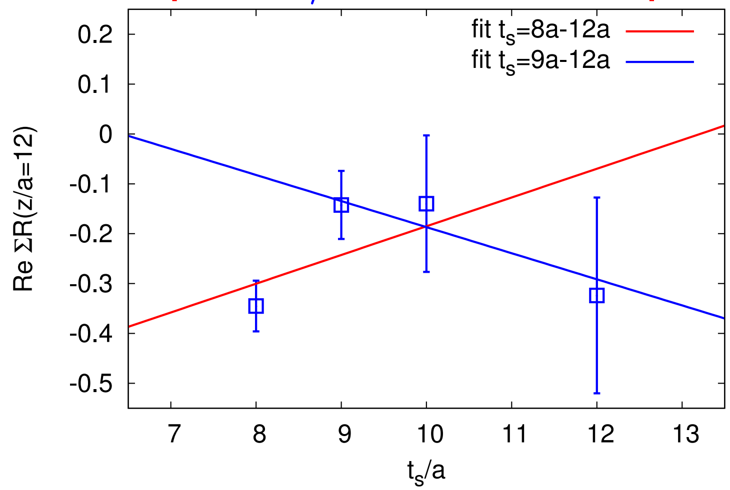
- $t_s = 8a$ – 4320 measurements,
- $t_s = 9a$ – 8820 measurements,
- $t_s = 10a$ – 9000 measurements,
- $t_s = 12a$ – 72990 measurements.

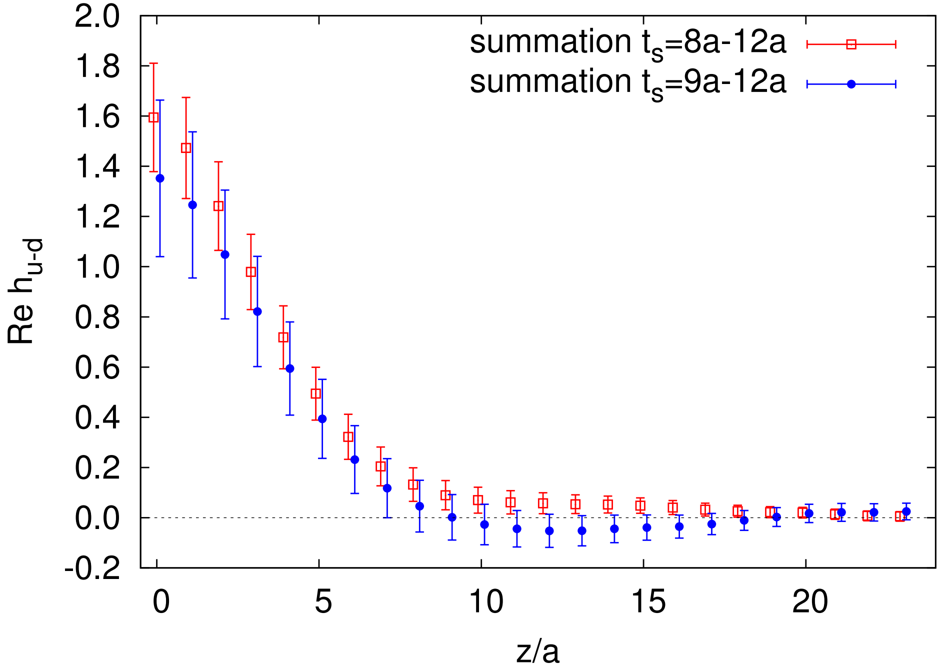
Increasing t_s by 1 lattice spacing worsens the signal by a factor 2-3!

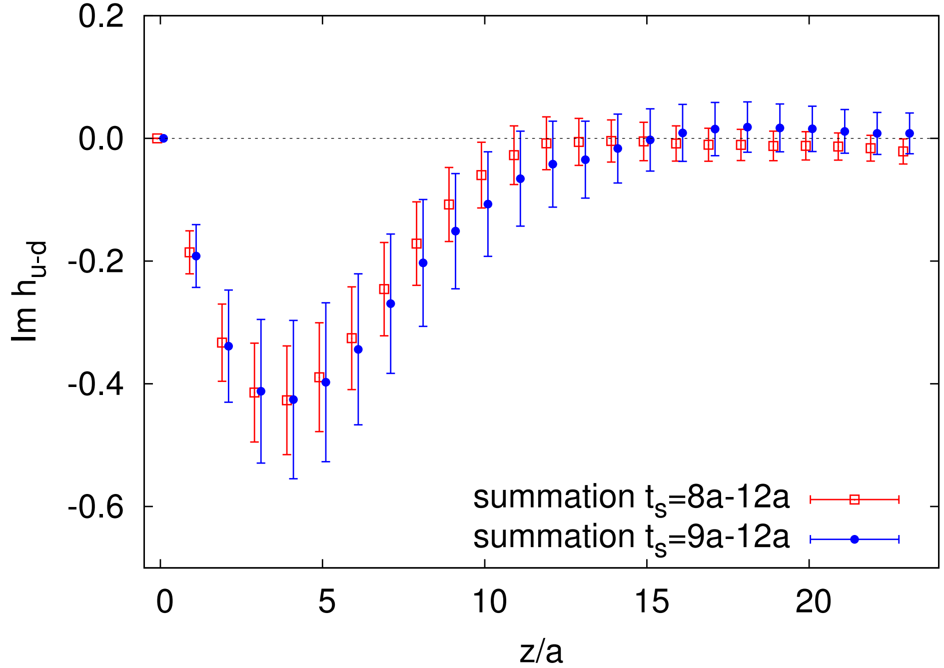
$$\mathcal{R}(P;t_s) \equiv \sum_{\tau=a}^{t_s-a} \frac{C^{3\text{pt}}(P;t_s,\tau)}{C^{2\text{pt}}(P_i;t_s)} =$$

$$= C + \mathcal{M}\, t_s + \mathcal{O}\left(e^{-(E_1-E_0)t_s}\right)$$

Example: $z/a = 12$, real part

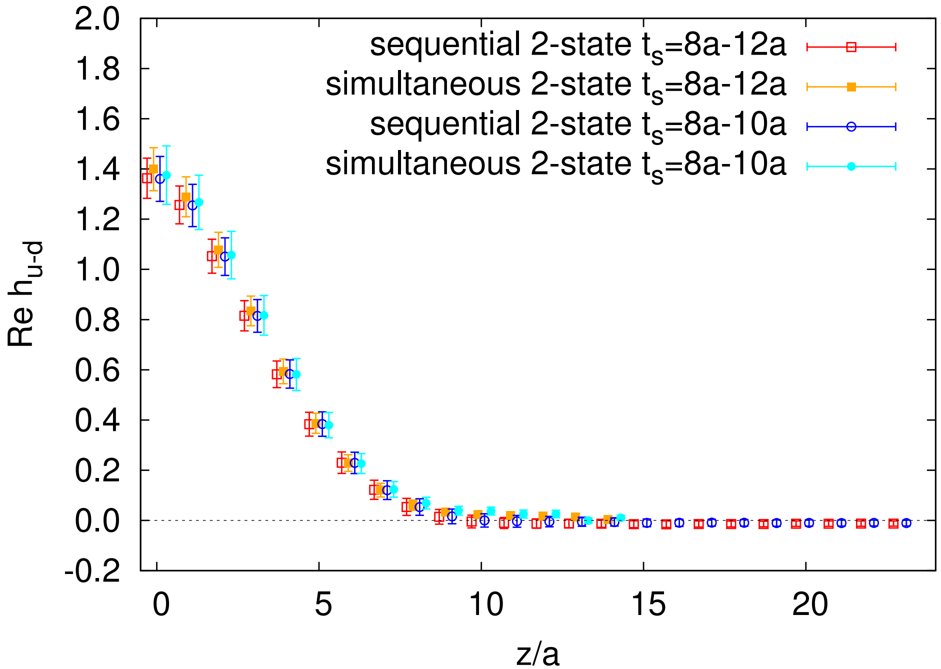


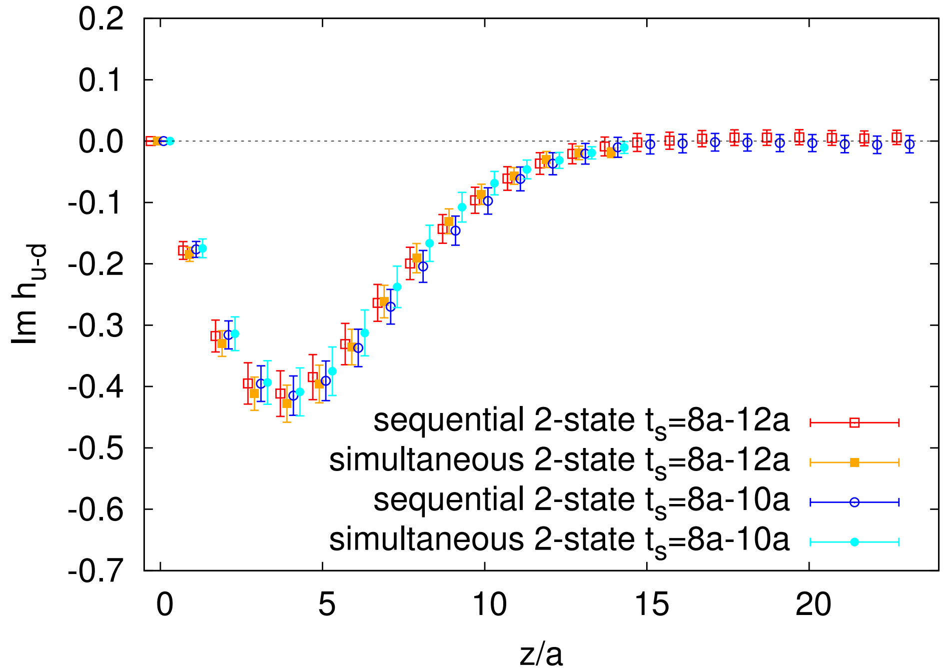




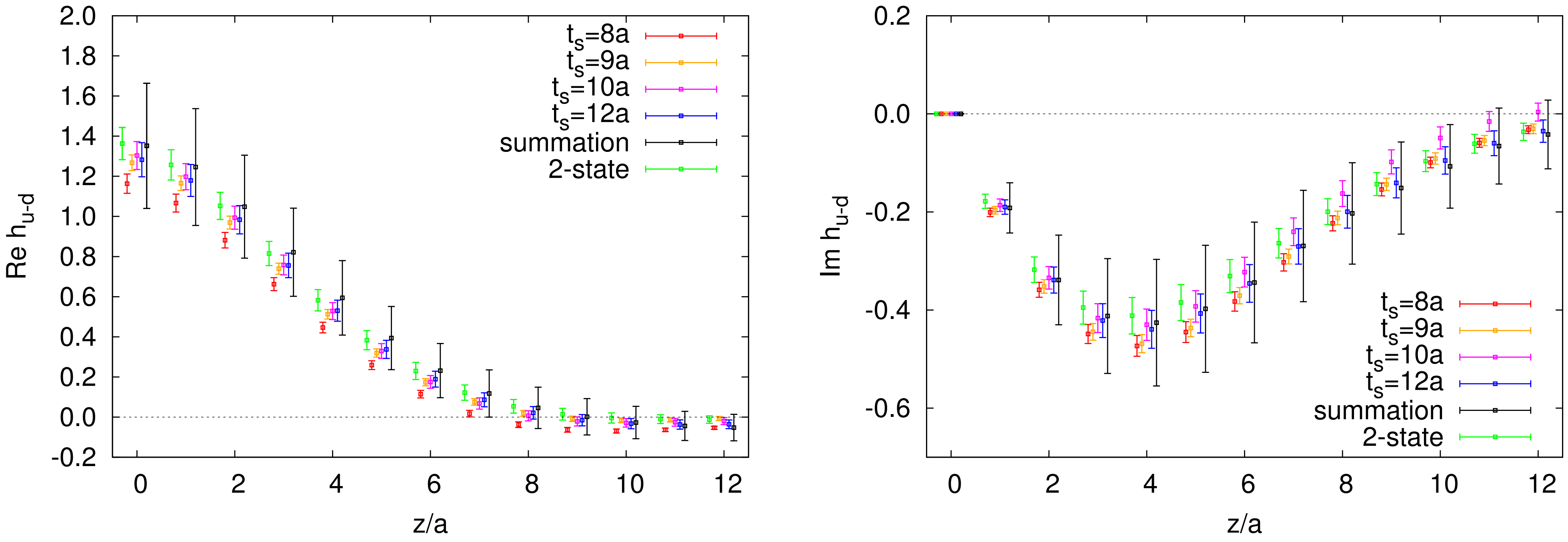
$$C^{\mathrm{2pt}}(P;t) = |A_0|^2 e^{-E_0 t} + |A_1|^2 e^{-E_1 t}$$

$$\begin{aligned} C^{\mathrm{3pt}}(P;t_s,\tau) &= |A_0|^2 \langle 0|\mathcal{O}|0\rangle e^{-E_0 t_s} + A_0^* A_1 \langle 1|\mathcal{O}|0\rangle e^{-E_1\tau} e^{-E_0(t_s-\tau)} \\ &+ A_0 A_1^* \langle 0|\mathcal{O}|1\rangle e^{-E_0\tau} e^{-E_1(t_s-\tau)} + |A_1|^2 \langle 1|\mathcal{O}|1\rangle e^{-E_1 t_s} \end{aligned}$$





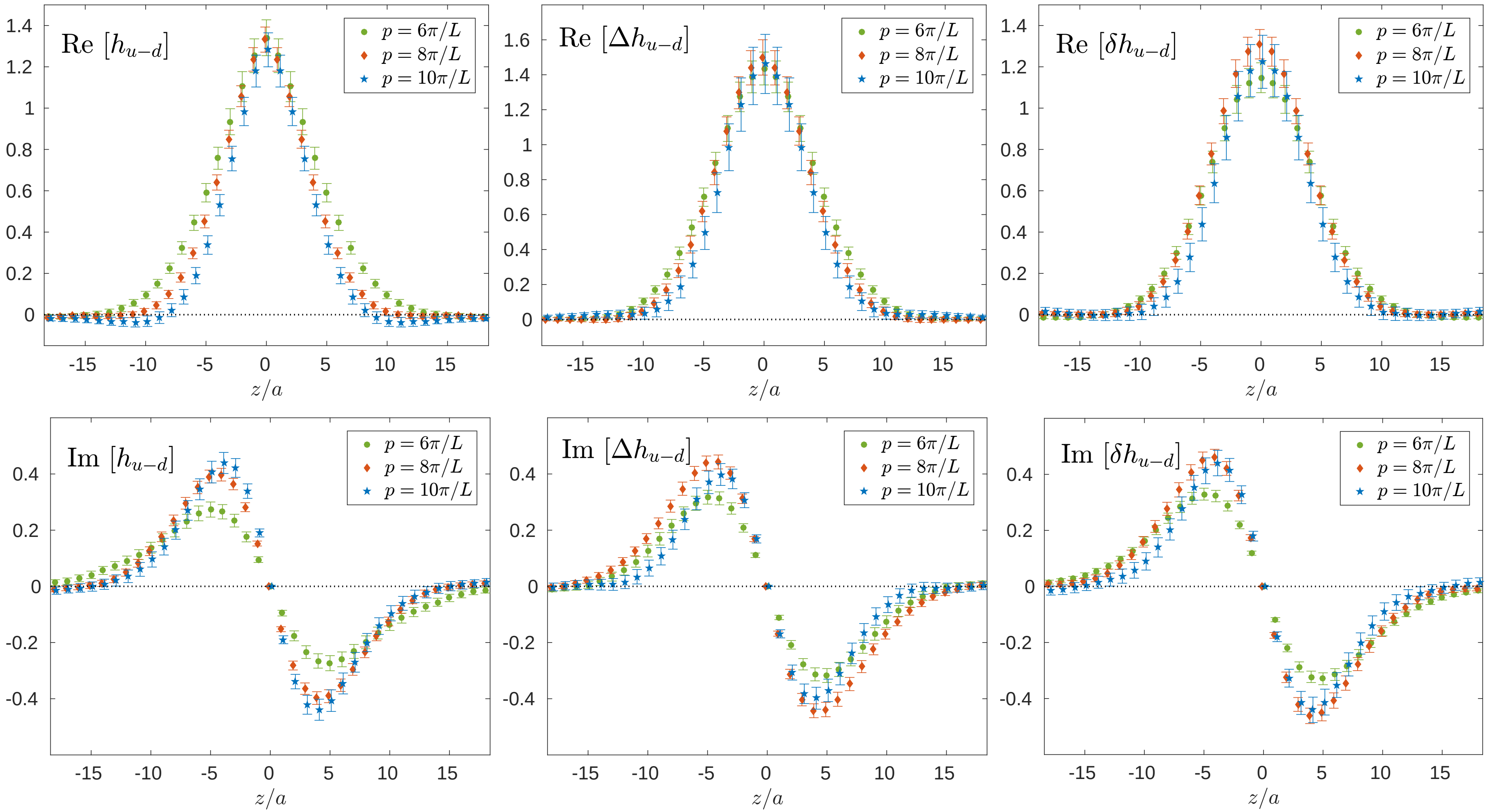
Excited states – comparison



- $t_s = 8a$ clearly off, excited states totally uncontrolled
- $t_s = 9a,\ 10a$ also show some tension
- $t_s = 12a \approx 1.1$ fm seems to be the best justifiable choice, i.e. it should be safe from excited states at the $\sim 10\%$ level.
- Robust statements about excited states because of **consistency between all methods**.
- Careful analysis needs to be repeated when aiming for larger momenta (increased excited states contamination!) or better precision.

Bare matrix elements at $t_s = 12a$

unpolarized (γ_0) — helicity ($\gamma_5\gamma_3$) — transversity (σ_{3i})



STATISTICS:

unpolarized	helicity	transversity
$P_3 = \frac{6\pi}{L}$ – 4800 meas.	$P_3 = \frac{6\pi}{L}$ – 6240 meas.	$P_3 = \frac{6\pi}{L}$ – 9600 meas.
$P_3 = \frac{8\pi}{L}$ – 38250 meas.	$P_3 = \frac{8\pi}{L}$ – 38250 meas.	$P_3 = \frac{8\pi}{L}$ – 38250 meas.
$P_3 = \frac{10\pi}{L}$ – 72990 meas.	$P_3 = \frac{10\pi}{L}$ – 72990 meas.	$P_3 = \frac{10\pi}{L}$ – 72990 meas.

To be continued in next talk

Thank you for your attention!