$K\pi$ scattering with partial wave mixing & isovector excited meson spectroscopy

Ruairí Brett

Carnegie Mellon University

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with John Bulava, Jacob Fallica, Andrew Hanlon, Ben Hörz, Colin Morningstar

Motivations & Overview

- study low-lying meson spectrum using lattice QCD:

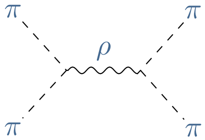
- use $2 \to 2$ Lüscher formalism to calculate hadron-hadron scattering amplitudes
  - P-wave $K\pi$ scattering: $K^*(892)$ resonance parameters
  - S-wave $K\pi$ scattering: $K_0^*(800)/\kappa$ resonance parameters
  - include partial wave mixing for $\ell \leq 2$

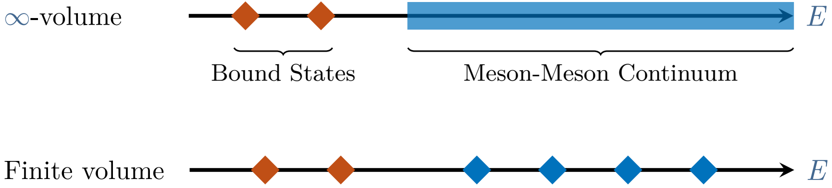
- qualitative spectrum extraction with large operator bases
  - single- and multi-hadron interpolating operators in large volumes
  - identify $\overline{q}q$-dominated states, mixed states, etc.

Finite Volume Spectra

Scattering process: eg.

$I = 1 \quad \pi\pi \to \pi\pi$





Momentum quantised $\to$ No continuum of scattering states

$$\boldsymbol{p} = \frac{2\pi}{L}\boldsymbol{d}$$

Lüscher Quantisation

Infinite volume physics from LQCD

Lüscher: Relationship between finite volume spectrum and infinite volume scattering matrix

- Quantisation condition:

$$\det[\,\widetilde{K}^{-1} - B\,] = 0$$

- For each E_{cm} in spectrum, determinant gives single relation to entire scattering matrix

  $\Rightarrow$ Exactly solvable for single channel, single partial wave

  $\Rightarrow$ ℓ mixing/coupled decay channels requires parameterisation of $\widetilde{K}$ and a fit (determinant residual method)

[Morningstar et al.; Nucl. Phys. B 924 (2017)]

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$$\widetilde{K}_\ell^{-1} = \left(\frac{q_{\mathrm{cm}}}{m_\pi}\right)^{2\ell+1} \cot\delta_l \qquad \det[\boxed{\widetilde{K}^{-1}} - B] = 0$$

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box matrix: known function of (E_{cm}, L)

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$$\widetilde{K}_\ell^{-1} = \left(\frac{q_{\mathrm{cm}}}{m_\pi}\right)^{2\ell+1} \cot\delta_l \qquad \det[\boxed{\widetilde{K}^{-1}} - \boxed{B}] = 0$$

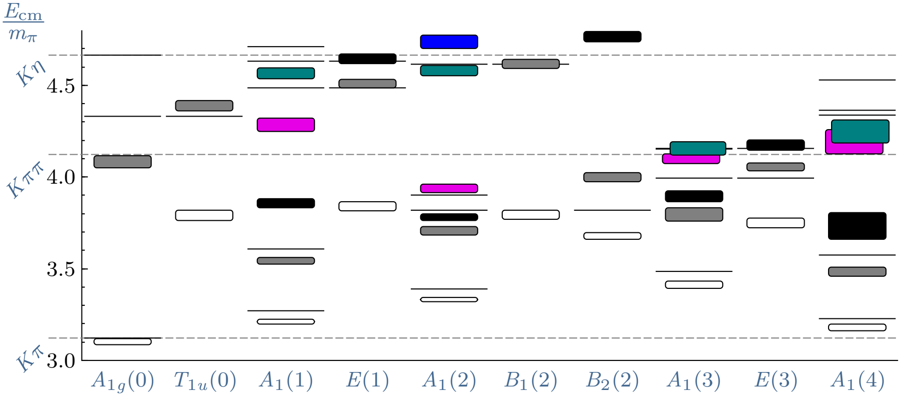
box matrix: known function of (E_{cm}, L)

- For given $\boldsymbol{P}$, $(\widetilde{K}^{-1} - B)$ block diagonal in little group irrep, simplifies determinant calculation
  - $\Rightarrow$ All infinite volume physics in $\widetilde{K}^{-1}$, finite volume in B
  - $\Rightarrow$ B describes how partial waves fit into cubic volume
  - $\Rightarrow$ Software available containing B elements up to $\ell = 6$

[Morningstar et al.; Nucl. Phys. B 924 (2017)]

$K\pi$ energies in finite volume

- ensemble: $32^3 \times 256$, $m_\pi \approx 230$ MeV, $m_\pi L \sim 4.4$
- 13 single-hadron (K) and 33 two-hadron ($K\pi$) interpolating operators
- all-to-all propagation using stochastic LapH method



[RB, J Bulava, J Fallica, A Hanlon, B Hörz, C Morningstar; Nucl.Phys. B932 (2018)]

Decay of $K^*(892)$

- large $32^3 \times 256$ anisotropic lattice, $m_\pi \approx 230$ MeV
- included $\ell = 0, 1, 2$ partial waves
- fit forms

$$(\widetilde{K}^{-1})_{11} = \frac{6\pi E_{\rm cm}}{g^2 m_\pi}\left(\frac{m_{K^*}^2}{m_\pi^2} - \frac{E_{\rm cm}^2}{m_\pi^2}\right) \qquad (\widetilde{K}^{-1})_{22} = \frac{-1}{m_\pi^5 a_2}$$

$$(\widetilde{K}^{-1})_{00}^{\rm lin} = a_{\rm l} + b_{\rm l} E_{\rm cm}, \quad (\widetilde{K}^{-1})_{00}^{\rm quad} = a_{\rm q} + b_{\rm q} E_{\rm cm}^2,$$

$$(\widetilde{K}^{-1})_{00}^{\rm ERE} = \frac{-1}{m_\pi a_0} + \frac{m_\pi r_0}{2}\frac{\boldsymbol{q}_{\rm cm}^2}{m_\pi^2}, \qquad (\widetilde{K}^{-1})_{00}^{\rm BW}$$

- results

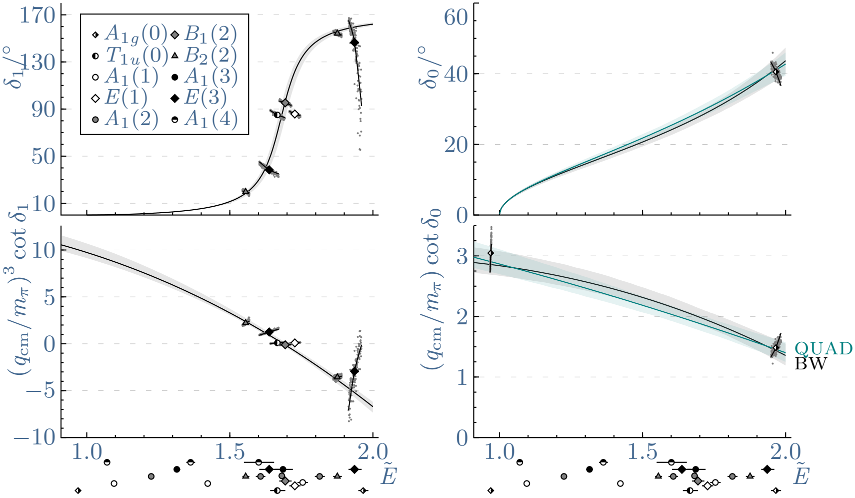
$$\frac{m_{K^*}}{m_\pi} = 3.808(18), \quad g = 5.33(20), \quad m_\pi a_0 = -0.353(25),$$

$$m_\pi^5 a_2 = -0.0013(68), \qquad \chi^2/\text{dof} = 1.42$$

[RB, J Bulava, J Fallica, A Hanlon, B Hörz, C Morningstar; Nucl.Phys. B932 (2018)]

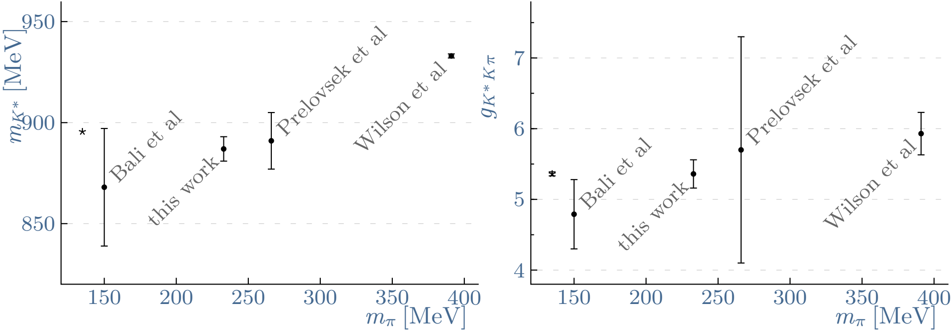
Decay of $K^*(892)$

- plots of P-wave and S-wave phase shift ($\tilde{E} = (E_{\rm cm} - m_K)/m_\pi$)
- $\kappa(\ell = 0)$ fit: Breit-Wigner or effective range



Decay of $K^*(892)$

- summary of lattice calculations of $K^*(892)$ resonance parameters
- phenomenological values shown as astericks



Excited meson spectroscopy

- As $m_\pi^{\text{lat}} \to m_\pi^{\text{phys}}$, fewer resonances lie below 3- & 4-particle thresholds
  - $\Rightarrow$ three particle quantisation condition in development (see talks by Mai (Mon) & Sharpe (Today))

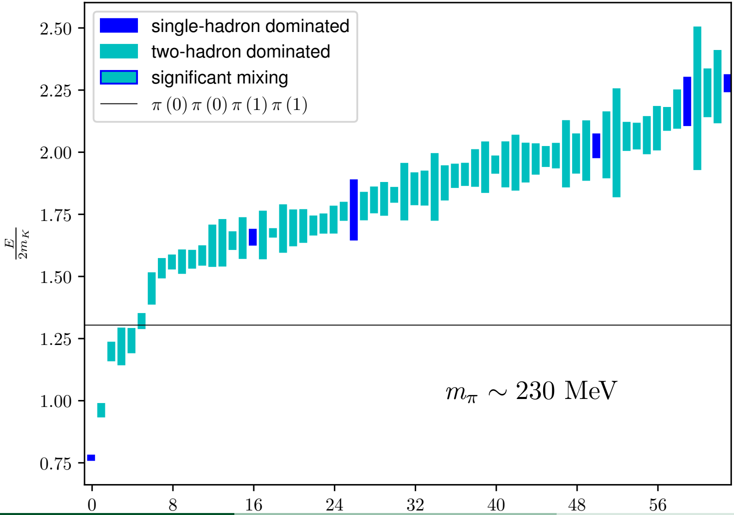
- For now, want to get a qualitative look at the excited meson spectrum
  - $\Rightarrow$ Using large bases of single- and two-hadron interpolating operators, extract excited spectrum
  - $\Rightarrow$ Identify $\overline{q}q$-like states by operator overlaps

- Using anisotropic $N_f = 2 + 1$ ensembles with clover fermions:
  - $\Rightarrow$ $(32^3|230)$: 412 configs $32^3 \times 256$, $m_\pi \approx 230$ MeV, $m_\pi L \sim 4.4$
  - $\Rightarrow$ $(24^3|390)$: 551 configs $24^3 \times 128$, $m_\pi \approx 390$ MeV, $m_\pi L \sim 5.7$

$I = 1, S = 0$ meson spectroscopy (*preliminary*)

eg. T_{1u}^{+} with $m_\pi \sim 230$ MeV

- 73 (9 SH + 64 MH) interpolating operators: full spectrum



$I = 1, S = 0$ meson spectroscopy (*preliminary*)

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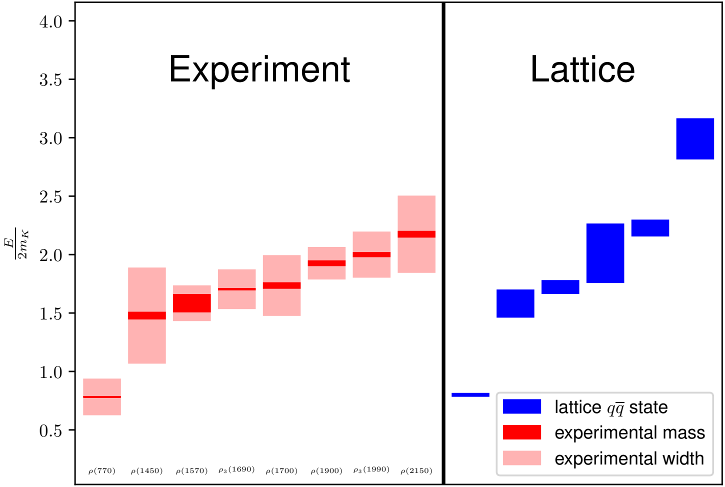
- (9) SH operators only: experiment vs $\overline{q}q$



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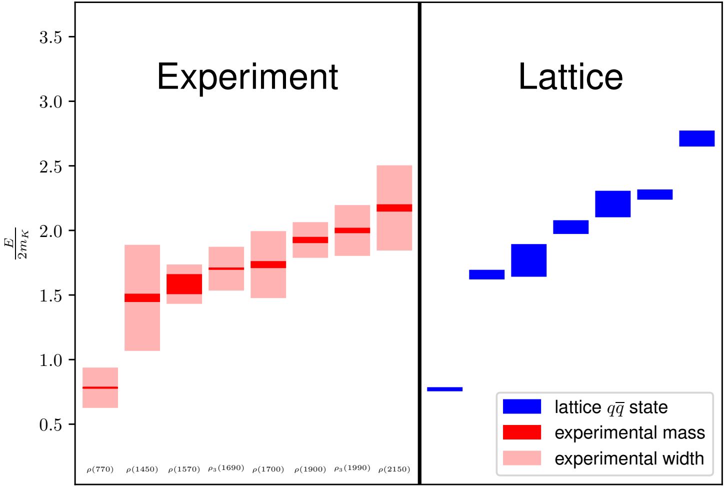
- full operator basis: experiment vs $\overline{q}q$



Conclusions - Scattering

- meson-meson scattering at a mature stage
  - moving towards physical point results - large volumes required

- meson-baryon scattering now possible: eg. $N\pi$: PRD **97**, 014506 (2018)

- successful results depend on time-slice to time-slice quark propagators needed for temporal correlators involving two-hadron operators
  - Stochastic LapH method (minimal volume scaling)

- *box matrix* formulation/software handles partial wave mixing & coupled channels

Conclusions - Spectroscopy

- goal: *qualitative* description of resonant spectrum
- high computational cost for large operator bases
  - Stochastic LapH method
- $\overline{q}q$ states straightforward but many interesting states not well described
  - $\Rightarrow$ hybrids
  - $\Rightarrow$ molecular states
  - $\Rightarrow$ ...
- effective Hamiltonian models to further explore content of finite volume QCD spectrum

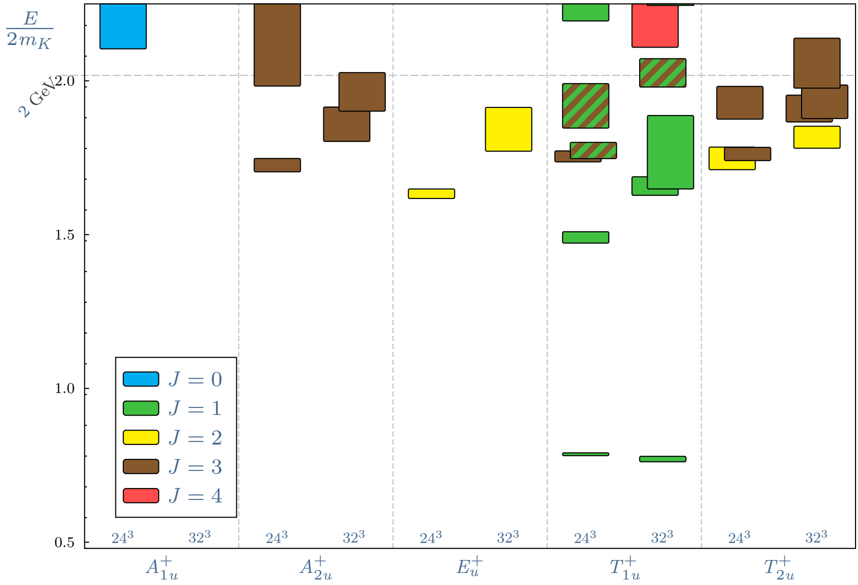
S-wave $K\pi$ amplitude: $K_0^*(800)$

$$\frac{m_{K^*}}{m_\pi} = 3.808(18), \quad g = 5.33(20), \quad m_\pi a_0 = -0.353(25),$$
$$m_\pi^5 a_2 = -0.0013(68), \qquad \chi^2/\mathrm{dof} = 1.42$$

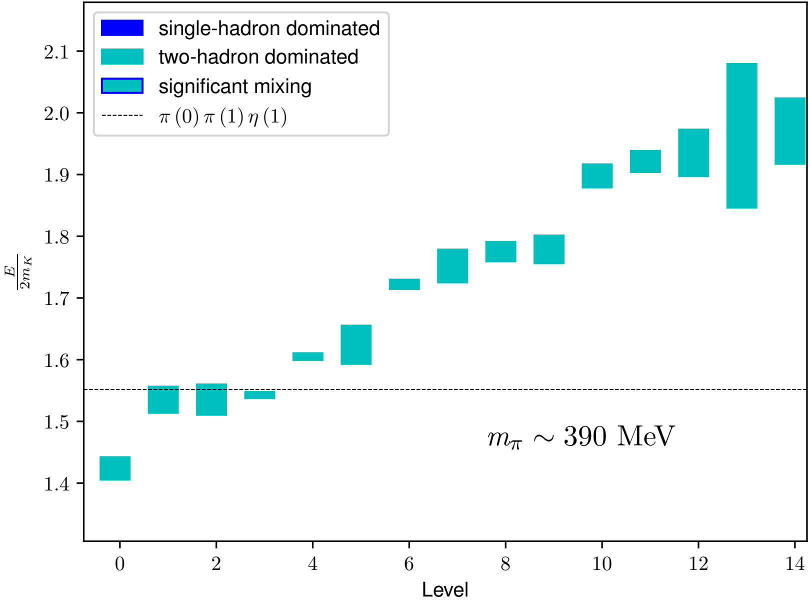
- based on LO ERE, $m_\pi a_0 < 0$ suggests virtual bound state
- however, NLO parameters give $1 - 2r_0/a_0 = -8.9(2.4)$ which must be > 0 for a (real or virtual) bound state
- zeros of $\boldsymbol{q}_{\mathrm{cm}} \cot\delta_0 - i\boldsymbol{q}_{\mathrm{cm}}$: $m_R/m_\pi = 4.66(13) - 0.87(18)i$
  - consistent with BW fit
- better energy resolution & careful analytic continuation required

[RB, J Bulava, J Fallica, A Hanlon, B Hörz, C Morningstar; Nucl.Phys. B932 (2018)]

$I = 1, S = 0$ meson spectroscopy (*preliminary*)



$I = 1, S = 0, A_{1u}^{+}$ spectrum (*preliminary*)



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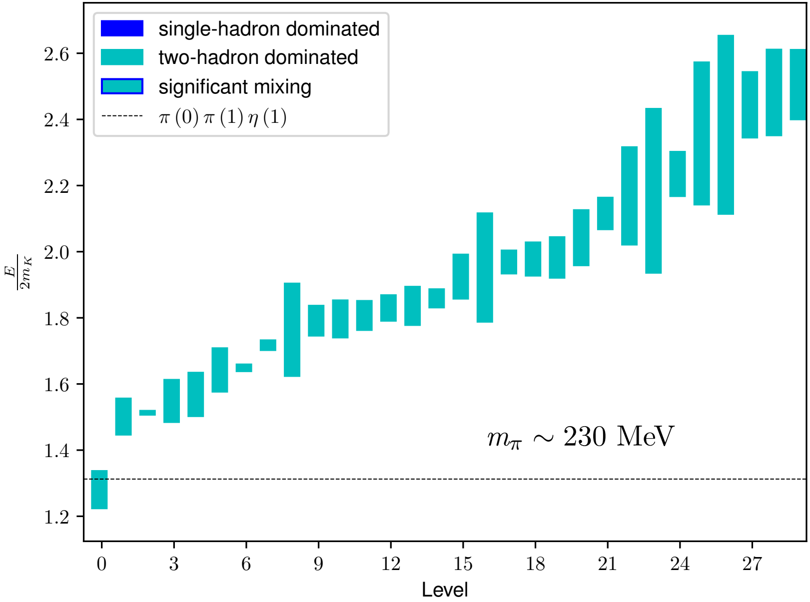
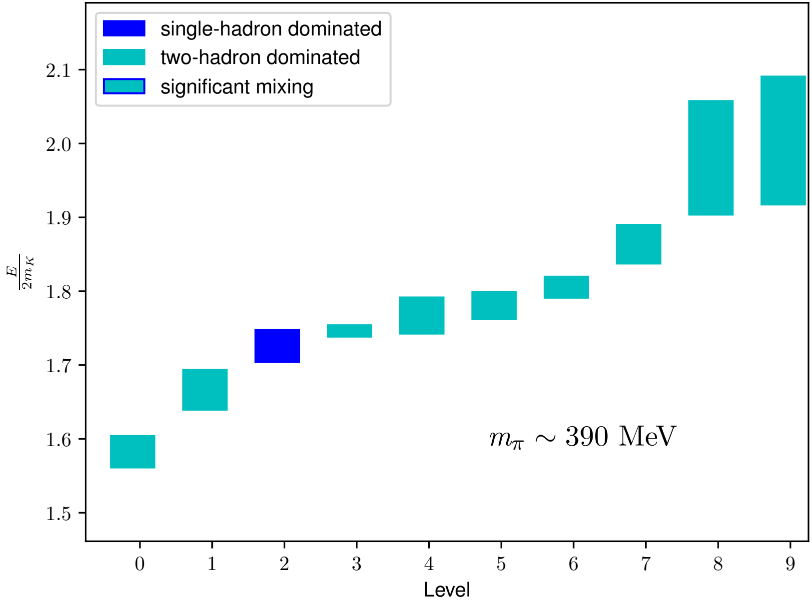


$I = 1, S = 0, A_{2u}^{+}$ spectrum (*preliminary*)



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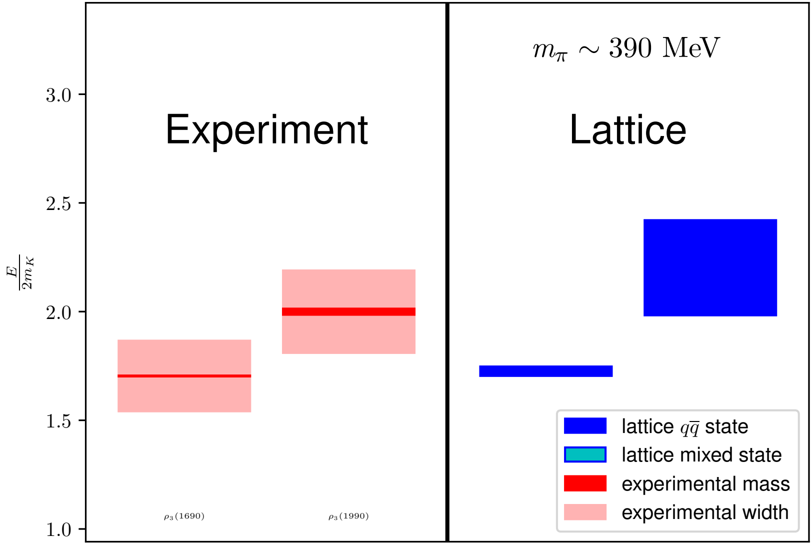

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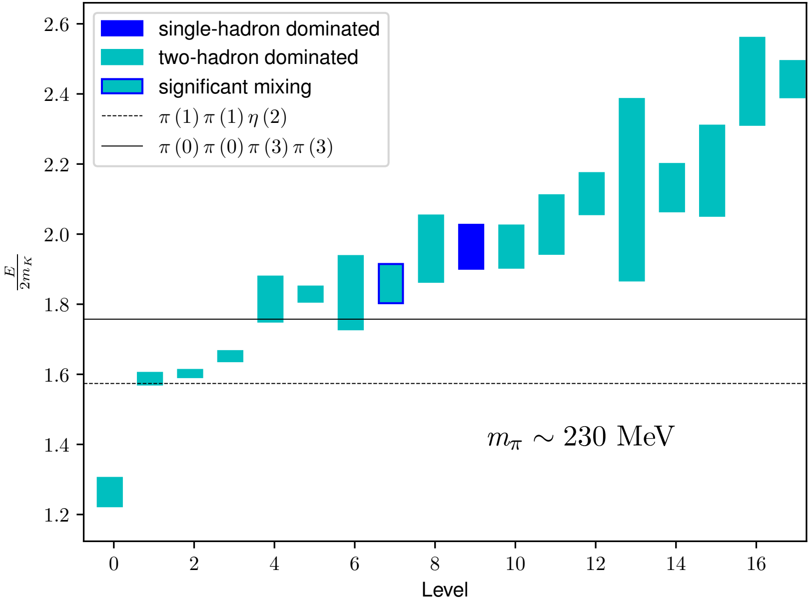

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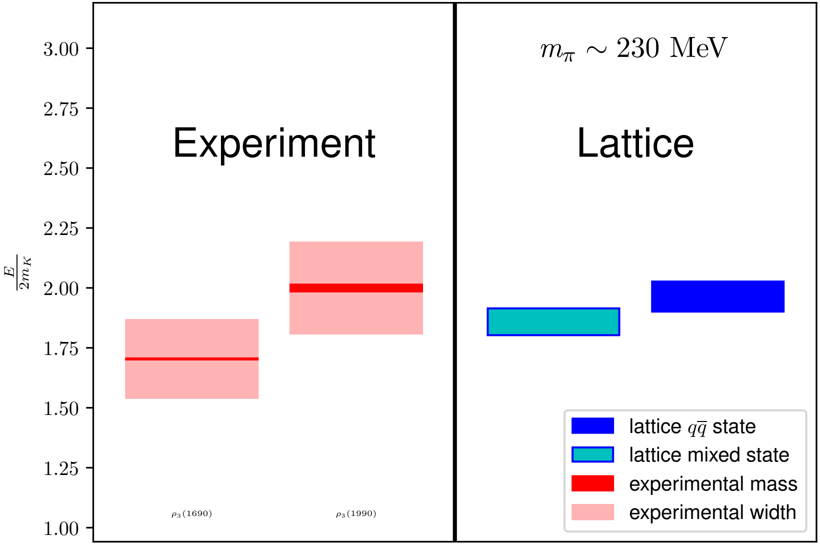
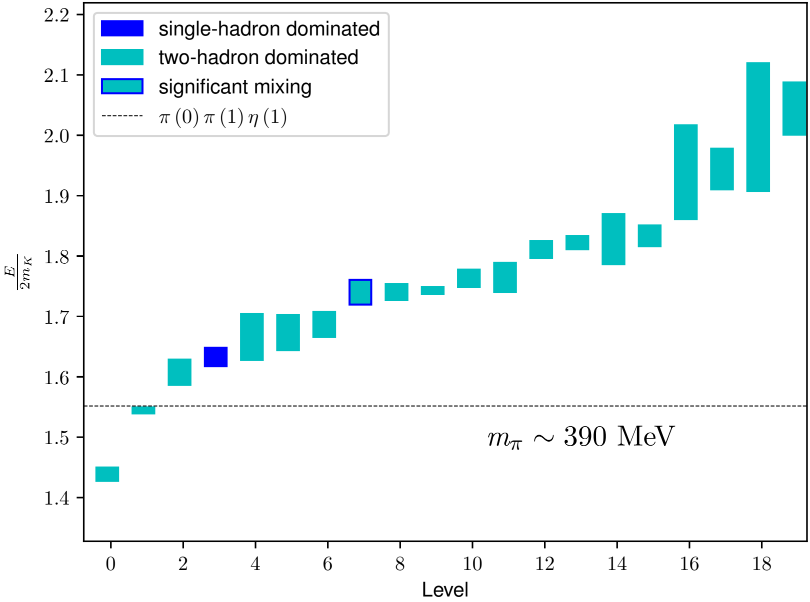


$I = 1, S = 0, E_u^+$ spectrum (*preliminary*)



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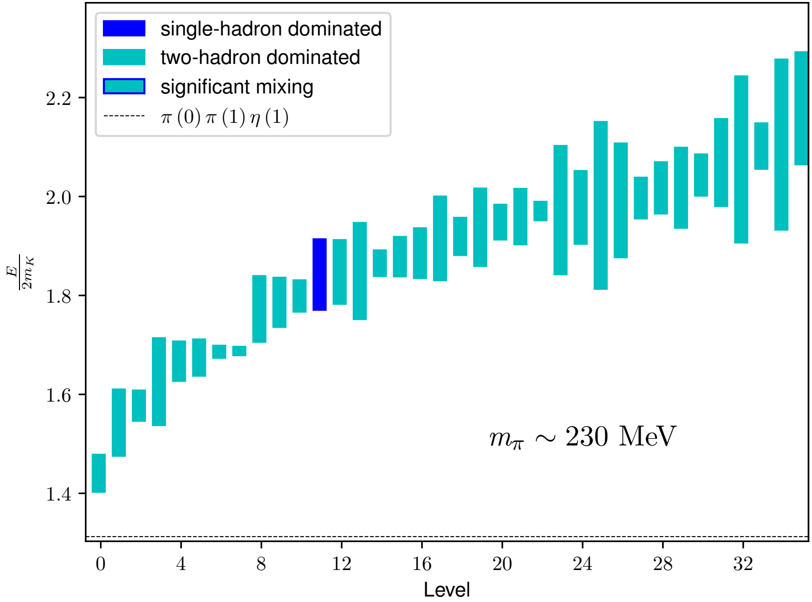
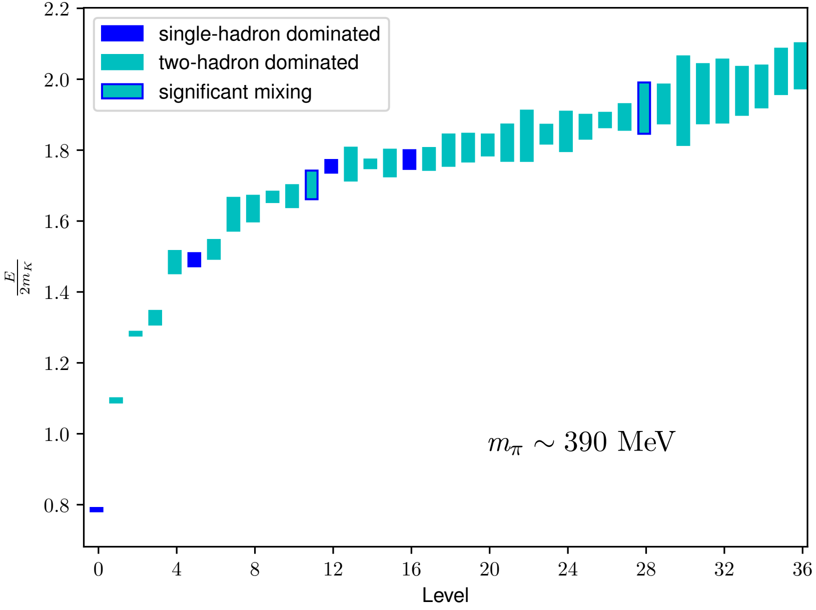


$I = 1, S = 0, T_{1u}^{+}$ spectrum (*preliminary*)



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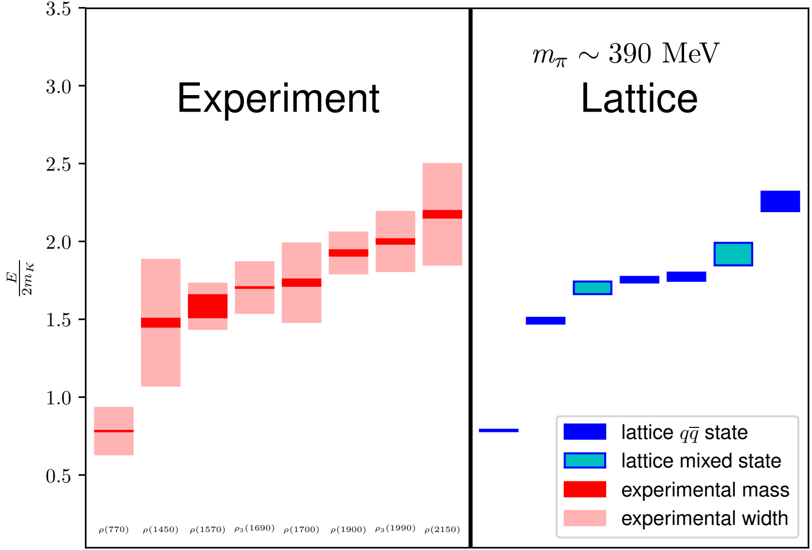

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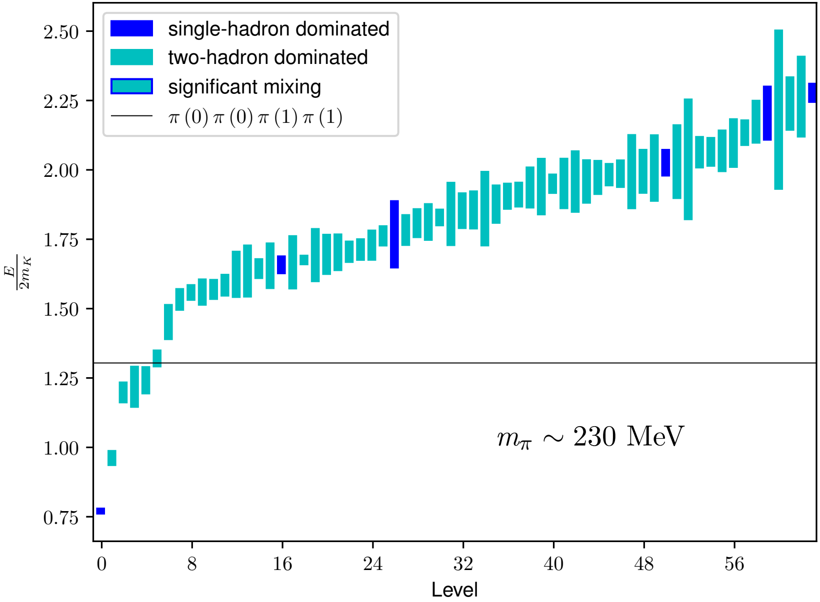

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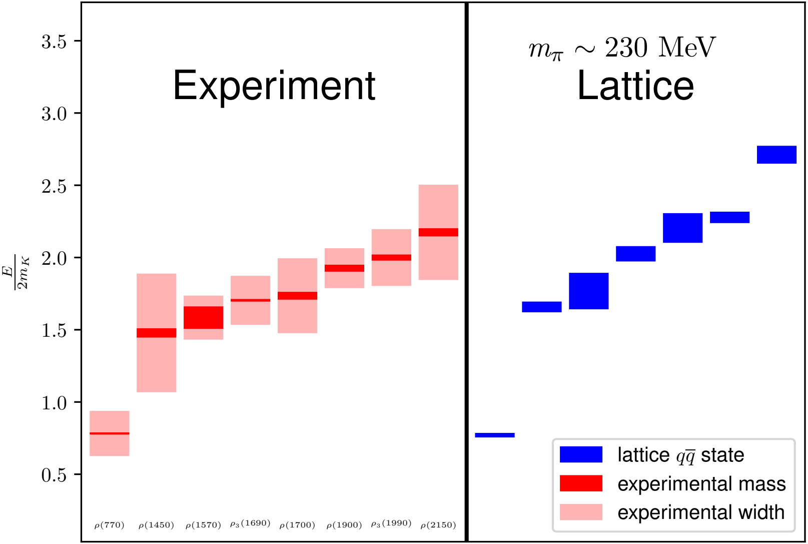

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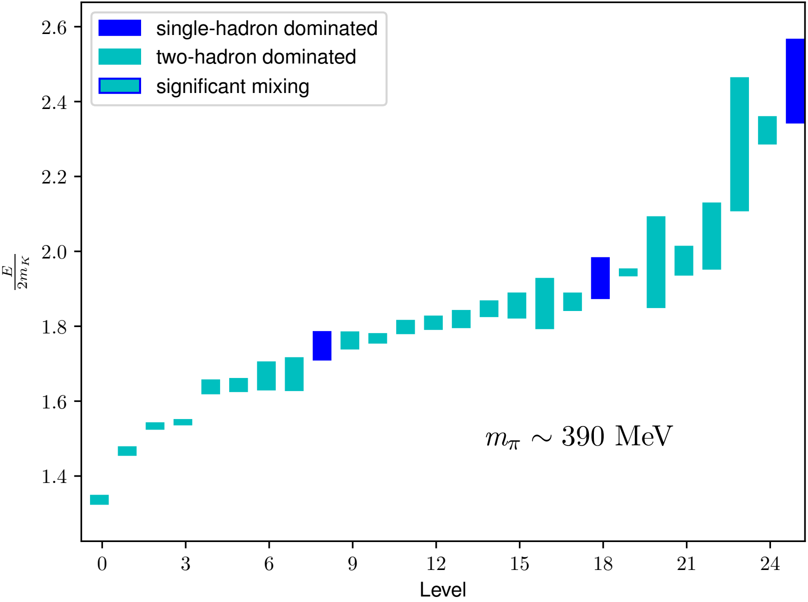

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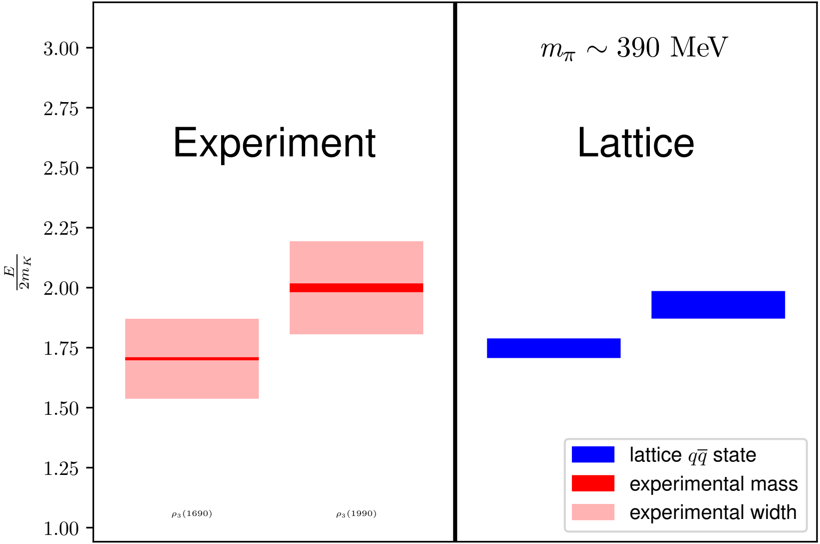

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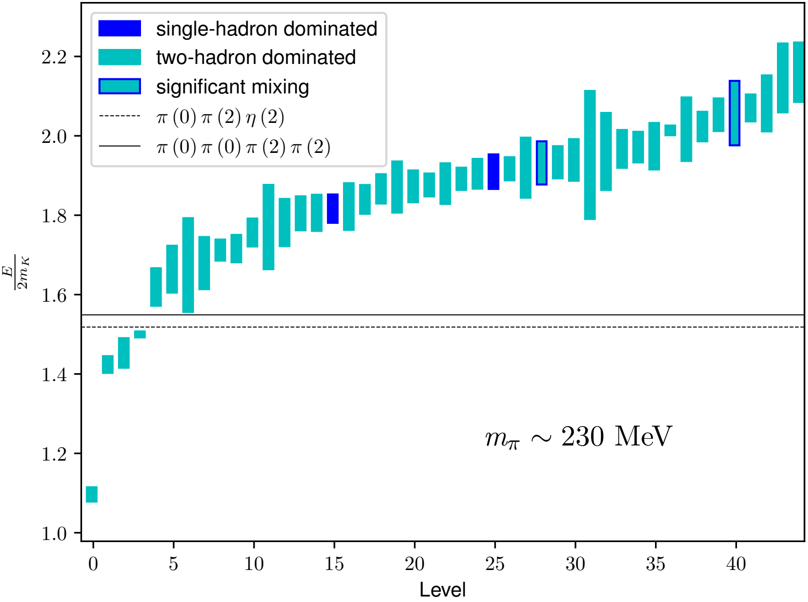

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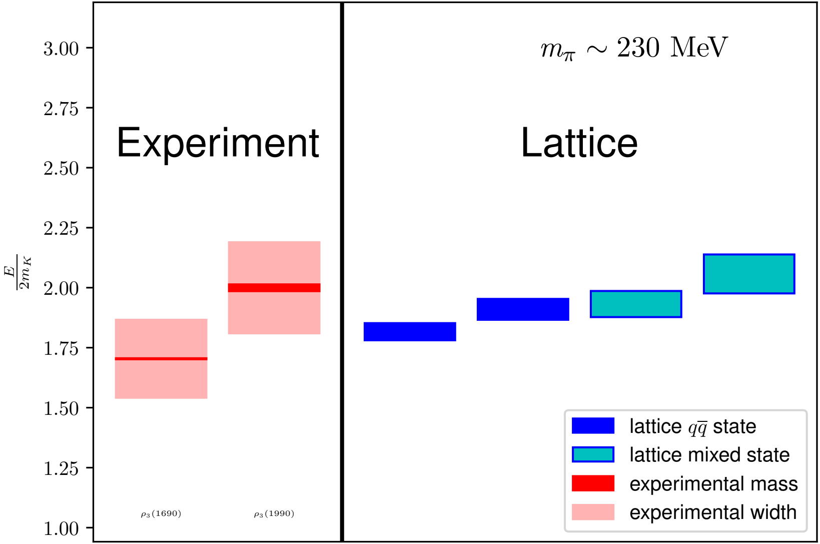


Extracting finite volume spectra

- Signal of interest: deviation of finite-volume two hadron levels from non-interacting counterparts
  - $\Rightarrow$ Extract energy difference from

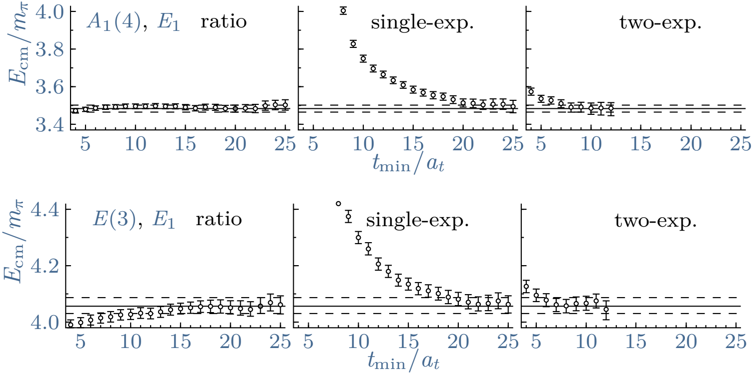
$$R_n(t) = \frac{\widetilde{C}_n(t)}{C_\pi(\boldsymbol{d}_\pi^2, t)\, C_K(\boldsymbol{d}_K^2, t)} \to A_n e^{-\Delta E_n t}$$

- Reconstruct:

$$a_t E_n = a_t \Delta E_n + \sqrt{a_t^2 m_\pi^2 + \left(\frac{2\pi a_s}{\xi L}\right)^2 \boldsymbol{d}_\pi^2} + \sqrt{a_t^2 m_K^2 + \left(\frac{2\pi a_s}{\xi L}\right)^2 \boldsymbol{d}_K^2}.$$

- Where ΔE_n is small, these ratio fits generally have smaller excited state contamination than direct fits to $\widetilde{C}_n(t)$

Ratio fits



Each row corresponds to the three fits for a single level specified in the left column as '$\Lambda(\boldsymbol{d}^2)$, E_n', denoting the nth level in finite volume irrep Λ with total momentum $\boldsymbol{d}^2$.

Decay of $\rho(770)$

- initially applied to P-wave $I = 1$ $\rho \to \pi\pi$ system
- now have included $\ell = 1, 3, 5$ partial waves
- large $32^3 \times 256$ anisotropic lattice, $m_\pi \approx 230$ MeV
- fit forms (first ever inclusion of $\ell = 5$ in lattice QCD):

$$(\widetilde{K}^{-1})_{11} = \frac{6\pi E_{\rm cm}}{g^2 m_\pi}\left(\frac{m_\rho^2}{m_\pi^2} - \frac{E_{\rm cm}^2}{m_\pi^2}\right)$$

$$(\widetilde{K}^{-1})_{33} = \frac{1}{m_\pi^7 a_3} \qquad (\widetilde{K}^{-1})_{55} = \frac{1}{m_\pi^{11} a_5}$$

- results

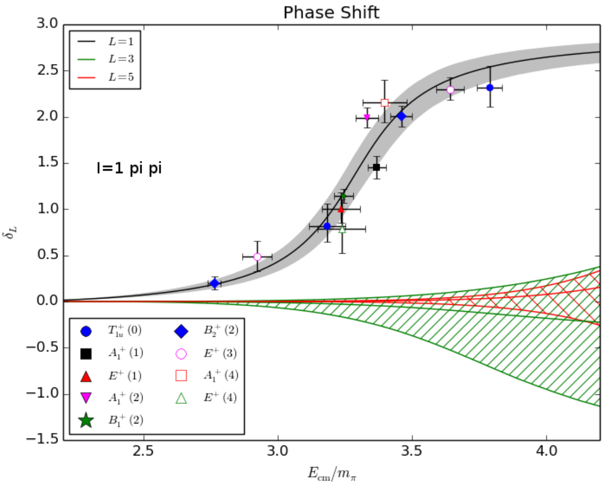
$$\frac{m_\rho}{m_\pi} = 3.349(25),\ g = 5.97(27),\ m_\pi^7 a_3 = -0.00021(100),$$
$$m_\pi^{11} a_5 = -0.00006(24),\ \chi^2/\text{dof} = 1.15$$

[J Bulava, B Fahy, B Hörz, K J Juge, C Morningstar, CH Wong; NPB 910, 842 (2016)]

[C Morningstar, J Bulava, B Singha, RB, J Fallica, A Hanlon, B Hörz; NPB 924, 477 (2017)]

Decay of $\rho(770)$

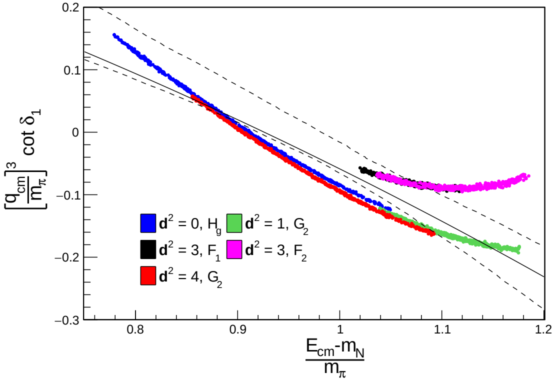
- $\ell = 1, 3, 5$ phase shifts



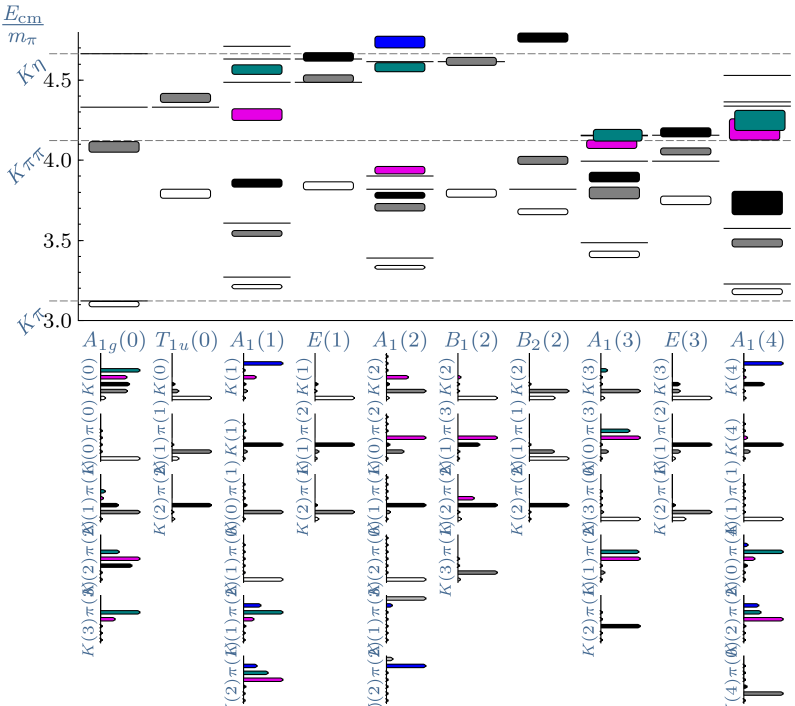
[J Fallica, PhD Thesis (2017)]

Decay of $\Delta(1232)$

- included $\ell = 1$ wave only (for now)
- large $48^3 \times 128$ isotropic lattice, $m_\pi \approx 280$ MeV, $a \sim 0.076$ fm
- Breit-Wigner fit gives $g_{\Delta N\pi} = 19.0(4.7)$ in agreement with experiment ~ 16.9



[CW Andersen, J Bulava, B Hörz, C Morningstar; PRD **97**, 014506 (2018)]



$\frac{E_{\rm cm}}{m_\pi}$
$K\eta$
$K\pi\pi$
$K\pi$
4.5
4.0
3.5
3.0
$A_{1g}(0)$
$T_{1u}(0)$
$A_1(1)$
$E(1)$
$A_1(2)$
$B_1(2)$
$B_2(2)$
$A_1(3)$
$E(3)$
$A_1(4)$