# String breaking with 2+1 dynamical fermions using the stochastic LapH method

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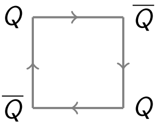
- The static potential $V(r)$, is defined as the energy of the ground state containing a static quark Q and static anti-quark $\overline{Q}$ separated by distance $\mathbf{r} = \mathbf{y} - \mathbf{x}$


- As a consequence of confinement, the energy between the quark-antiquark pair is contained inside a color flux tube, the so called string
- The time correlation function yields the Wilson loop:

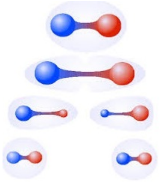
$$\langle W(T, r)\rangle \propto e^{-V(r,T)}$$




- If no pair creation from vacuum is allowed, V(r) can be parametrized by:

$$V(r) = A + \frac{B}{r} + \sigma r$$

In the full theory with dynamical quarks in the fundamental representaion, the string will break due to creation of a pair of light quarks $q\bar{q}$, which combine into two static light mesons B=$q\overline{Q}$.



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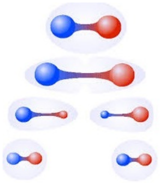

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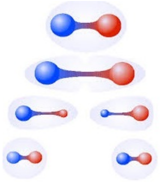

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**Problem**: This behavior could not be seen in early lattice simulations

**Reasons**:

- Lattice size has to be $> 2r_b$
- weak signal-to-noise ratio for distances $> 1fm$
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**Solution**:

- SB is a mixing phenomenon
- $\left|Q\overline{Q}\right\rangle, \left|B\overline{B}\right\rangle$ are not QCD eigenstates
- the ground state $|1\rangle$ and first excited state $|2\rangle$ are superpositions of $\left|Q\overline{Q}\right\rangle, \left|B\overline{B}\right\rangle \rightarrow$ avoided level crossing

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Using this Ansatz, evidence for SB was found for the SU(2) Higgs model
Knechtli and Sommer [arXiv:hep-lat/9807022], Philipsen and Wittig [arXiv:hep-lat/arXiv:hep-lat/9807020]
and later for $n_f = 2$ QCD Bali et al.[arXiv:hep-lat/0505012]

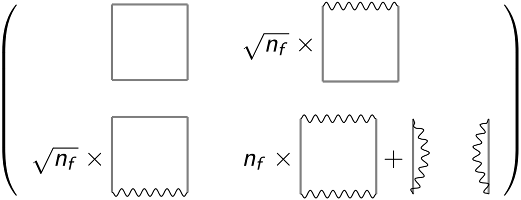
The ground state and first excited state of the static potential are now determined by a variational technique from a correlation matrix $C(t)$

The interpolators $\mathcal{O}_Q$ and $\mathcal{O}_B$ corresponding to the string and the state consisting of two staticlight mesons are given by:

$$\mathcal{O}_Q(t) \equiv \overline{Q}(\mathbf{y},t)\Gamma\mathcal{W}(\mathbf{y},\mathbf{x},t)Q(\mathbf{x},t)$$

$$\mathcal{O}_B(t) \equiv \overline{Q}(\mathbf{y},t)\Gamma q^i(\mathbf{y},t)\ \overline{q^i}(\mathbf{x},t)\Gamma Q(\mathbf{x},t)$$

$$C(t) = \begin{pmatrix} C_{Q\overline{Q}} = \langle \mathcal{O}_Q(t)\overline{\mathcal{O}}_Q(0)\rangle & C_{B\overline{Q}} = \langle \mathcal{O}_B(t)\overline{\mathcal{O}}_Q(0)\rangle \\ C_{Q\overline{B}} = \langle \mathcal{O}_Q(t)\overline{\mathcal{O}}_B(0)\rangle & C_{B\overline{B}} = \langle \mathcal{O}_B(t)\overline{\mathcal{O}}_B(0)\rangle \end{pmatrix}$$

$=$ 

## Our aim

Analyze SB using state of the art methods in order to

- examine SB on a bigger lattice with smaller sea quark mass
- examine $n_f = 2 + 1$ flavors to observe effect of including strange quark

## Challenges

- need for 'all-to-all propagators'
- large lattice size needed
- large set of off-axis distances needed for good resolution of SB

# Methods and setup

- use of stochastic LapH method for 'all to all propagators'
- use of suitable set of ensembles with $n_f = 2 + 1$ generated by the CLS effort Bruno et al.[arXiv:1411.3982]

| id | a[fm] | $N_s$ | $N_t$ | $m_\pi$ [MeV] | $m_K$ [MeV] | $m_\pi L$ |
|---|---|---|---|---|---|---|
| B105 | 0.086 | 32 | 64 | 280 | 460 | 3.9 |
| H101 | 0.086 | 32 | 96 | 420 | 420 | 5.8 |
| H102 | 0.086 | 32 | 96 | 350 | 440 | 4.9 |
| H105 | 0.086 | 32 | 96 | 280 | 460 | 3.9 |
| C101 | 0.086 | 48 | 96 | 220 | 470 | 4.7 |
| D100 | 0.086 | 64 | 128 | 130 | 480 | 3.7 |
| H200 | 0.064 | 32 | 96 | 420 | 420 | 4.4 |
| N200 | 0.064 | 48 | 128 | 280 | 460 | 4.4 |
| D200 | 0.064 | 64 | 128 | 200 | 480 | 4.2 |
| N300 | 0.05 | 48 | 128 | 420 | 420 | 5.1 |
| N301 | 0.05 | 48 | 128 | 410 | 410 | 4.9 |
| J303 | 0.05 | 64 | 192 | 260 | 470 | 4.1 |

- Distillation is a form of quark smearing that facilitates all-to-all propagators    Peardon et al.[arXiv:0905.2160]
- the important contributions to the quark propagator are encoded in smaller subspace, spanned by eigenvectors of covariant 3D Laplace operator

$\rightarrow$ smearing matrix S is a projector into LapH subspace

$$S_{xy}(t) = \sum_{k}^{N_{ev}} v_x^k(t) v_y^k(t)^\dagger \equiv VV^\dagger.$$

A smeared quark propagator $\mathcal{Q}$ now reads:

$$\mathcal{Q} = S\Omega^{-1}S = V(V^\dagger\Omega^{-1}V)V^\dagger,$$

where $\Omega^{-1} = \gamma_4 D^{-1}$

- computation and storage of the much smaller matrix $(V^\dagger\Omega^{-1}V)$
- to get constant physical smearing: $N_{inv} \propto N_{ev} \sim V$

- Stochastically estimate the inverse of the large matrix using random noise vectors $\eta$

$$E(\eta_i) = 0 \qquad E(\eta_i \eta_j^*) = \delta_{ij}$$

- path integrals evaluated using MC, statistical errors for the correlators limited by the statistical fluctuations from gauge-field sampling
- $\rightarrow$ propagators only have to be estimated to a comparable accuracy

If for $N_R$ noise sources, $\Omega X^r = \eta^r$ is solved

$$\Omega^{-1}_{ij} \approx N_R^{-1} \sum_{r=1}^{N_R} X_i^r \eta_j^{r*}.$$

"Monte Carlo within a Monte Carlo"

Use dilution of the noise vectors to reduce variance

Foley et al. [arXiv:hep-lat/0505023]

A Dilution scheme amounts to the application of a complete set of projection operators $P^{(b)}$

Define diluted noise $\rho^{r[b]} = P^{(b)}\rho^r$

If $X^{r[b]}$ is solution of $\Omega X^{r[b]} = \rho^{r[b]}$ a 'better' MC estimate is:

$$\Omega_{ij}^{-1} \approx \frac{1}{N_R} \sum_{r=1}^{N_R} \sum_{b=1}^{N_b} X_i^{r[b]} \rho_j^{r[b]*}.$$

Noise $\rho$ is introduced only in the LapH-subspace

Morningstar et al. [arXiv:1104.3870]

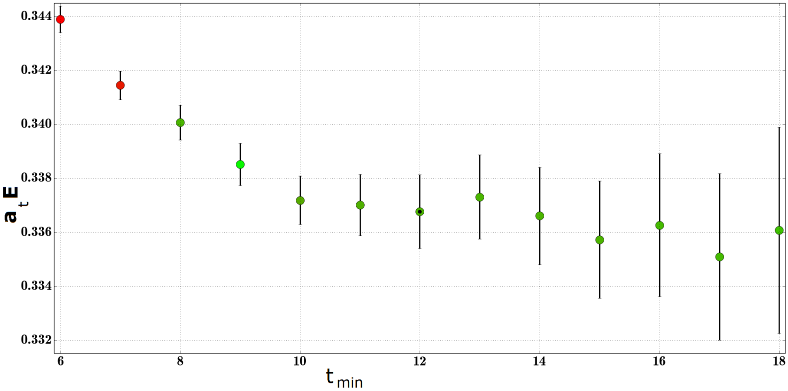
$\rightarrow N_{inv} \propto N_R N_b$ Morningstar, Bulava, Hörz arXiv:1710.04545 [hep-lat]

Steps towards string breaking with 2+1 dynamical fermions using the stochastic LapH-method:

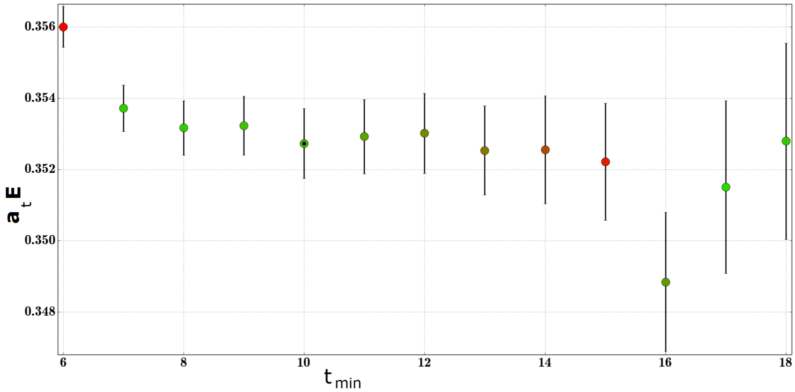
$$\text{N200 } (128{\times}48^3),\ n_f = 2+1$$

| id | a[fm] | $m_\pi$[MeV] | $m_K$[MeV] | $m_\pi L$ |
|---|---|---|---|---|
| N200 | 0.064 | 280 | 460 | 4.4 |

1. test stochastically estimated light quark propagators in correlation functions involving a static color source
2. extract $E_B$, the mass of the static-light meson



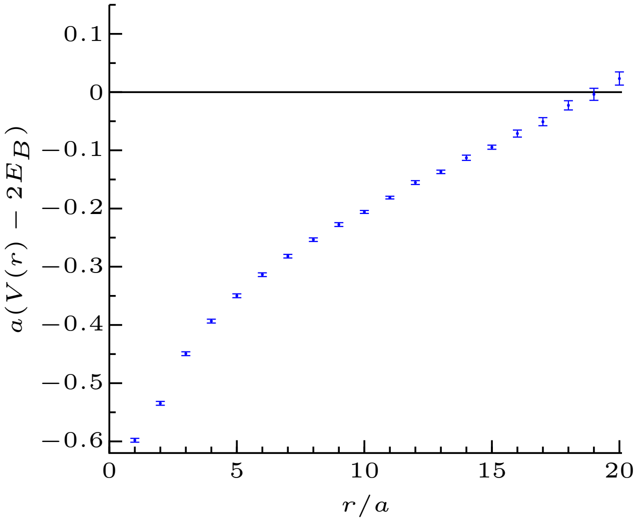

$t_{min}$-plot for the static-light meson on 200 configurations of N200, $t_{min}$ indicates starting point of exponential fit.The black dot represents the chosen value with a relative uncertainty of 0.4%. arXiv:1511.04029 [hep-lat]




$t_{min}$-plot for the static-strange meson, on 100 configurations of N200, $t_{min}$ indicates starting point of exponential fit. The black dot represents the chosen value with a relative uncertainty of 0.3%. arXiv:1511.04029 [hep-lat]

Steps towards string breaking with 2+1 dynamical fermions using the stochastic LapH-method:

1. test stochastically estimated light quark propagators in correlation functions involving a static color source $\surd$
2. extract $E_B$, the mass of the static-light meson $\surd$
3. calculate the static potential $V(r)$ using wilson loops to estimate the string breaking distance $r_b$ to observe where the string breaks




Ground state potential $V(r)$ on 1600 configurations of N200, following the method presented in Donnellan et al. [arXiv:1012.3037]

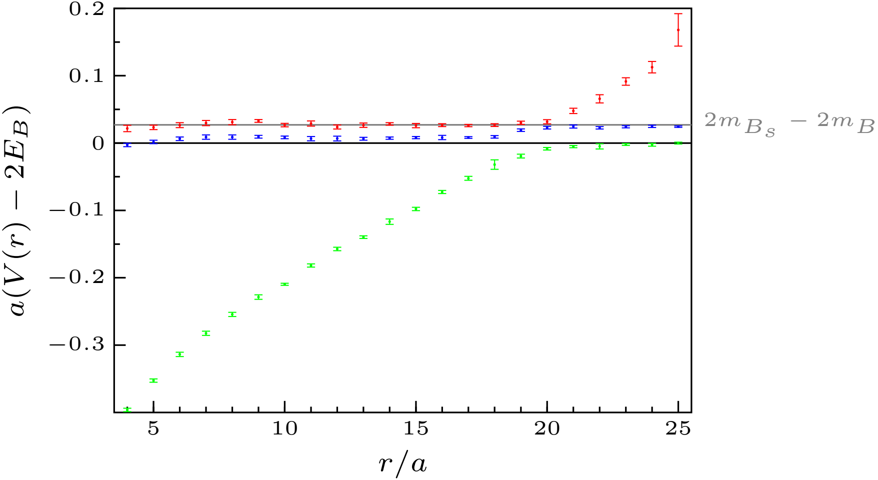
## Next step: perform full mixing analysis

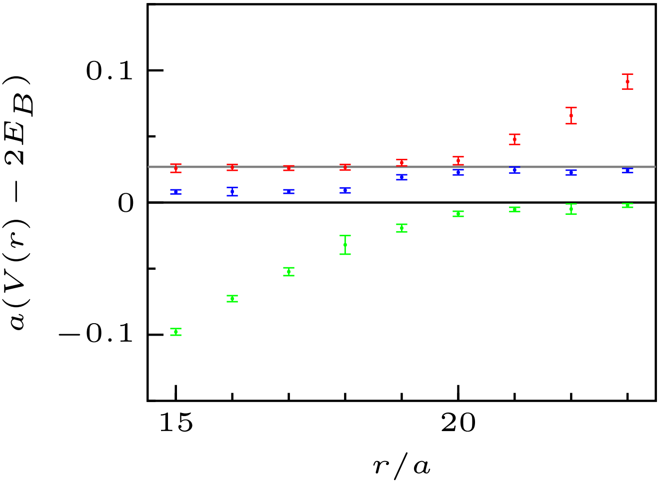
Inclusion of the strange quark yields $3 \times 3$ correlation matrix, larger if different levels of smearing are used

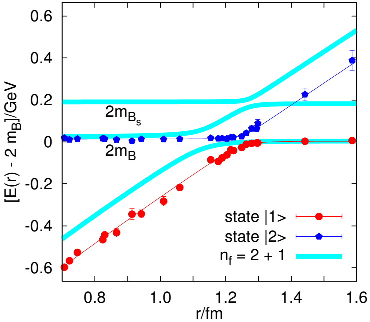
$$C(t) = \begin{pmatrix} C_{Q\overline{Q}}(t) & C_{B\overline{Q}}(t) & C_{B_s\overline{Q}}(t) \\ C_{Q\overline{B}}(t) & C_{BB}(t) & C_{B_s\overline{B}}(t) \\ C_{Q\overline{B_s}}(t) & C_{B\overline{B_s}}(t) & C_{B_s\overline{B_s}}(t) \end{pmatrix}$$

| id | $N_{ev}$ | line type | dilution scheme | $N_r$ light/strange | source time |
|---|---|---|---|---|---|
| N200 | 192 | fixed | (TF,SF,LI8) | 5 / 2 | 32,52 |
| | | relative | (TI8,SF,LI8) | 2 / 1 | - |

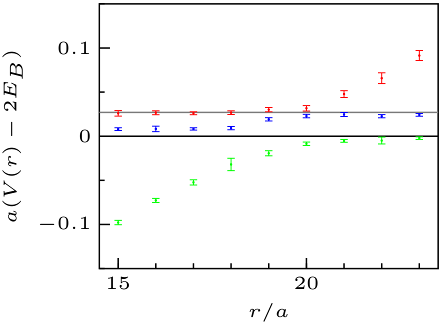
Preliminary results on 52 configurations (1800 for Wilson loops) of N200, only on-axis distances




0.1
0
−0.1
15
20
$r/a$
$a(V(r) - 2E_B)$




[Bali et al. 2005]



## Summary

- stochastic LapH method allows for accurate determinations of temporal correlations involving static quarks
- we see the effect of the strange sea-quark flavor, which results in a second mixing-phenomenon due to the formation of two static-strange mesons

## Outlook

- analysis of N200 (and D200) with full data set (105 configurations) and off-axis distances
- investigate dependence of string breaking distance and shape of gap on sea quark mass

$\longrightarrow$ repeat calculation on other ensembles