# Kaon Matrix Elements from Coarse Lattices

Lattice 2018
Michigan State University
July 26, 2018

Robert Mawhinney and Jiqun Tu
Columbia University
RBC and UKQCD Collaborations

# The RBC & UKQCD collaborations


*BNL and BNL/RBRC*

Yasumichi Aoki (KEK)
Mattia Bruno
Taku Izubuchi
Yong-Chull Jang
Chulwoo Jung
Christoph Lehner
Meifeng Lin
Aaron Meyer
Hiroshi Ohki
Shigemi Ohta (KEK)
Amarjit Soni

*UC Boulder*

Oliver Witzel

*Columbia University*

Ziyuan Bai
Norman Christ
Duo Guo
Christopher Kelly
Bob Mawhinney
Masaaki Tomii
Jiqun Tu
Bigeng Wang
Tianle Wang
Evan Wickenden
Yidi Zhao

*University of Connecticut*

Tom Blum
Dan Hoying (BNL)
Luchang Jin (RBRC)
Cheng Tu

*Edinburgh University*

Peter Boyle
Guido Cossu
Luigi Del Debbio
Tadeusz Janowski
Richard Kenway
Julia Kettle
Fionn O'haigan
Brian Pendleton
Antonin Portelli
Tobias Tsang
Azusa Yamaguchi

*KEK*

Julien Frison

*University of Liverpool*

Nicolas Garron

*MIT*

David Murphy

*Peking University*

Xu Feng

*University of Southampton*

Jonathan Flynn
Vera Guelpers
James Harrison
Andreas Juettner
James Richings
Chris Sachrajda

*Stony Brook University*

Jun-Sik Yoo
Sergey Syritsyn (RBRC)

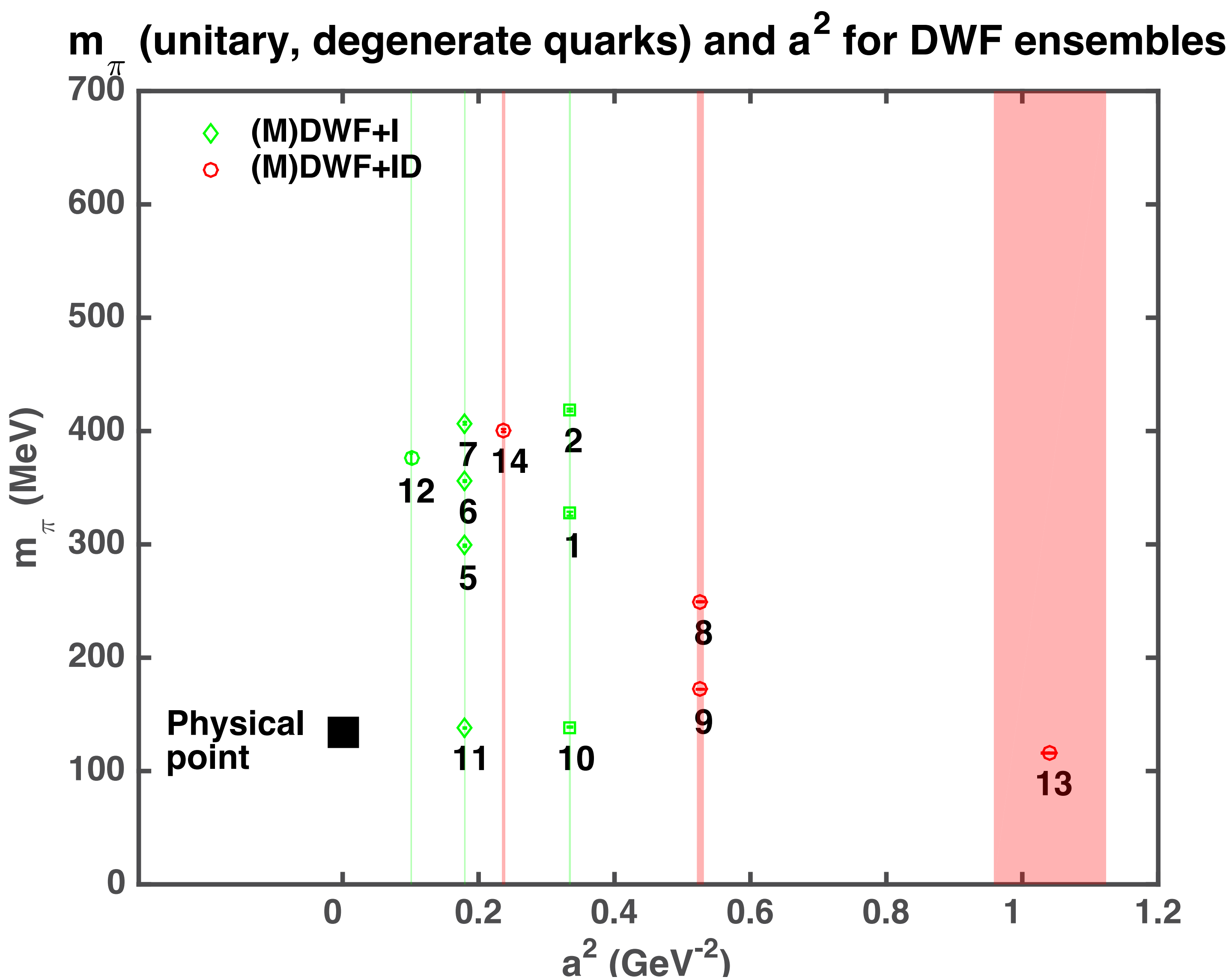
*York University (Toronto)*

Renwick Hudspith

# Outline

- We have been generating coarse ensembles ($1/a \approx 1$ GeV) with the Iwasaki+DSDR (ID) gauge action with physical pion and kaon masses.
- Ideal testing ground for algorithms and physics measurements
  * Large physical volumes from modest lattice volumes
  * Physical u,d and s quark masses.
  * Easier to study finite volume effects than at weak coupling
  * With MDWF, no ensemble generation problems from large lattice spacing.
- For many quantities $O(a^2)$ scaling errors have generally been small.
  * Possibility of accurate continuum limit, with at least two lattice spacings.
  * Are $O(a^4)$ errors visible or large?
  * For quantities which are difficult to measure, statistical errors may be more important than scaling errors
- Will report on measurements of kaon matrix elements on these ensembles and their continuum limit.
  * Relevant to seeking a continuum limit for $\varepsilon'/\varepsilon$

# RBC/UKQCD 2+1 Flavor DWF Ensembles

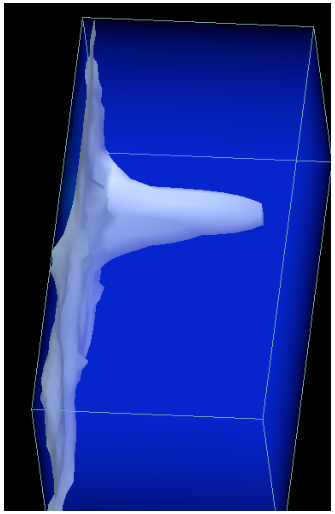


# Balancing $m_{res}$ and Topological Tunneling for DWF

- The propagation of light modes between the five-dimensional boundaries is controlled by the eigenvalues of the transfer matrix, $H_T$

$$H_T = \gamma_5 D_W(M) \frac{1}{2 + (b_i - c_i) D_W(M)}$$

- Zeros of $D_w(M)$ produce modes not bound to the five-dimensional boundaries
- These zeros occur when the gauge fields are changing topology (picture from PRD 77 (2008) 014509)



- Refer to this type of localized fluctuation in the gauge fields as a dislocation.
- For a given $L_s$, dislocations increase the size of the residual mass, $m_{res}$.

# Choices of Action

- For 1/a in range 1.5 - 2.5 GeV, Iwasaki gauge action suppresses dislocations sufficiently with 2+1 flavors of fermions to allow physical light quark masses to be reached.

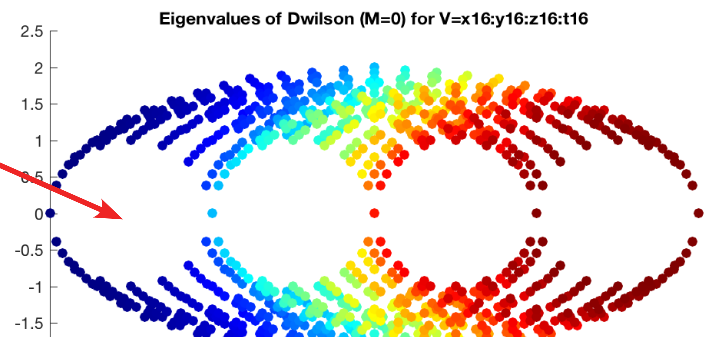
  *   1/a = 1.73 GeV: $L_s = 24$ for MDWF (b+c=2) gives $m_{res} = 0.45\, m_{ud}$

  *   1/a = 2.31 GeV: $L_s = 12$ for MDWF (b+c=2) gives $m_{res} = 0.32\, m_{ud}$

- For stronger couplings, add the Dislocation Suppressing Determinant Ratio (DSDR) to suppress topological tunneling

$$\det\left(\frac{D_W^\dagger(M)D_W(M)+\varepsilon_f^2}{D_W^\dagger(M)D_W(M)+\varepsilon_b^2}\right)=\prod_\lambda\frac{\lambda^2+\varepsilon_f^2}{\lambda^2+\varepsilon_b^2} \qquad \varepsilon_f < \varepsilon_b$$

choose $M = -M_5$




*   1/a = 1.35 GeV: $L_s = 12$ for MDWF (b+c=32/12) gives $m_{res} = 0.95\, m_{ud}$

# 2+1 Flavor Iwasaki + DSDR (ID) (M)DWF ensembles

- Original DSDR ensemble had 1/a = 1.37(1) GeV, $m_\pi$ = 170 MeV and V = (4.7 fm)$^3$
  - * Another ensemble, with G-parity boundary conditions, has been generated for $K \to \pi\pi$ matrix elements calculations with $m_\pi$ = 143 MeV
- Global fits (chiral and continuum) show small $O(a^2)$ errors for quantities studied for ID ensembles, even at 1/a = 1 GeV.
- We are generating 3 ensembles with 1/a = 1 GeV, physical pions and kaons
  - * $24^3$: physical volume is (4.8 fm)$^3$, $m_\pi L$ = 3.4, currently ~3000 MD time units
  - * $32^3$: physical volume is (6.4 fm)$^3$, $m_\pi L$ = 4.5, currently ~1200 MD time units
  - * $48^3$: physical volume is (9.6 fm)$^3$, $m_\pi L$ = 6.7, currently ~800 MD time units
- We are generating 1 ensemble with 1/a = 1 GeV, physical pions and $m_K$ ~ 300 MeV
  - * $32^3$: physical volume is (4.8 fm)$^3$, $m_\pi L$ = 3.4, currently ~800 MD time units
- We are generating 1 ensemble with 1/a = 1.37 GeV, physical pions and kaons
  - * $32^3$: physical volume is (4.7 fm)$^3$, $m_\pi L$ = 3.4, currently ~800 MD time units

# SU(2) ChPT Fits to $m_{PS}$ and $f_{PS}$

- We can simultaneously fit lattice data for different lattice spacings, actions and volumes using expansions of the form (SU(2) NLO example):

$$(m^{\mathbf{e}}_{ll})^2 = \chi^{\mathbf{e}}_l + \chi^{\mathbf{e}}_l \cdot \left\{ \frac{16}{f^2}\Big((2L^{(2)}_8 - L^{(2)}_5) + 2(2L^{(2)}_6 - L^{(2)}_4)\Big)\chi^{\mathbf{e}}_l + \frac{1}{16\pi^2 f^2}\chi^{\mathbf{e}}_l \log\frac{\chi^{\mathbf{e}}_l}{\Lambda^2_\chi} \right\}$$

$$f^{\mathbf{e}}_{ll} = f\left[1 + c_f (a^{\mathbf{e}})^2\right] + f \cdot \left\{ \frac{8}{f^2}(2L^{(2)}_4 + L^{(2)}_5)\chi^{\mathbf{e}}_l - \frac{\chi^{\mathbf{e}}_l}{8\pi^2 f^2}\log\frac{\chi^{\mathbf{e}}_l}{\Lambda^2_\chi} \right\}$$

with

$$\chi^{\mathbf{e}}_l = \frac{Z^{\mathbf{e}}_l}{R^{\mathbf{e}}_a}\frac{B^{\mathbf{1}}\widetilde{m}^{\mathbf{e}}_l}{(a^{\mathbf{e}})^2}$$

- At NNLO order, using codes from Bijnens and collaborators, we fit to

$$X(\tilde{m}_q, L, a^2) \simeq X_0\left(1 + \underbrace{X^{\rm NLO}(\tilde{m}_q) + X^{\rm NNLO}(\tilde{m}_q)}_{\text{NNLO Continuum PQChPT}} + \underbrace{\Delta^{\rm NLO}_X(\tilde{m}_q, L)}_{\text{NLO FV corrections}} + \underbrace{c_X a^2}_{\text{Lattice spacing}}\right)$$

- For SU(2), we use $m_\pi$, $m_K$ and $m_\Omega$ to set the scale.
- There are different $a^2$ corrections to the decay constants for I and ID actions.
- Heavy quark ChPT used for light quark extrapolation of kaon.
- $t_0^{1/2}$ and $w_0$ are also fit using a linear chiral ansatz.

# Scaling Errors for $f_\pi$ and $f_K$

- Fits use different $O(a^2)$ coefficients for Iwasaki and Iwasaki+DSDR actions
- Results for these coefficients from PRD 93 054502 (2016):

| | NLO (370 MeV cut) | NNLO (450 MeV cut) |
|---|---|---|
| Iwasaki $f_\pi$ $a^2$ coeff. | 0.059(47) $GeV^2$ | 0.065(45) $GeV^2$ |
| DSDR $f_\pi$ $a^2$ coeff. | -0.013(17) $GeV^2$ | 0.012(16) $GeV^2$ |
| Iwasaki $f_K$ $a^2$ coeff. | 0.049(39) $GeV^2$ | 0.069(36) $GeV^2$ |
| DSDR $f_K$ $a^2$ coeff. | -0.005(15) $GeV^2$ | 0.019(15) $GeV^2$ |

- For 1/a = 1 GeV, percent scaling error:

| | NLO (370 MeV cut) | NNLO (450 MeV cut) |
|---|---|---|
| Iwasaki $f_\pi$ | 6 ± 5% | 7 ± 5% |
| DSDR $f_\pi$ | -1 ± 2% | 1 ± 2% |
| Iwasaki $f_K$ | 5 ± 4% | 7 ± 4% |
| DSDR $f_K$ | -1 ± 2% | 2 ± 2% |

- Canonical scaling errors should be $(a\Lambda_{QCD}^{(3)})^2 \sim (330\,\mathrm{MeV}/980\,\mathrm{MeV})^2 \sim 0.11$.
- 2+1 flavor physical quark mass simulations at strong coupling well behaved.

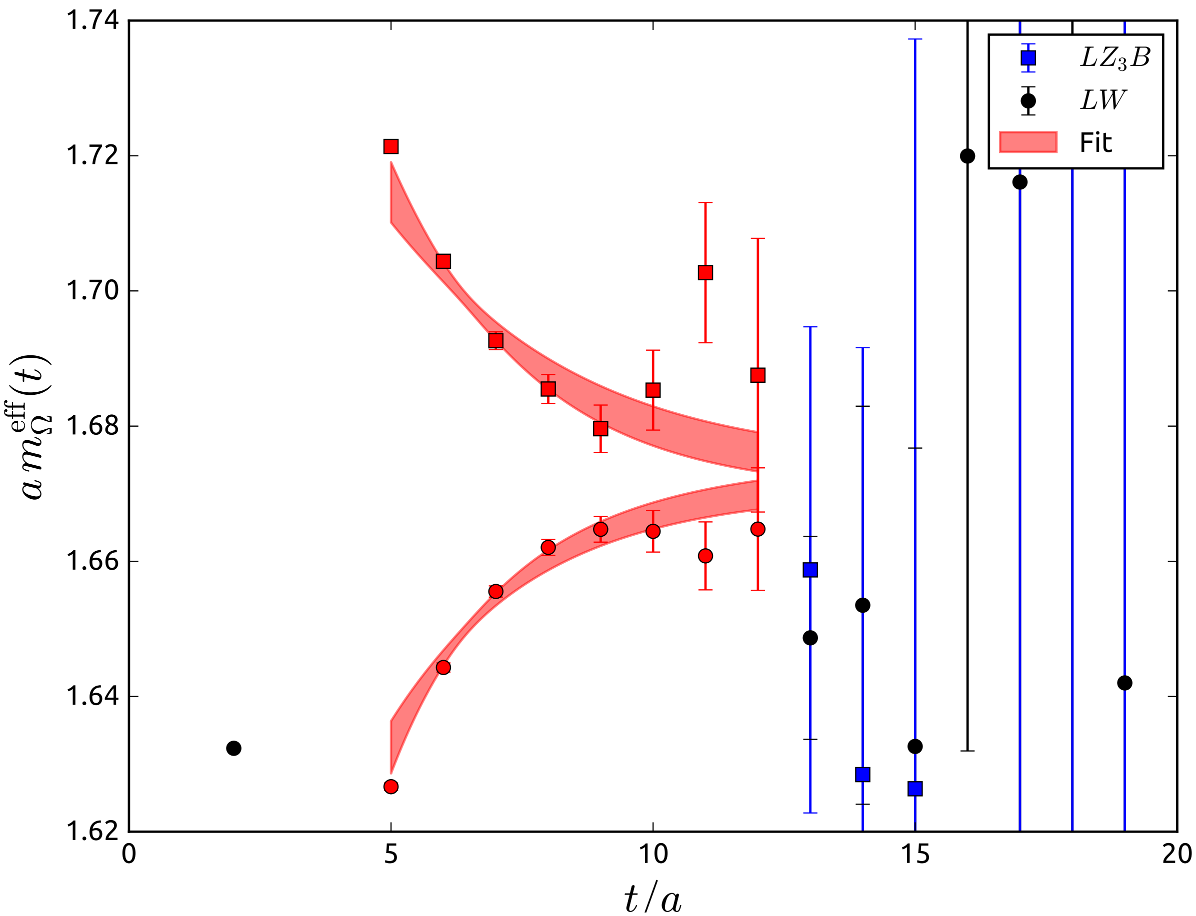
# Scaling Errors For More Observables

- We have preliminary fits with more observables, including the $\pi\pi$ I=2 scattering length (David Murphy)
- Show results for SU(2) NNLO fits with pseudoscalar masses below 450 MeV

| | Iwasaki $a^2$ coefficient | DSDR $a^2$ coefficient |
|---|---|---|
| $f_\pi$ | 0.070±0.041 | 0.022±0.017 |
| $f_K$ | 0.079±0.034 | 0.030±0.014 |
| $t_0^{1/2}$ | -0.017±0.041 | -0.021±0.020 |
| $w_0$ | -0.117±0.360 | -0.039±0.018 |
| $a_0^2$ (I=2 pi-pi scattering) | -0.15±0.33 | -0.04±0.45 |

# Omega Baryon Effective Mass on $24^3$ 1 GeV Ensemble

- Two sources: Coulomb gauge fixed wall source and 8 smaller Coulomb gauge fixed wall sources.
- Fit to common ground and excited states.



# $B_K$ from (M)DWF Ensembles

- Combined continuum and chiral fit (global fit) to 2+1 flavor I and ID ensembles
  - * Use $m_\pi$, $m_K$ and $m_\Omega$ to set the scale and quark masses values for each ensemble
  - * Lattice scales are used to find $Z_{B_K}$ to renormalize to SMOM($\not{q},\not{q},\mu$=3 GeV)
  - * A combined continuum and chiral fit is then done to $B_K(\not{q},\not{q},\mu=3\ \text{GeV})$
  - * Result from Iwasaki ensembles plus the ID ensembles with 1/a = 1.37 GeV. (PRD 91 (2015) 074502)

$$B_K(\overline{\text{MS}}, \mu=3\ \text{GeV}) = 0.5293 \pm 0.0017_{\text{stat}} \pm 0.0150_{\text{sys}}$$

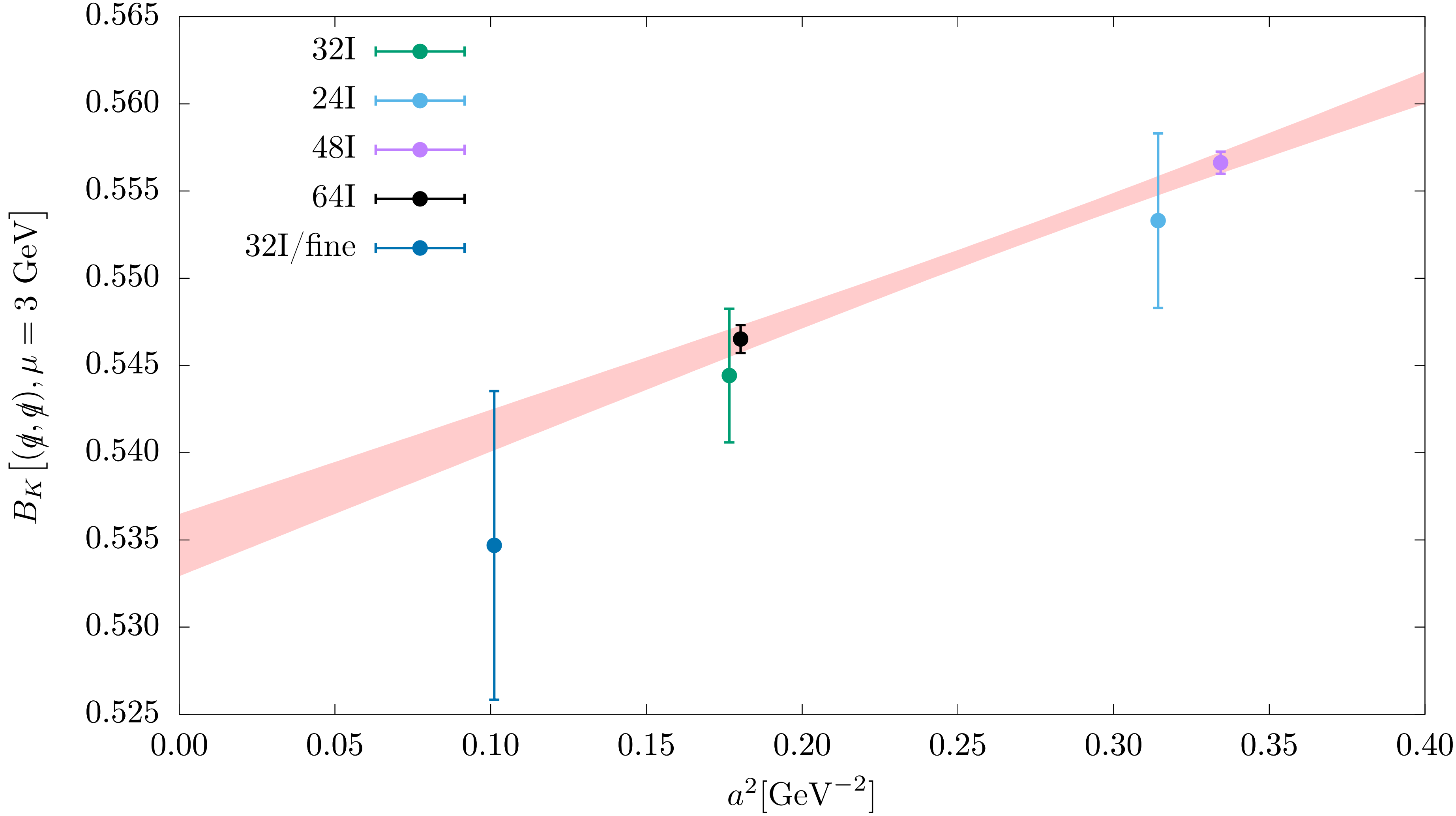
- I and ID have separate $O(a^2)$ scaling errors for $B_K$
  - * For Iwasaki ensembles: $0.125(12) \times a^2$ ($a^2$ in $\text{GeV}^{-2}$)
  - * For ID enesmbles: $0.148(15) \times a^2$
- Get $a^2$ scaling coefficient from single ID ensemble by requiring a common continuum limit.
- Is $a^2$ scaling for $B_K$ justified on ID ensembles even for $1/a \approx 1$ GeV?

# $a^2$ Scaling for $B_K$ from ID (M)DWF Ensembles

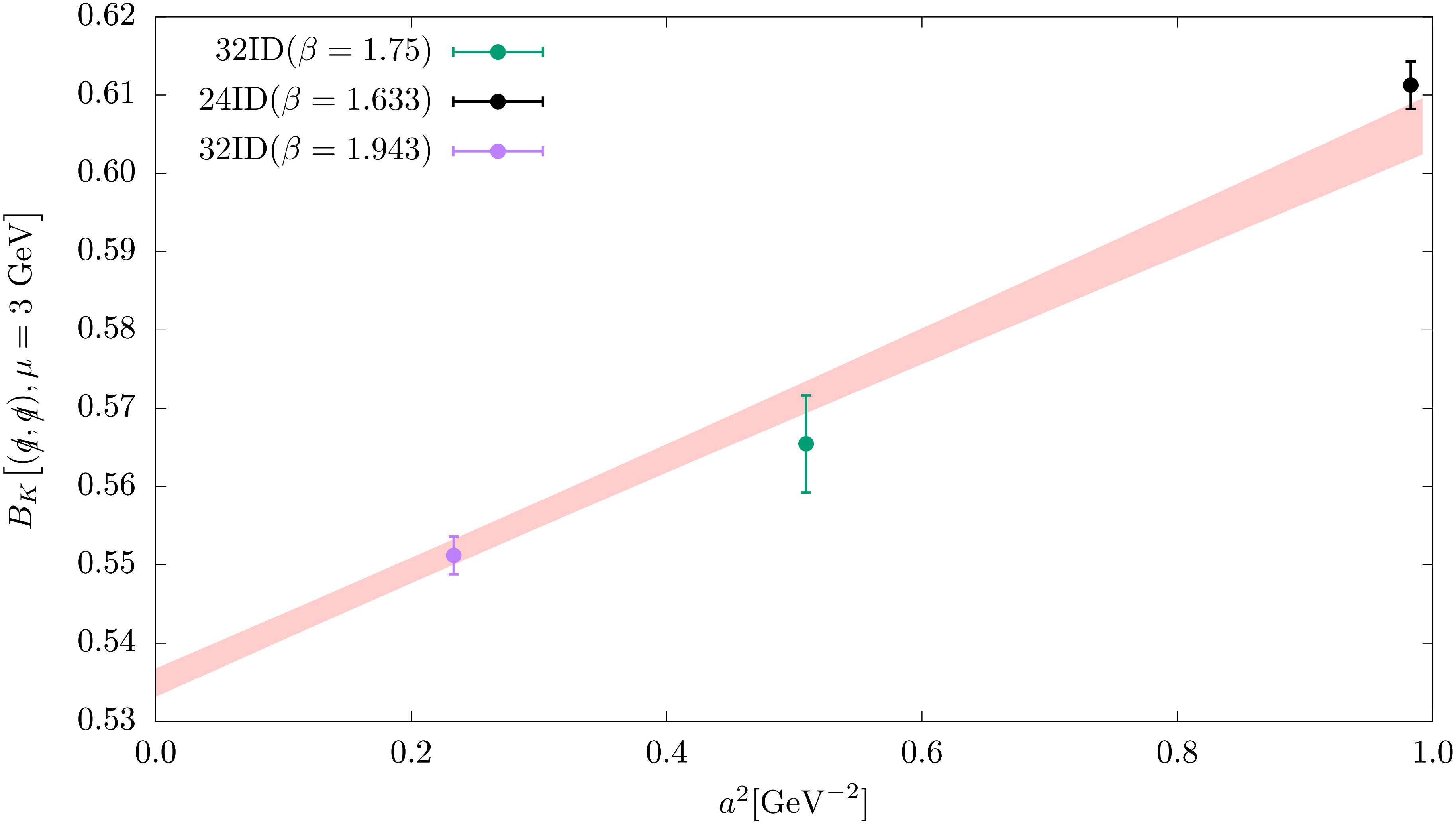
- Have measured $B_K$ on 1/a = 1 GeV ID ensemble
  - * NPR done with $\mu_1$ = 1.4363, $\mu$ = 3.0 GeV.
  - * Step scaling to connect ($\mu_1$, $\mu$).
- Have also remeasured $B_K$ on 1/a = 1.35 GeV ensemble
  - * AMA plus EigCG deflation markedly reduces statistical errors
- Updated global fit show similar $a^2$ coeffecients with smaller statistical errors.

| | ChPTFV | ChPTFV[3] |
|---|---|---|
| $\chi^2$/dof | 0.50(34) | – |
| $B_K^{phys}$ | 0.5350(18) | 0.5341(18) |
| $B_K^0$ | 0.5282(17) | 0.5278(16) |
| $c^{\mathrm{I}}_{B_K,a^2}$ | 0.114(11) | 0.128(12) |
| $c^{\mathrm{ID}}_{B_K,a^2}$ | 0.1262(72) | 0.153(15) |
| $c_{B_K,m_l}$ | −0.0075(10) | −0.00728(95) |
| $c_{B_K,m_x}$ | 0.00439(66) | 0.00420(64) |
| $c_{B_K,m_h}$ | −0.09(18) | −0.06(18) |
| $c_{B_K,m_y}$ | 1.218(29) | 1.324(32) |

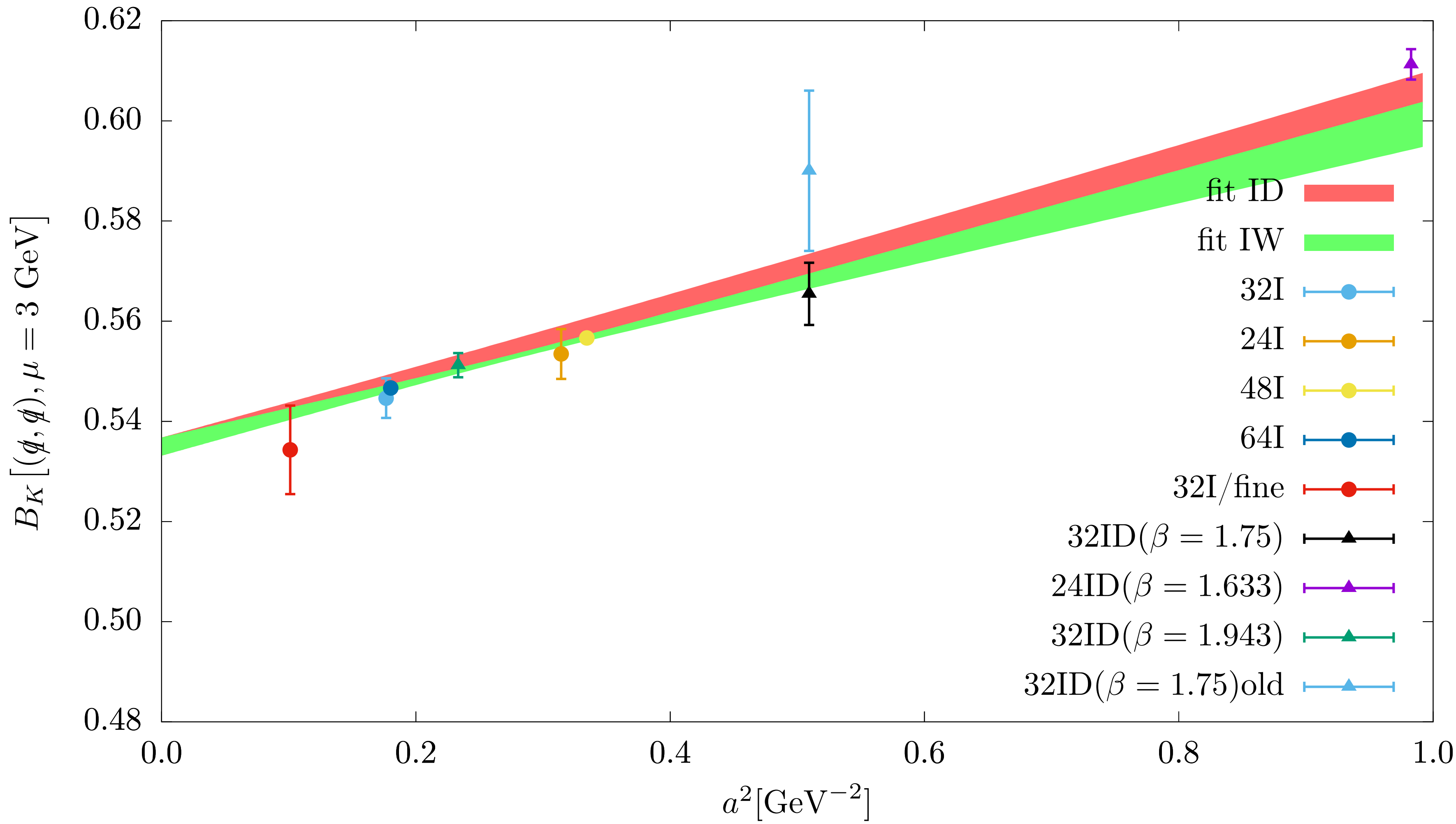
# $a^2$ Scaling for $B_K$ from Iwasaki (M)DWF Ensembles



# $a^2$ Scaling for $B_K$ from Iwasaki+DSDR (M)DWF Ensembles



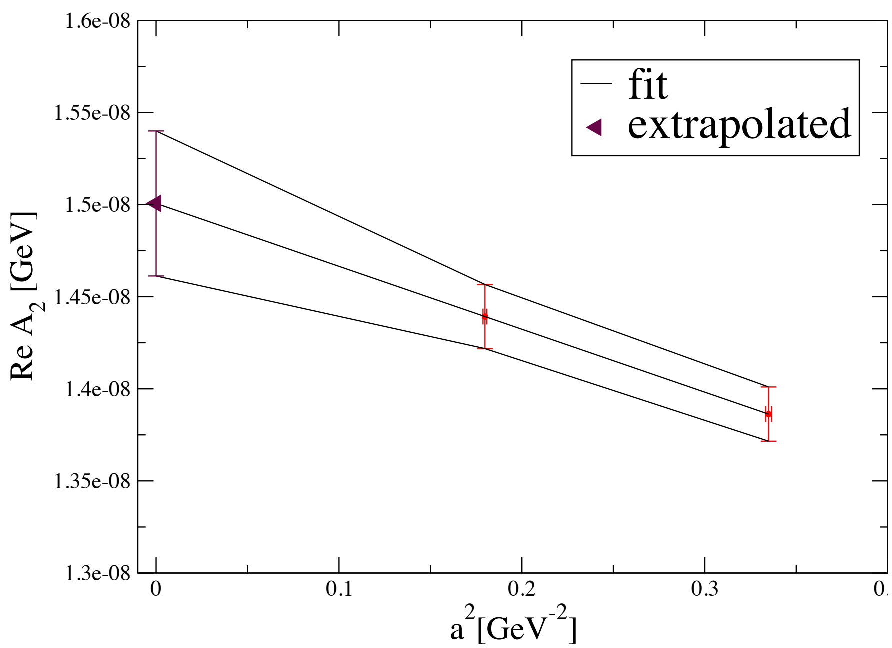
# $a^2$ Scaling for $B_K$

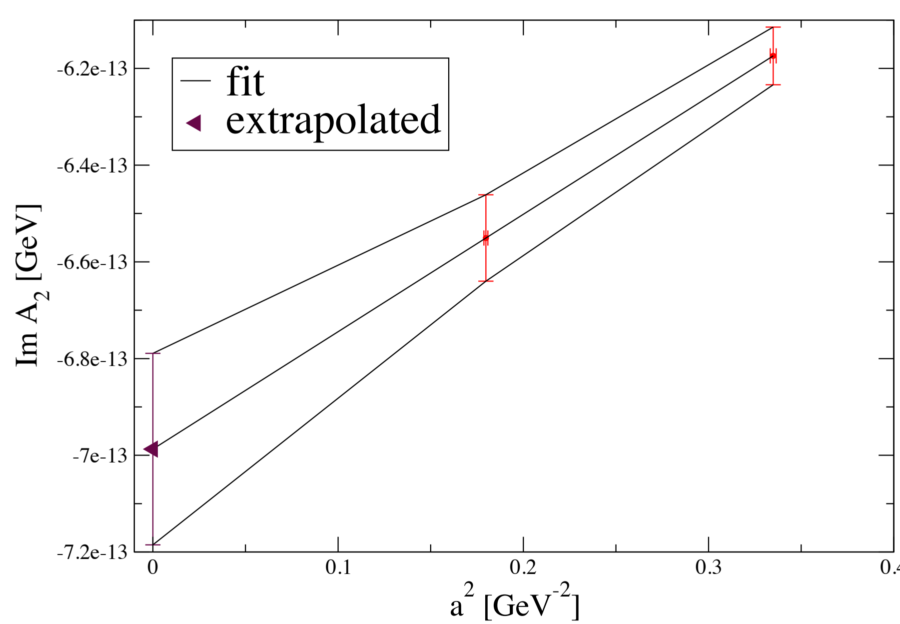


# $\Delta I = 3/2$ K →ππ Matrix Elements from Iwasaki (M)DWF

- RBC-UKQCD has calculated Re($A_2$) and Im($A_2$) on Iwasaki ensembles with 1/a = 1.73 and 2.35 GeV and taken the continuum limit. PRD 91 (2015) 074502

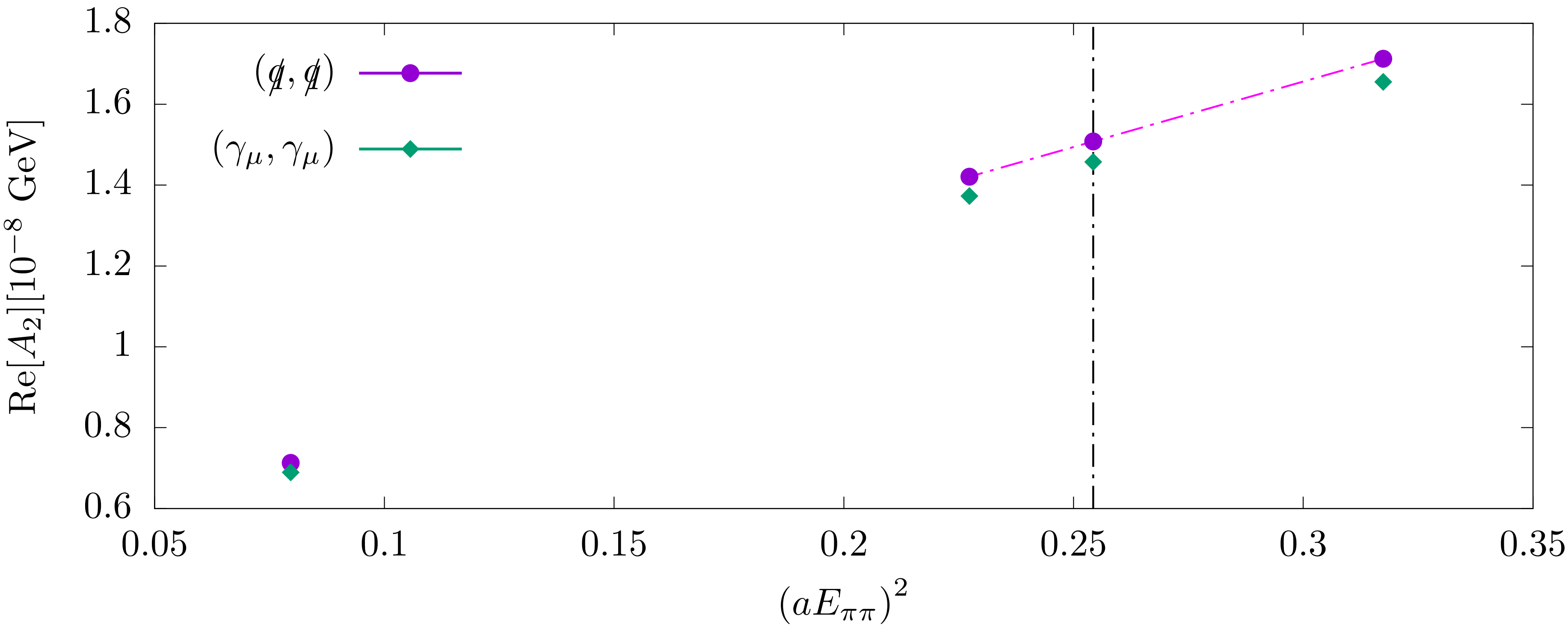
| | $m_\pi$ | $m_K$ | $E_{\pi\pi}$ | $m_K - E_{\pi\pi}$ |
|---|---|---|---|---|
| $48^3$ (lattice units) | $8.050(13) \times 10^{-2}$ | $2.8867(15) \times 10^{-1}$ | $2.873(13) \times 10^{-1}$ | $1.4(14) \times 10^{-3}$ |
| $64^3$ (lattice units) | $5.904(14) \times 10^{-2}$ | $2.1531(14) \times 10^{-1}$ | $2.1512(68) \times 10^{-1}$ | $9(10) \times 10^{-4}$ |
| $48^3$ (MeV) | 139.1(2) | 498.82(26) | 496.5(16) | 2.4(24) |
| $64^3$ (MeV) | 139.2(3) | 507.4(4) | 507.0(16) | 2.1(26) |



# $\Delta I = 3/2$ K →ππ Matrix Elements from ID ensembles

- Measured on 1/a = 1 GeV ensembles
  - * NPR done with $\mu_1 = 1.4363$, $\mu = 3.0$ GeV.
  - * Step scaling to connect $(\mu_1, \mu)$.
- 2 calculations done, one with 2 anti-periodic spatial directions and the other with 3.
- Can extrapolate to physical kinematics



# Correcting for kinematics for ΔI = 3/2 K →ππ Matrix Elements

- For 1/a = 1 GeV ID ensembles, interpolate to physical kinematics

| $n_{tw}$ | $am_K$ | $aE_{\pi\pi}^{I=2}$ | $\mathrm{Re}[A_2][10^{-8}\ \mathrm{GeV}]$ | $\mathrm{Im}[A_2][10^{-13}\ \mathrm{GeV}]$ |
|---|---|---|---|---|
| 3 | 0.50425(49) | 0.5634(40) | $1.7125(68)_{\mathrm{stat.}}(575)_{\mathrm{NPR}}$ | $-5.27(15)_{\mathrm{stat.}}(41)_{\mathrm{NPR}}$ |
| 2 | 0.50425(49) | 0.4768(17) | $1.4206(57)_{\mathrm{stat.}}(476)_{\mathrm{NPR}}$ | $-5.98(16)_{\mathrm{stat.}}(45)_{\mathrm{NPR}}$ |
| 0 | 0.50425(49) | 0.28221(70) | $0.7132(39)_{\mathrm{stat.}}(233)_{\mathrm{NPR}}$ | $-8.32(20)_{\mathrm{stat.}}(58)_{\mathrm{NPR}}$ |
| $*$ | 0.50425(49) | $am_K$ | $1.5079(80)_{\mathrm{stat.}}(505)_{\mathrm{NPR}}$ | $-5.77(13)_{\mathrm{stat.}}(43)_{\mathrm{NPR}}$ |

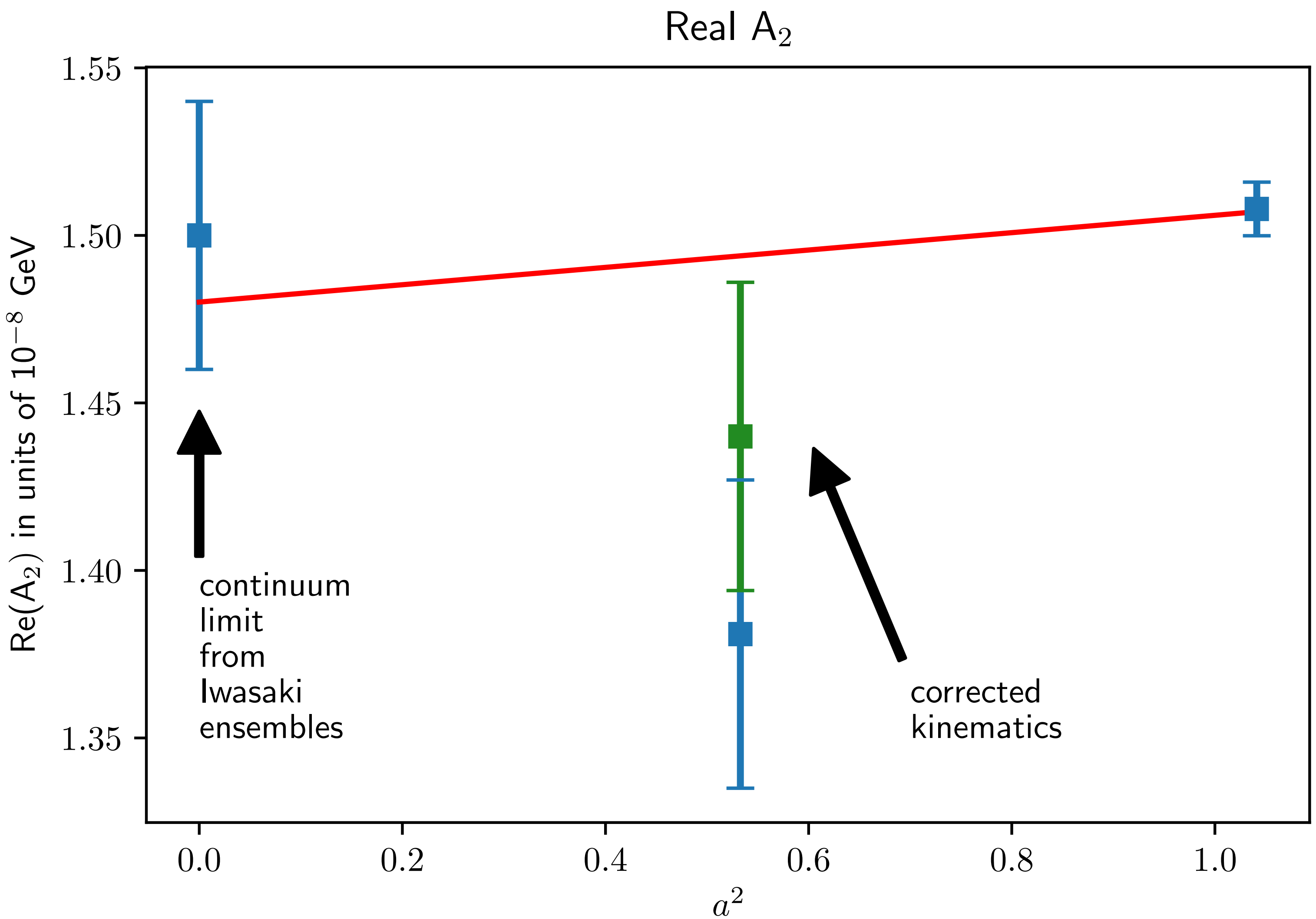
Table 3: NPR is done in $\overline{\mathrm{MS}}$, $(\not{q}, \not{q})$ scheme and $\mu = 3$ GeV. $a^{-1} = 1.0083$ GeV. The NPR error is taken as the difference between $(\not{q}, \not{q})$ and $(\gamma_\mu, \gamma_\mu)$ scheme. $*$ is the result from linear extrapolation in $E_{\pi\pi}^2$ to physical kinematics.

- Previous 1/a = 1.35 GeV ID ensemble result (PRD 86 (2012) 074513).

$$\mathrm{Re}A_2 = 1.381(46)_{\mathrm{stat}}(258)_{\mathrm{syst}}\,10^{-8}\,\mathrm{GeV}, \quad \mathrm{Im}A_2 = -6.54(46)_{\mathrm{stat}}(120)_{\mathrm{syst}}10^{-13}\,\mathrm{GeV}\,.$$

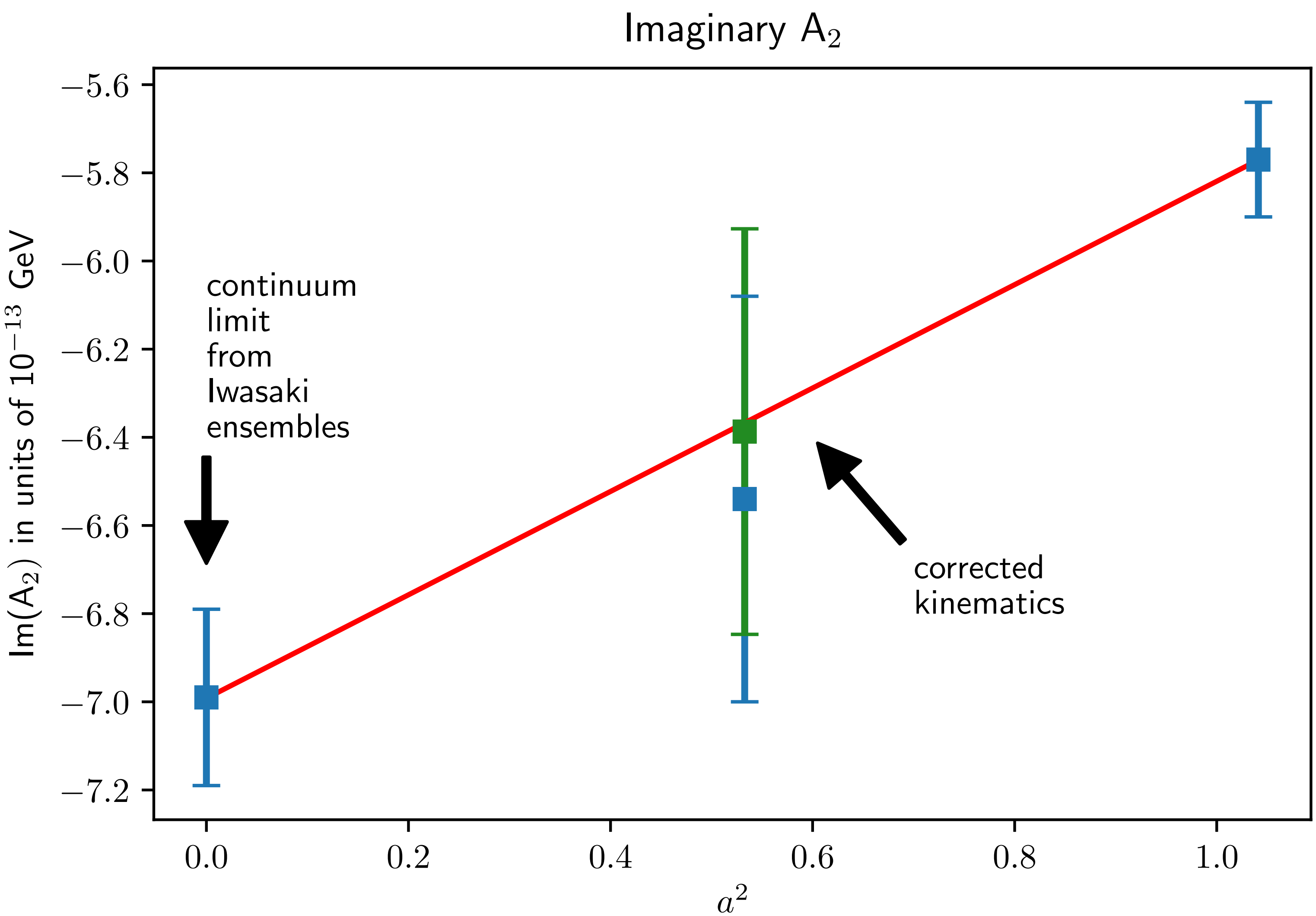
| units | $m_\pi$ | $m_K$ | $E_{\pi,2}$ | $E_{\pi\pi,0}$ | $E_{\pi\pi,2}$ | $m_K - E_{\pi\pi,2}$ |
|---|---|---|---|---|---|---|
| lattice | 0.10421(22) | 0.37066(68) | 0.17386(91) | 0.21002(43) | 0.3560(23) | 0.0146(23) |
| MeV | 142.11(94) | 505.5(3.4) | 237.1(1.8) | 286.4(1.9) | 485.5(4.2) | 20.0(3.1) |

# Scaling for Re($A_2$) on Iwasaki+DSDR Ensembles




- Only statistical errors plotted, not errors in conversion from RI-SMOM to $\overline{\text{MS}}$

# Scaling for Im($A_2$) on Iwasaki+DSDR Ensembles




- Only statistical errors plotted, not errors in conversion from RI-SMOM to $\overline{\mathrm{MS}}$

# Scaling of Local Vector Current Matrix Elements

- HVP and HLBL measured on these ensembles, as part of RBC effort on this quantity
  * Useful for determining finite volume effects present on weaker coupling Iwasaki ensembles
- Two different values for $Z_V$ have been measured
  * From charge of pion: $Z_V^{\pi} = 0.72672$
  * From ratio of local to conserved current: $Z_V^{lc} = 0.6333$
- HVP on 1 GeV ensembles agrees with Iwasaki ensemble results using $Z_V^{lc}$
- HLBL gives better agreement with weak coupling with $Z_V^{\pi}$
- Appears to be large scaling error.

# Scaling of Operators versus Masses

- Christoph Lehner has used the phase shift from the I=1 spectrum on the 32ID physical ensemble and the Gounaris-Sakurai model to predict the spectrum and matrix elements on the 24 ID model.
- Good agreement for the energies of the lowest 3 states

| | Measured on 24ID | Predicted from 32ID |
|---|---|---|
| $E_0$ | 0.5746(7) | 0.577(2) |
| $E_1$ | 0.716(3) | 0.718(8) |
| $E_2$ | 0.841(9) | 0.846(7) |

- Matrix elements of the local vector current show 10-20% differences, attributed to scaling errors

Amplitudes $c_n = \langle 0 | V_i^{\mathrm{loc}} | n \rangle$

| | Measured on 24ID | Pred.fr. 32ID w/ $Z_V^{\mathrm{lc}}$ | Pred.fr. 32ID w/$Z_V^{\pi}$ |
|---|---|---|---|
| $c_0$ | 0.0524(7) | 0.052(3) | 0.045(3) |
| $c_1$ | 0.123(2) | 0.118(9) | 0.103(9) |
| $c_2$ | 0.120(6) | 0.09(1) | 0.08(1) |

Lower states prefer $Z_V^{\mathrm{lc}}$ but in general $O(10\% - 20\%)$ discretization errors on coefficients possible.

# Conclusions

- $B_K$ accurately measured on coarse ID ensembles and shows only $O(a^2)$ errors for lattice spacings ≥ 1 $GeV^{-1}$
- $Re(A_2)$ and $Im(A_2)$ well fit with $a^2$ term to lattice spacings ≥ 1 $GeV^{-1}$
  - * Measurement of corrections from unphysical kinematics done on coarse lattices
  - * Important correction for scaling plot.
- Coupling of local vector current to I=1 states shows 10-20% discretization errors in vacuum ot I=1 matrix elements.
- Spectrum has (so far) not shown any large scaling errors.
- In terms of Symanzik improvement, our results to date are consistent with the ID ensembles having
  - * Very small $O(a^2)$ errors in the action
  - * Possible canonically sized (10-20%) $O(a^2)$ errors for matrix elements.
- These are empirical results. There is no theorem of systematic $O(a^2)$ improvement in the action.