Phase structure of $\mathcal{N}$=1 Super Yang-Mills theory from the gradient flow

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Motivation

- Study the confinement-deconfinement phase transition and chiral symmetry restoration in minimal supersymmetric Yang-Mills theory on the lattice
- Both phenomena seem to be related in QCD with fundamental matter ($T_{dec} \sim T_c$)
- In QCD with $N_f = 2$ adjoint quarks (AdjQCD) crit. temperatures do not seem to match: $T_c \sim 4T_{dec}$ for SU(2) and $T_c \sim 7.8T_{dec}$ for SU(3) (with staggered fermions)
- $\mathcal{N} = 1$ SYM is AdjQCD with $N_f = 1/2$:
  - Do critical temperatures coincide in SYM?
  - Are the underlying non-pert phenomena related?
- First investigations support this [Bergner et al. arXiv:1405.3180, '14]

Take advantage of renormalisaton properties of the gradient flow to get

a better signal of the gluino condensate

Confinement

- Confining properties similar to YM:
  - Unbroken centre symmetry at low temperatures
  - Confinement of static external fundamental colour charges
  - Zero asymptotic string tension for adjoint charges
  - Gluons and gluinos found in bound colourless states (supermultiplets)
- Deconfinement phase transition: Spontaneous breaking of centre symmetry at some critical temperature
- Bound colourless states melt down near T_c
- Non-perturbative SSB by non-trivial gauge field topology

Confinement

- Possible order parameter: fundametal Polyakov loop

$$P_L(\vec{x}) = \mathrm{Tr}\mathcal{P}\exp\left(\int_0^{\tau} dx_0 A_0(\vec{x}, x_0)\right) \rightarrow P_L = \frac{1}{V}\sum_x \mathrm{Tr}\left\{\prod_{x_0=0}^{N_t} U_0(x)\right\}$$

- Polyakov loop related to free energy of isolated *fundamental* colour sources

$$\langle P_L\rangle = 0 \Rightarrow F_q \rightarrow \infty$$

- It transforms non trivialy w.r.t the centre of the gauge group

$$P_L \rightarrow P'_L = \exp\left(2\pi i\frac{n}{N_c}\right) P_L$$

- A non-zero vev signals the breaking of the centre symmetry and thus deconfinement phase transition

Confinement

- Effective 3-d theory of Polyakov loop:
  * SU(2): Real scalar field with Z_2-invariant action. Universality class of 3-D Ising model $\rightarrow$ second order phase transition
  * SU(3): Complex scalar field with Z_3 (and charge conj.) invariant action with three degenerated vacua at phase transition $\rightarrow$ first order phase transition. Similar to 3-d 3-state Potts model
- Adjoint fields do not break centre symmetry $\rightarrow$ exact phase transition (at $V \rightarrow \infty$ limit) for every gluino mass.

Chiral symmetry

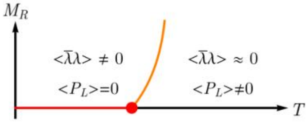
- Clasically, *massless* $\mathcal{N} = 1$ SYM has a U(1)$_R$-symmetry $\lambda \rightarrow \lambda' = \exp(-i\omega\gamma_5)\lambda$
- On the quantum level, U(1)$_R$-symmetry is broken as $\partial_\mu J_5^\mu \sim N_c g^2 \epsilon_{\mu\nu\rho\sigma} F^{\mu\nu} F^{\rho\sigma}$
- Remanent symmetry is Z_{2N_c}
- At zero temperature the gluino condensate $< \bar{\lambda}\lambda > \neq 0 \rightarrow$ vacuum is not invariant under Z_{2N_c} only under $\lambda \rightarrow -\lambda$
- Thus, Z_{2N_c} broken down to Z_2:

$$\mathrm{U}(1)_R \rightarrow Z_{2N_c} \rightarrow Z_2$$

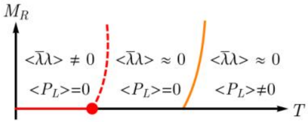
Chiral symmetry

- SU(2)
  * First order phase transition line at $m_g^R = 0, T = 0$ (jump in vev of $< \bar{\lambda}\lambda >$) Existence of 2 degenerated ground states of the condensate
  * Second order phase transition point at T_c with $< \bar{\lambda}\lambda >= 0$ and restoration of Z_4 chiral symmetry
- SU(3)
  * Richer phase structure
  * First order phase transition line. Similar to Potts model
  * Three degenerated vacua at first order phase transition

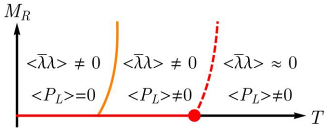
- **Do both critical temperatures coincide or is there a deconfined phase with broken chiral symmetry?**



(a) Coincident phase transitions



(b) Mixed phases allowed



(c) Mixed phases allowed

The condensate on the lattice

- The bare condensate can be computed as

$$\langle\bar{\lambda}\lambda\rangle_B = -\frac{T}{V}\frac{\partial}{\partial m}\log\left(Z(\beta,m)\right) = -\frac{T}{V}\left\langle\frac{1}{2}\mathrm{tr}(D_W^{-1})\right\rangle$$

- Additive renormalisation constant necessary when using Wilson fermions (due to explicit chiral violation).

$$\langle\bar{\lambda}\lambda\rangle_R = Z_{\bar{\lambda}\lambda}(\beta)\left(\langle\bar{\lambda}\lambda\rangle_B - b_0\right)$$

- Multiplicative constant avoided when choosing a fixed scale approach (fix β and κ, vary N_t)
- Use the gradient flow to directly compute the renormalised condensate on the lattice

Flow equations

- Fields are evolved along a trajectory on field space through the differential equations

$$\partial_t B_\mu = D_\nu G_{\nu\mu}, \quad B_\mu|_{t=0} = A_\mu : \; \texttt{flow of gauge fields}$$
$$\partial_t \chi = D_\mu D^\mu \chi, \quad \chi|_{t=0} = \psi : \; \texttt{flow of fermion fields}$$

[Lüscher and Weisz,arXiv:1405.3180]

- t is called *flow time* and parametrises the trajectory
- At leading order in g_0 the flow kernel is $K_t(x) = -(4\pi t)^{-D/2} \mathrm{e}^{-x^2/4t}$
  * Represents UV cut-off
  * Field is spread on a spherical region with radius $\sqrt{8t}$
  * Smoothing operation. Similar to stout smearing

Flow equations

- D+1 dimensional field theory developed by Lüscher
- Two point functions:
  - Automatically finite for flowed gauge fields. No counter-terms
  - For fermions: wave function renormalisation constant required beyond tree level
- Regularisation scheme independence: easy to implement on the lattice
- Useful to compute composite local operators, e.g. energy densitiy
- On the lattice:
  - Wilson (action) flow: used to set scale with t_0
    $\dot{V}_t(x,\nu) = -g_0^2 \left\{\partial_{x,\mu} S_{\mathrm{w}}(V_t)\right\} V_t(x,\mu), \quad V_t(x,\mu)|_{t=0} = U(x,\mu)$
  - Fermion flow: $\partial_t \chi_t = \Delta(V_t)\chi_t$

Gluino condensate from the gradient flow

- The condensate can be measured on the lattice as

$$\langle \chi_t(x) \rangle = -\sum_{v,w} \left\langle \mathrm{tr} \left\{ \underbrace{K(t,x;0,v)}_{\text{diff eq kernel}} \overbrace{S(v,w)}^{\text{propagator}} K(t,x;0,w)^\dagger \right\} \right\rangle$$

- Introducing random sources η_k and averaging over the position x

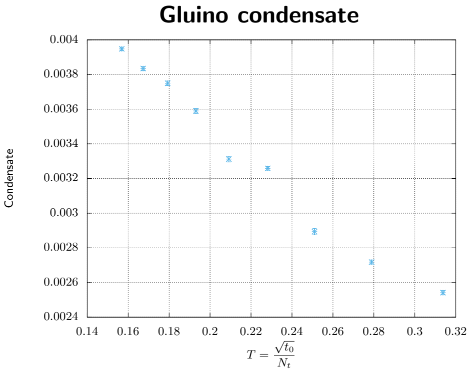
$$\frac{1}{N_\Gamma} \sum_{x\in\Gamma} \langle \chi_t(x) \rangle = -\frac{1}{N_\Gamma} \sum_{v,w} \left\langle \xi_k(t;0,v)^\dagger S(v,w) \xi_k(t;0,w) \right\rangle$$

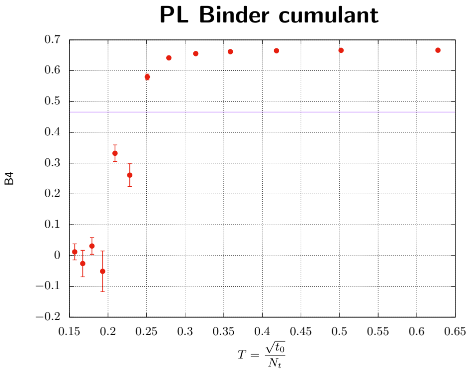
$$\xi_k(t;s,w) = \sum_x K(t,x;s,w)^\dagger \eta_k(x)$$

- The random source satisfy the *adjoint flow equation*, is integrated from $s = t$ down to $s = 0$ with a third-order Runge-Kutta
[Taniguchi et al.,Phys.Rev. D96 (2017) no.1, 014509]

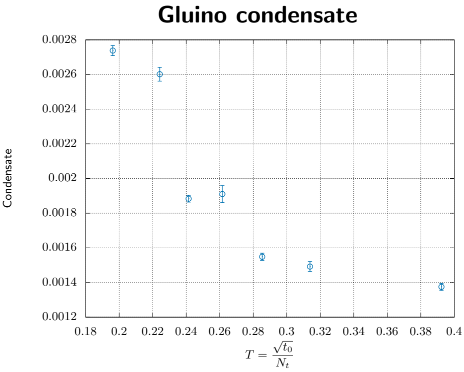
$$(\partial_s + \Delta)\xi_k(t;s,w) = 0,\ \xi_k(t;t,w) = \eta_k$$

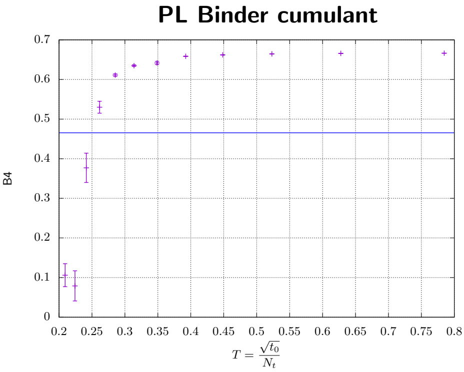
SU(2), $24^3 \times N_t$, $\beta = 1.75$, $\kappa = 0.1480$, $t = 6.3$



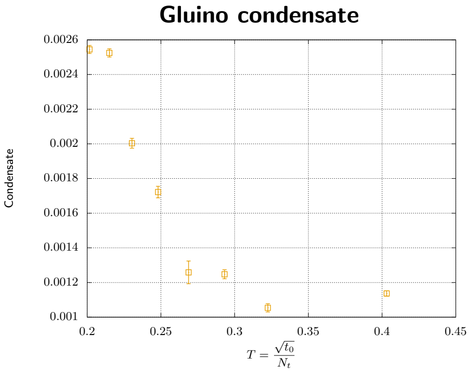


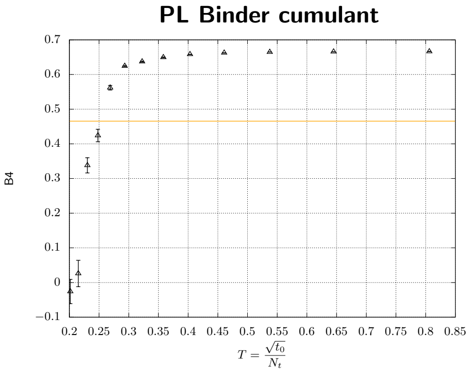
SU(2), $24^3 \times N_t$, $\beta = 1.75$, $\kappa = 0.1490$, $t = 9.85$



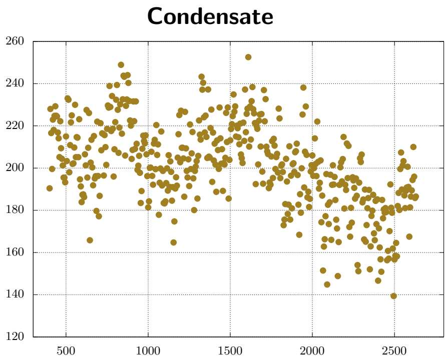


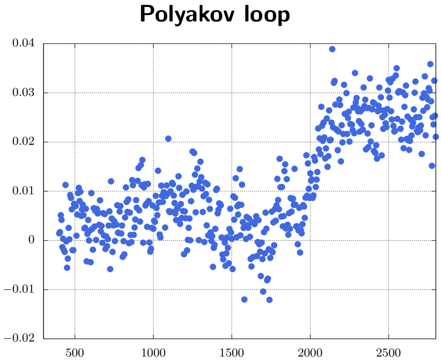
SU(2), $24^3 \times N_t$, $\beta = 1.75$, $\kappa = 0.14925$, $t = 10.5$



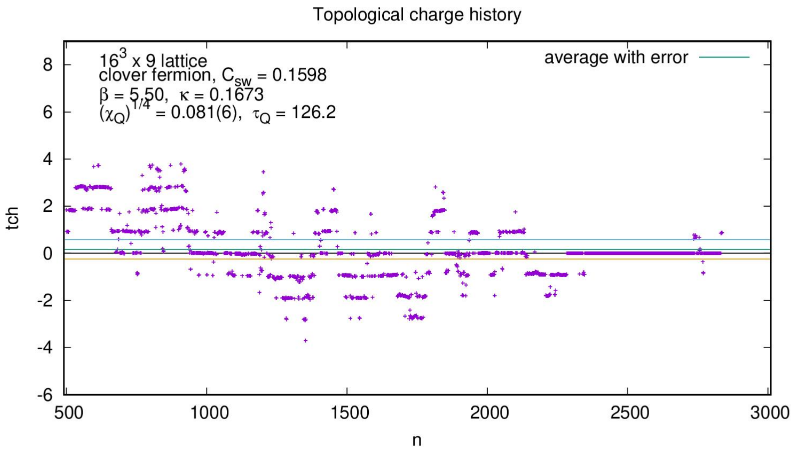


SU(3) results, $16^3 \times 9$, $\beta = 5.50$, $\kappa = 0.1673$





- Correlation of real part of Polyakov loop with and the scalar condensate. Pearson coefficient $\rho = -0.565$.
- Statiscally significant correlation between confinement and chiral restoration



Topological charge history
16^3 x 9 lattice
clover fermion, C_sw = 0.1598
β = 5.50, κ = 0.1673
(χ_Q)^1/4 = 0.081(6), τ_Q = 126.2
average with error
tch
n
8
6
4
2
0
-2
-4
-6
500
1000
1500
2000
2500
3000

Summary

- The gradient flow simplifies the measurement of local densities like the gluino condensate, on the lattice
- Better signal of chiral symmetry restoration as in previous studies
- Centre symmetry breaking and chiral restoration seem to happen at the same critical temperature for both SU(2) and SU(3)
- Correlation at phase transition suggests the existence of a common non-perturbative origin.

Thank you for your attention