

# Meson correlations at high T QCD: $SU(2)_{CS}$ symmetry vs. free quarks

C. Rohrhofer, Y. Aoki, G. Cossu,  
L. Glozman, S. Hashimoto, S. Prelovsek

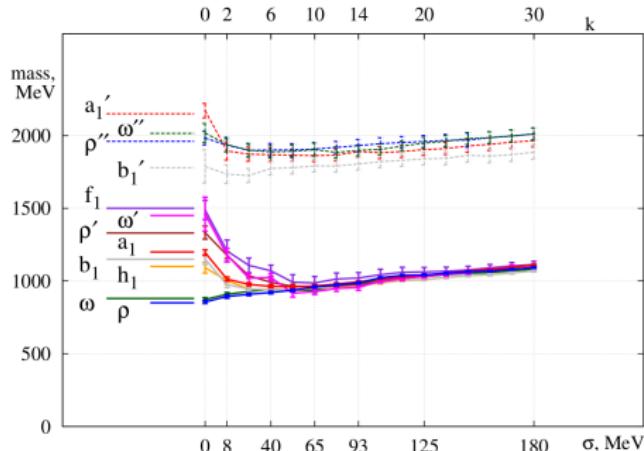
July 2018, Lattice 2018, East Lansing, MI



## Motivation: a numerical experiment

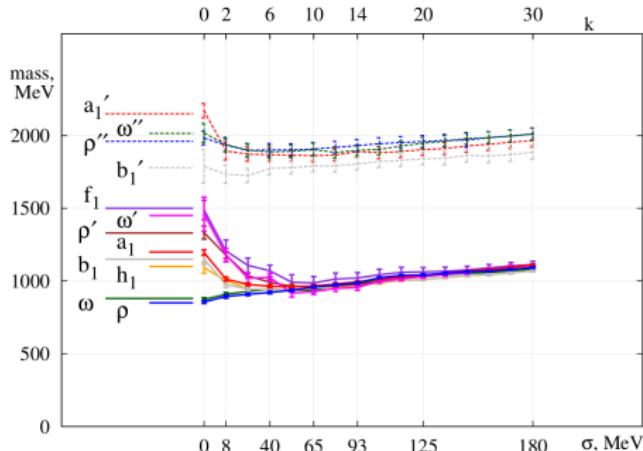
# Motivation: a numerical experiment

*Numerical studies of Hadron spectrum upon Dirac low-mode truncation  
(Chiral condensate  $\leftrightarrow$  Banks-Casher  $\leftrightarrow$  low modes)*



# Motivation: a numerical experiment

Numerical studies of Hadron spectrum upon Dirac low-mode truncation  
(Chiral condensate  $\leftrightarrow$  Banks-Casher  $\leftrightarrow$  low modes)



Chiral spin  $SU(2)_{CS}$  and  $SU(2n_f)$  symmetries derived

similarity due to suppression of low modes in high  $T$  QCD?

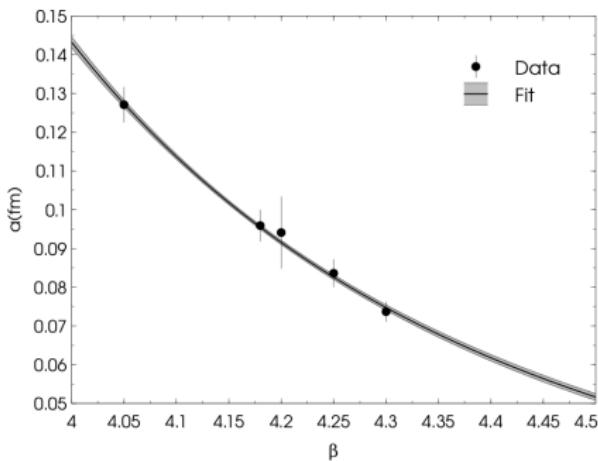
# High T study: Vector channel spatial correlators

CR, Y.Aoki, G.Cossu, H.Fukaya, L.Glozman, S.Hashimoto, C.B.Lang, S.Prelovsek (Phys.Rev.D96,094501)

# High T study: Vector channel spatial correlators

CR, Y.Aoki, G.Cossu, H.Fukaya, L.Glozman, S.Hashimoto, C.B.Lang, S.Prelovsek (Phys.Rev.D96,094501)

- $n_f = 2$  Möbius DW fermions, Symanzik gauge action
- $32^3 \times 8$  lattices,  $T_c = 175\text{MeV}$
- local isovectors  $\mathcal{O}_\Gamma(x) = \bar{q}(x)\Gamma q(x)$
- measuring spatial correlations in  $z$ -direction



$\beta$	$m_{ud}$	$T [\text{MeV}]$	
4.10	0.001	220	$1.3T_c$
4.18	0.001	260	$1.5T_c$
4.30	0.001	330	$1.9T_c$
4.37	0.005	380	$2.2T_c$

# Components of the Dirac algebra

# Components of the Dirac algebra

Fix direction of propagation (*z-direction*):

$$C_\Gamma(n_z) = \sum_{n_x, n_y, n_t} \langle \mathcal{O}_\Gamma(n_x, n_y, n_z, n_t) \mathcal{O}_\Gamma(0, 0, 0, 0)^\dagger \rangle$$

# Components of the Dirac algebra

Fix direction of propagation (*z-direction*):

$$C_\Gamma(n_z) = \sum_{n_x, n_y, n_t} \langle \mathcal{O}_\Gamma(n_x, n_y, n_z, n_t) \mathcal{O}_\Gamma(0, 0, 0, 0)^\dagger \rangle$$

Using  $\partial_\mu j^\mu = \partial_\mu j_5^\mu = 0$  we identify the Gamma structures for the Vectors:

$$\mathbf{v} = \begin{pmatrix} \gamma_1 & = Vx \\ \gamma_2 & = Vy \\ \gamma_4 & = Vt \end{pmatrix} \quad \mathbf{A} = \begin{pmatrix} \gamma_1 \gamma_5 & = Ax \\ \gamma_2 \gamma_5 & = Ay \\ \gamma_4 \gamma_5 & = At \end{pmatrix}$$

$$\mathbf{T} = \begin{pmatrix} \gamma_1 \gamma_3 & = Tx \\ \gamma_2 \gamma_3 & = Ty \\ \gamma_4 \gamma_3 & = Tt \end{pmatrix} \quad \mathbf{X} = \begin{pmatrix} \gamma_1 \gamma_3 \gamma_5 & = \gamma_2 \gamma_4 & = Xx \\ \gamma_2 \gamma_3 \gamma_5 & = \gamma_4 \gamma_1 & = Xy \\ \gamma_4 \gamma_3 \gamma_5 & = \gamma_1 \gamma_2 & = Xt \end{pmatrix}$$

$\gamma_3$  &  $\gamma_3 \gamma_5$  no propagation due to current conservation!

+ Pion, Scalar

# Components of the Dirac algebra

Fix direction of propagation (*z-direction*):

$$C_\Gamma(n_z) = \sum_{n_x, n_y, n_t} \langle \mathcal{O}_\Gamma(n_x, n_y, n_z, n_t) \mathcal{O}_\Gamma(0, 0, 0, 0)^\dagger \rangle$$

Using  $\partial_\mu j^\mu = \partial_\mu j_5^\mu = 0$  we identify the Gamma structures for the Vectors:

$$\mathbf{v} = \begin{pmatrix} \gamma_1 & = Vx \\ \gamma_2 & = Vy \\ \gamma_4 & = Vt \end{pmatrix} \quad \mathbf{A} = \begin{pmatrix} \gamma_1 \gamma_5 & = Ax \\ \gamma_2 \gamma_5 & = Ay \\ \gamma_4 \gamma_5 & = At \end{pmatrix}$$

$$\mathbf{T} = \begin{pmatrix} \gamma_1 \gamma_3 & = Tx \\ \gamma_2 \gamma_3 & = Ty \\ \gamma_4 \gamma_3 & = Tt \end{pmatrix} \quad \mathbf{X} = \begin{pmatrix} \gamma_1 \gamma_3 \gamma_5 & = \gamma_2 \gamma_4 & = Xx \\ \gamma_2 \gamma_3 \gamma_5 & = \gamma_4 \gamma_1 & = Xy \\ \gamma_4 \gamma_3 \gamma_5 & = \gamma_1 \gamma_2 & = Xt \end{pmatrix}$$

$\gamma_3$  &  $\gamma_3 \gamma_5$  no propagation due to current conservation!

+ Pion, Scalar

# What to expect from $\mathcal{L}_{QCD}$ and $\chi S?$

	<i>Pseudoscalar</i>	<i>Scalar</i>
<b>PS</b>	$\bar{q}(\vec{\tau} \otimes \gamma_5)q$	$\bar{q}(\vec{\tau} \otimes \mathbb{1}_D)q$
	<i>Vector</i>	<i>Axial Vector</i>
<b>V</b>	$\bar{q}(\vec{\tau} \otimes \gamma_k)q$	$\bar{q}(\vec{\tau} \otimes \gamma_5\gamma_k)q$
	<i>Tensor Vector</i>	<i>Axial Tensor V.</i>
<b>T</b>	$\bar{q}(\vec{\tau} \otimes \gamma_3\gamma_k)q$	$\bar{q}(\vec{\tau} \otimes \gamma_5\gamma_3\gamma_k)q$

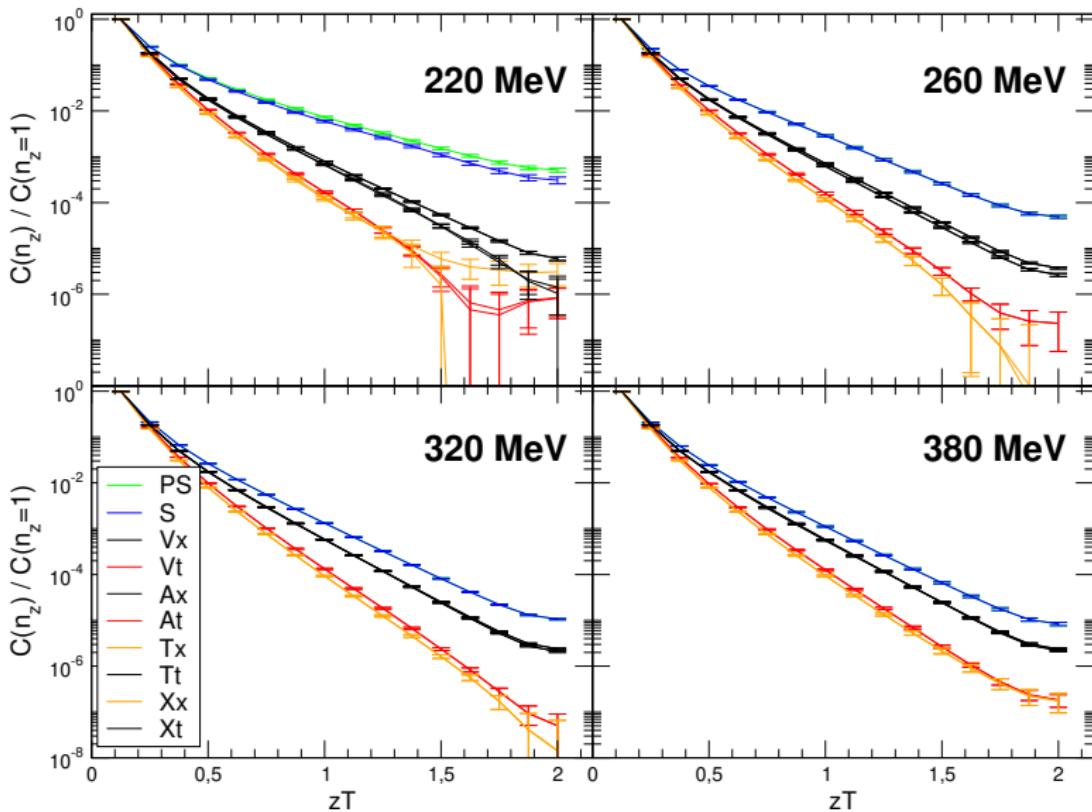
# What to expect from $\mathcal{L}_{QCD}$ and $\chi S?$

<b>PS</b>	<i>Pseudoscalar</i>	$\xleftrightarrow{U(1)_A}$	<b>S</b>	<i>Scalar</i>
	$\bar{q}(\vec{\tau} \otimes \gamma_5)q$			$\bar{q}(\vec{\tau} \otimes \mathbb{1}_D)q$
<b>V</b>	<i>Vector</i>	$\xleftrightarrow{SU(2)_A}$	<b>A</b>	<i>Axial Vector</i>
	$\bar{q}(\vec{\tau} \otimes \gamma_k)q$			$\bar{q}(\vec{\tau} \otimes \gamma_5\gamma_k)q$
<b>T</b>	<i>Tensor Vector</i>	$\xleftrightarrow{U(1)_A}$	<b>X</b>	<i>Axial Tensor V.</i>
	$\bar{q}(\vec{\tau} \otimes \gamma_3\gamma_k)q$			$\bar{q}(\vec{\tau} \otimes \gamma_5\gamma_3\gamma_k)q$

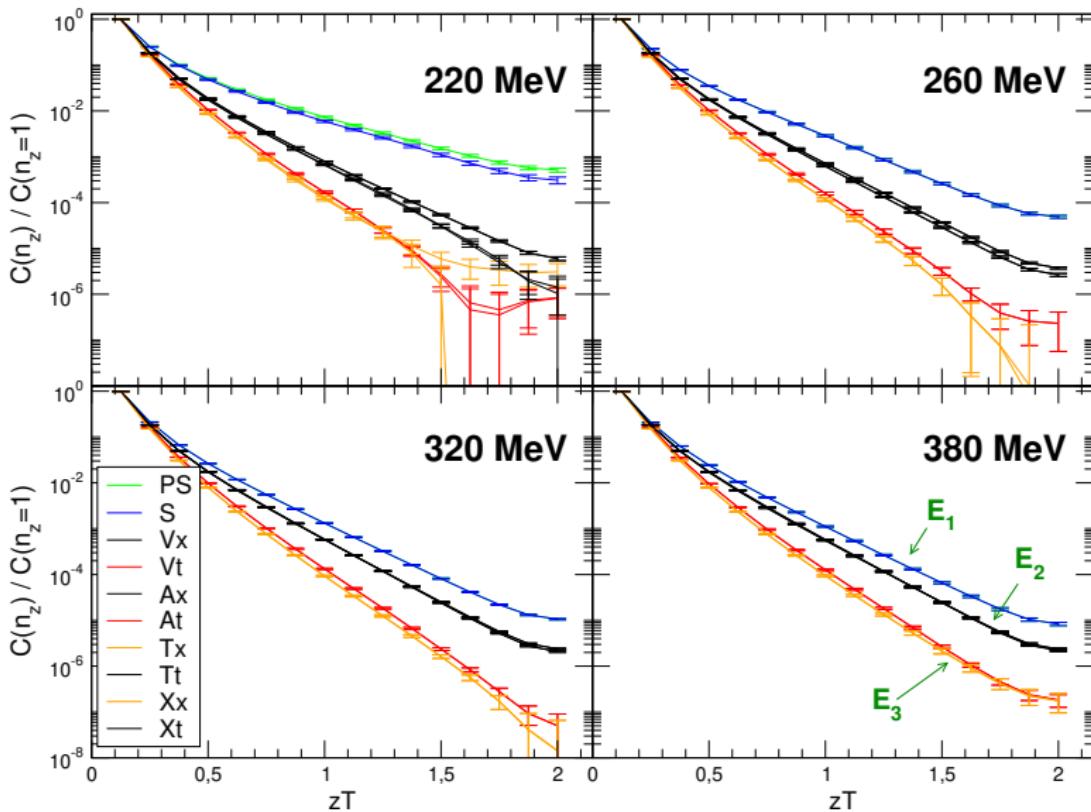
- $U(1)_A$  broken: *by  $\langle \bar{q}q \rangle$  and axial anomaly*
- $SU(2)_L \times SU(2)_R$  broken: *by  $\langle \bar{q}q \rangle$*

## Finite T. spatial correlations

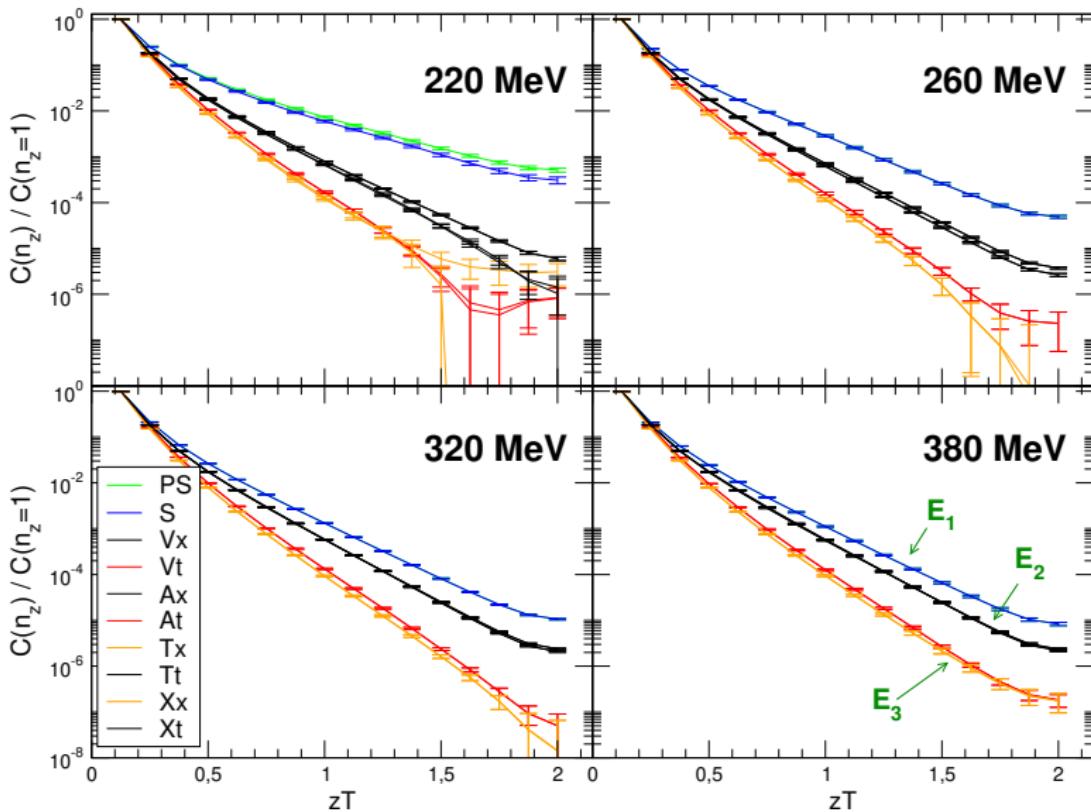
# Finite T. spatial correlations



# Finite T. spatial correlations



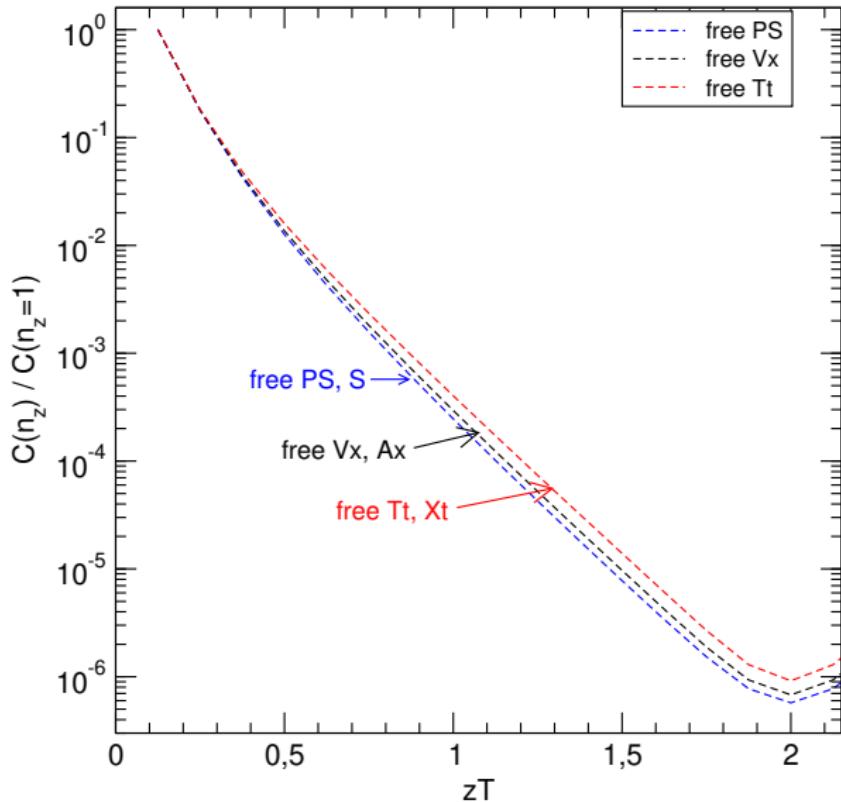
# Finite T. spatial correlations



Well pronounced multiplet structure: indication for symmetry!

## $E_1$ and $E_2$ multiplets

# $E_1$ and $E_2$ multiplets



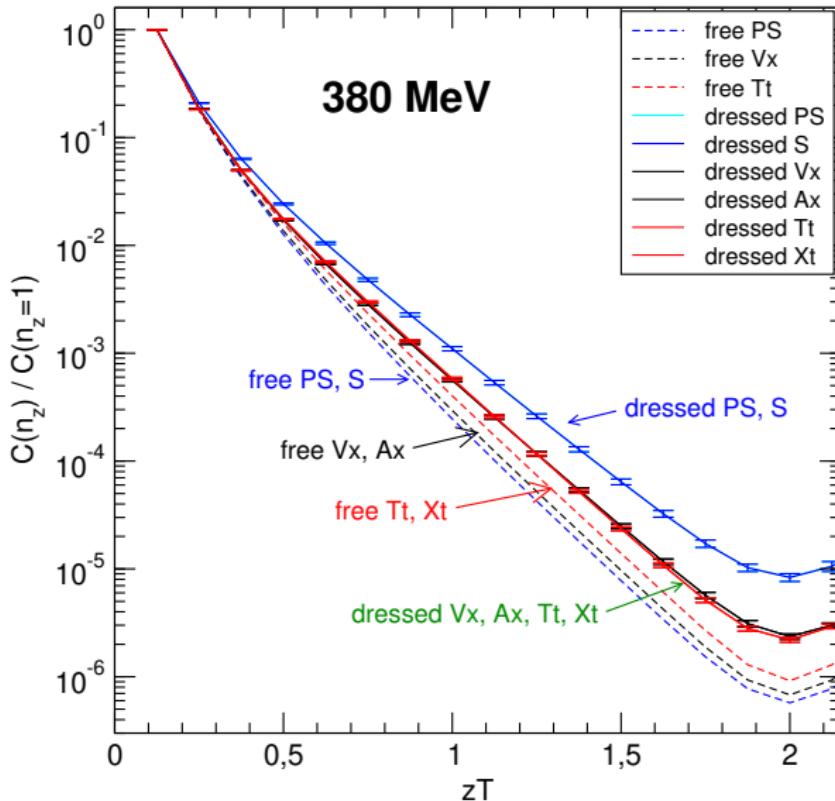
free ( $U(x)_\mu = \mathbb{1}$ ),  
non-interacting quarks:  
**chiral symmetry**

$$U(1)_A : S \leftrightarrow PS$$

$$SU(2)_A : V_x \leftrightarrow A_x$$

$$U(1)_A : T_t \leftrightarrow X_t$$

# $E_1$ and $E_2$ multiplets



free ( $U(x)_\mu = \mathbb{1}$ ),  
non-interacting quarks:  
**chiral symmetry**

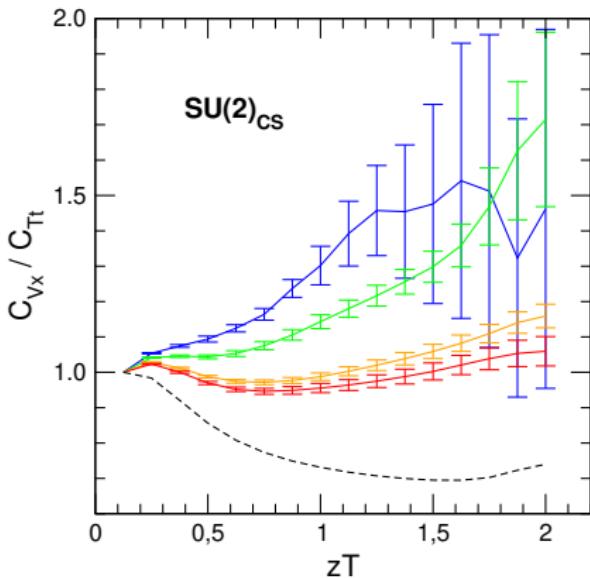
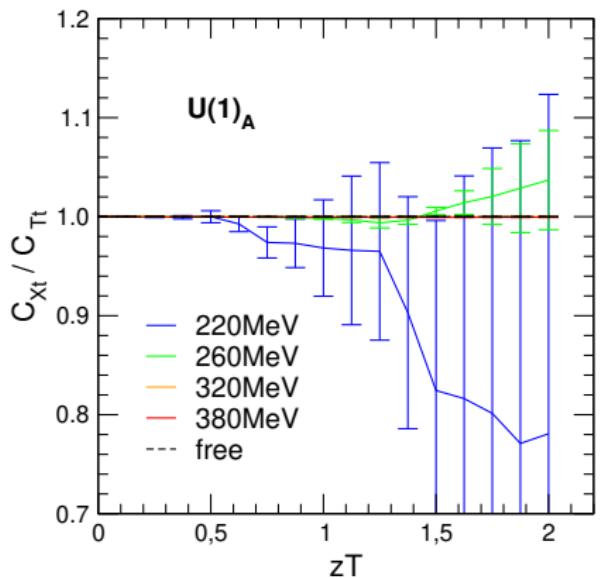
$$U(1)_A : S \leftrightarrow PS$$

$$SU(2)_A : V_x \leftrightarrow A_x$$

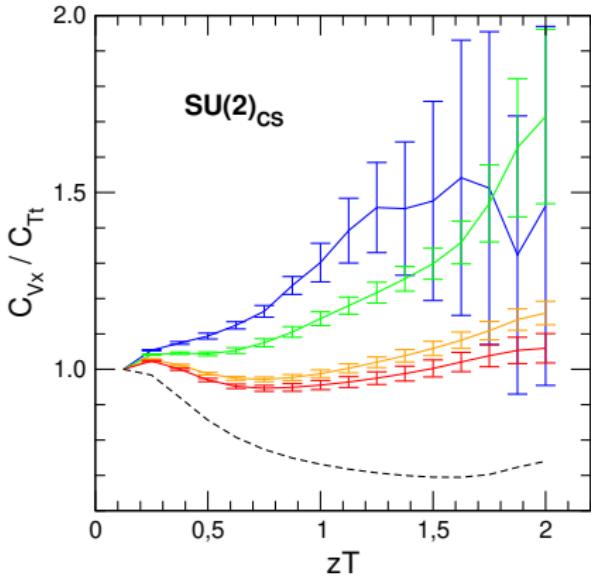
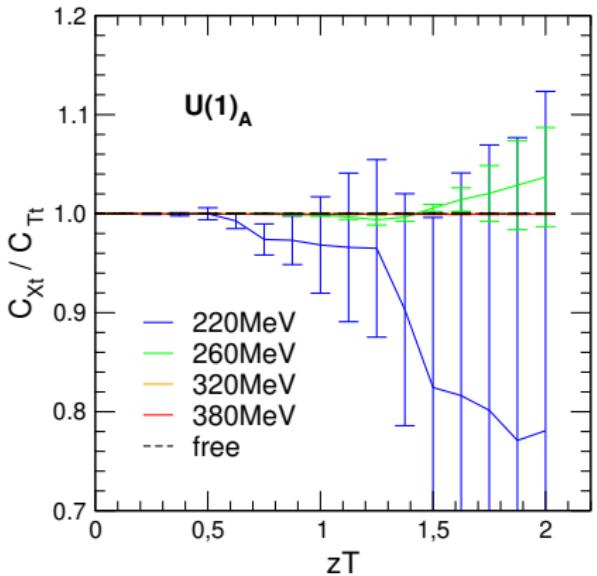
$$U(1)_A : T_t \leftrightarrow X_t$$

dressed meson  
correlators:  
**larger symmetry**

# $U(1)_A$ and $SU(2)_{CS}$ detailed ratios



# $U(1)_A$ and $SU(2)_{CS}$ detailed ratios



$SU(2)_{CS}$  breaking at 5% level for  $\sim 2T_c$

# $SU(2)_{CS}$ and $SU(4)$ symmetries

## $SU(2)_{CS}$ and $SU(4)$ symmetries

- ◊ for spatial  $z$ -correlators generated by representations:

$$\begin{array}{lll} R_1 : & \{\gamma_1, -i\gamma_5\gamma_1, \gamma_5\} & \Rightarrow \\ R_2 : & \{\gamma_2, -i\gamma_5\gamma_2, \gamma_5\} & V_y \leftrightarrow T_t \leftrightarrow X_t \\ & & V_x \leftrightarrow T_t \leftrightarrow X_t \end{array}$$

## $SU(2)_{CS}$ and $SU(4)$ symmetries

- ◊ for spatial  $z$ -correlators generated by representations:

$$\begin{aligned} R_1 : \quad & \{\gamma_1, -i\gamma_5\gamma_1, \gamma_5\} & \Rightarrow & \quad V_y \leftrightarrow T_t \leftrightarrow X_t \\ R_2 : \quad & \{\gamma_2, -i\gamma_5\gamma_2, \gamma_5\} & & V_x \leftrightarrow T_t \leftrightarrow X_t \end{aligned}$$

- ◊ Minimal group containing  $SU(2)_{CS}$  and  $\chi S$  is  $SU(4)$ :

$$\begin{aligned} V_x &\leftrightarrow T_t \leftrightarrow X_t \leftrightarrow A_x \\ V_y &\leftrightarrow T_t \leftrightarrow X_t \leftrightarrow A_y \\ V_t &\leftrightarrow T_x \leftrightarrow X_x \leftrightarrow A_t \\ V_t &\leftrightarrow T_y \leftrightarrow X_y \leftrightarrow A_t \end{aligned}$$

## $SU(2)_{CS}$ and $SU(4)$ symmetries

- ◊ for spatial  $z$ -correlators generated by representations:

$$\begin{aligned} R_1 : \quad & \{\gamma_1, -i\gamma_5\gamma_1, \gamma_5\} & \Rightarrow & \quad V_y \leftrightarrow T_t \leftrightarrow X_t \\ R_2 : \quad & \{\gamma_2, -i\gamma_5\gamma_2, \gamma_5\} & & V_x \leftrightarrow T_t \leftrightarrow X_t \end{aligned}$$

- ◊ Minimal group containing  $SU(2)_{CS}$  and  $\chi S$  is  $SU(4)$ :

$$\left. \begin{array}{l} V_x \leftrightarrow T_t \leftrightarrow X_t \leftrightarrow A_x \\ V_y \leftrightarrow T_t \leftrightarrow X_t \leftrightarrow A_y \end{array} \right\} E_2$$
$$\left. \begin{array}{l} V_t \leftrightarrow T_x \leftrightarrow X_x \leftrightarrow A_t \\ V_t \leftrightarrow T_y \leftrightarrow X_y \leftrightarrow A_t \end{array} \right\} E_3$$

## $SU(2)_{CS}$ and $SU(4)$ symmetries

- ◊ for spatial  $z$ -correlators generated by representations:

$$\begin{aligned} R_1 : \quad & \{\gamma_1, -i\gamma_5\gamma_1, \gamma_5\} \\ R_2 : \quad & \{\gamma_2, -i\gamma_5\gamma_2, \gamma_5\} \end{aligned} \quad \Rightarrow \quad \begin{aligned} V_y &\leftrightarrow T_t \leftrightarrow X_t \\ V_x &\leftrightarrow T_t \leftrightarrow X_t \end{aligned}$$

- ◊ Minimal group containing  $SU(2)_{CS}$  and  $\chi S$  is  $SU(4)$ :

$$\left. \begin{aligned} V_x &\leftrightarrow T_t \leftrightarrow X_t \leftrightarrow A_x \\ V_y &\leftrightarrow T_t \leftrightarrow X_t \leftrightarrow A_y \end{aligned} \right\} E_2$$
$$\left. \begin{aligned} V_t &\leftrightarrow T_x \leftrightarrow X_x \leftrightarrow A_t \\ V_t &\leftrightarrow T_y \leftrightarrow X_y \leftrightarrow A_t \end{aligned} \right\} E_3$$

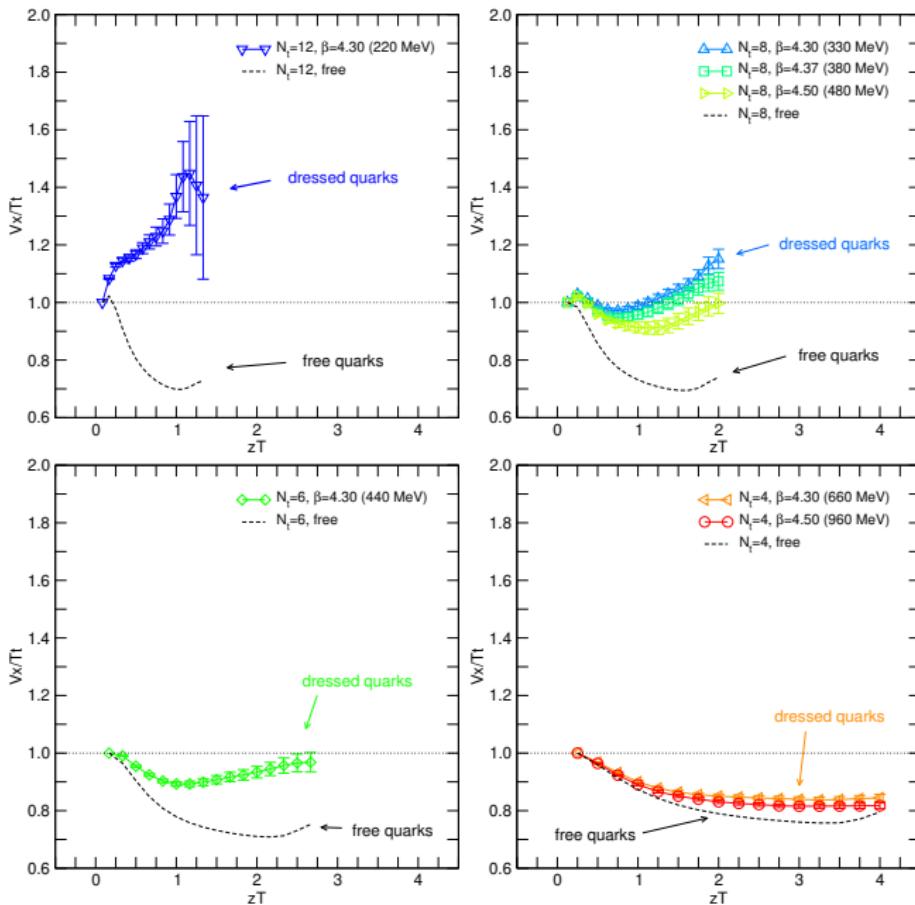
- ◊ Physical interpretation:  $\begin{pmatrix} u_L \\ u_R \\ d_L \\ d_R \end{pmatrix}$  *all components of fundamental vector mix!*

## Increasing temperature..

## Increasing temperature..

- by varying  $\beta$  and geometry
- using same simulation as in first study
- for  $m_{ud} = 0.001, 0.005, 0.01$
- temperature range  $1.25 - 5.5T_c$

$T$ [MeV]	$32^3 \times 12$	$32^3 \times 8$	$32^3 \times 6$	$32^3 \times 4$
$\beta = 4.10$		220		
$\beta = 4.18$		260		
$\beta = 4.30$	220	330	440	660
$\beta = 4.37$		380		
$\beta = 4.50$		480		960



$V_x/T_t$  ratio measures  $SU(2)_{cs}$  symmetry breaking

## Symmetries of the Lagrangian

$$\Psi \xrightarrow{SU(2)_{CS}} e^{i\vec{\Sigma}\vec{\theta}/2} \Psi \quad \vec{\Sigma} = \{\gamma_0, -i\gamma_5\gamma_0, \gamma_5\}$$

## Symmetries of the Lagrangian

$$\Psi \xrightarrow{SU(2)_{CS}} e^{i\vec{\Sigma}\vec{\theta}/2} \Psi \quad \vec{\Sigma} = \{\gamma_0, -i\gamma_5\gamma_0, \gamma_5\}$$

Free, massless Lagrangian:  $\mathcal{L} = \bar{\Psi} i\cancel{D} \Psi$

Covariant derivative:  $D_\mu = \partial_\mu - igA_\mu$

Massless (fermionic) Lagrangian:  $\mathcal{L} = \bar{\Psi} i\cancel{D} \Psi = \bar{\Psi} i\gamma^0 D_0 \Psi + \bar{\Psi} i\gamma^i D_i \Psi$

# Symmetries of the Lagrangian

$$\Psi \xrightarrow{SU(2)_{CS}} e^{i\vec{\Sigma}\vec{\theta}/2} \Psi \quad \vec{\Sigma} = \{\gamma_0, -i\gamma_5\gamma_0, \gamma_5\}$$

Free, massless Lagrangian:

$$\mathcal{L} = \bar{\Psi} i\cancel{\partial} \Psi$$

breaks  $SU(2)_{CS}$

Covariant derivative:

$$D_\mu = \partial_\mu - igA_\mu$$

Massless (fermionic) Lagrangian:  $\mathcal{L} = \bar{\Psi} i\cancel{\partial} \Psi = \bar{\Psi} i\gamma^0 D_0 \Psi + \bar{\Psi} i\gamma^i D_i \Psi$

$SU(2)_{CS}$  invariant

- Kinetic term breaks  $SU(2)_{CS}$
- ‘Magnetic’ term breaks  $SU(2)_{CS}$
- ‘Electric’ term is  $SU(2)_{CS}$  symmetric

# Symmetries of the Lagrangian

$$\Psi \xrightarrow{SU(2)_{CS}} e^{i\vec{\Sigma}\vec{\theta}/2} \Psi \quad \vec{\Sigma} = \{\gamma_0, -i\gamma_5\gamma_0, \gamma_5\}$$

Free, massless Lagrangian:

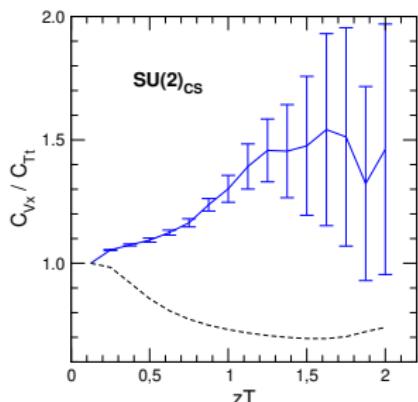
$$\mathcal{L} = \bar{\Psi} i\cancel{\partial} \Psi$$

breaks  $SU(2)_{CS}$

Covariant derivative:

$$D_\mu = \partial_\mu - igA_\mu$$

Massless (fermionic) Lagrangian:  $\mathcal{L} = \bar{\Psi} i\cancel{\partial} \Psi = \bar{\Psi} i\gamma^0 D_0 \Psi + \bar{\Psi} i\gamma^i D_i \Psi$



$SU(2)_{CS}$  invariant

- Kinetic term breaks  $SU(2)_{CS}$
- ‘Magnetic’ term breaks  $SU(2)_{CS}$
- ‘Electric’ term is  $SU(2)_{CS}$  symmetric

# Symmetries of the Lagrangian

$$\Psi \xrightarrow{SU(2)_{CS}} e^{i\vec{\Sigma}\vec{\theta}/2} \Psi \quad \vec{\Sigma} = \{\gamma_0, -i\gamma_5\gamma_0, \gamma_5\}$$

Free, massless Lagrangian:

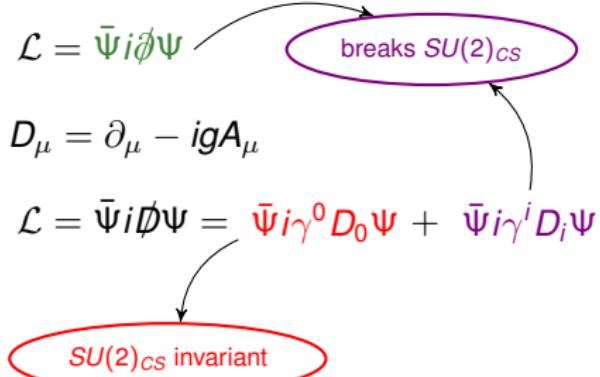
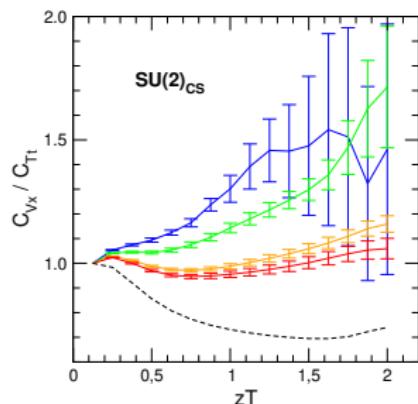
$$\mathcal{L} = \bar{\Psi} i\cancel{\partial} \Psi$$

breaks  $SU(2)_{CS}$

Covariant derivative:

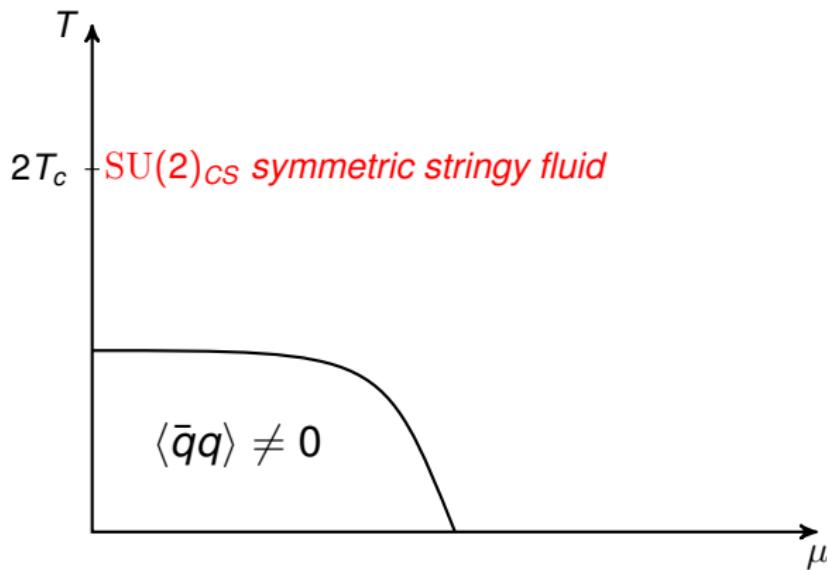
$$D_\mu = \partial_\mu - igA_\mu$$

Massless (fermionic) Lagrangian:  $\mathcal{L} = \bar{\Psi} i\cancel{\partial} \Psi = \bar{\Psi} i\gamma^0 D_0 \Psi + \bar{\Psi} i\gamma^i D_i \Psi$

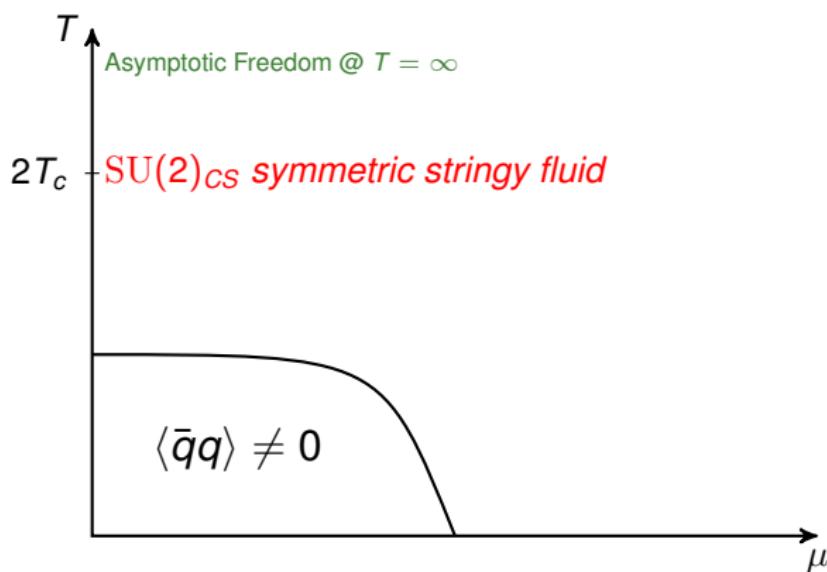


- Kinetic term breaks  $SU(2)_{CS}$
- ‘Magnetic’ term breaks  $SU(2)_{CS}$
- ‘Electric’ term is  $SU(2)_{CS}$  symmetric

## Effects of chemical potential

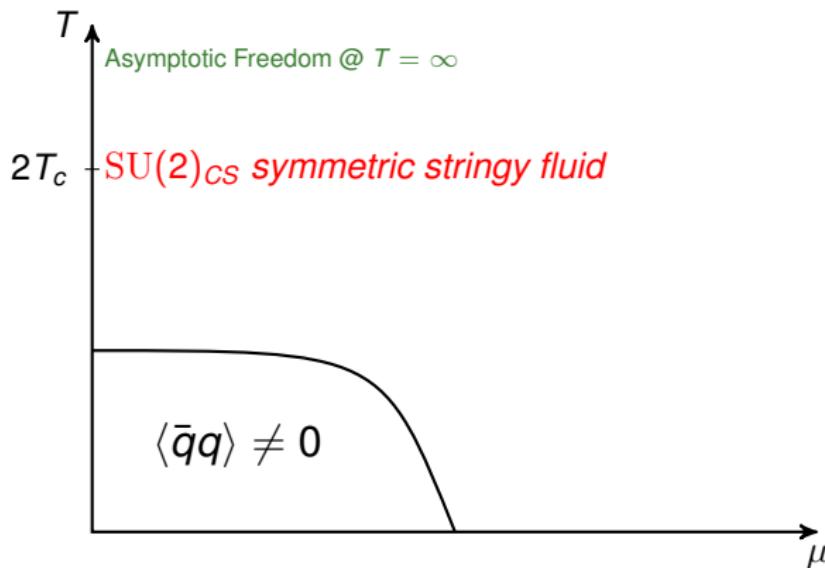


# Effects of chemical potential



# Effects of chemical potential

$$S = \int_0^\beta \int d^3x \bar{\Psi} [\gamma_\mu D_\mu + \mu \gamma_4] \Psi$$



# Summary

## Summary

- ✓ spatial correlations at temperatures  $1.25 - 5.5 T_c$
- ✓ approximate  $SU(2)_{CS}$  symmetric region  $\rightarrow SU(4)$

## Summary

- ✓ spatial correlations at temperatures  $1.25 - 5.5 T_c$
  - ✓ approximate  $SU(2)_{CS}$  symmetric region  $\rightarrow SU(4)$
- $\Rightarrow SU(2)_{CS}$  a tool to distinguish  
*color-electric* and *color-magnetic* contributions

# Summary

- ✓ spatial correlations at temperatures  $1.25 - 5.5 T_c$
- ✓ approximate  $SU(2)_{CS}$  symmetric region  $\rightarrow SU(4)$

$\Rightarrow SU(2)_{CS}$  a tool to distinguish

*color-electric* and *color-magnetic* contributions

*chiral quarks connected by color-electric field  
as elementary objects at high  $T$*

*strings?*