

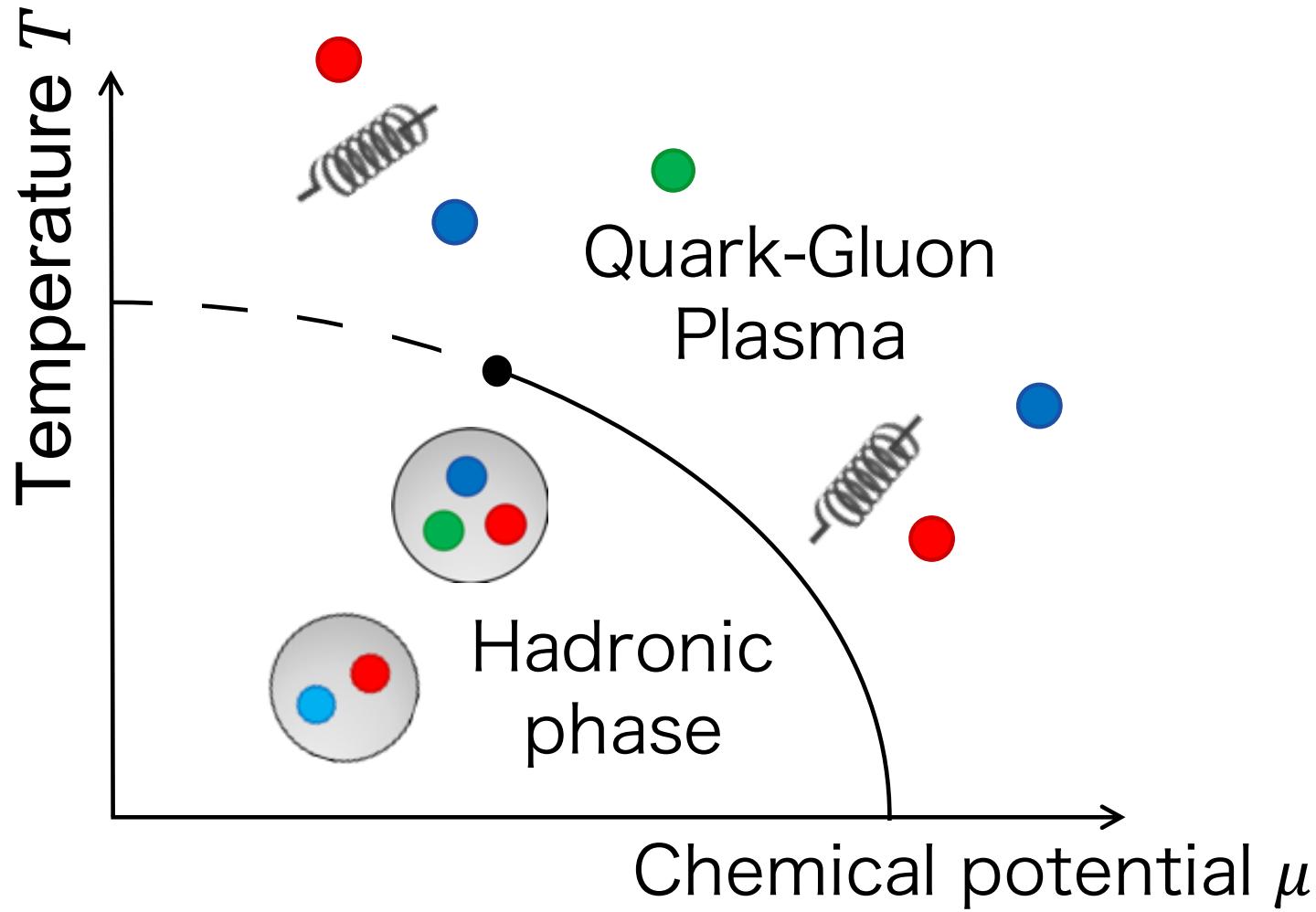
Measuring of chiral susceptibility using gradient flow

Atsushi Baba (Univ. of Tsukuba)

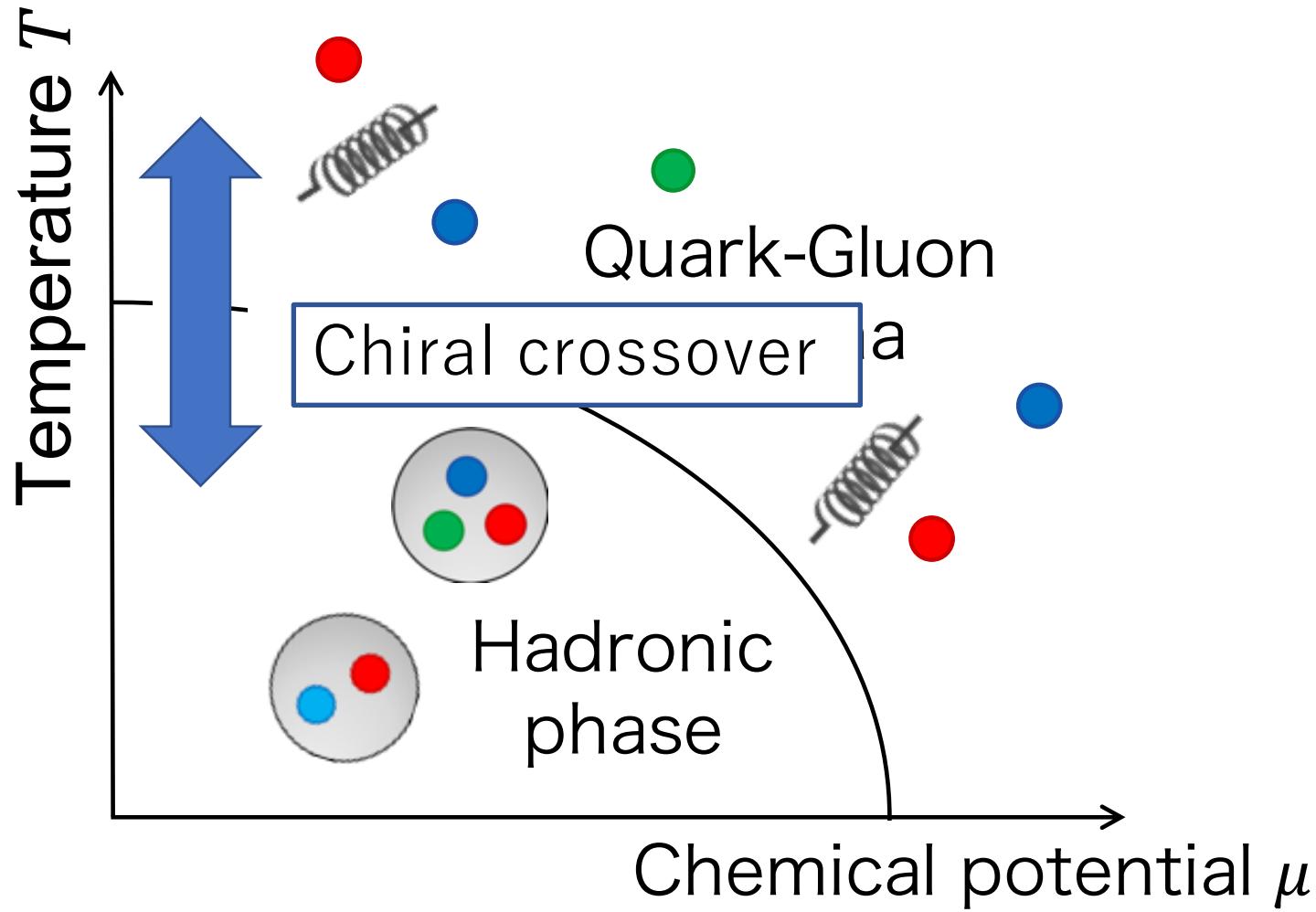
For
WHOT-QCD Collaboration

S. Ejiri, K. Kanaya, M. Kitazawa, T. Shimojo,
H. Suzuki, T. Umeda, Y. Taniguchi, A. Suzuki

Conjectured QCD phase diagram



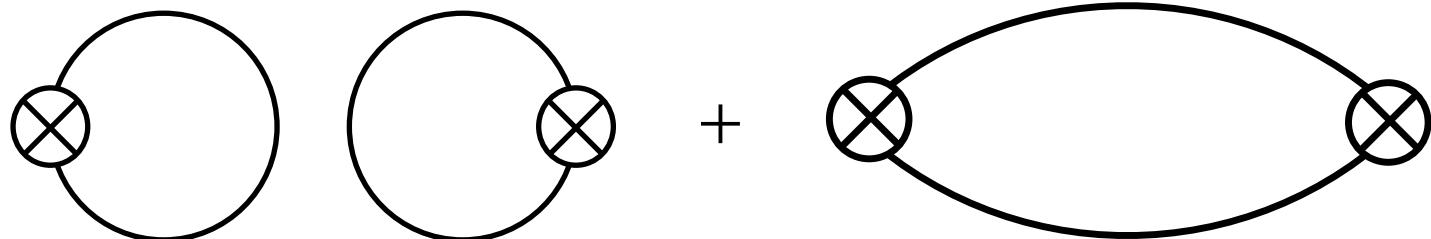
Conjectured QCD phase diagram



To explore the transition temperature

Chiral susceptibility

$$\chi_{\bar{f}f}^{\text{full}} = \underbrace{\langle (\bar{\psi}(x)\psi(x))^2 \rangle}_{\text{disc}} - (\langle \bar{\psi}(x)\psi(x) \rangle)^2$$



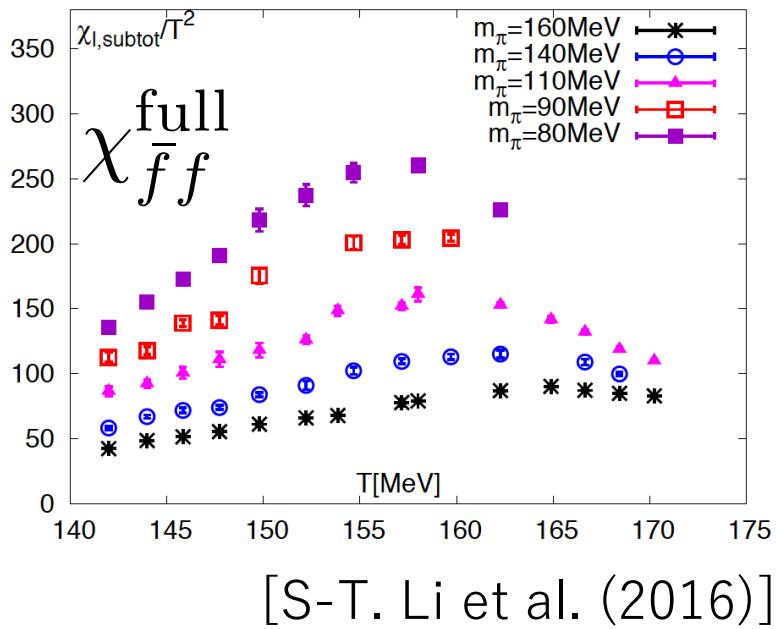
$$\langle (\bar{\psi}_f(x)\psi_f(x))^2 \rangle_{\text{disc}} + \langle (\bar{\psi}_f(x)\psi_f(x))^2 \rangle_{\text{conn}}$$

→

$$\chi_{\bar{f}f}^{\text{disc}} = \langle (\bar{\psi}(x)\psi(x))^2 \rangle_{\text{disc}} - (\langle \bar{\psi}(x)\psi(x) \rangle)^2$$
$$\chi_{\bar{f}f}^{\text{conn}} = \langle (\bar{\psi}(x)\psi(x))^2 \rangle_{\text{conn}}$$

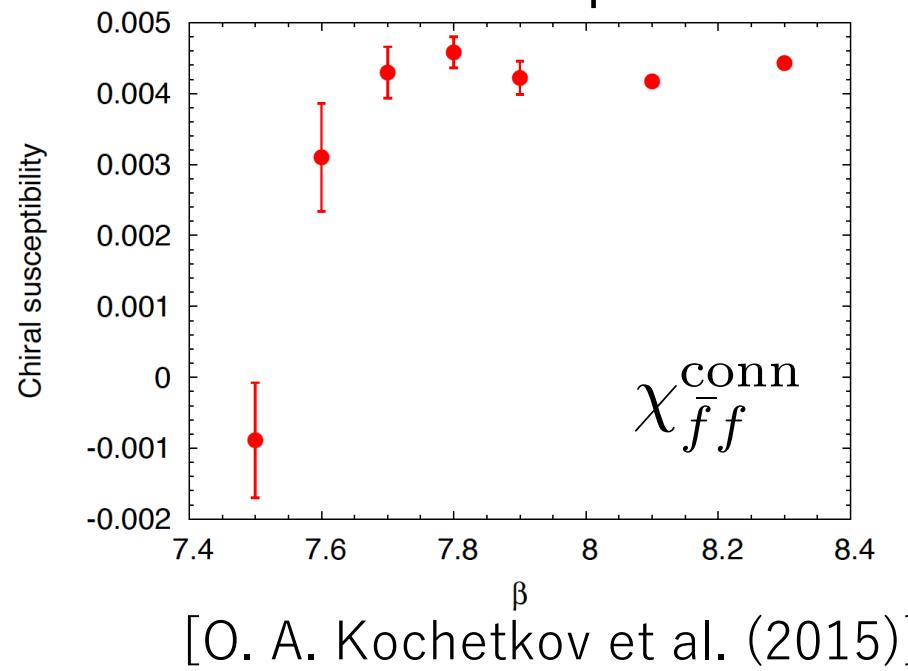
Numerical result from chiral lattice fermions

Staggered



Have a smeared peak

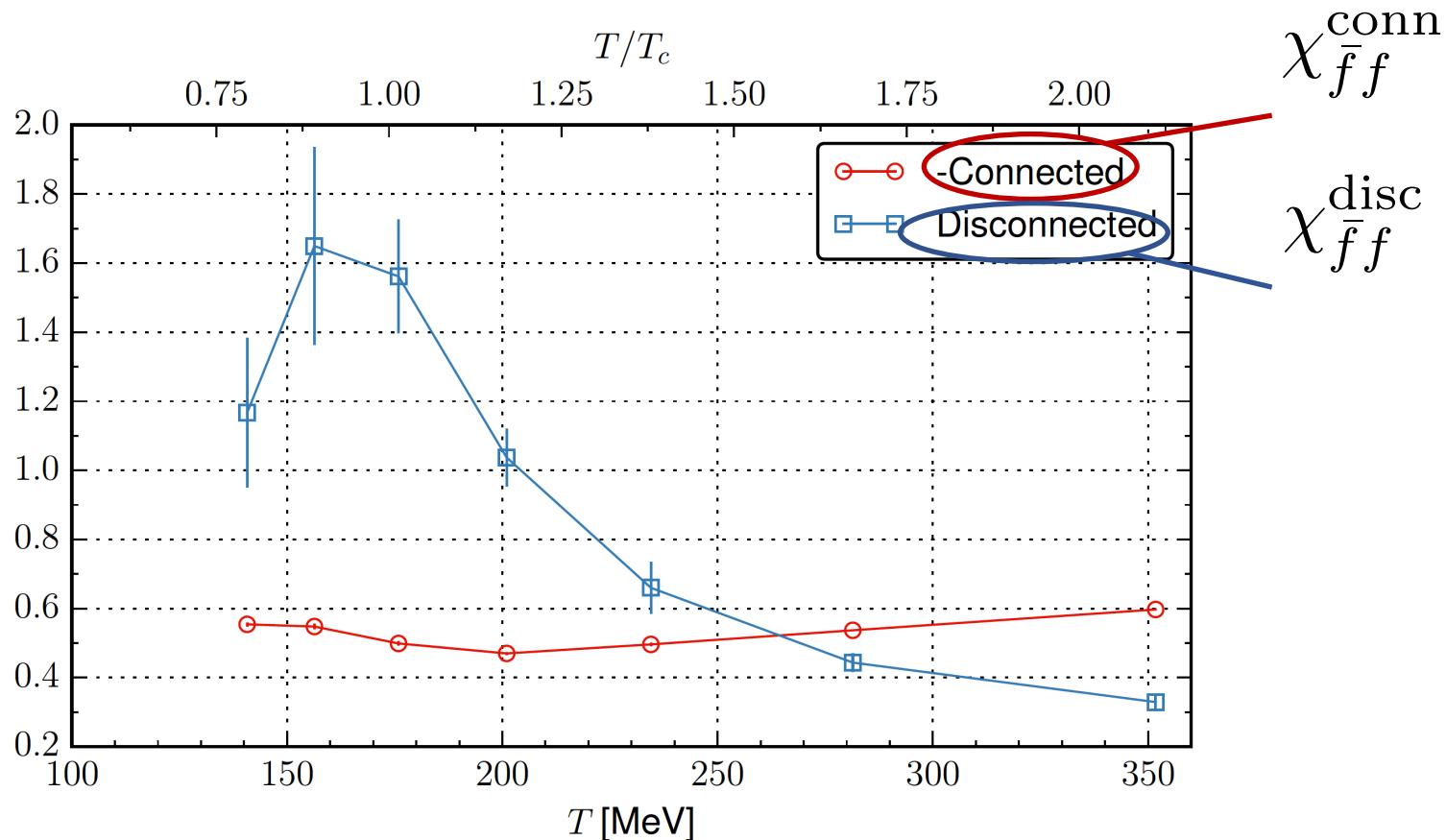
Overlap



No pronounced peak
Only hint peak on the plateau

Numerical result of Wilson fermion [G, Aarts et al. (2015)]

But unrenormalized



Connected part is NOT singular to temperature

We want to use correctly renormalized operator with Wilson fermion

Gradient flow [Luscher (2010)]

Initial condition
 $B_\mu(0, x) = A_\mu(x)$

Gauge flow for flowed field $B_\mu(t, x)$

$$\partial_t B_\mu(t, x) = D_\nu G_{\mu\nu}(t, x)$$

$$\sim \partial^2 B_\mu(t, x) + \text{interaction}$$

Flowed field strength

$$G_{\mu\nu}(t, x) = \partial_\mu B_\nu(t, x) - \partial_\nu B_\mu(t, x) + [B_\mu, B_\nu]$$

Formal solution

$$B_\mu(t, x) = \int d^D y K_t(x - y) A_\mu(y) + \text{interaction}$$

$$K_t(x) = \int_p e^{ipx} e^{-tp^2} = \frac{e^{-x^2/4t}}{(4\pi t)^2}$$

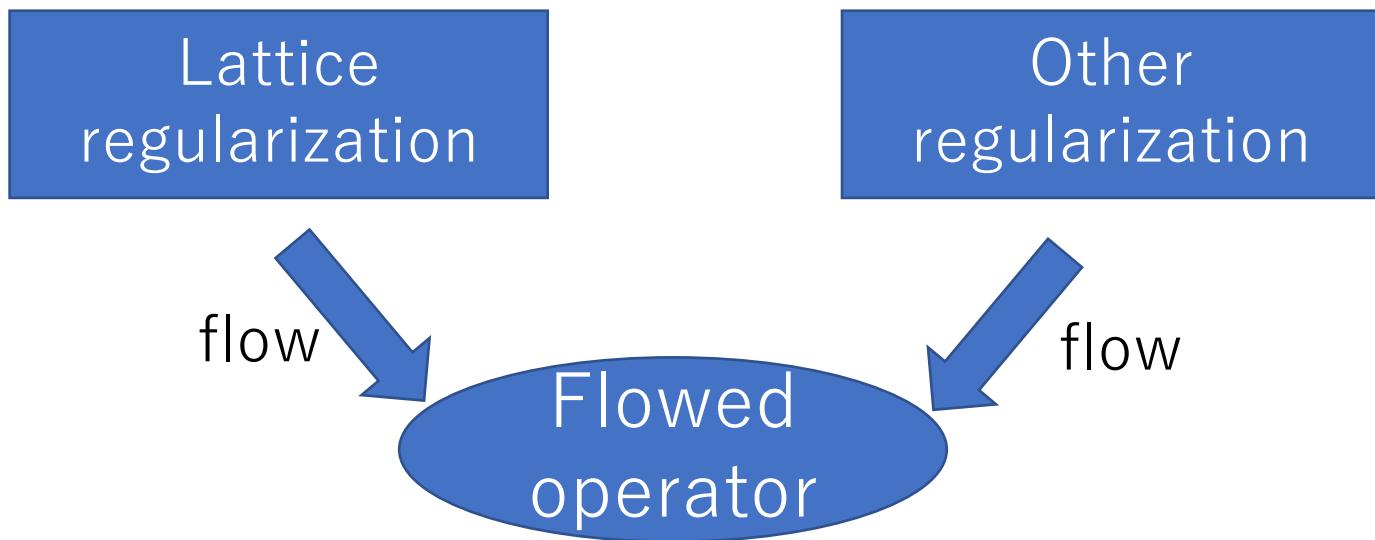
Smearing within $\sqrt{8t}$

Use of gradient flow

$$K_t(x) = \int_p e^{ipx} \boxed{e^{-tp^2}} = \frac{e^{-x^2/4t}}{(4\pi t)^2}$$

This factor dumps high energy mode at $t > 0$

Flowed operators are UV finite !



for $a \rightarrow 0$ (in lattice regularization)
 $\varepsilon \rightarrow 0$ (in dimensional regularization) 8

Gradient flow

Fermion flow equation [Luscher2013]

$$\begin{aligned}\partial_t \chi(t, x) &= D^2 \chi(t, x) && \text{Initial condition} \\ \partial_t \bar{\chi}(t, x) &= \bar{\chi}(t, x) \overset{\leftarrow}{D}^2 && \chi(0, x) = \psi(x) \\ &&& \bar{\chi}(0, x) = \bar{\psi}(x)\end{aligned}$$

Formal solution

$$\chi(t, x) = \int d^D y \boxed{K_t(x - y)} \psi(x) + \text{interaction}$$

Fermion renormalization

$$\varphi_f(t) \equiv -\frac{6}{(4\pi)^2 t^2 \langle \bar{\chi}_f(t, x) \overset{\leftrightarrow}{D} \chi_f(t, x) \rangle_0}$$

Small flow time expansion [Luscher-Weisz (2011)]

[Suzuki (2013)]

Flowed operator

Renormalized operator

$$\tilde{\mathcal{O}}(t, x) = \{\mathcal{O}\}_R(x) + \underline{O(t)} \sim tA + t^2B + \dots$$

Irrelevant higher dimension operators

To remove irrelevant operators, we need $t \rightarrow 0$

$$\bar{g}(\mu = 1/\sqrt{8t}) \rightarrow 0 \text{ (in flow scheme)}$$

→ we can evaluate renormalization constant perturbatively

Simulation set up

$N_f = 2+1$ QCD

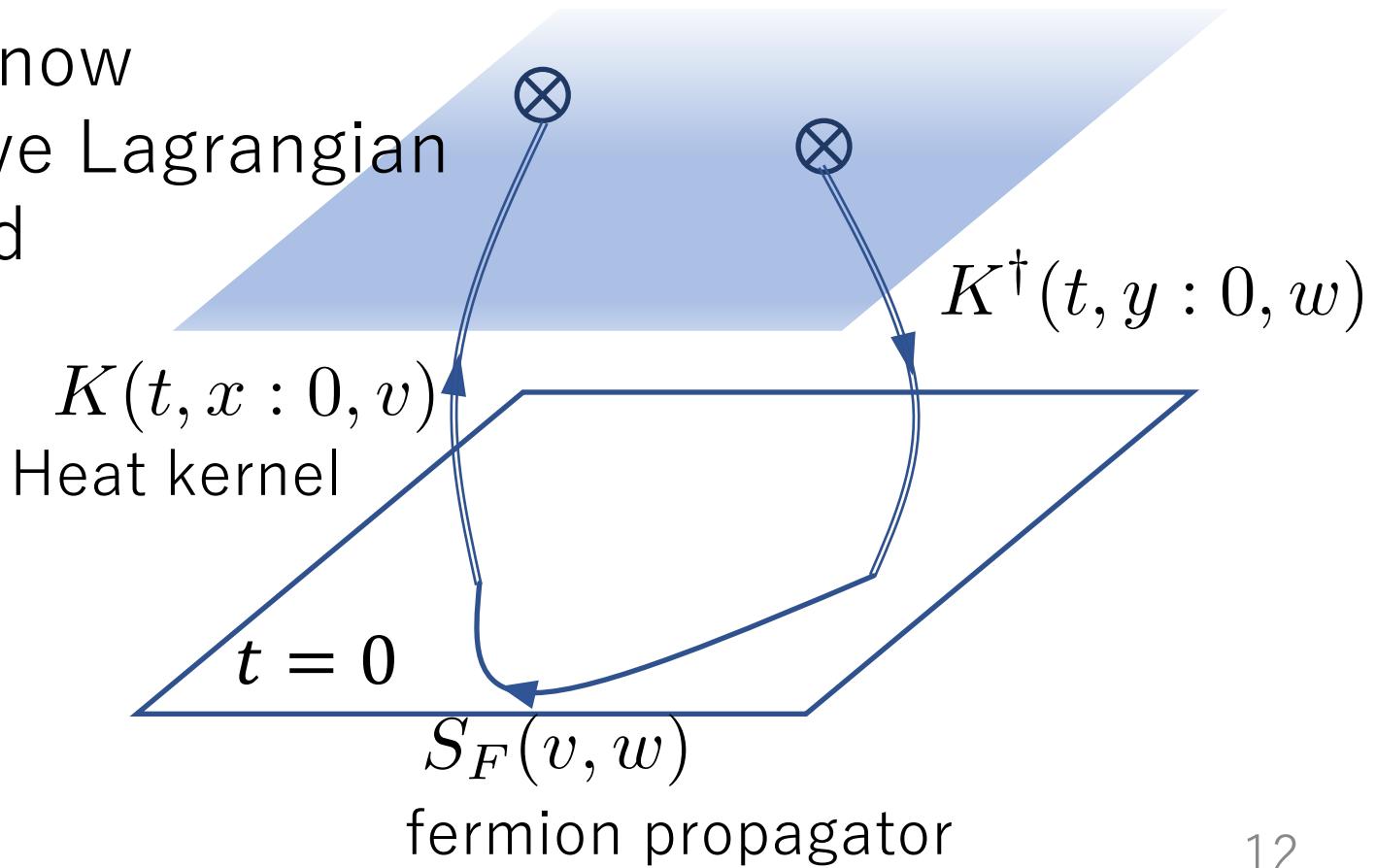
- Wilson fermion + NP clover $c_{sw} = 1.628$
- Iwasaki gauge action
- $a \sim 0.07$ fm, $\beta = 2.05$
- $m_\pi/m_\rho \sim 0.63$, $m_{\eta_{ss}}/m_\phi \sim 0.74$
- $\kappa_{ud} = 0.1356$, $\kappa_s = 0.1351$

Ns	Nt	T [MeV]	conf
28	56	0	650
	16	174	1440
	14	199	1270
32	12	232	1290
	10	279	780
	8	348	510

Flowed fermion propagator

$$\overline{\chi}(t, y) \chi(t, x) \neq (\not{D}[B_\mu(t, x)] + m)^{-1}$$

We don't know
the effective Lagrangian
in bulk field



Take $t \rightarrow 0$ limit

In gradient flow method $\sim tA + t^2B + t^3C + \dots$

$$\chi^{\text{full}}(t) = \{\chi^{\text{full}}\}_R + O(t)$$

1. Find linear window

$$\underline{\chi^{\text{full}}(t) = \{\chi^{\text{full}}\}_R + tA}$$

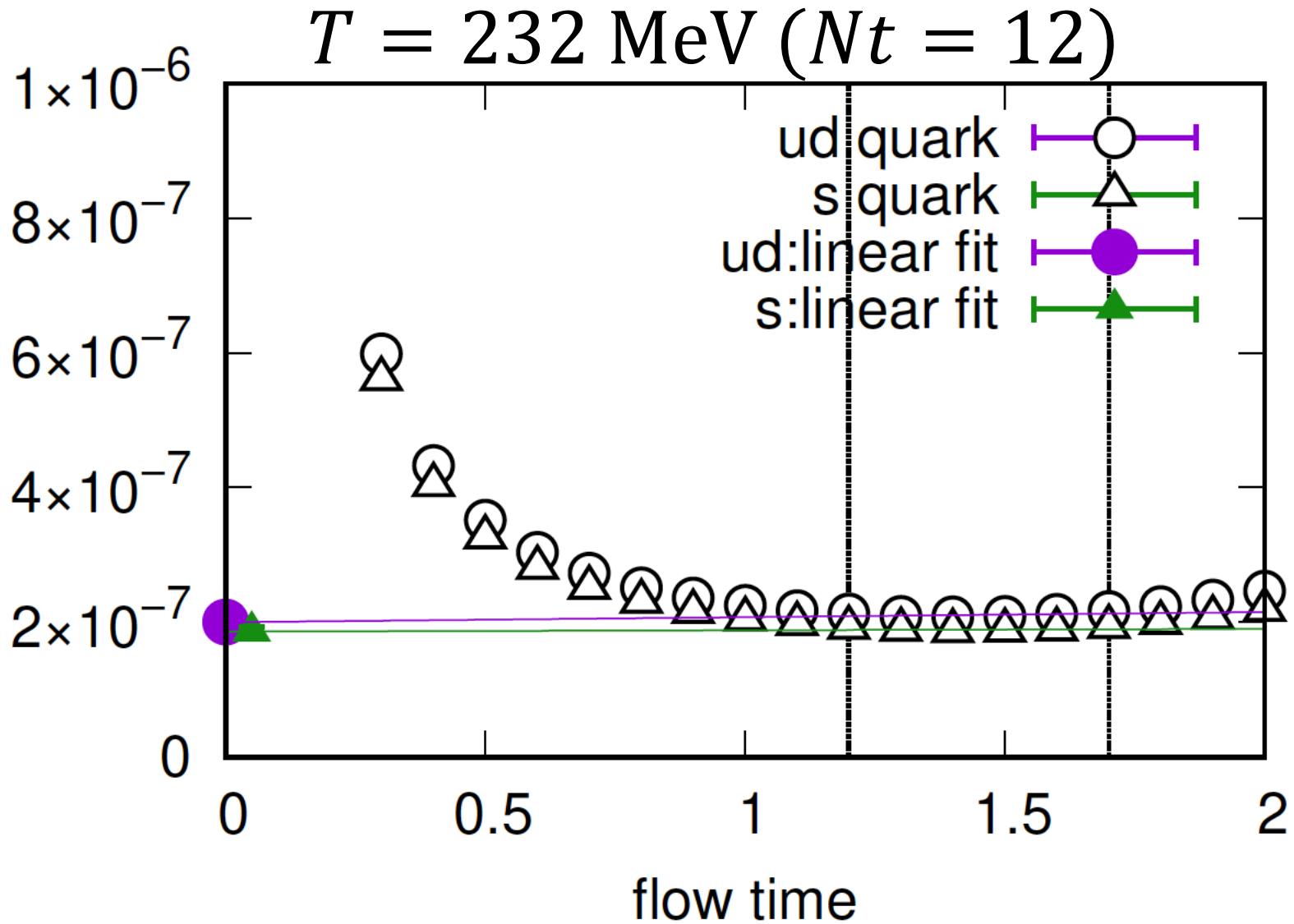
Include lattice artifact,

$$\chi^{\text{full}}(t) = \{\chi^{\text{full}}\}_R + O(t)$$

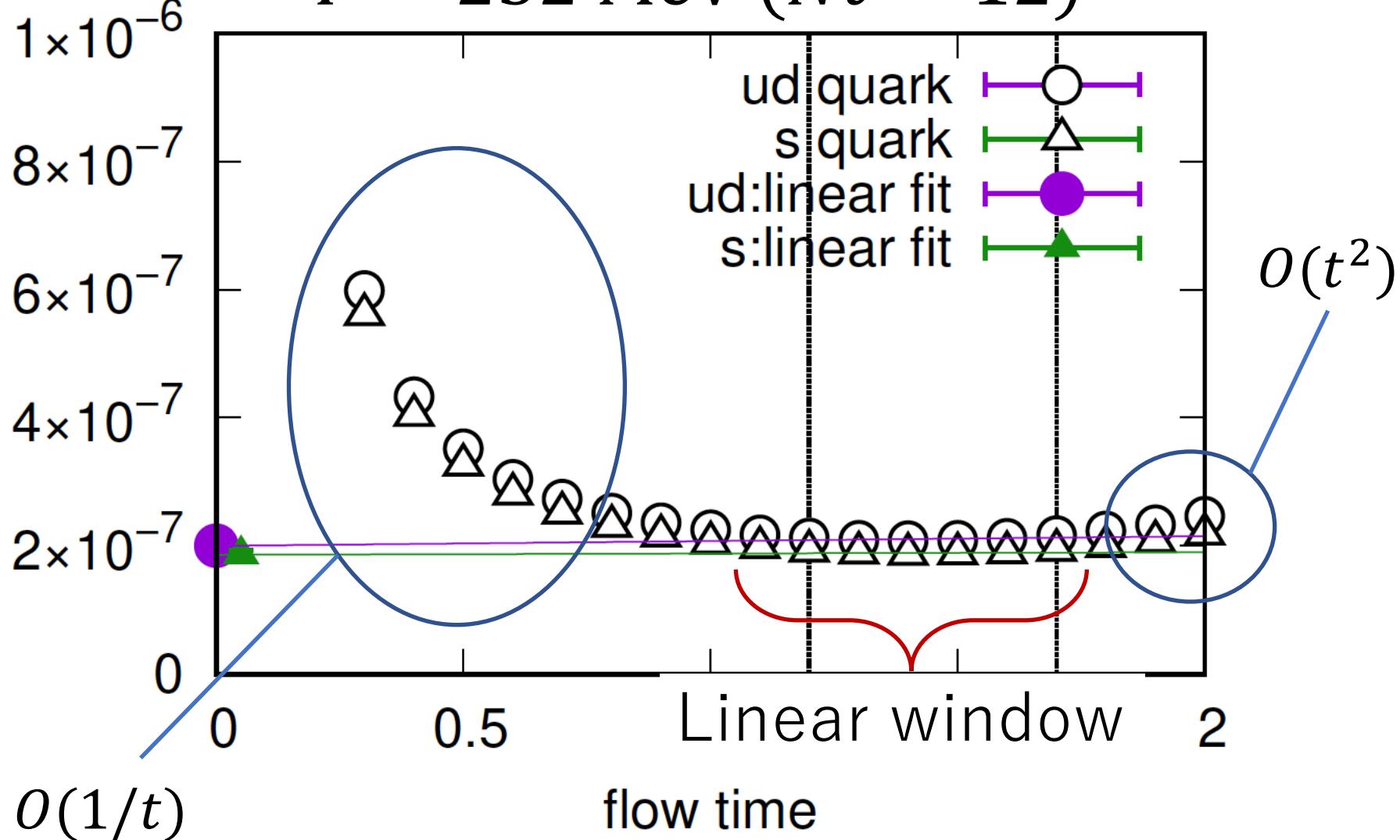
$$+ O(a^2/t, a^2T^2, a^2m^2, a^2\Lambda_{QCD}^2)$$

2. Let's include nonlinear term

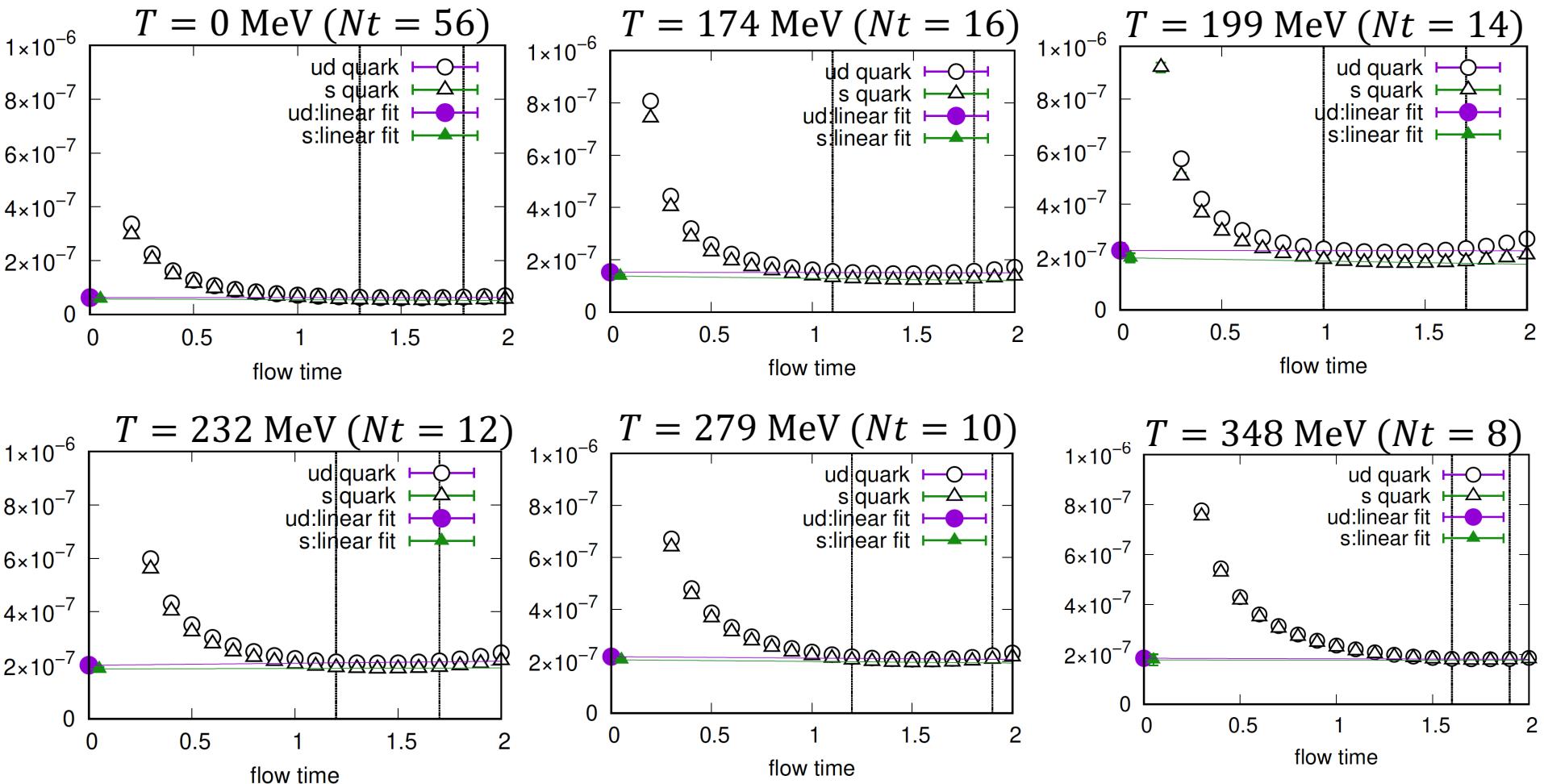
$$\chi^{\text{full}}(t) = \{\chi^{\text{full}}\}_R + tA + t^2B + \frac{a^2}{t}C$$



$T = 232 \text{ MeV} (Nt = 12)$

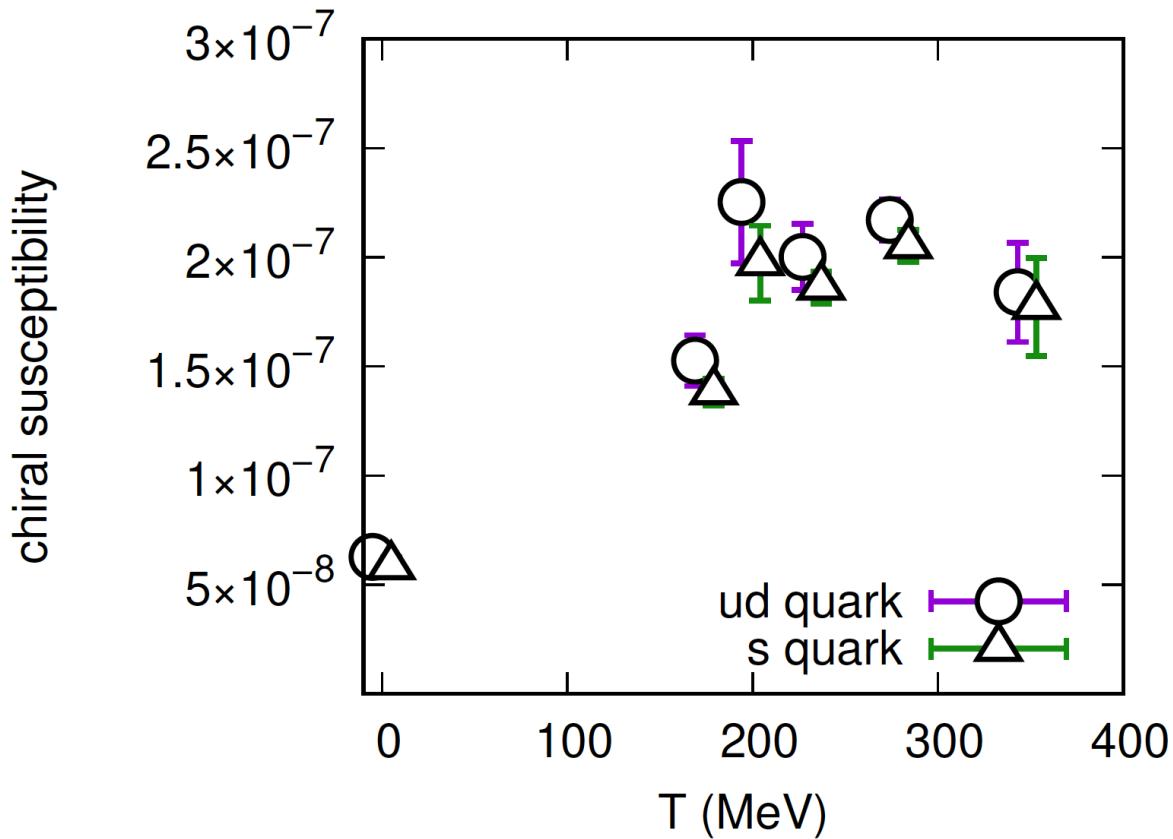


$t \rightarrow 0$ extrapolation



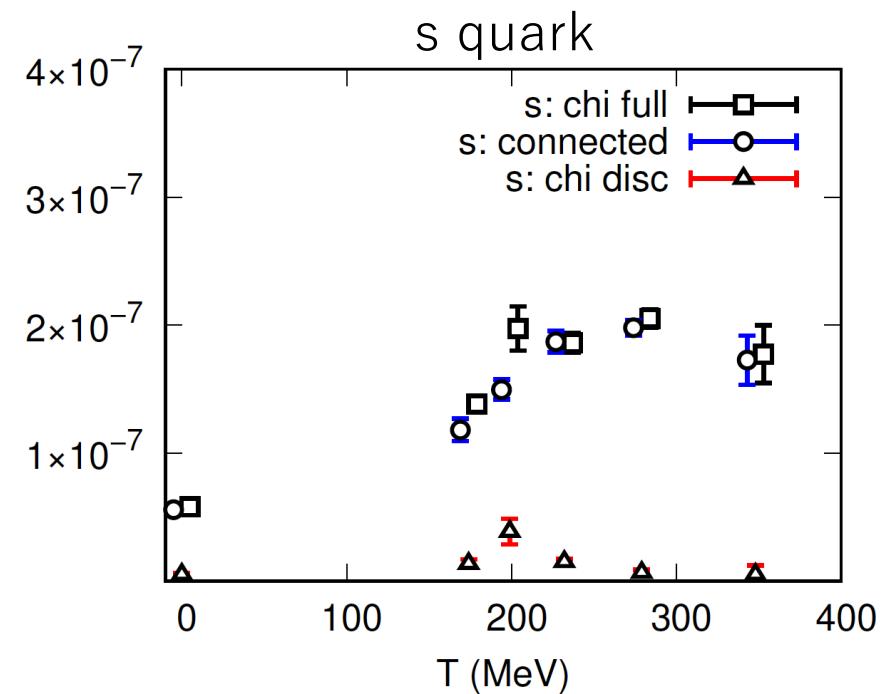
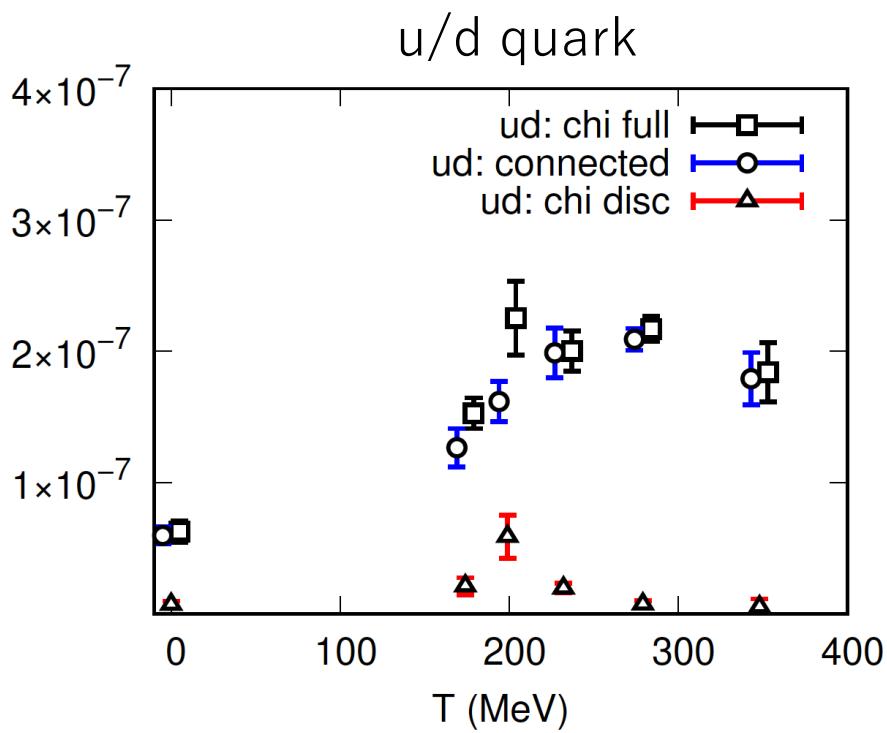
We find linear window in all temperature

Chiral susceptibility $\chi_{f\bar{f}}^{\text{full}}$



Have NO peak within error bars
Slight peak on plateau

Disconnected vs Connected



Connected parts does not singular

Connected contribution is very large:
 $\chi_{f\bar{f}}^{\text{conn}}/\chi_{f\bar{f}}^{\text{disc}} \sim 20$ to each temperature

Conclusion

We use gradient flow method to calculate Chiral susceptibility with Wilson fermion

Behavior of $\chi_{\bar{f}f}^{\text{full}}$ is very similar to chiral lattice fermions

Our connected part contribution is much larger than disconnected part.

Future work

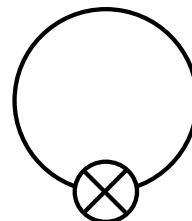
Try nonlinear fit

Take continuum limit

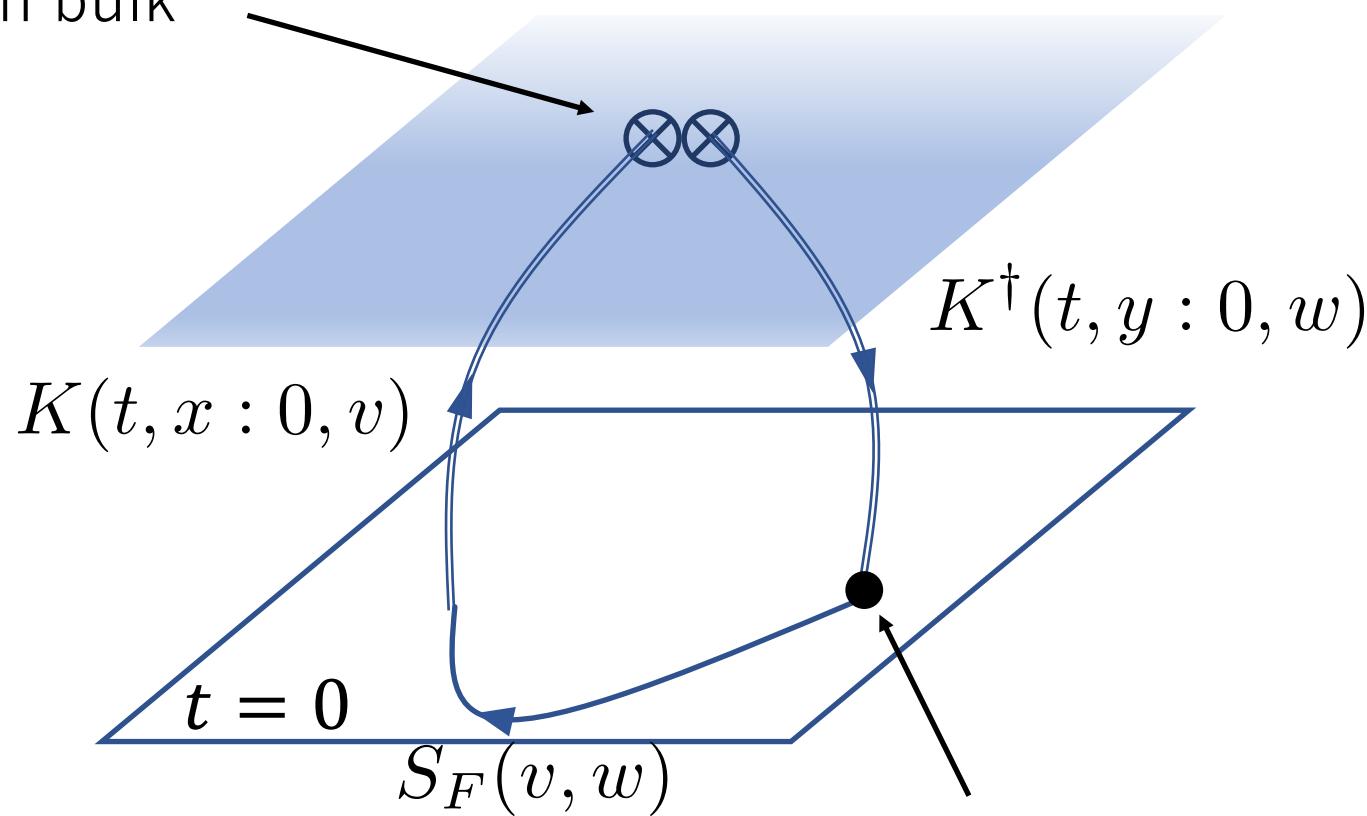
Check chiral WT identity

Back up

Disconnected part: 1-pt function

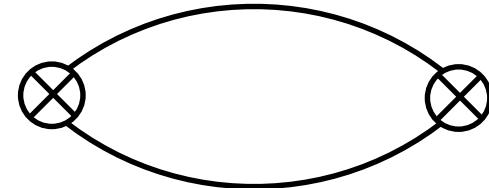


Put the sink
on bulk

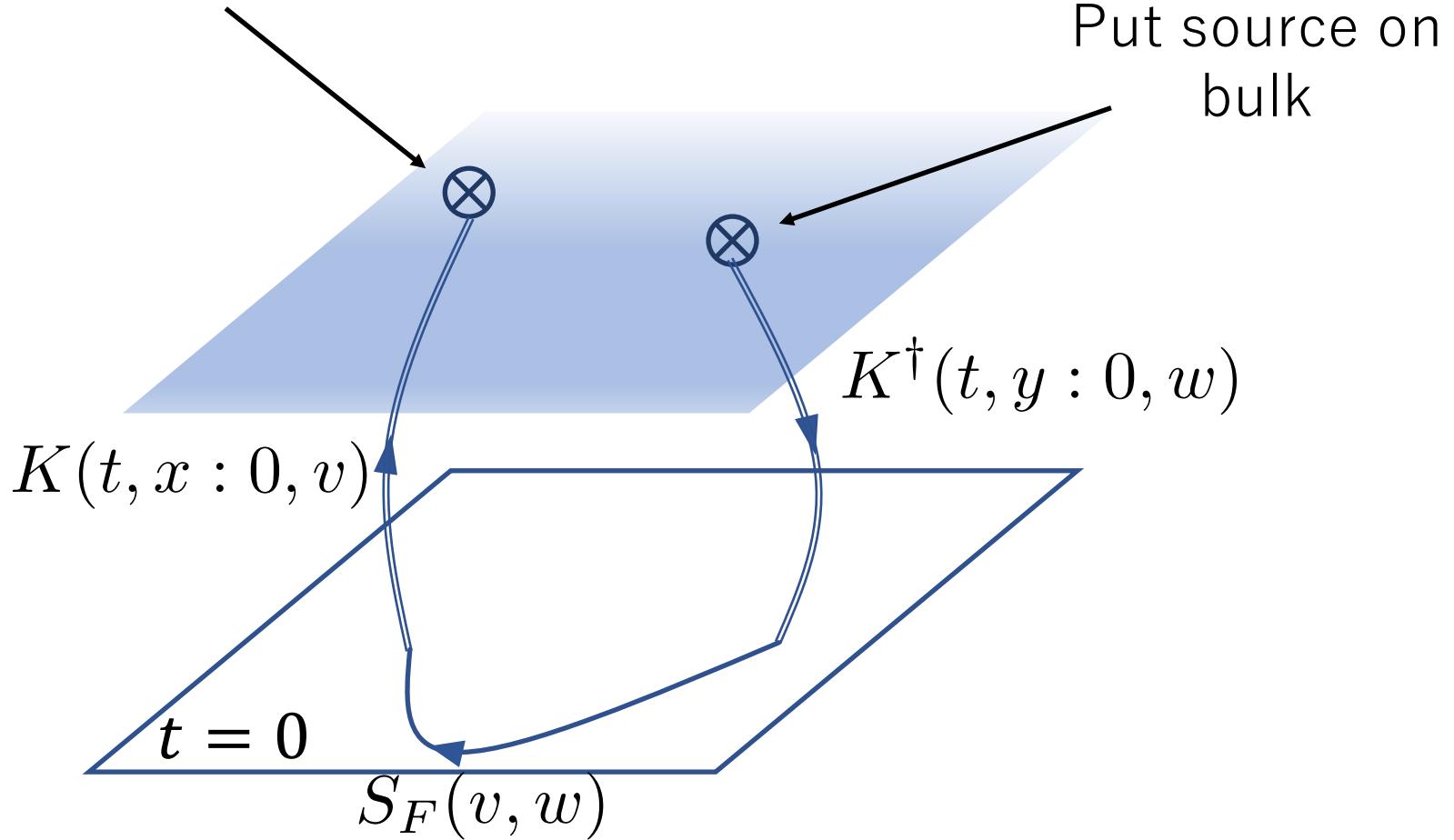


Put on random source
on 4-dim spacetime

Connected part: 2-pt function

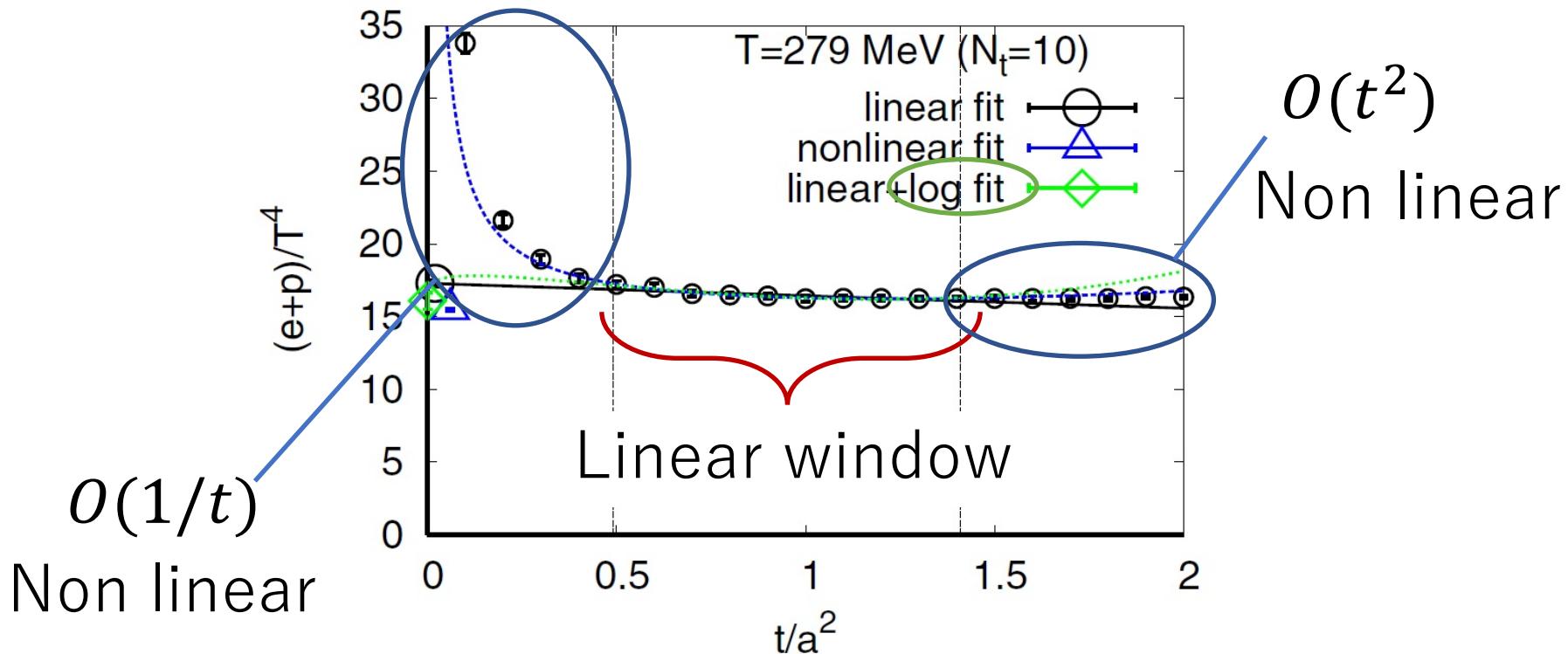


Put the sink on bulk



Example (for full QCD)

WHOT-QCD collaboration (2017)



Log term: possible higher loop coefficient

$$\chi(t) = c_s^2(t) \varphi^4(t) \left[\langle (\bar{\chi}(t, x) \chi(t, x))^2 \rangle - (\langle \bar{\chi}(t, x) \chi(t, x) \rangle)^2 \right]$$