

Topology of two-color QCD at low temperature and high density

Etsuko Itou (Kochi U./ RCNP, Osaka U.)
collaboration with K.Iida and T.-G. Lee

Motivation

Understand the physics in QCD phase diagram
but there is the sign problem….

Our Work:

Avoid the sign problem (consider 2color 2flavor QCD)

Focus on the low-T and high-density regime:

Problem: Numerical instability $\mu/m_{PS} \geq 1/2$ in low-T
→ Introduce the **diquark source** in the action

Qualitatively understand the physics in finite density

Phase structure

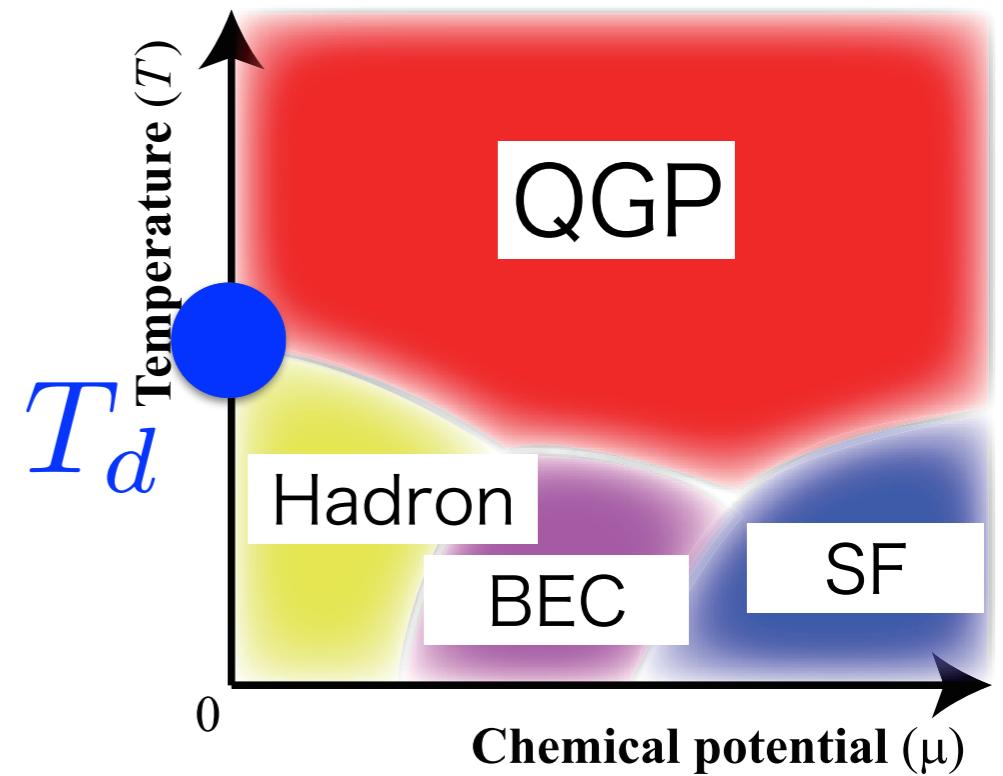
Topological objects

Superfluid density from energy-momentum tensor

Mass gap (BCS gap) from diquark condensate

Plan of talk

1. Two-color finite-density QCD
Action with diquark source



2. Simulation results
Determine the phase diagram in low-T ($T \sim T_d/2$)
Topological susceptibility

3. Summary and Outlook

Action with diquark source term

Fermion action in continuum limit

$$S_F^{cont.} = \frac{\int d^4x \bar{\psi}(x) (\gamma_\mu D_\mu + m) \psi(x) + \mu \hat{N}}{\text{QCD}} - \frac{j}{2} (\bar{\psi}_1 K \bar{\psi}_2^T - \bar{\psi}_2^T K \psi_1)$$

Number op. diquark source

Related works on Nc=2 with even # flavor

Kogut et al. NPB642 (2002) 18, Alles et al. NPB752 (2006) 124,

Hands et al. NPB752 (2006) 124, PRD81 (2010) 091502, EPJ. A47 (2011) 60, PRD87 (2013) 034507, Kotov et al. PRD94 (2016) 114510, JHEP 1803 (2018) 161

The QCD phase diagram appears in the $j \rightarrow 0$ limit

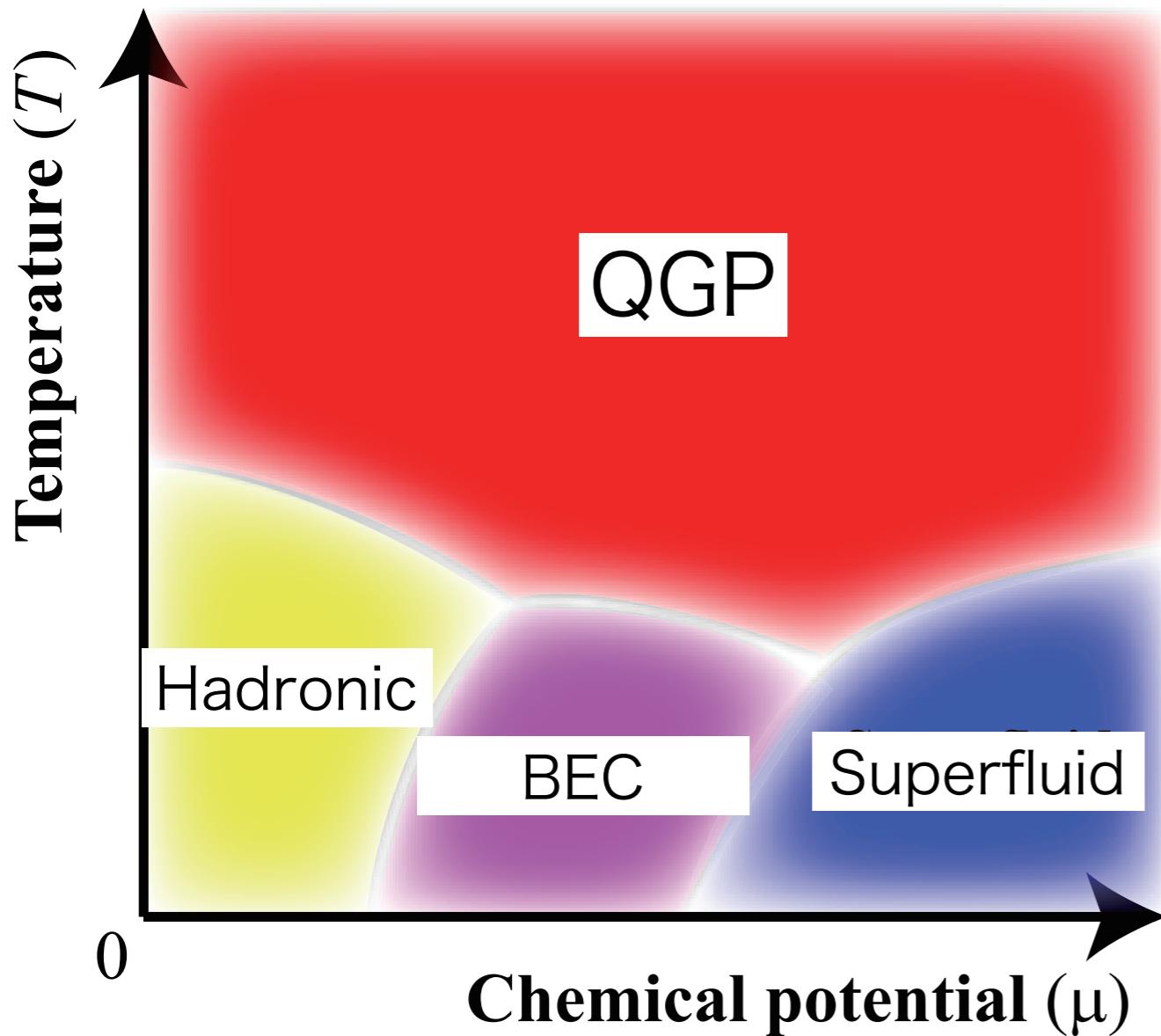
cf.) diquark $\rightarrow \pi^-$ in 3-color QCD with isospin chemical

Fermion action on the lattice

$$\det[\mathcal{M}^\dagger \mathcal{M}]^{1/2} = \det[\Delta^\dagger(\mu) \Delta(\mu) + |\bar{J}|^2]^{1/2} \det[\Delta^\dagger(-\mu) \Delta(-\mu) + |J|^2]^{1/2}$$

j-source lifts the eigenvalue of Dirac op. up

Expected phase diagram in Two-color QCD



Order parameters

* Polyakov loop

$\langle |L| \rangle \sim 0$ confined

$\langle |L| \rangle \neq 0$ deconfined

* (Isoscalar) diquark cond.

$$\langle qq \rangle = \frac{\kappa}{2} \langle \bar{\psi}_1 K \bar{\psi}_2^T - \psi_1 K \psi_2^T \rangle$$

$$K \equiv C \gamma_5 \tau_2$$

$\langle qq \rangle = 0$ no superfluidicity

$\langle qq \rangle \neq 0$ superfluidicity

Hadron	confined	no diquark condensate
QGP	deconfined	no diquark condensate
SuperFluid	deconfined	diquark condensate
Bose-Einstein	confined	diquark condensate

Lattice setup

Lattice action:

Iwasaki gauge action + Nf=2 Wilson fermion

Include quark chemical potential + diquark source term

RHMC algorithm

Lattice parameter: beta=0.8, (1.0)

mass para. is tuned to be $m_{PS}/m_V = 0.8$
 $\text{@} \mu = 0$

$a\mu \leq 1.0$ to avoid a lattice artifact

Lattice size: 16^4 : corresponds to $T \sim T_d/2$

(T_d : critical temperature $\text{@} \mu = 0$)

Parameter regime of chemical potential

$\mu/T \leq 16, \mu/m_{PS} \leq 2.5$

preliminary

Results

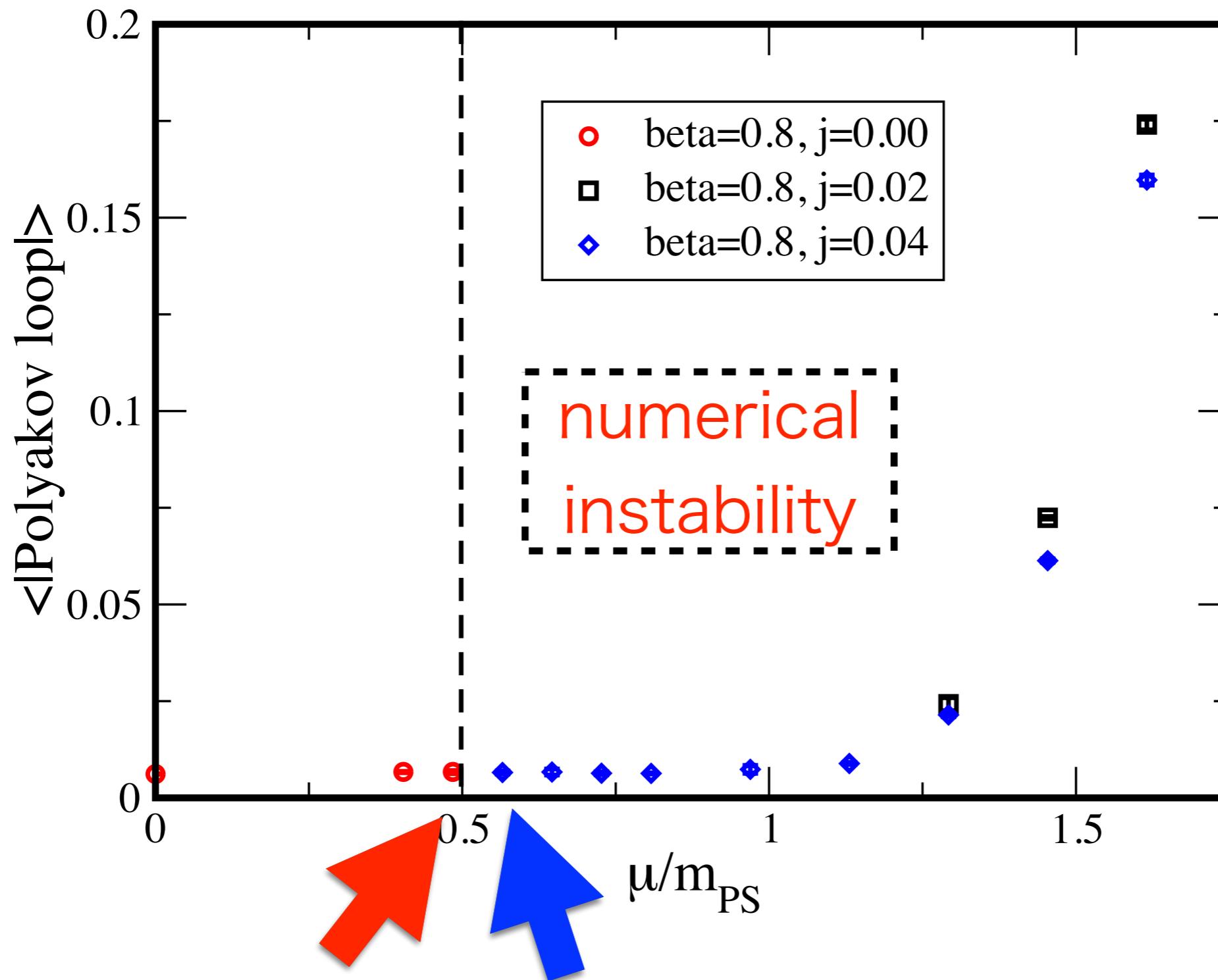
@ $\mu \neq 0$, $T \sim T_d/2$

Polyakov loop

(approximate order param. of confinement)

$$L = \frac{1}{V} \sum_x \frac{1}{N_c} \text{tr} \prod_i^{N_\tau} U_\tau(x, i)$$

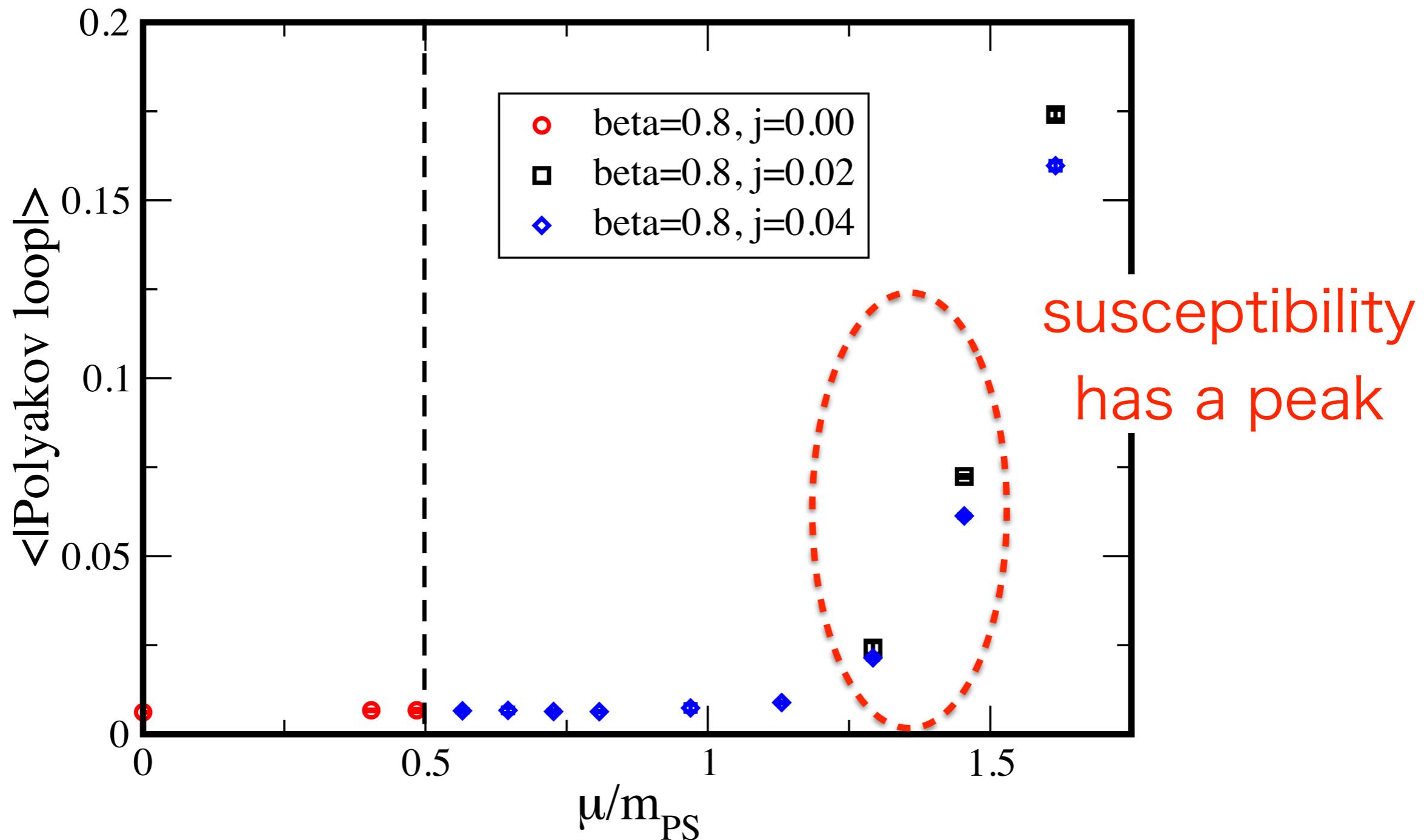
Polyakov loop @ $T \sim T_d/2$



HMC without j is doable
(minimum MC step $\sim 1/800$)

HMC without j cannot run even with
a tiny MC step($\sim 1/1000$)

Polyakov loop @ $T \sim T_d/2$



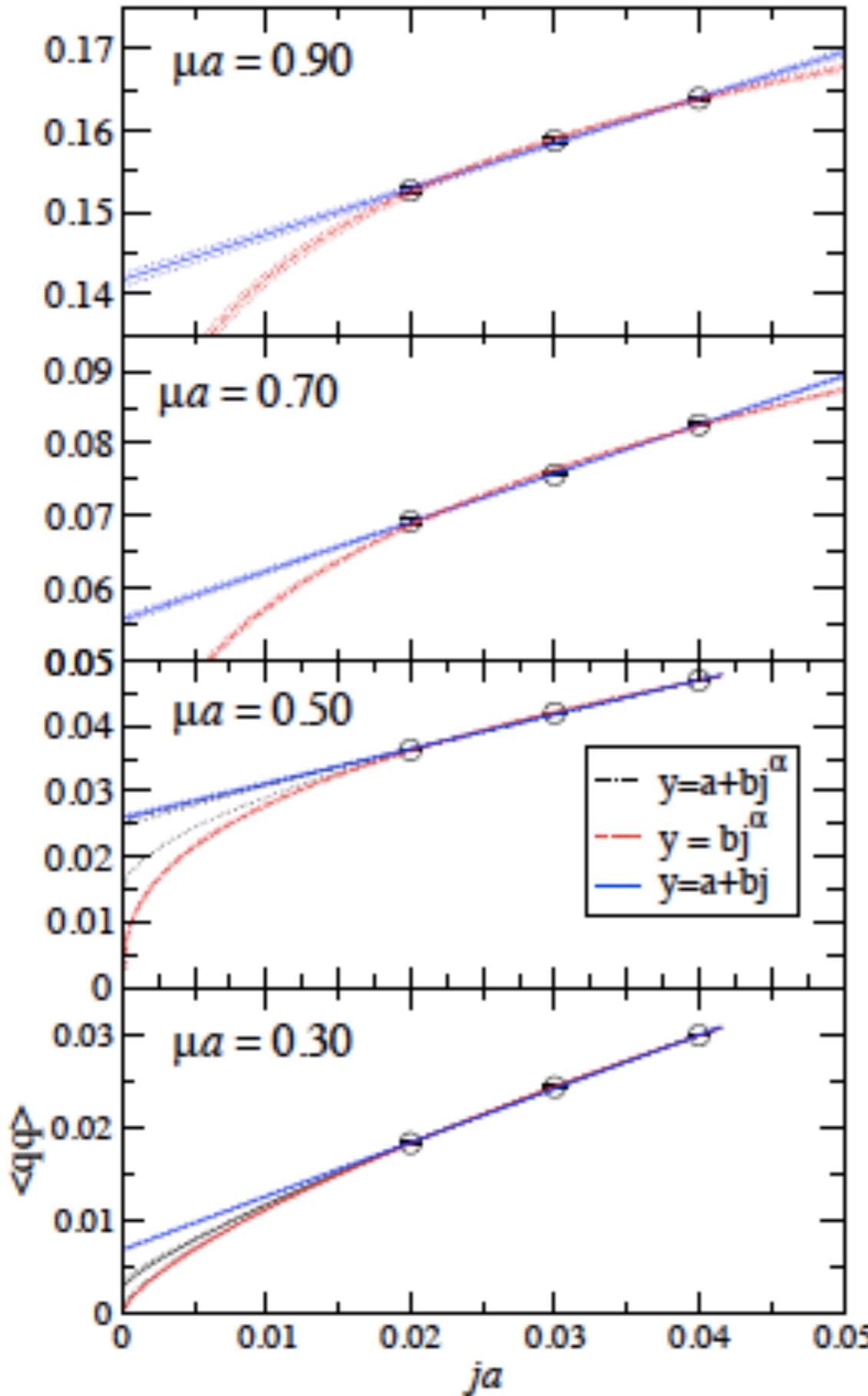
diquark condensate
using the noise method and reweighting

$$\langle qq \rangle = \frac{\kappa}{2} \langle \bar{\psi}_1 K \bar{\psi}_2^T - \psi_1 K \psi_2^T \rangle$$

$$K \equiv C \gamma_5 \tau_2$$

(order param. of superfluidicity)

diquark cond. with $j=0.02, 0.03, 0.04$



S.Cotter et al. Phys.Rev. D87 (2013) 034507

$j \rightarrow 0$ extrapolation
is a hard task

To find diquark cond. in $j=0$ limit

— reweighting —

$$\mu/m_{PS} \lesssim 0.5$$

config. generation @ $j=0$

measure by introducing

small j -source as a probe

J (measurement) > J0 (sampling)

$$\mu/m_{PS} > 0.5$$

config. generation @ $j=0.02, 0.04$

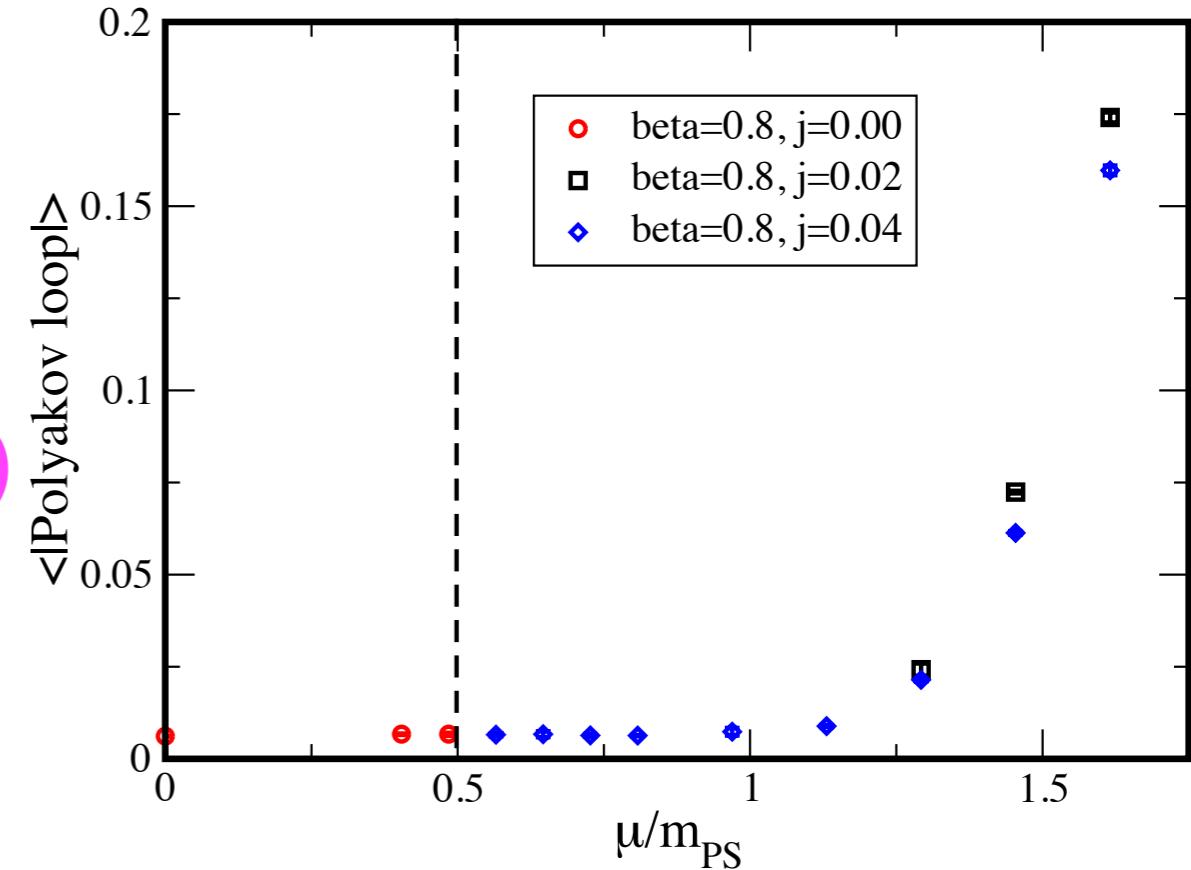
measure by introducing a probe j -source

@ $j=0.001, 0.005, 0.01, 0.02, 0.04$

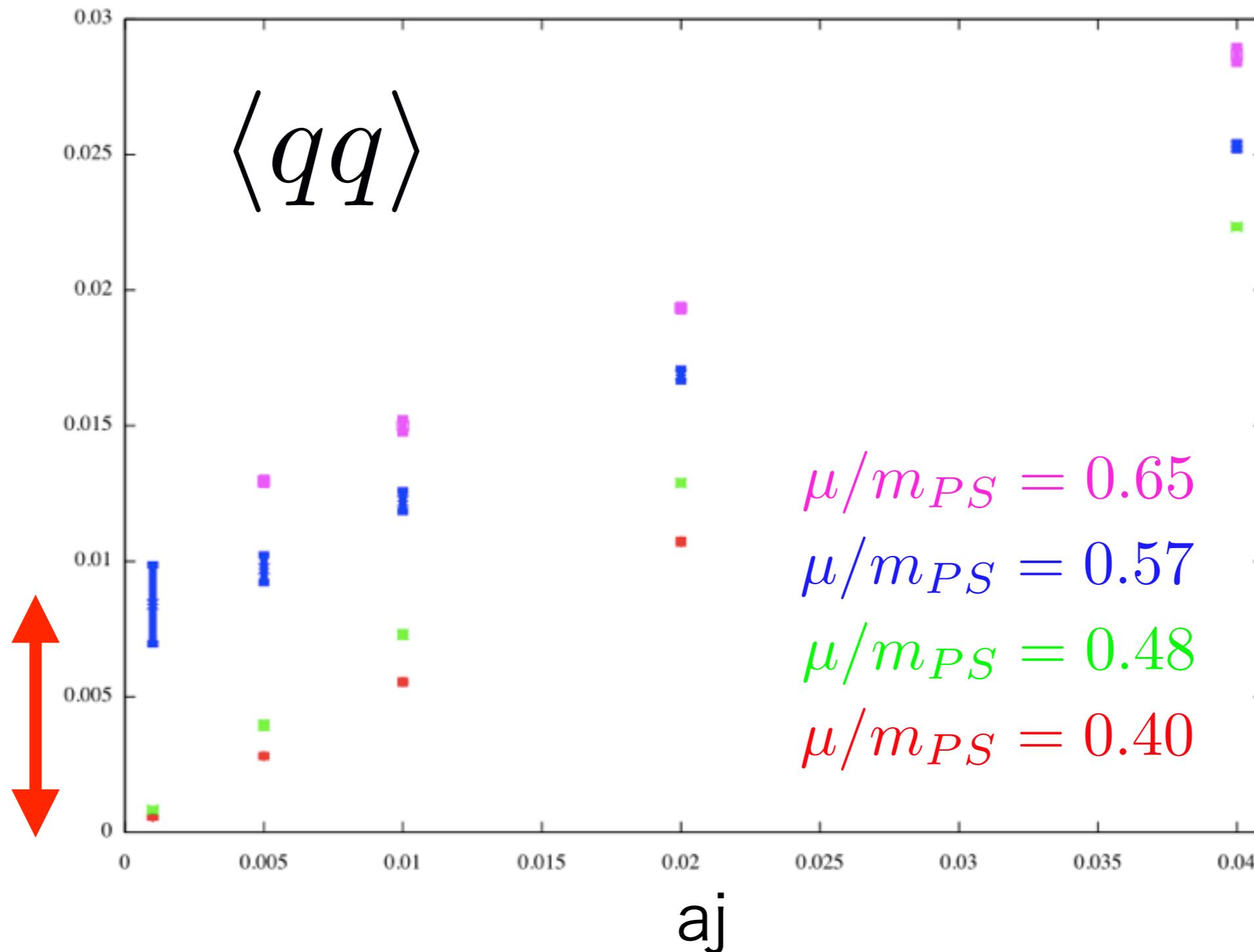
J (measurement) < J0 (sampling)

Both reweighting works very well. $(J^2 - J_0^2) \sim O(10^{-6})$

$$R = \det[1 + (J^2 - J_0^2)(\Delta^\dagger(\mu)\Delta(\mu) + J_0^2)^{-1}]^{1/2} \times (\mu \rightarrow -\mu)$$

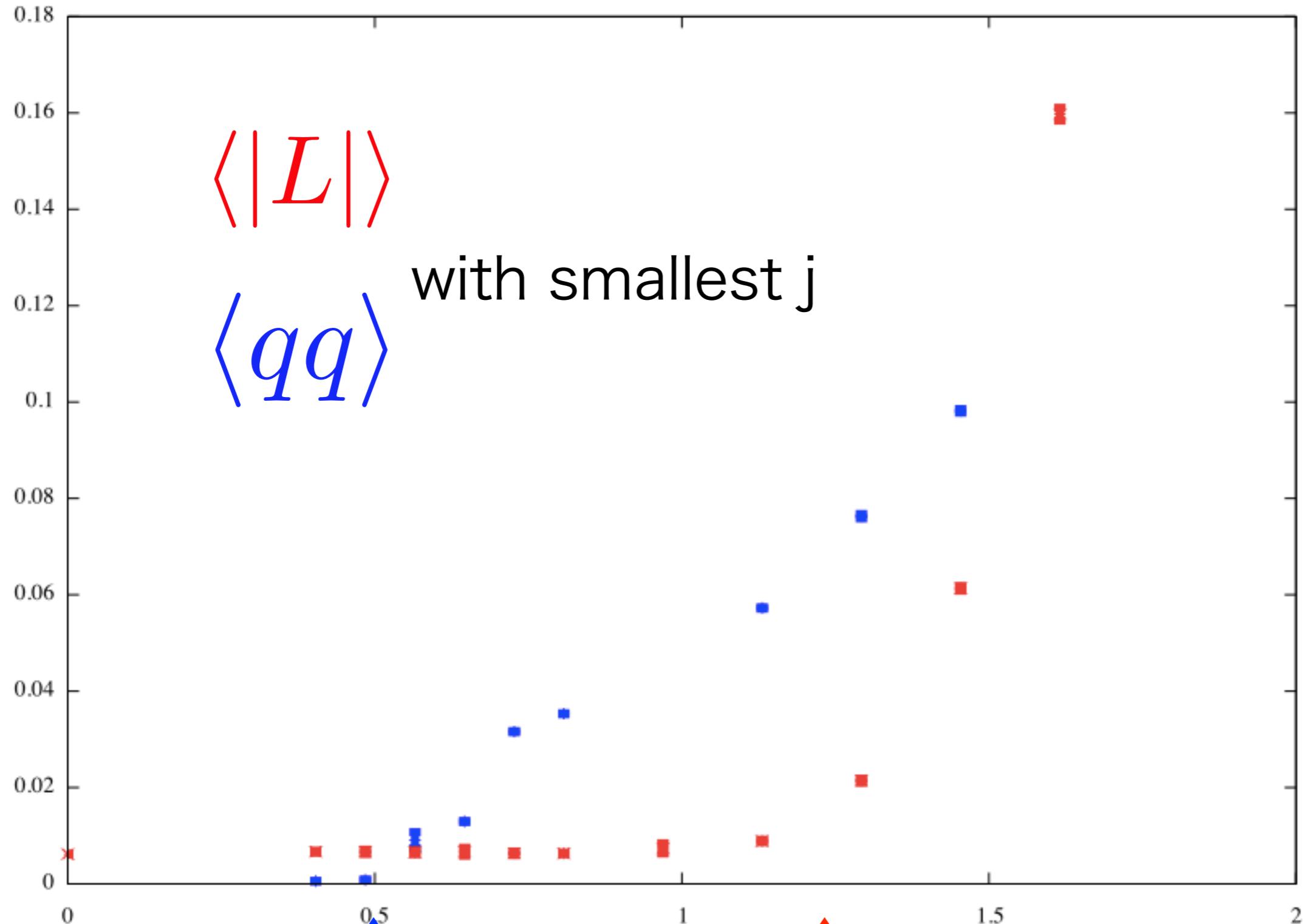


j dependence of diquark condensate



In $j=0$ limit, the gap exists around $\mu/m_{PS} \simeq 0.5$

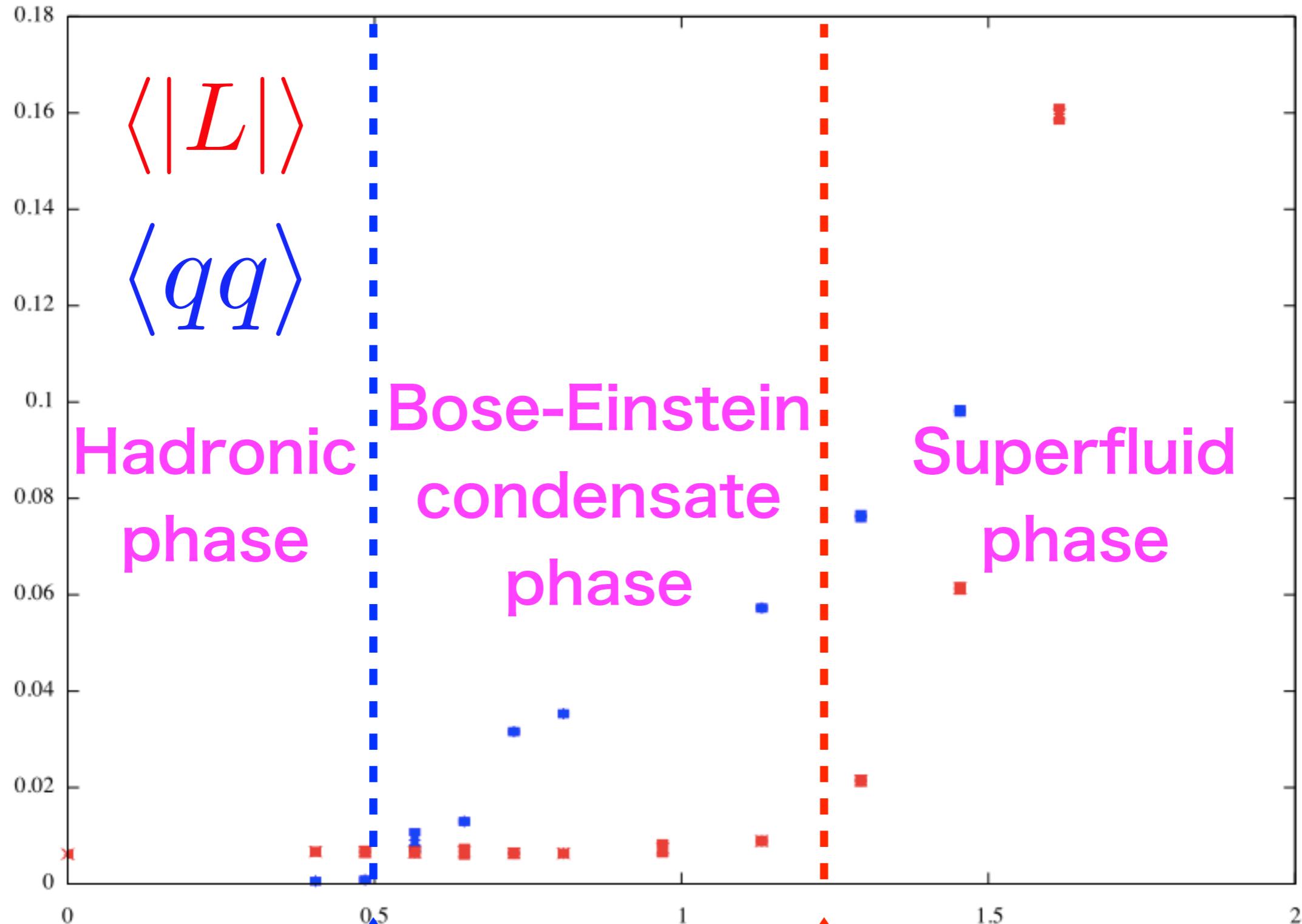
Phase diagram @ $T \sim T_d/2$



$\mu_Q/m_{PS} \sim 0.5$

$\mu_D/m_{PS} \sim 1.0$

Phase diagram @ $T \sim T_d/2$

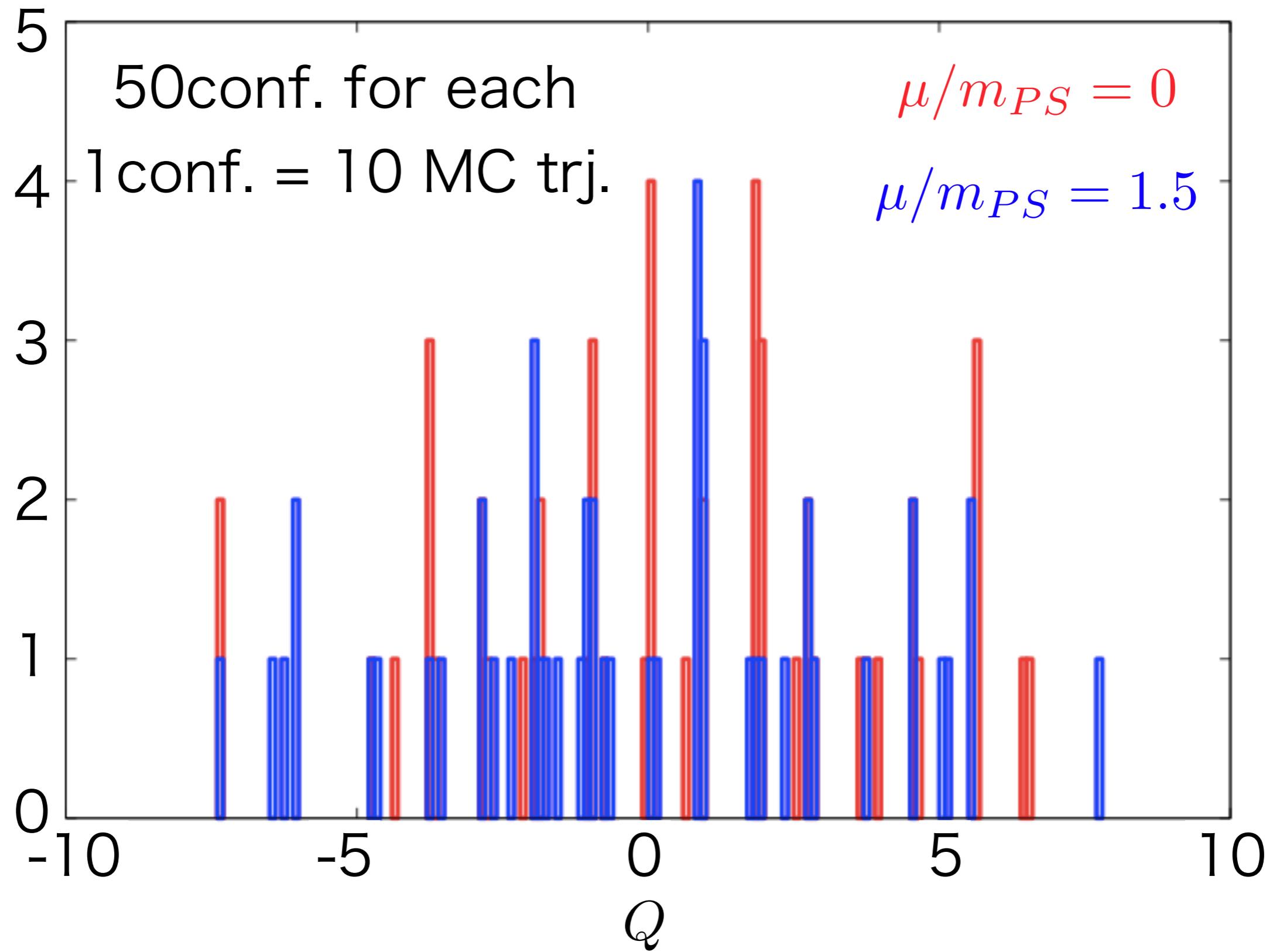


$\mu_Q/m_{PS} \sim 0.5$ $\mu_D/m_{PS} \sim 1.0$

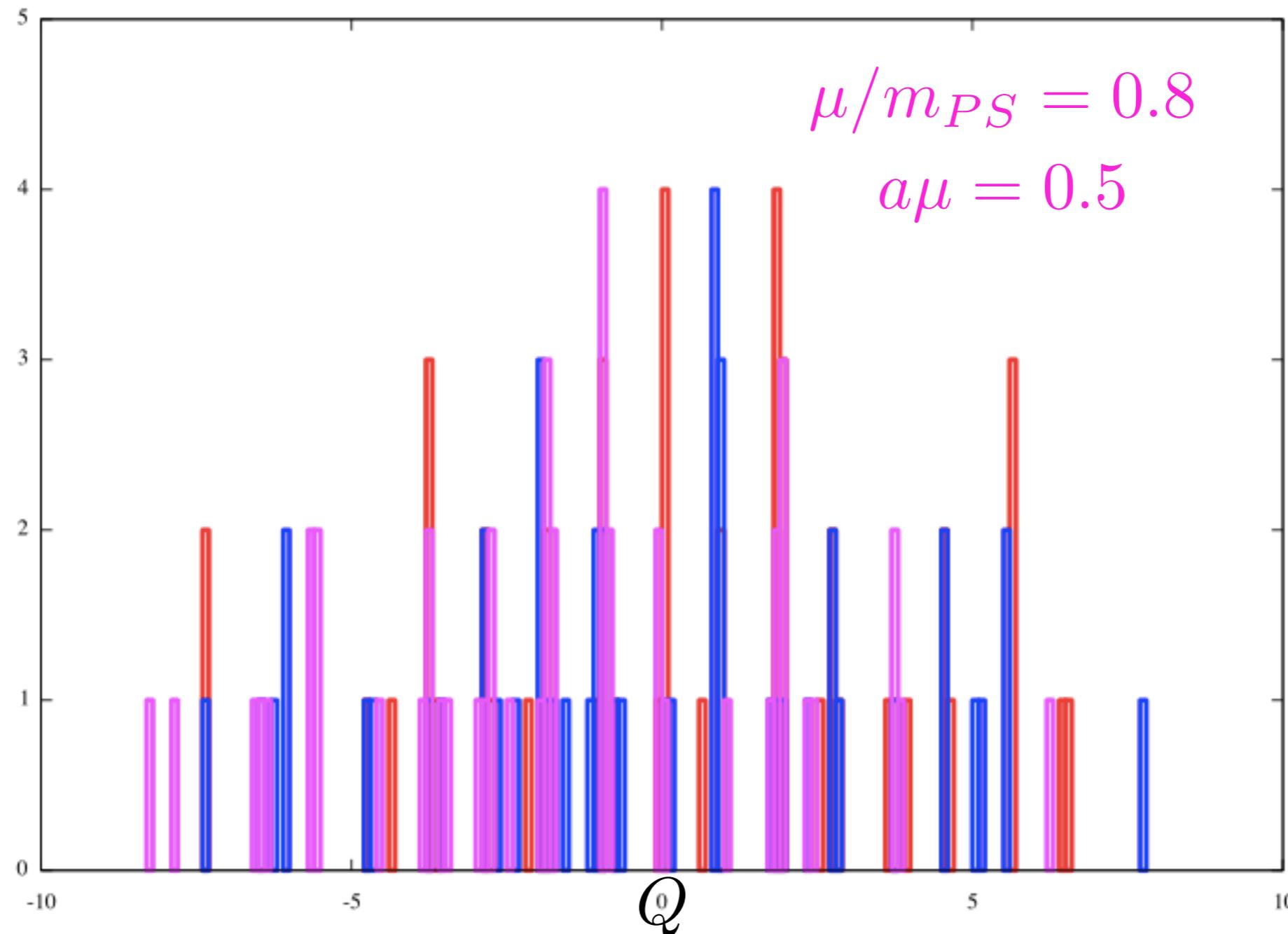
topological charge using gradient flow

$$Q = \frac{1}{32\pi^2} \int d^4x \epsilon_{\mu\nu\rho\sigma} \text{Tr} G_{\mu\nu}^a G_{\rho\sigma}^a$$

histogram of Q



topological charge distribution



$\mu=0.0$

$\mu=0.5$

$\mu=0.9$

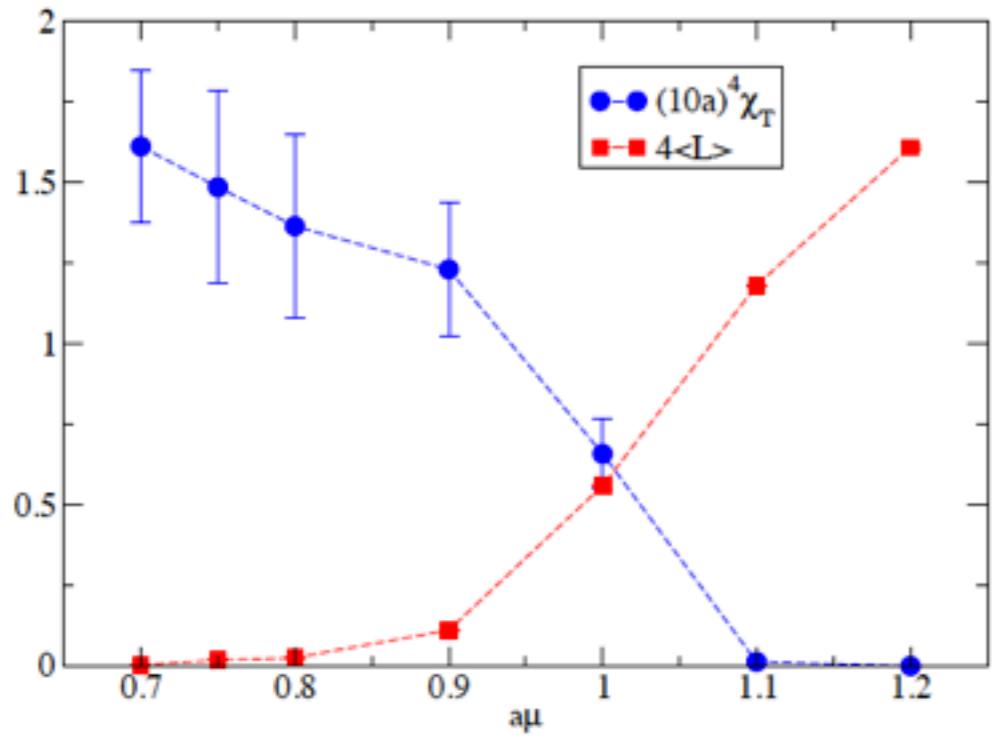
χ_Q

$0.00093(17)$

$0.00104(16)$

$0.00100(18)$

Hands. (1104.0522)
Nf=4, T=0 (12 ^ 3x24)



Alles-D'elia-Lombardo(0602022)
Nf=8 staggered, finite T (14 ^ 3x6)

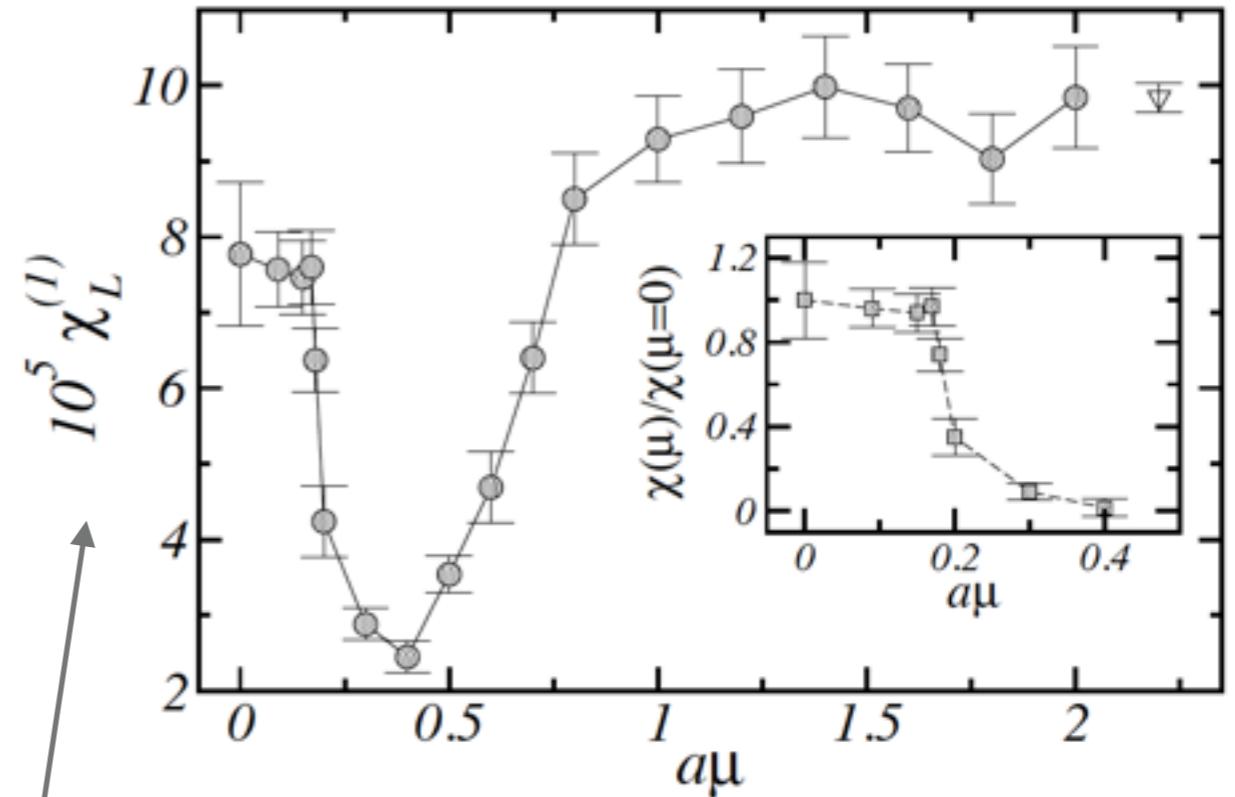
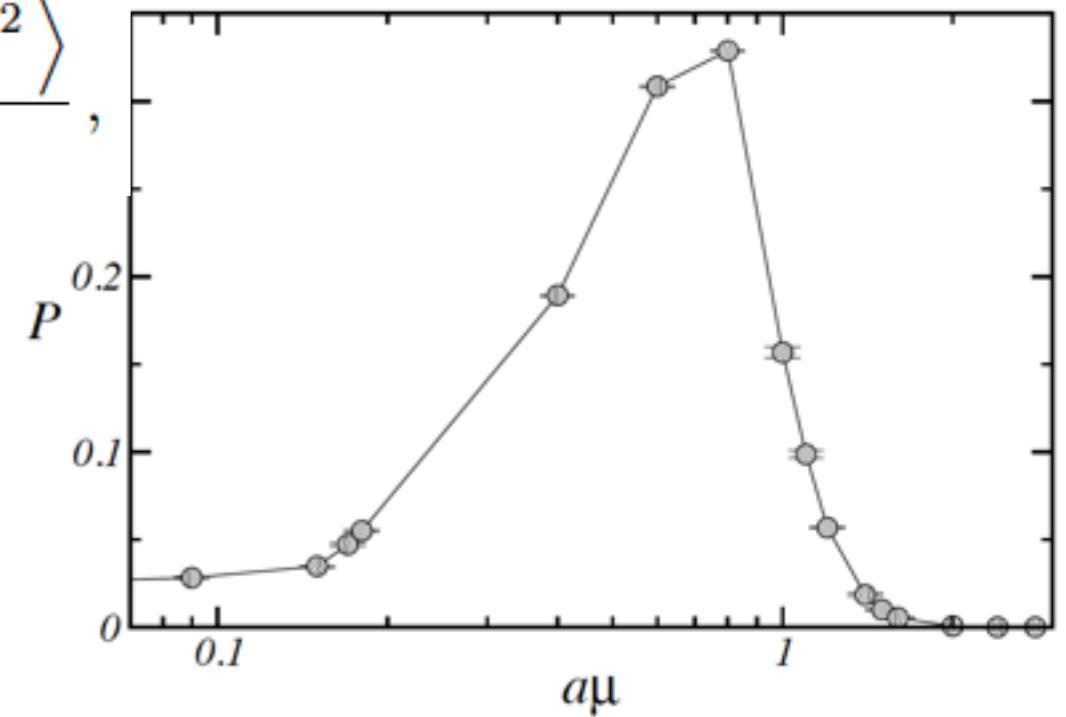


Figure 2: The suppression of χ_T coinciding with the rise in $\langle L \rangle$ for $N_f = 4$. Note $\langle L \rangle$ has been rescaled for clarity.

$$\chi_L \equiv \frac{\langle (Q_L)^2 \rangle}{V},$$

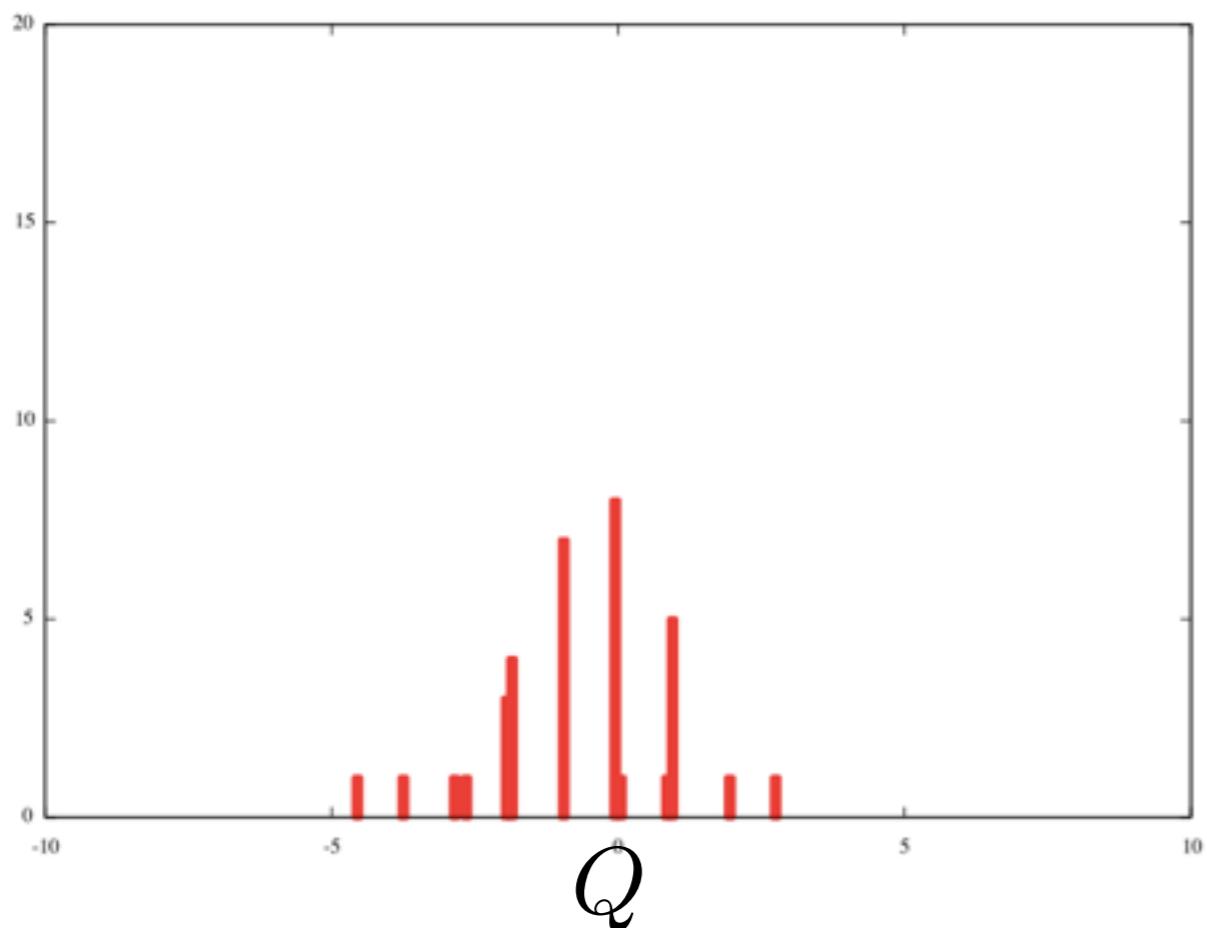


Polyakov loop increasing
 ||
 Topological suscep. decreasing

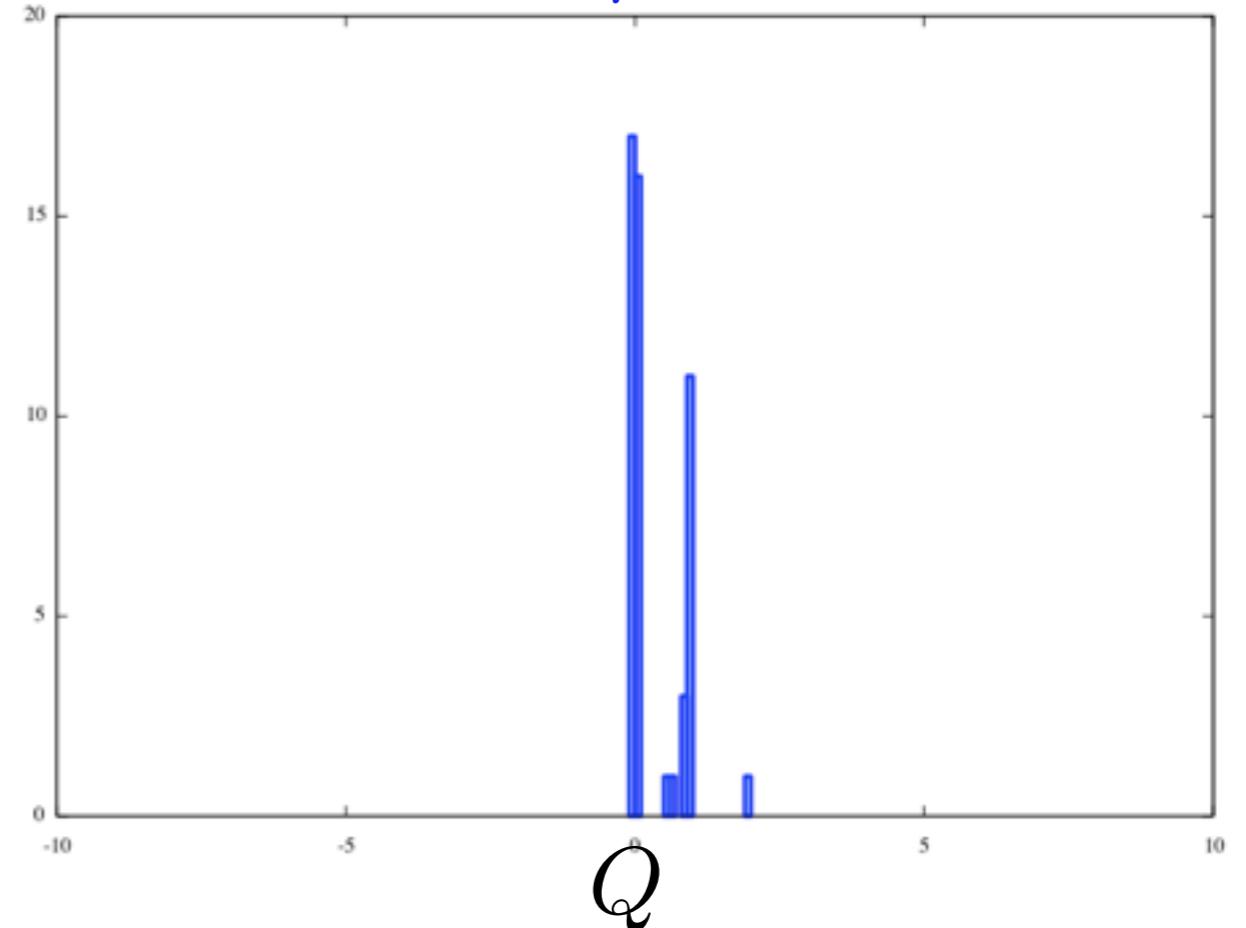
FIG. 2. Polyakov loop P as a function of $a\mu$. The logarithmic scale allows to disentangle the data obtained in the vicinity of the transition point. Points are joined by a line to guide the eye.

beta=1.0 , lattice size 16^4
 (half lattice spacing of beta=0.8) $T \sim T_d$

$$\mu/m_{PS} = 0$$



$$\begin{aligned}\mu/m_{PS} &= 2.5 \\ a\mu &= 0.9\end{aligned}$$



Clearly, susceptibility is small in high mu.

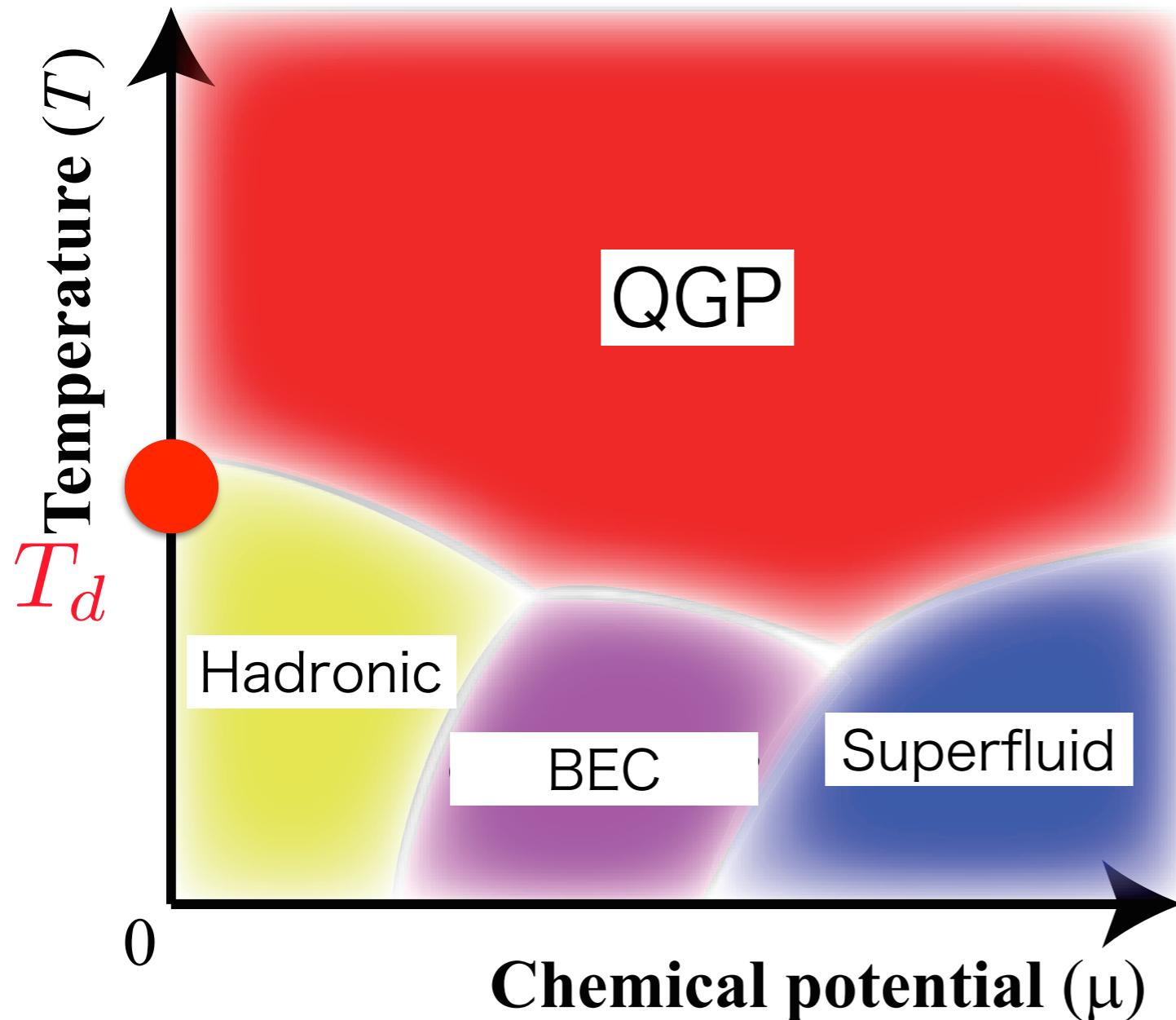
Temperature dependence?

Finite Volume effect? (beta=0.8, Ns=8, Nt=16 shows small chi_Q)

Finite vol. effect might be strong in high mu regime.

summary of phase diagram

Phase diagram in Two-color QCD



Observables to determine
the phase diagram

- * Polyakov loop
- * diquark cond.

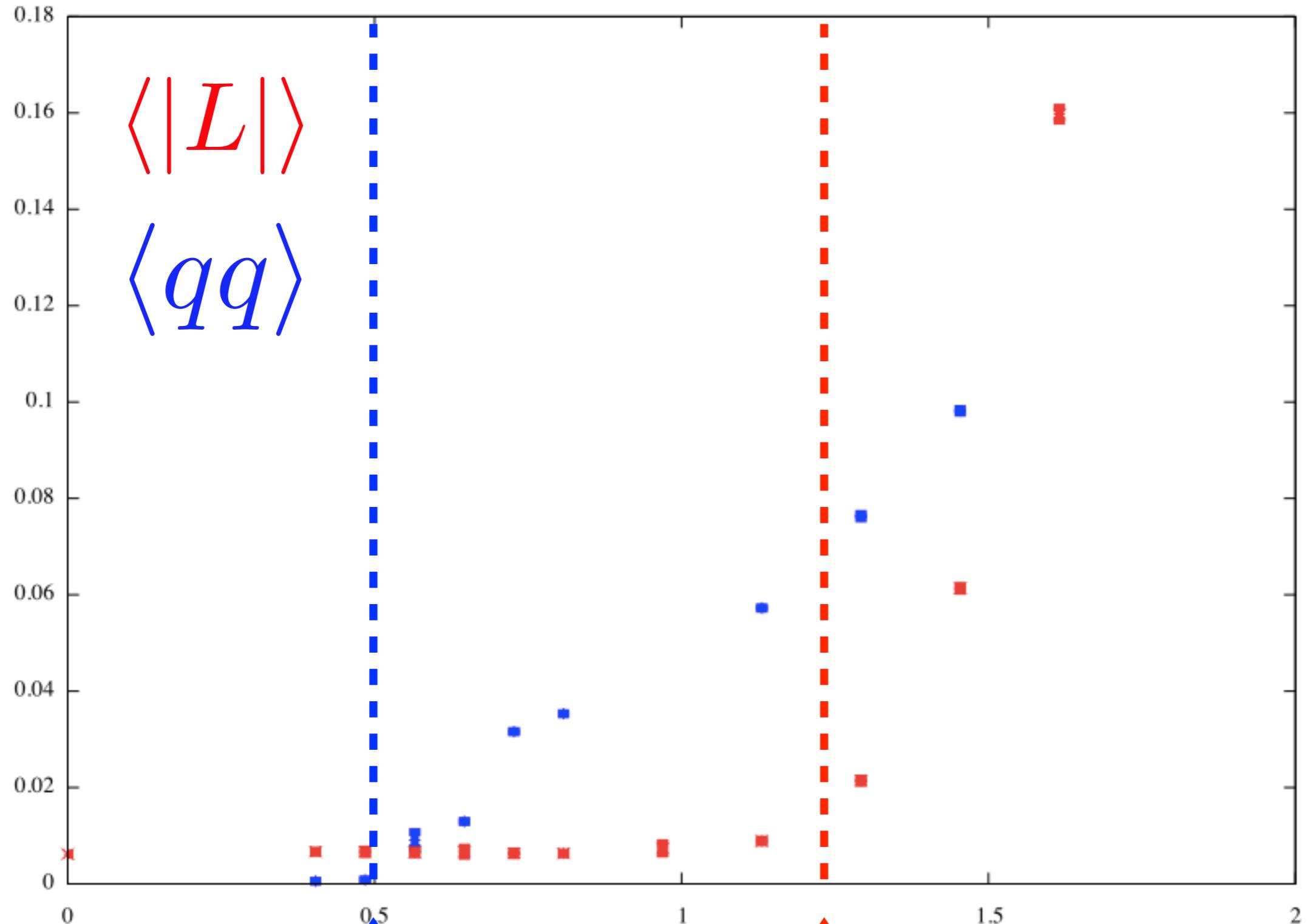
critical points

$$T_d @ \mu = 0$$

Force on

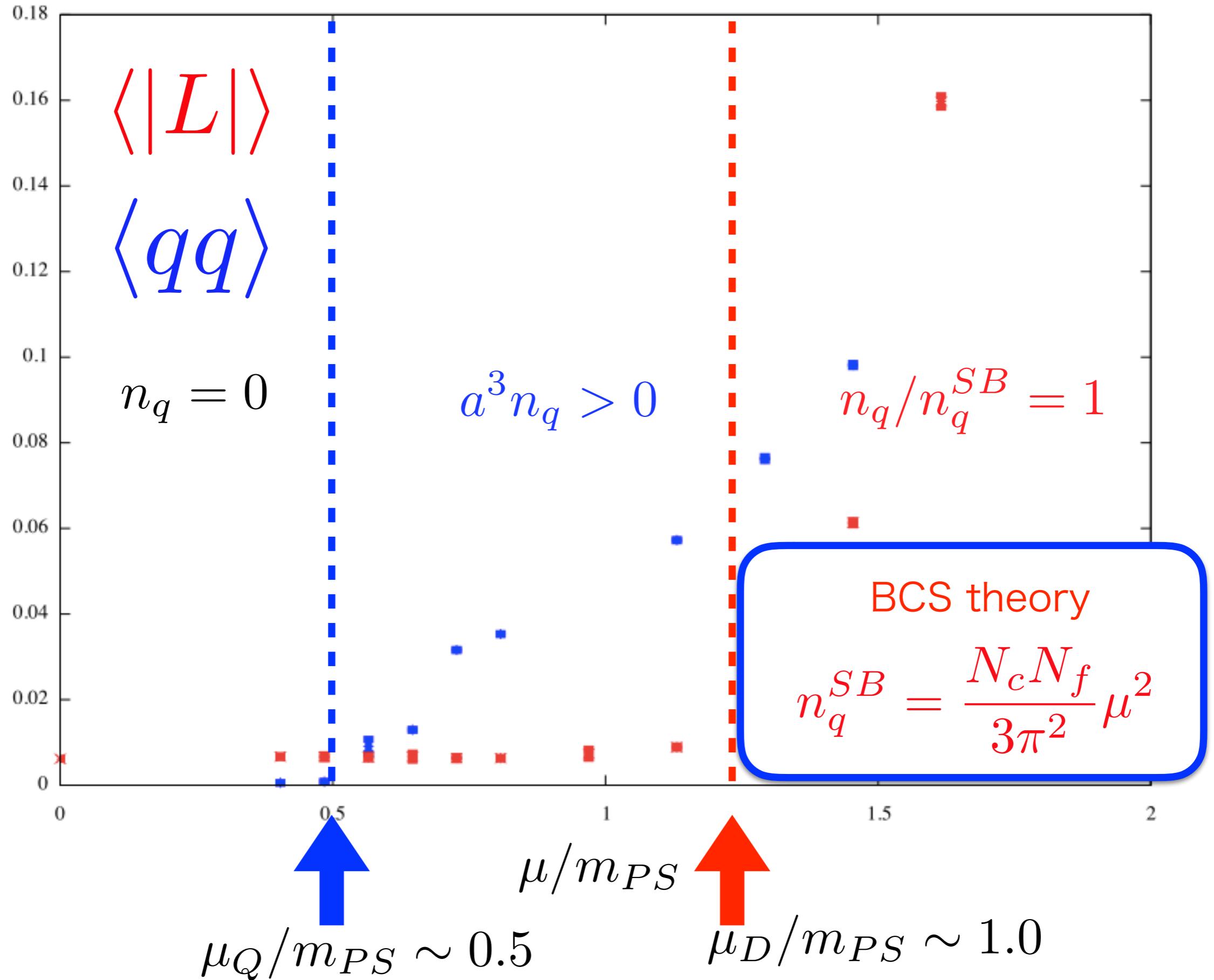
$$T \sim T_d/2 @ \mu \neq 0$$

Phase diagram @ $T \sim T_d/2$

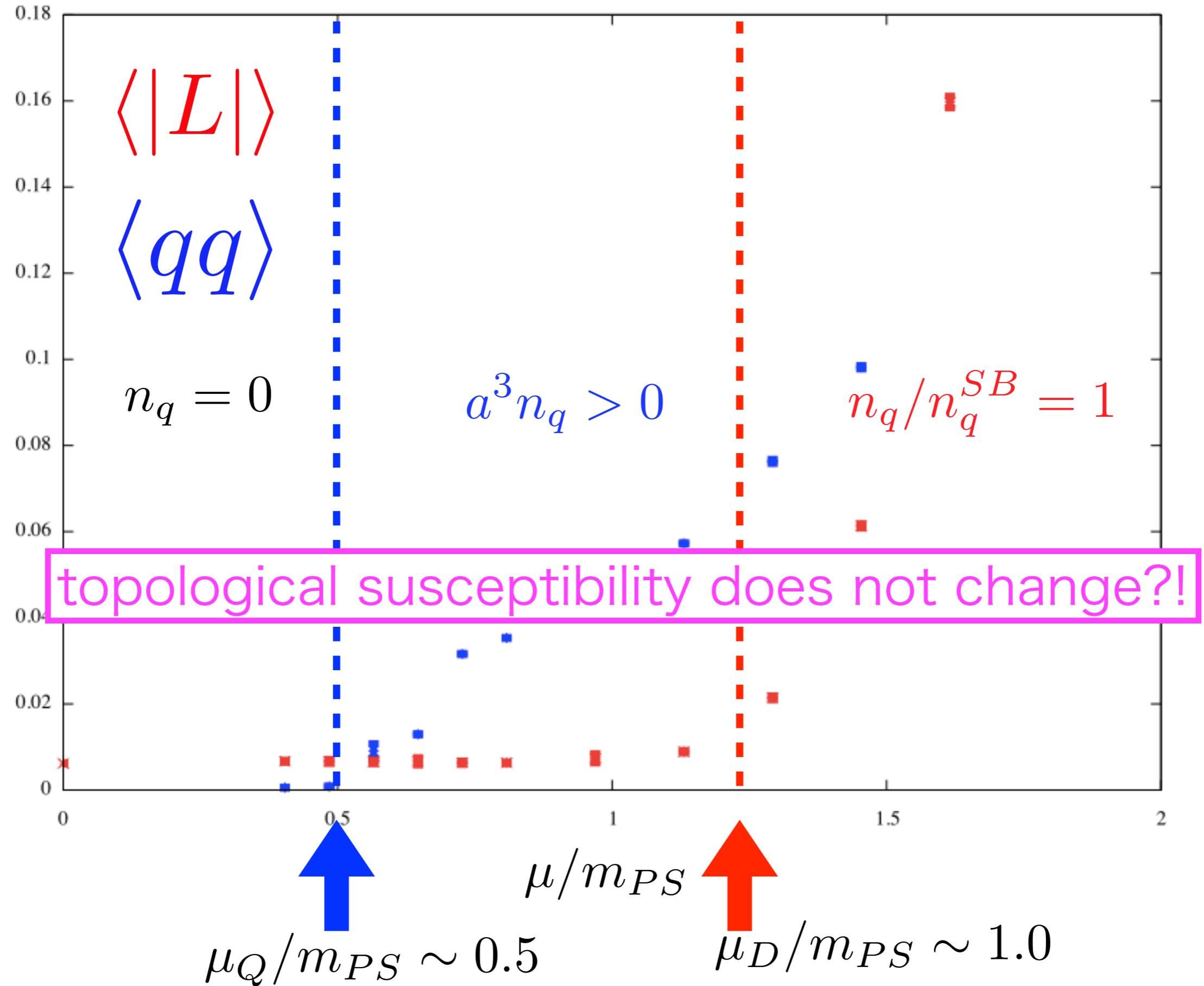


$\mu_Q/m_{PS} \sim 0.5$ $\mu_D/m_{PS} \sim 1.0$

Phase diagram @ $T \sim T_d/2$



Phase diagram @ $T \sim T_d/2$



Future directions

- * Property of Superfluid phase
role of instanton (Rapp et al.) and axial Ward identity
(Kanazawa, Wittig, Yamamoto Eur.Phys.J. **A49** (2013) 88)
superfluid density from energy-momentum tensor
- * Phase diagram in finite-T and finite-mu
order of each phase transition and its mu-dependence
- * Toward 3-color QCD
light diquark condensate in high-density nucleus?

