Control of SU(3) symmetry breaking effects in calculations of B meson decay constant

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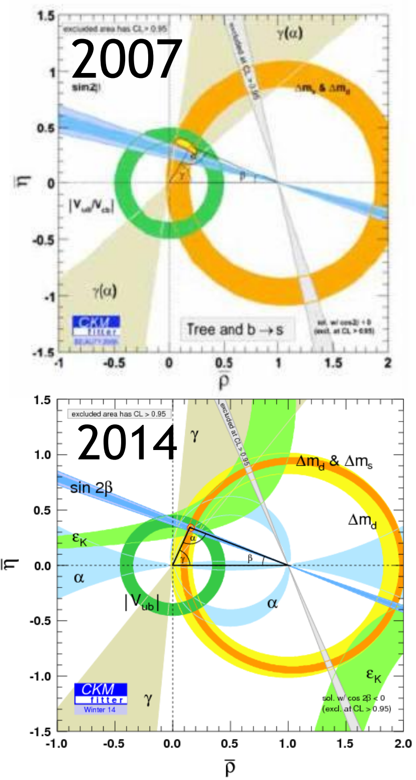
Why B decay constants?



- New experiments and the CKM matrix:
  - Need to reduce error in theoretical calculations to reduce error on CKM matrix elements ahead of new experimental results from Belle II
  - Decay constant f_B could be used alongside measurement of $B \rightarrow \tau\nu$ to pinpoint $|V_{ub}|$
  - f_B, f_{Bs} also important to $|V_{td}|, |V_{ts}|$ through $B^0\overline{B^0}$ oscillations
- We want to learn more about the way SU(3) breaking in the lightest quarks affects heavy B mesons
  - Need a strategy for studying SU(3) breaking effects in u,d,s quarks on the lattice

Choosing light and strange quarks

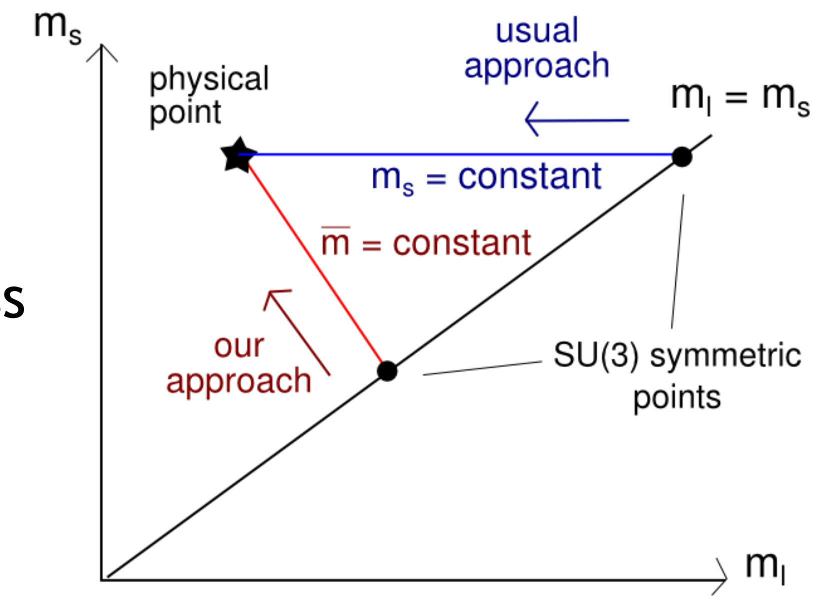
- We choose to study SU(3) breaking in a controlled way, by keeping the average mass of these three lightest quarks constant.
  - Lattice configurations for this method are produced by the QCDSF Collaboration. These configurations are simplified with $m_u = m_d$, (called m_{light})

- Choose constant average mass

$$\overline{m} = \tfrac{1}{3}\ (\ 2m_l + m_s\ )$$

  matching the physical average mass

- Produces controlled breaking of SU(3) symmetry
- Flavour singlet quantities remain approx. constant ($O(\delta m)$ removed)



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Light flavour singlets on QCDSF configurations, including:

$$X_\pi^2 = \frac{1}{6}(M_{K^+}^2 + M_{K^0}^2 + M_{\pi^+}^2 + M_{\pi^-}^2 + M_{\bar{K}^0}^2 + M_{K^-}^2)\,,$$

ss

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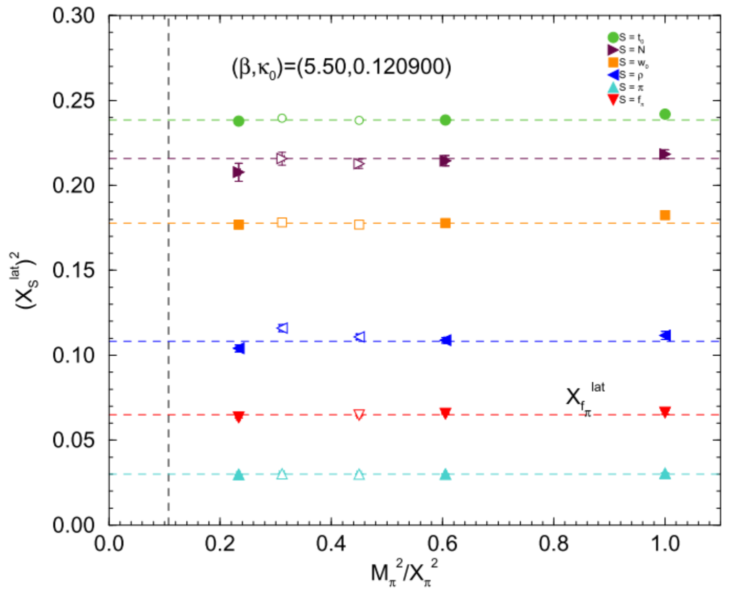

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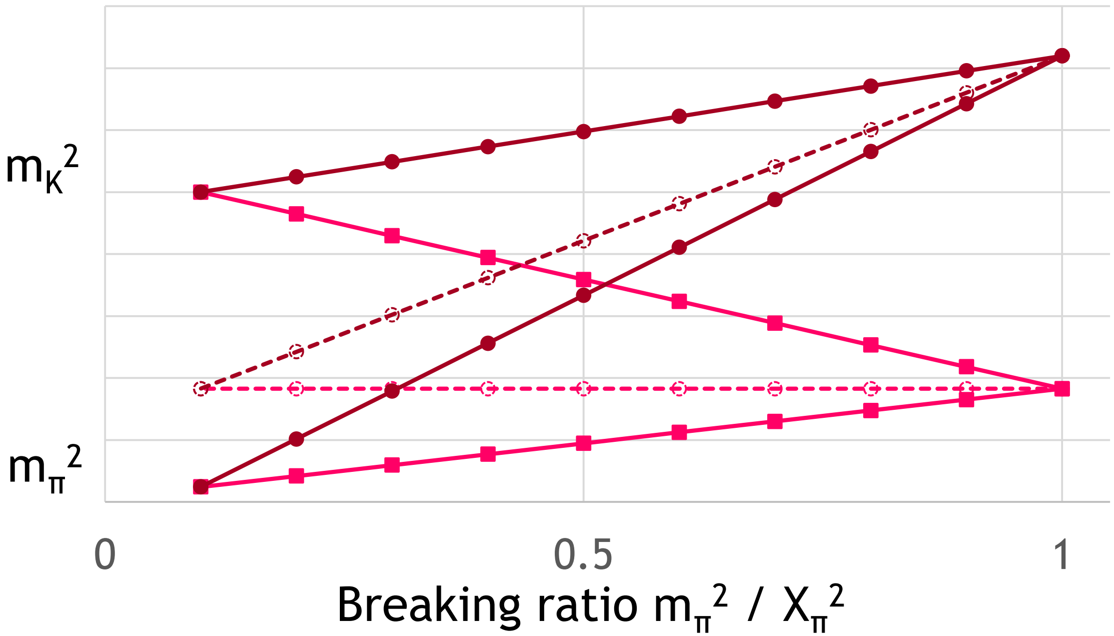

m_s = constant

The kaon is light + strange, so its mass still changes when m_s is constant

SU(3) breaking effects and effects from simulating a heavier vacuum occur together

$$\overline{m} = \tfrac{1}{3}\ (2m_l + m_s)$$

The average quark mass in the vacuum is constant

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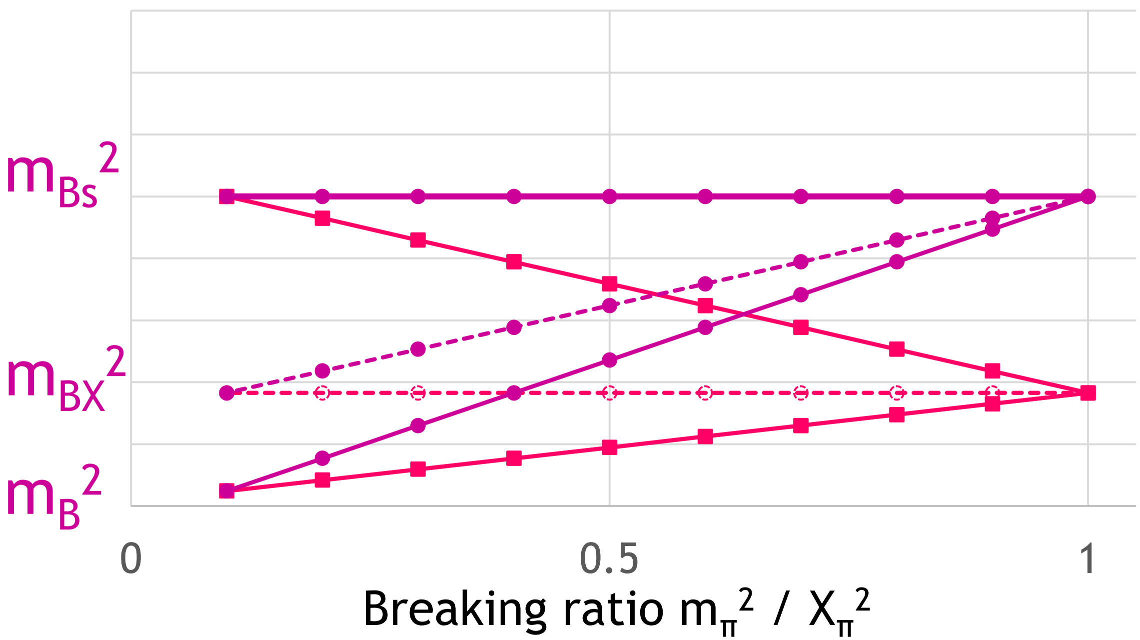

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Generating *b*-quarks

- *b*-quarks are heavy and “fall through” the lattice if a standard quark action is used.
- We use an anisotropic, clover-improved action (Relativistic Heavy Quark Action), and then tune the free parameters to physical quantities for the B meson.

$$S_{lat} = a^4 \sum_{x,x'} \overline{\psi}(x') \left(\underbrace{m_0}_{\text{bare mass}} + \gamma_0 D_0 + \underbrace{\zeta}_{\text{anisotropy}} \vec{\gamma} \cdot \vec{D} - \frac{a}{2}(D^0)^2 - \frac{a}{2}\underbrace{\zeta}_{\text{anisotropy}}(\vec{D})^2 + \sum_{\mu,\nu} \frac{ia}{4} \underbrace{c_P}_{\text{clover coefficient}} \sigma_{\mu\nu} F_{\mu\nu} \right)_{x,x'} \psi(x)$$ [1]

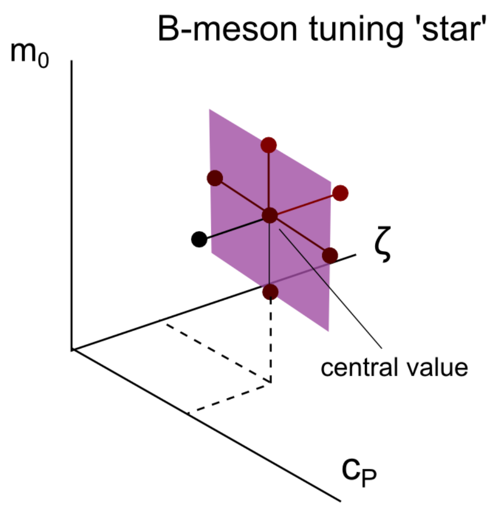
bare mass → spin-averaged meson mass

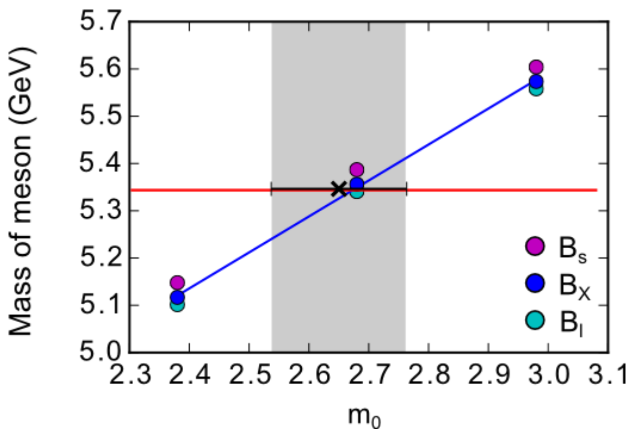
anisotropy → dispersion relation

clover coefficient → hyperfine splitting between B* and B

1 Aoki, Y et al (2012). “Nonperturbative tuning of an improved relativistic heavy-quark action with application to bottom spectroscopy.” *Physical Review D, 86*(11), 116003. doi:10.1103/PhysRevD.86.116003

Generating b-quarks

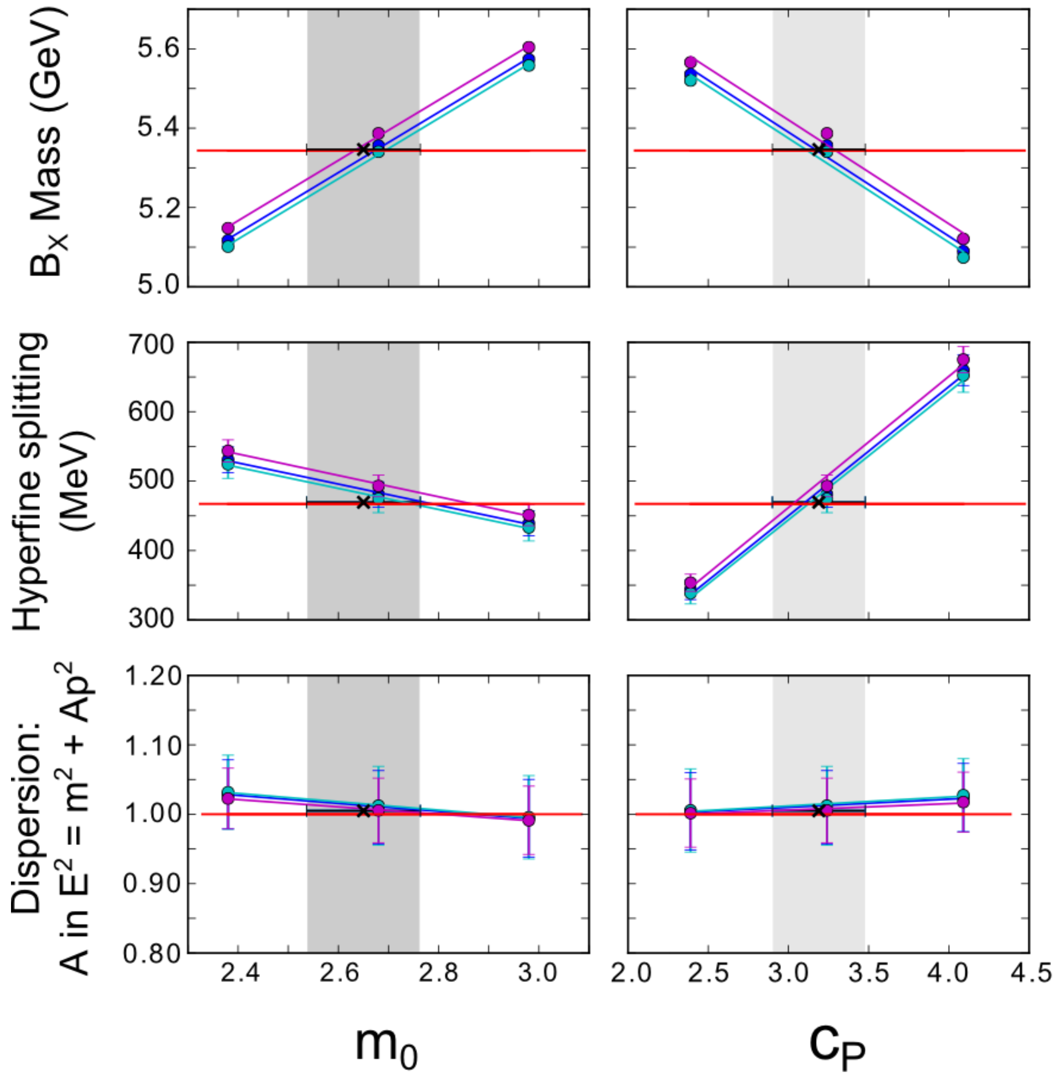




METHOD:

1. On every set of configurations, generate one "central" b-quark and six other b-quarks in a "parameter star" by changing our three free variables.
2. Make a B_{light} and $B_{strange}$ meson for each b quark
3. Calculate the "singlet" B meson, $B_X = (2/3)\, B_l + (1/3)\, B_s$ for each of our seven b-quarks.
4. Compare the calculated B_X mesons to the physical B_X meson, and find the set of parameters matching the physical B.

Tuning B mesons



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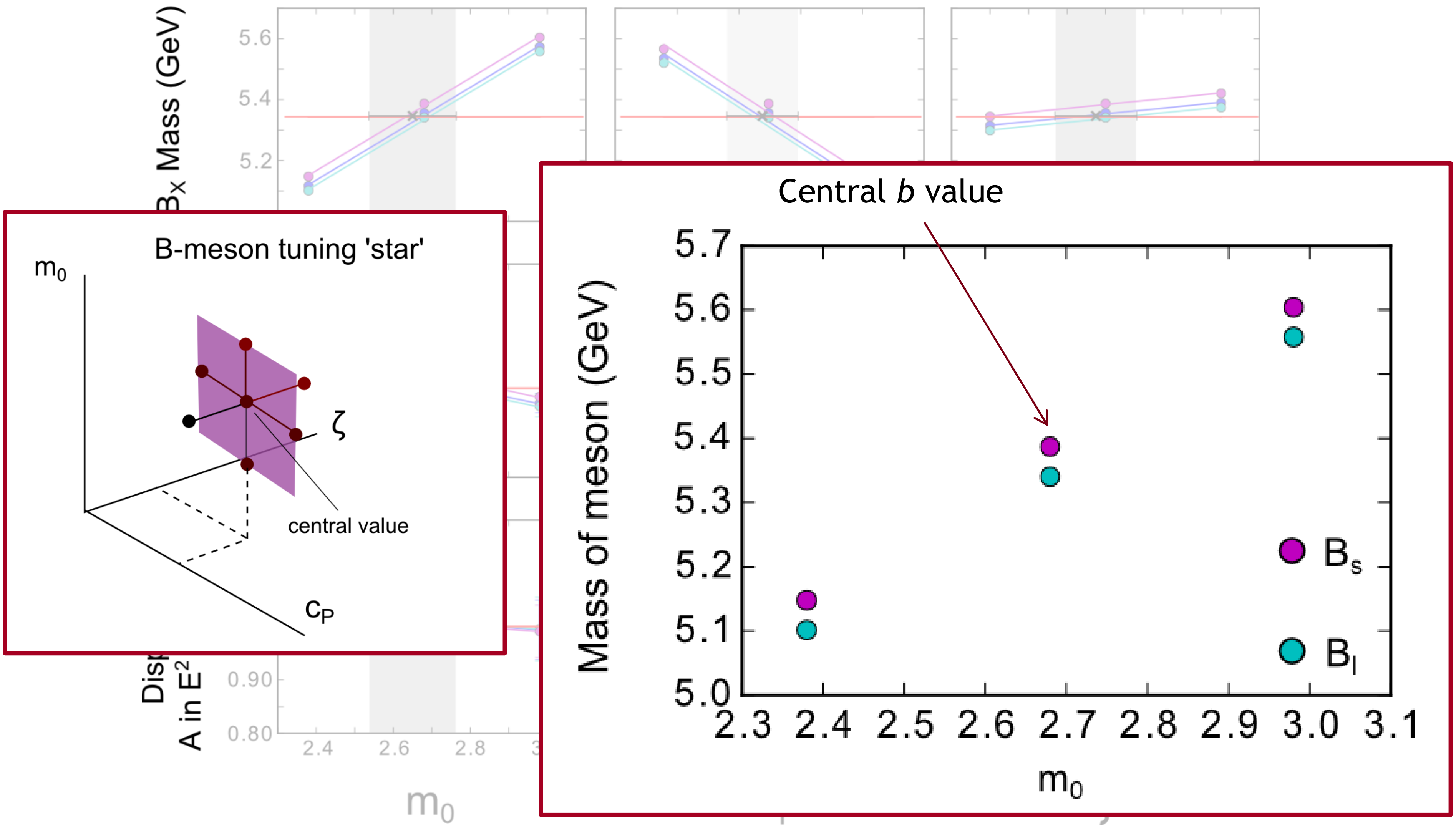

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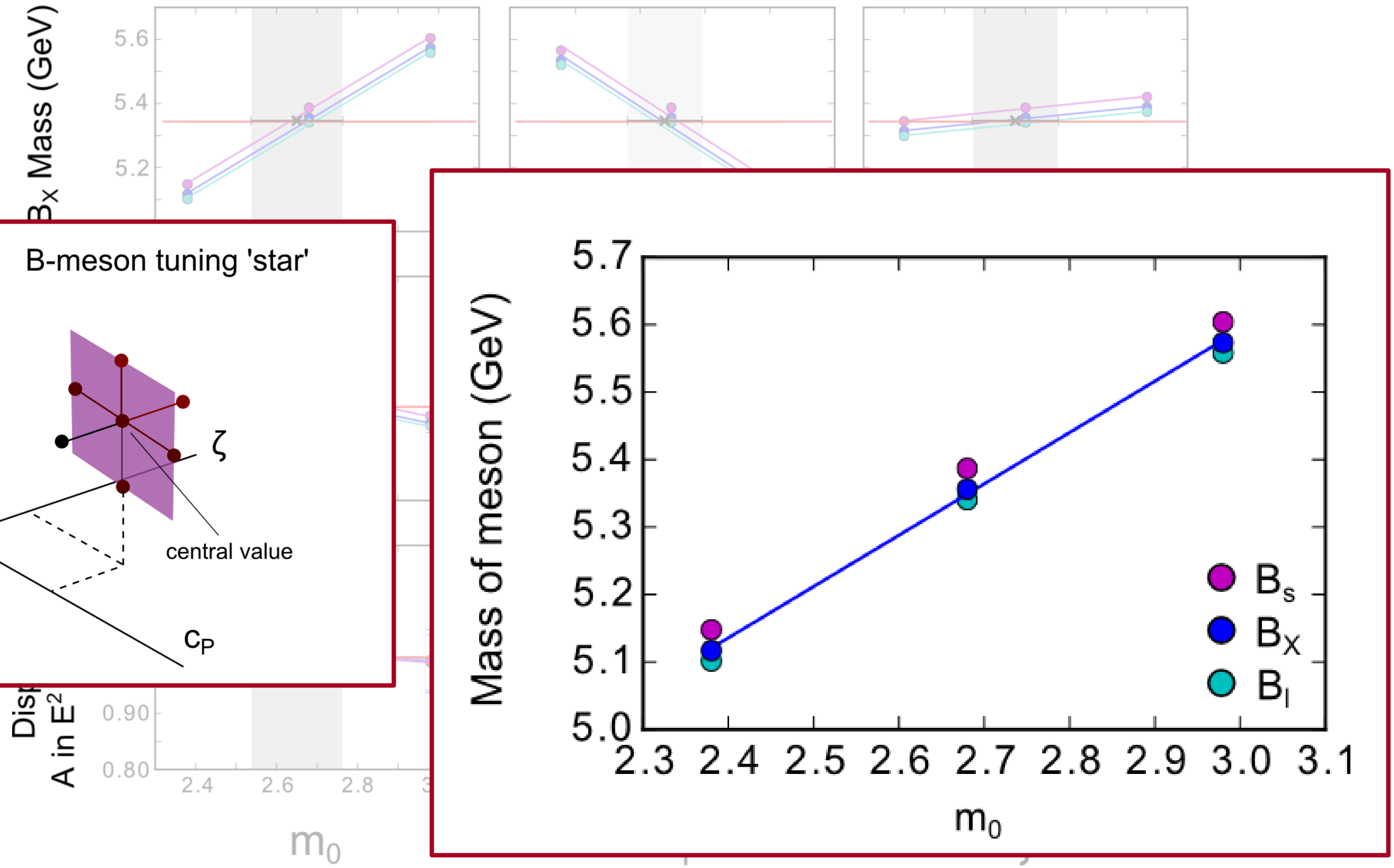

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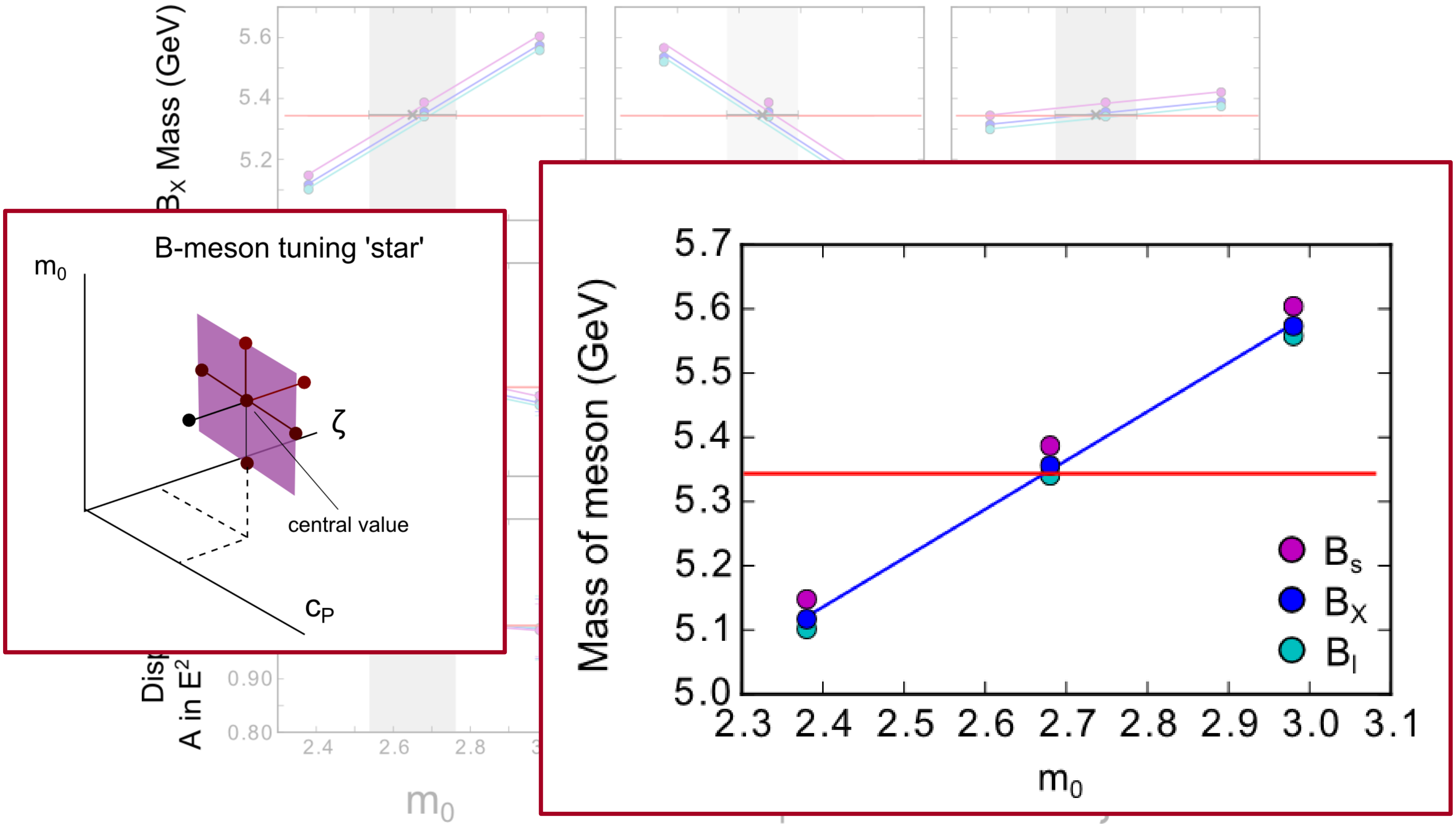

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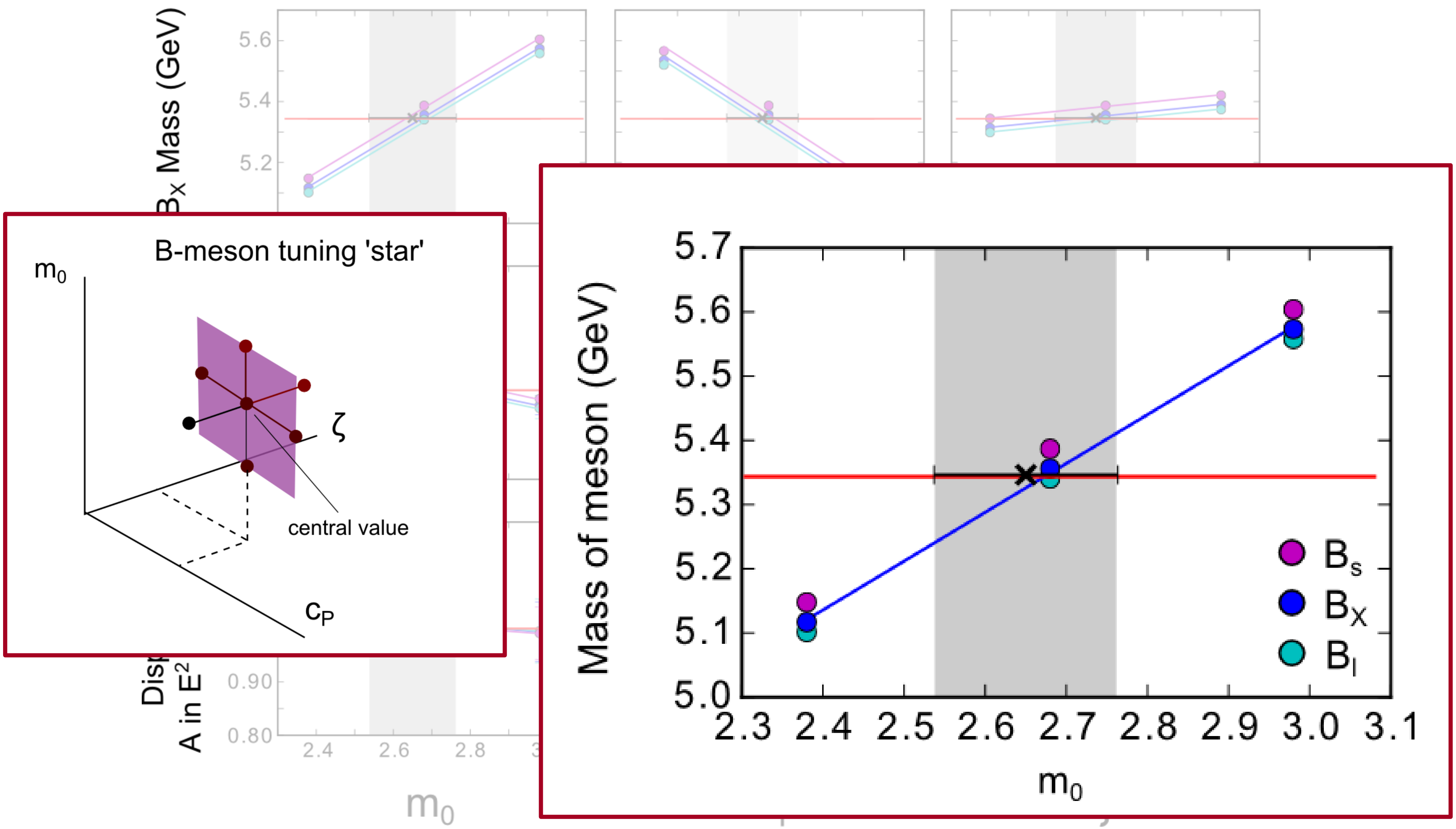

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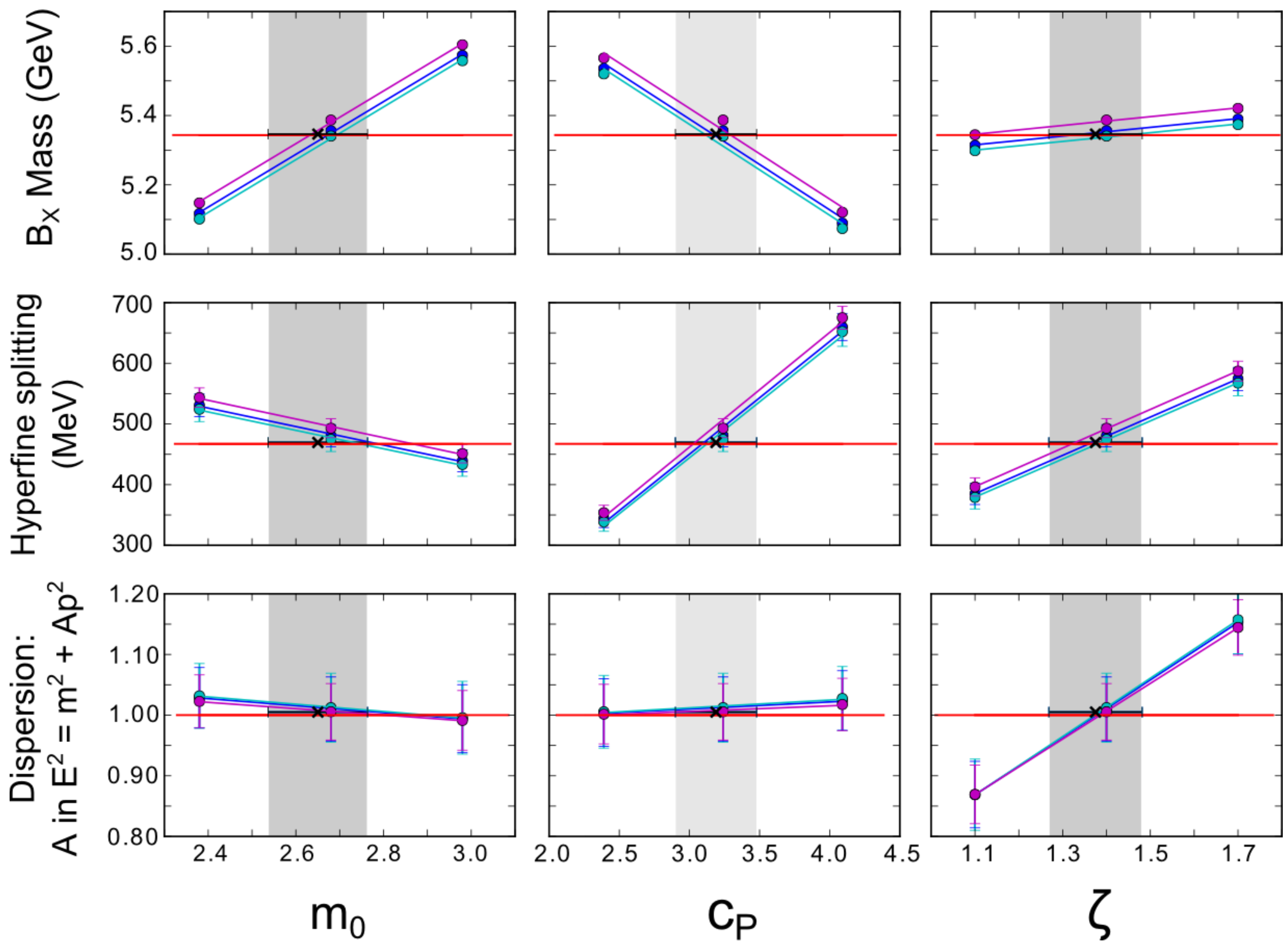
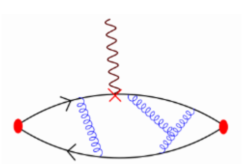


Calculating the decay constant f_{Bq}

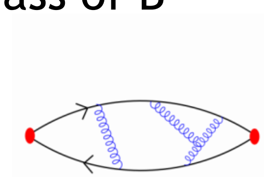
- Once we have chosen the appropriate quarks, the decay constant is calculated mostly using two point functions

$$f_B = \frac{\hbar c}{a} \underline{Z_\Phi} \left[\underline{\Phi_B^0} + c_A \Phi_B^1 \right]$$

Renormalisation factor:
Ratio of 2 point and 3 point functions with constant coefficient ρ=1



Lattice decay constant:
2 point functions with different operators in the quark propagators, and mass of B

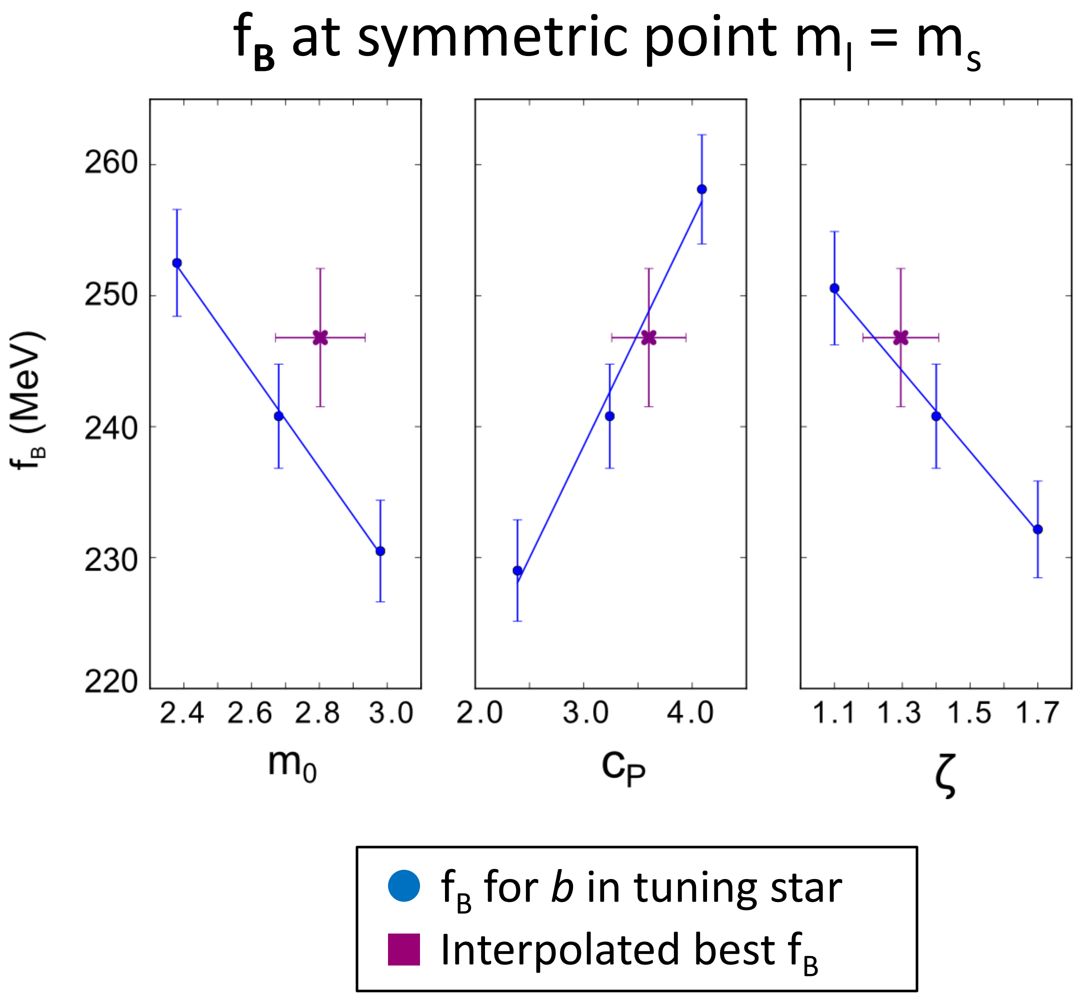


Improvement term:
2 point correlators & coefficient c_A

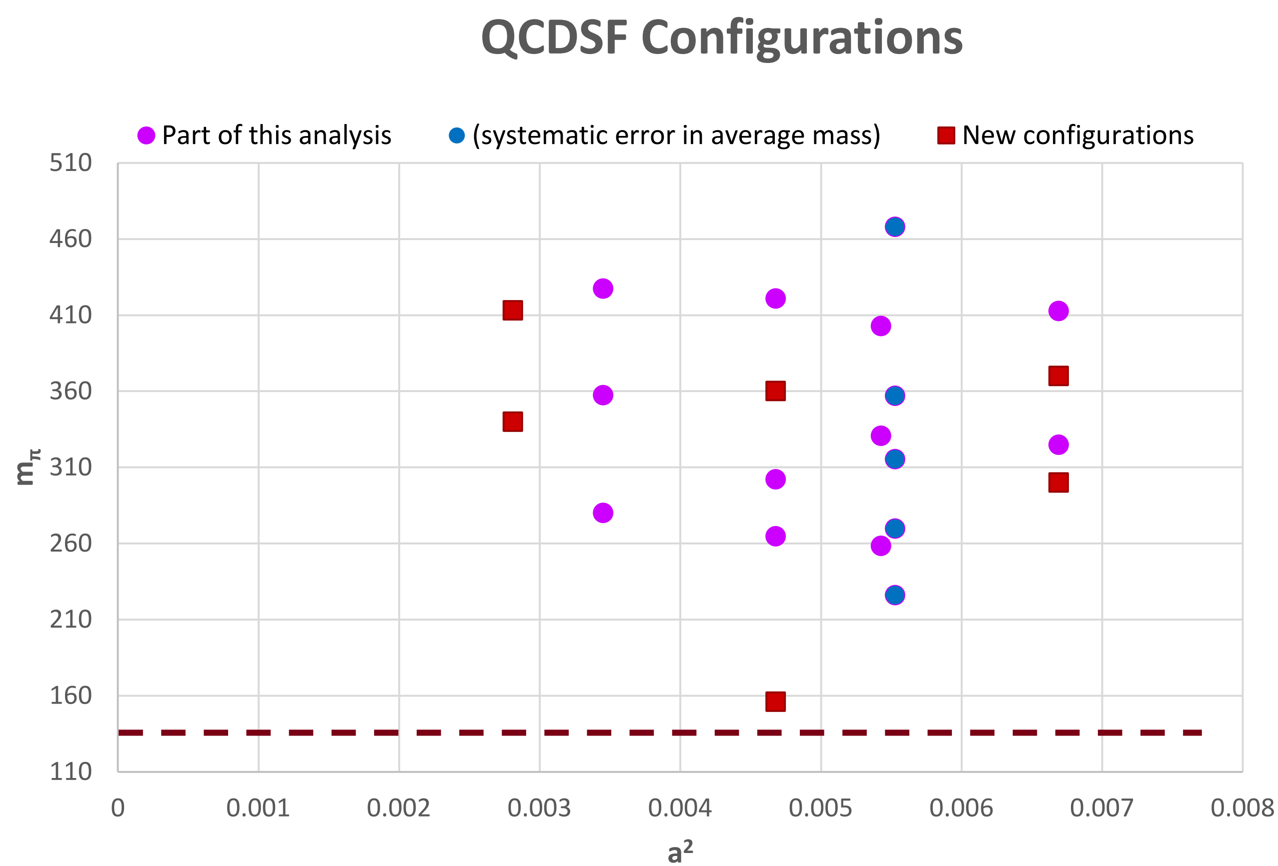
Currently take c_A=0,
Exact value can be calculated using perturbative QCD

Calculating the decay constant f_{Bq}

1. Calculate Φ_B and Φ_{Bs} for each of the *b*-quarks in the tuning "star"
2. For each set of lattice configurations, collect the "best" tuning parameters matching the physical properties of the B_X meson (as seen earlier)
3. Use these parameters to interpolate to a "best" Φ_B and thus calculate "best" f_B
4. Repeat at other light quark masses and lattice spacings!



Configurations used



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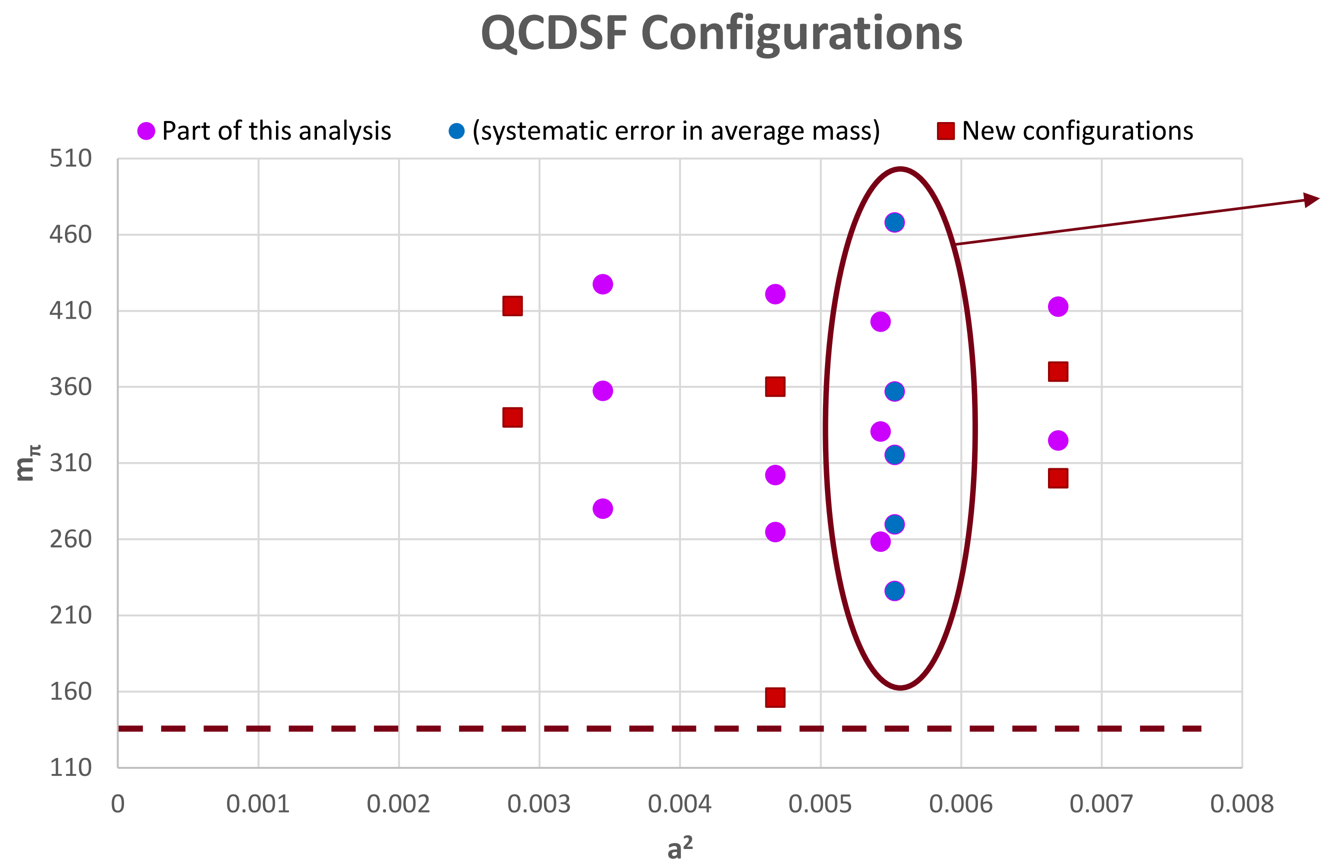
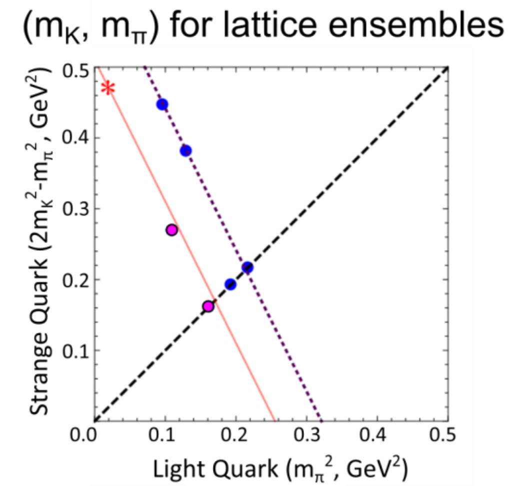
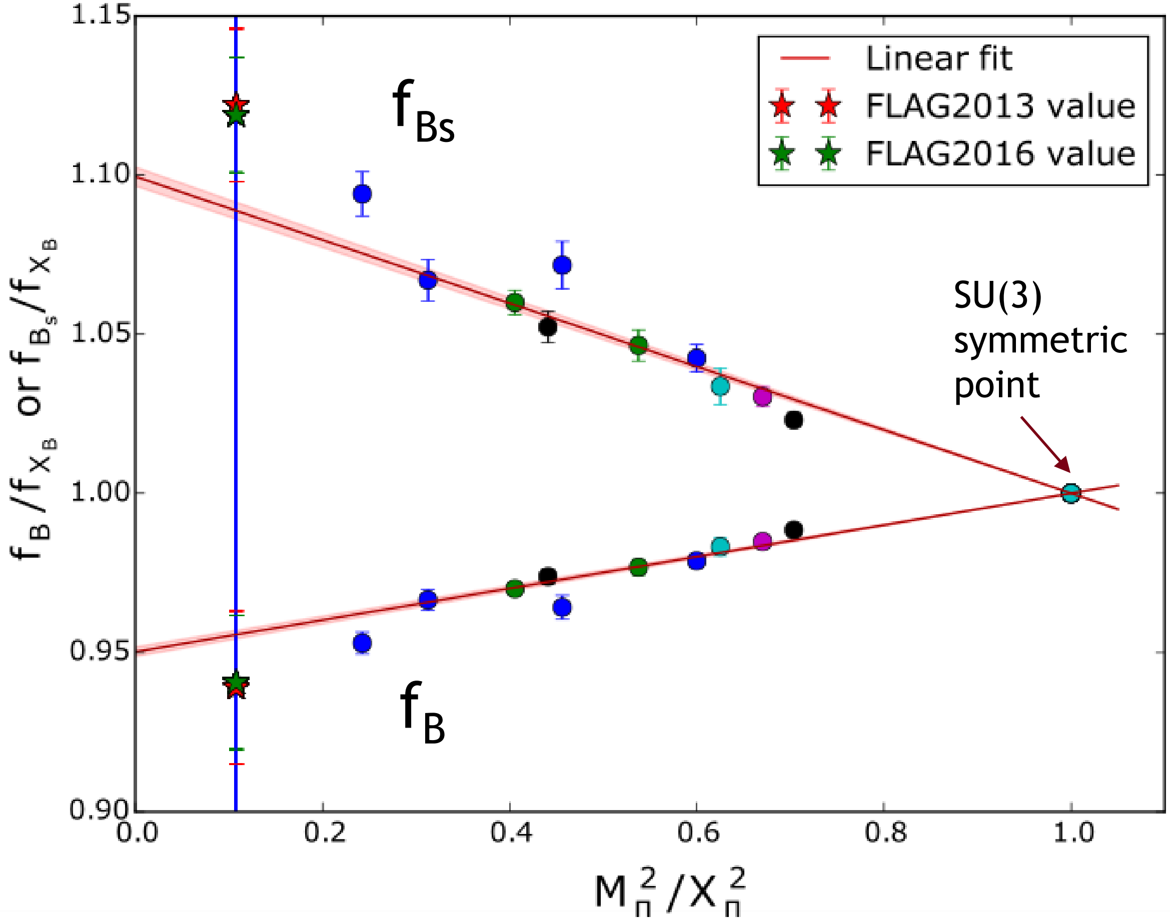




BLUE configurations have a systematic error in the SU(3) symmetric point value compared to the physical point, so we need a more careful approach

SU(3) breaking of f_{Bq}



- On each configuration, calculate f_{Bl} and f_{Bs} and the average f_{Bx} to cancel most systematic errors from calculation method
  - Visible errors are almost entirely from extrapolation to best B meson
- Linear fit is not sufficient!

Toward physical f_B and f_{Bs}

- If we take an SU(3) expansion of f_{Bq} / f_{BX} to NLO, and include quenched light quarks (*q*) and ignore the *b* quark in the SU(3) breaking, we can write: [2]

$$\frac{f_B(q\bar{b})}{f_{B_X}} = 1 + G(\delta\mu_q) + (H_1 + H_2)\delta\mu_q^2$$
$$- (\tfrac{2}{3}H_1 + H_2)(\delta m_u^2 + \delta m_d^2 + \delta m_s^2)$$
$$+ \ldots$$

Difference between valence quark mass and SU(3) quark mass
($\delta\mu_b = 0$, not part of SU(3))

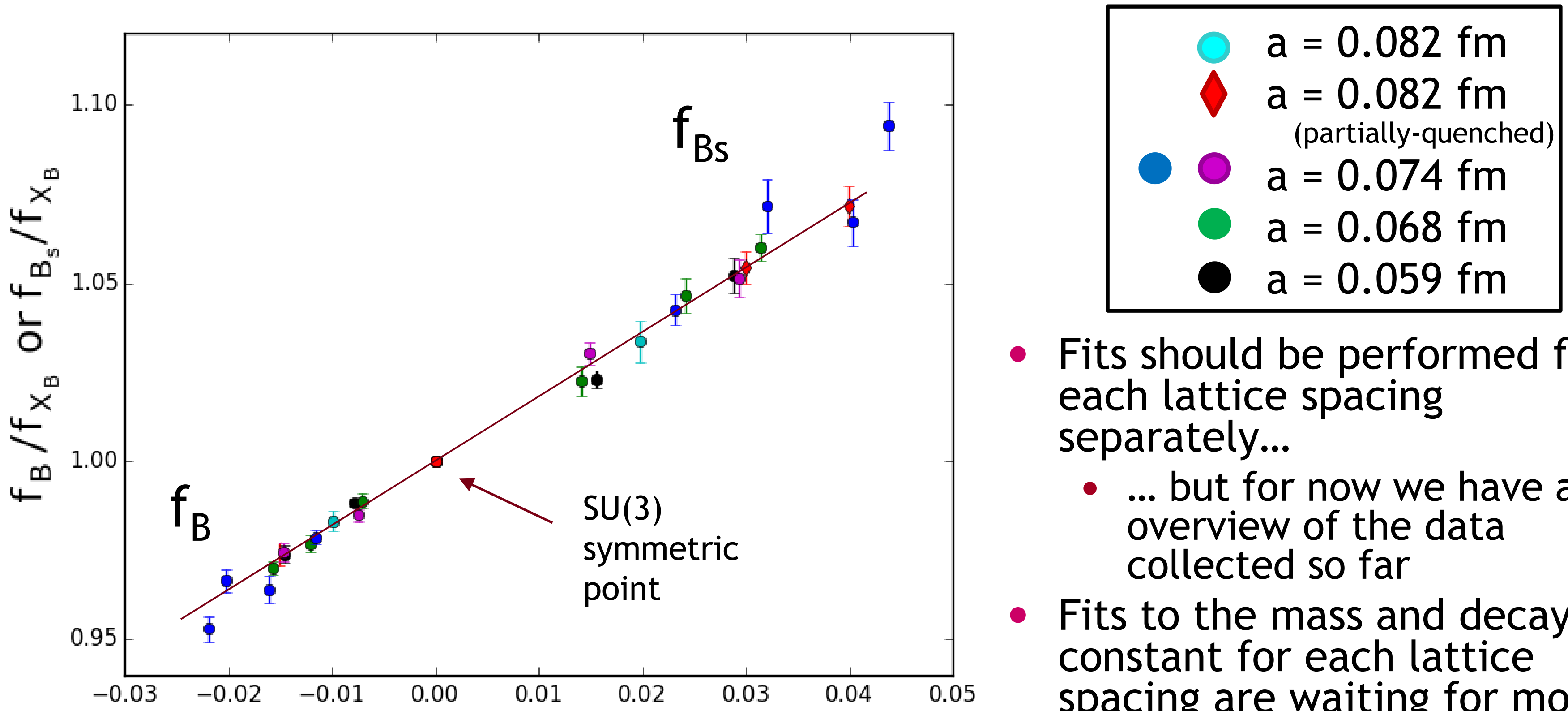
Differences between sea quark masses and SU(3) quark mass

with a similar equation governing the mass of the B mesons.

- By using lattice data to fit the coefficients for both f and M, we can:
  - Extrapolate to a value of f_{Bq} at the physical point for each lattice spacing
  - Perform a continuum extrapolation for each f_{Bq}

2 Based on equation in Bornyakov, V. G. et al (2017). "Flavour breaking effects in the pseudoscalar meson decay constants." *Physics Letters B*, *767*(3), 366–373. doi:10.1016/j.physletb.2017.02.018

Toward physical f_B and f_{Bs}



δm_q
change in quark mass from SU(3) symmetric point(s)

- Fits should be performed for each lattice spacing separately…
  - … but for now we have an overview of the data collected so far
- Fits to the mass and decay constant for each lattice spacing are waiting for more lattice configurations to be processed.
- Next: extrapolate from finite lattice spacing to continuum QCD

Summary and future work

- f_B and f_{Bs} calculated for a large number of lattice spacings and SU(3) splittings
  - Additional configurations to be included soon
  - Adding more partially-quenched light quarks
  - Improvement coefficients
- Future plans include
  - Measurement of f_{B*}
  - Semileptonic form factors $B \rightarrow D^{(*)} l\nu$
  - Studies of Λ_b