Determination of nucleon sigma terms I

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Nucleon sigma terms are defined as

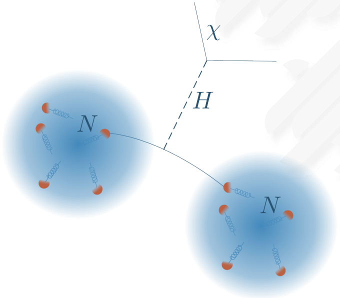
$$\sigma_{qN} = m_q \langle N \mid \bar{q}q \mid N \rangle - m_q \langle 0 \mid \bar{q}q \mid 0 \rangle.$$

They can be related — via the Feynmann Hellmann theorem — to the quark mass derivative of the nucleon mass:

$$\sigma_{qN} = m_q \frac{\partial M_N}{\partial m_q}.$$

They are closely related to the contribution of individual quark flavors to the nucleon masses.

The are relevant for dark matter detection experiments.



$$\sigma_{udN} = m_{ud}\langle N \mid \bar{u}u + \bar{d}d \mid N\rangle - m_{ud}\langle 0 \mid \bar{u}u + \bar{d}d \mid 0\rangle = m_{ud} \left.\frac{\partial M_N}{\partial m_{ud}}\right|_{m_s,a}$$

$$\sigma_{sN} = m_s\langle N \mid \bar{s}s \mid N\rangle - m_s\langle 0 \mid \bar{s}s \mid 0\rangle = m_s \left.\frac{\partial M_N}{\partial m_s}\right|_{m_{ud},a}$$

Light and strange sigma terms can be related to "mesonic" sigma terms via a transformation matrix:

$$\begin{pmatrix} \sigma_{udN} \\ \sigma_{sN} \end{pmatrix} = \begin{pmatrix} \frac{m_{ud}}{M_\pi^2} \left.\frac{\partial M_\pi^2}{\partial m_{ud}}\right|_{m_s,a} & \frac{m_{ud}}{M_{K_\chi}^2} \left.\frac{\partial M_{K_\chi}^2}{\partial m_{ud}}\right|_{m_s,a} \\ \frac{m_s}{M_\pi^2} \left.\frac{\partial M_\pi^2}{\partial m_s}\right|_{m_{ud},a} & \frac{m_s}{M_{K_\chi}^2} \left.\frac{\partial M_{K_\chi}^2}{\partial m_s}\right|_{m_{ud},a} \end{pmatrix} \begin{pmatrix} \sigma_{\pi N} \\ \sigma_{K_\chi N} \end{pmatrix}$$

with

$$\sigma_{\pi N} = M_\pi^2 \left.\frac{\partial M_N}{\partial M_\pi^2}\right|_{M_{K_\chi}^2,a} \quad \text{and} \quad \sigma_{K_\chi N} = M_\pi^2 \left.\frac{\partial M_N}{\partial M_{K_\chi}^2}\right|_{M_\pi^2,a} .$$

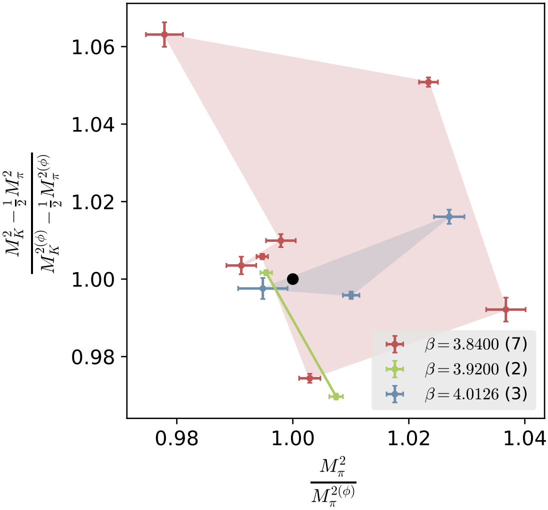
$$M_{K_\chi}^2 = 2M_K^2 - M_\pi^2$$

$$\begin{pmatrix}\sigma_{udN}\\ \sigma_{sN}\end{pmatrix} = \overbrace{\underbrace{\begin{pmatrix} \frac{m_{ud}}{M_\pi^2}\frac{\partial M_\pi^2}{\partial m_{ud}}\Big|_{m_s,a} & \frac{m_{ud}}{M_{K_\chi}^2}\frac{\partial M_{K_\chi}^2}{\partial m_{ud}}\Big|_{m_s,a} \\ \frac{m_s}{M_\pi^2}\frac{\partial M_\pi^2}{\partial m_s}\Big|_{m_{ud},a} & \frac{m_s}{M_{K_\chi}^2}\frac{\partial M_{K_\chi}^2}{\partial m_s}\Big|_{m_{ud},a}\end{pmatrix}}_{\text{this talk}}}^{=:J} \underbrace{\begin{pmatrix}\sigma_{\pi N}\\ \sigma_{K_\chi N}\end{pmatrix}}_{\text{next talk}}$$

Staggered fermions are well suited to determine J:

- Quark masses are easy to define
- Only pseudoscalar masses are required
- Available configurations bracket the physical point

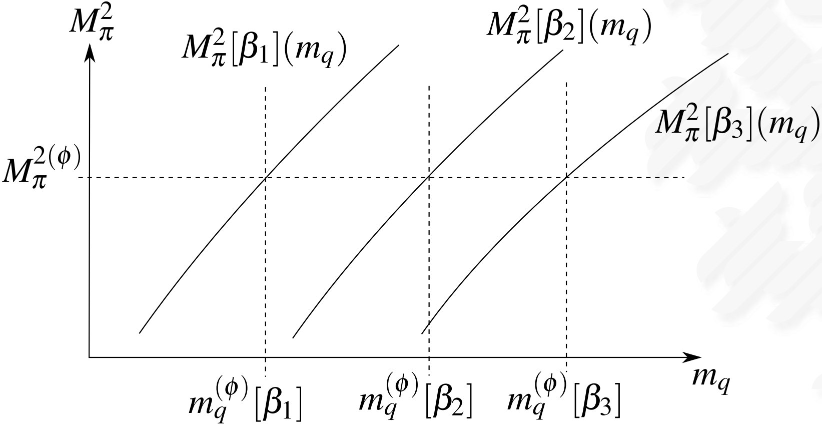
We used staggered $N_f = 2 + 1 + 1$ configurations with tree-level improved Symmanik gauge action and a 2-stout smeared fermion action.

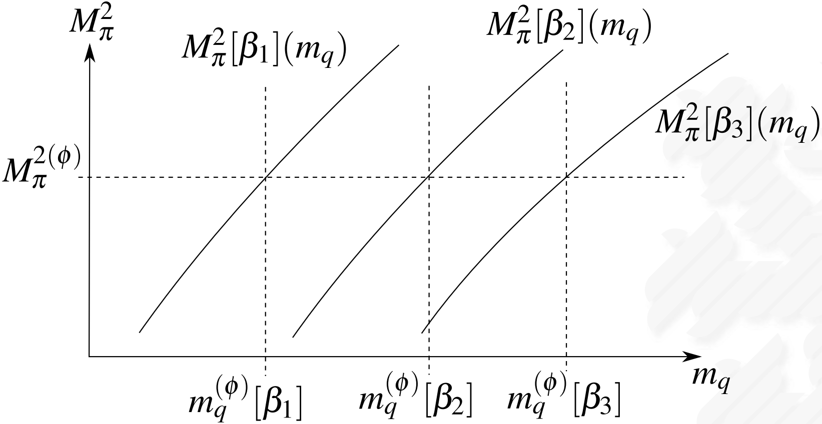


We expand the pion and reduced kaon mass around the physical point like

$$c_0 + c_{1,ud}(m_{ud} - m_{ud}^{(\phi)}) + c_{1,s}(m_s - m_s^{(\phi)}) + \dots$$

But how to define $m_{ud}^{(\phi)}$ and $m_s^{(\phi)}$?





Physical values of the $M_q^{(\phi)}$ depend on the gauge coupling:

$$c_0 + c_{1,ud}(m_{ud} - m_{ud}^{(\phi)}[\beta]) + c_{1,s}(m_s - m_s^{(\phi)}[\beta]) + \dots$$

The ratio $r = m_s/m_{ud}$ of strange and light quark masses is a physical observable. We use it to rewrite the expansion as

$$c_0 + c'_{1,ud}\left(\frac{m_{ud}\, r}{m_s^{(\phi)}[\beta]} - 1\right) + c'_{1,s}\left(\frac{m_s}{m_s^{(\phi)}[\beta]} - 1\right) + \dots$$

We treat $m_s^{(\phi)}[\beta]$ as a fit parameter per gauge coupling and assume

$$r = r_0 + r_1 a^2 + \mathcal{O}(a^4)$$

Up to higher order correction $c_{1,ud}$ and $c_{1,s}$ are the matrix elements of J.

Our ensambles feature a constant $m_c/m_s = 11.85$. Using the expansion of the form

$$c_0 + c'_{1,ud}\left(\frac{m_{ud}\, r}{m_s^{(\phi)}[\beta]} - 1\right) + c'_{1,s}\left(\frac{m_s}{m_s^{(\phi)}[\beta]} - 1\right) + \dots$$

allows to extract derivative like e.g.

$$m_s \frac{\partial M_\pi^2}{\partial m_s}\Big|_{m_{ud}, m_c/m_s, a}$$

Hence we had to introuce a term proportional to m_c/m_s to our fit function and use the relation

$$m_s \frac{\partial M_\pi^2}{\partial m_s}\Big|_{m_{ud}, m_c, a} = m_s \frac{\partial M_\pi^2}{\partial m_s}\Big|_{m_{ud}, m_c/m_s, a} - \frac{m_c}{m_s} \frac{\partial M_\pi^2}{\partial (m_c/m_s)}\Big|_{m_{ud}, m_s, a}$$

We generated a dedicated ensamble with $m_c/m_s = 11.45$ so that we are sensitive on the m_c/m_s direction.

We used the expansion up to quadratic order and included a^2 corrections on the leading terms:

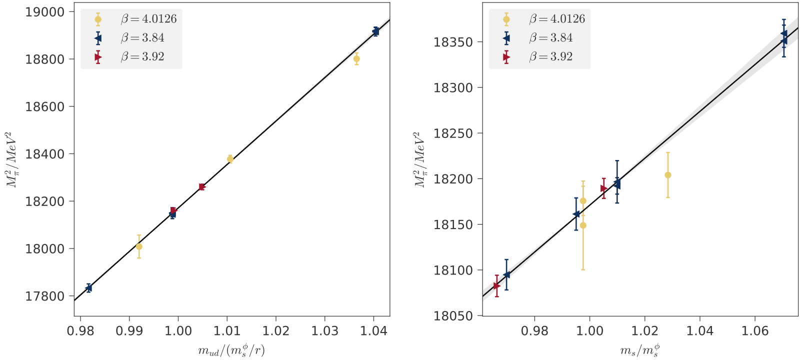
$$c_0 + (c'_{1,ud} + d_{1,ud}a^2)\Delta_{ud} + (c'_{1,s} + d_{1,s}a^2)\Delta_s + c_{2,ud,s}\Delta_{ud}\Delta_s + c_{2,ud}\Delta_{ud}^2 + c_{2,s}\Delta_s^2 + c_{c/s}\Delta_{c/s}$$

with

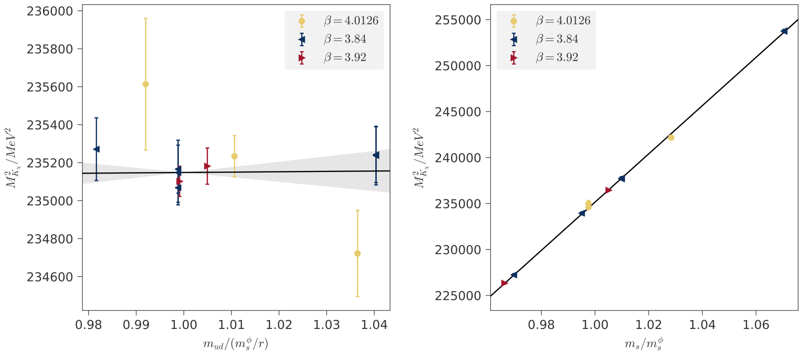
$$\Delta_{ud} = \frac{m_{ud}(r_0 + r_1 a^2)}{m_s^{(\phi)}[\beta]} - 1,$$
$$\Delta_s = \frac{m_s}{m_s^{(\phi)}[\beta]} - 1,$$
$$\Delta_{c/s} = \frac{m_c}{m_s} - \left(\frac{m_c}{m_s}\right)^{(\phi)}.$$

We use fit function of this for to simultaneous fit M_π^2, $M_{K_\chi}^2$ and for scale setting f_π.

Dependence of M_π^2 on the light (left) and strange (right) quark mass:



Dependence of $M^2_{K_\chi}$ on the light (left) and strange (right) quark mass:



For the systematic error we used the histogram method and varied the fit function in the following ways:

- A early and a late plateou for the extraction of M_π^2, $M_{K_\chi}^2$, and f_π.
- Quadratic or no quadratic terms in m_{ud} and m_s.
- All possible combinations of a^2 terms switched on and of.

We weight individual fits with their respective AIC weight.

We estimated the statistical error with the bootstrap procedured

A independent analysis was carried out where the $m_{ud}(M_\pi^2, M_{K_\chi}^2)$ and $m_s(M_\pi^2, M_{K_\chi}^2)$ instead of $M_\pi^2(m_{ud}, m_s)$ and $M_\pi^2(m_{ud}, m_s)$ was fitted.

The fits of this type allow for a direct determination of the inverse matrix J^{-1}.

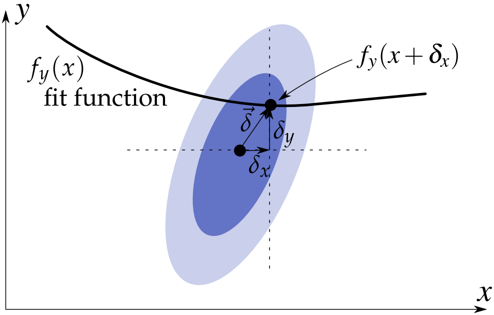
The two analysis methods are physically closely related but are technically quite different.

Both analysis procedures where implemented fully independently and show an excelent agreement.

Example: Treatment of x and y errors.

In one analysis there are only x-errors, in the other cases there are only y errors.

For x errors we use the following procedure:



Calculate χ^2 via

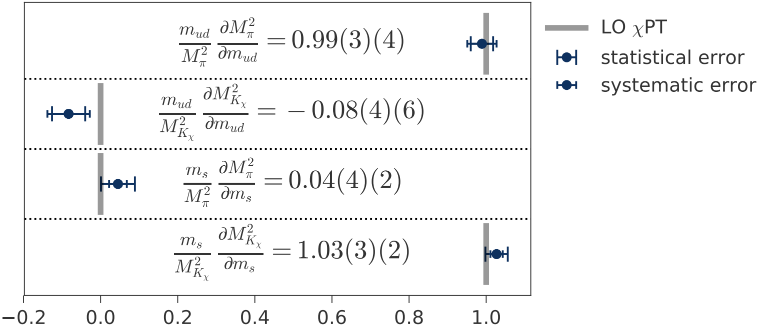
$$\vec{\delta} = \begin{pmatrix} f(x+\delta_x) - y \\ \delta_x \end{pmatrix}$$

$$\chi^2 = \sum_i \vec{\delta}^T C^{-1} \vec{\delta}$$

Generalizes to the case of several channels.

preliminary

The results for the mixing matrix are:



The result for the strange to light quark mass ratio are

$$\frac{m_s}{m_{ud}} = 27.293(33)(08)$$

(FLAG result: $\frac{m_s}{m_{ud}} = 27.30(34)$).