Progress on Parton Pseudo-Distributions II

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Introduction

- Lattice calculations of Distribution functions are maturing to the point of realistic comparison with experiment
- EIC will be able to measure more PDFs with more precision than ever before
- Project Goals
  - Long Term: Study methods of calculating parton distributions from ab initio Lattice QCD
  - Short Term: Understand systematic effects in the simple case of iso-vector quark unpolarized PDF
- Mellin moments and OPE
  - Restricted to low moments by reduced rotational symmetry
- Hadronic Tensor Methods A Chambers et.al (2017) 1703.01153
  - "Light-like" separated Hadronic Tensor K-F Liu et al Phys. Rev. Lett. 72 1790 (1994) , Phys. Rev. D62 (2000) 074501
  - Good lattice cross sections Y.-Q. Ma J.-W. Qiu (2014) 1404.6860 Y.-Q. Ma, J.-W. Qiu (2017) 1709.03018
- Ioffe Time Pseudo Distribution Methods
  - Quasi PDF X. Ji, Phys.Rev.Lett. 110, (2013) J.-W. Chen et.al. (2018) 1803.04393 C Alexandrou et.al. (2018) 1803.02685
  - Pseudo PDF A. Radyushkin Phys.Lett. B767 (2017) K. Orginos, A Radyushkin, JK, S Zafeiropoulos (2017) 1706.05373

Ioffe Time distribution

$\nu = p \cdot z$ B. L. Ioffe, Phys. Lett. 30B, 123 (1969)

- $$I(\nu, \mu^2) = \int_{-1}^{1} dx\, e^{i\nu x} f(x, \mu^2)$$

V. Braun, et. al Phys. Rev. D 51, 6036 (1995)
I.I. Balitsky and V.M. Braun, Nucl. Phys. B311, 541 (1988)

- CP Even/Odd combinations
  - Even: $q_-(x) = f(x) + f(-x) = q(x) - \bar{q}(x) \equiv q_V(x)$
  - Odd: $q_+(x) = f(x) - f(-x) = q(x) + \bar{q}(x) = 2\, q_V(x) + \bar{q}(x)$

$$Re\,[I(\nu)] = \int_0^1 dx\, \cos(\nu x)\, q_-(x) = \int_0^1 dx\, \cos(\nu x) q_V(x) \equiv I_V(\nu)$$

$$Im\,[I(\nu)] = \int_0^1 dx\, \sin(\nu x) q_+(x) = \int_0^1 dx\, \sin(\nu x)(q(x) + \bar{q}(x))$$

- Perturbative position space DGLAP evolution

$$I_V(\nu, \mu_2^2) = I_V(\nu, \mu_1^2) - \frac{C_F \alpha_S}{2\,\pi} \log\left(\frac{\mu_2^2}{\mu_1^2}\right) \int_0^1 du\, \left[\frac{1}{2}\delta(1-u) - (1-u) - 2\left(\frac{u}{1-u}\right)_+\right] I_V(u\nu, \mu_1^2)$$

What is a pseudo-distribution?

- Standard partonic distributions, particularly collinear distributions, are defined via matrix elements with light like separations
  - Describe probability distribution of quark states
  - Not suitable for lattice calculation
- Pseudo distributions are Lorentz invariant generalizations of partonic distributions defined via matrix elements with space like separations
  - Do not have probabilistic interpretation
  - Acceptable for lattice calculation
- In the limit that the space like separation goes to 0, the standard distribution is recovered

Pseudo Ioffe Time Distributions

- A general matrix element of interest

$$M^{\alpha}(z,p) = \langle h(p)|\bar{\psi}(z)\gamma^{\alpha} W(z;0)\psi(0)|\, h(p)\rangle$$

- Lorentz decomposition
    - Physicists love to use of symmetries
    - Choice of p, z, and α can remove higher twist term

$$M^{\alpha}(z,p) = 2\, p^{\alpha}\, M_p(\nu, z^2) + z^{\alpha} M_z(\nu, z^2)$$

- Relation to ITDF
    - Perturbatively calculable Wilson coefficients for each parton

$$M(\nu, -z^2) = \sum_i C_i(\, z^2\, \mu^2, \alpha_S) \otimes I_i(\nu, \mu^2) + H.T.$$

A. Radyushkin (2017) 1710.08813
J.-H. Zhang (2018) 1801.03023
T. Izubuchi (2018) 1801.03917

$$M^{\alpha}(z,p) = \langle h(p)|\bar{\psi}(z)\gamma^{\alpha}W(z;0)\psi(0)|\,h(p)\rangle$$

Special Cases

$$M^{\alpha}(z,p) = 2\,p^{\alpha}\,M_p(\nu,z^2) + z^{\alpha}M_z(\nu,z^2)$$

- Light cone PDF

$$p = (p^+,p^-,0_T) \quad z = (0,z^-,0_T) \quad \alpha = +$$

$$M_p\big((p^+z^-),0\big) = \int_{-1}^{1} dx\, e^{ix(p^+z^-)}\, f(x)$$

- Straight Link “Primordial” TMD

$$p = (p^+,p^-,0_T) \quad z = (0,z^-,z_T) \quad \alpha = +$$

$$M_p\left((p^+z^-),-z_T^2\right) = \int_{-1}^{1} dx\, e^{ix(p^+z^-)} \int d^2k_T\;\; e^{ik_T\cdot z_T}F(x,k_T^2)$$

- Pseudo PDF

$$p = (E,0,0,p\_3\,) \quad z = (0,0,0,z_3\,) \quad \alpha = 0$$

$$M_P\left((-z_3p_3),-z_3^2\right) = \int_{-1}^{1} dx\, e^{ix(-z_3p_3)}P(x,-z_3^2)$$

A. Radyushkin (2017) 1612.05170

Matching Lattice data to Ioffe distribution

- Matching between pseudo ITDF and MS bar scheme ITDF via factorization of IR divergences.
- At 1-loop, scale evolution and matching can be simultaneous
- Allows for a direct relationship between ITDF/PDF and pseudo ITDF
    - No more need for extrapolations in the scale
    - Does require scale to be in regime dominated by pertubative effects
- No real need for pseudo PDFs. Go directly from pseudo ITDF to PDF
- Only perturbative correction propotional to α_S (around 10%)

$$I(\nu,\mu^2) = \mathcal{M}(\nu,z^2) + \frac{C_F\alpha_S}{2\pi}\int_0^1 du\left[B(u)\left(\log\left(z^2\mu^2\frac{e^{2\gamma_E}}{4}\right)+1\right)+\left(\frac{4\log(1-u)}{1-u}-2(1-u)\right)_+\right]\mathcal{M}(u\nu,z^2)$$

Pseudo ITD as a Good Lattice Cross Section

- Good Experimental Cross Section - An experiment whose results, Form Factors or asymmetries, is sensitive to a particular PDF.
    - DIS, SIDIS, DY, ….

- Good Lattice Cross Section - A lattice QCD calculable matrix element whose result is sensitive to a particular PDF (Matrix element and not actually a cross section)
    - Vector-vector currents, Axial-vector currents, Quark fields separated by Wilson line, ….

Numerical Study

Both use Wilson-Clover Stout smeared Fermions

Quenched Wilson plaquette gauge action

K. Orginos, A Radyushkin, JK, S Zafeiropoulos (2017) 1706.05373

- $\beta = 6.0$ $m_\pi = 600$ MeV $32^3 \times 64$ $a = 0.1$ fm

Dynamical Tree level tadpole Symanzik improved gauge action (Preliminary)

Unpublished

- $a127m440:$ $\beta = 6.1$ $m_\pi = 440$ MeV $24^3 \times 64$ $a = 0.127$ fm
- $a127m440L:$ $\beta = 6.1$ $m_\pi = 440$ MeV $32^3 \times 96$ $a = 0.127$ fm

Summation method for Matrix element extraction

- Correlation functions

$$C_2(\vec{p},T) = \langle O_N(-\vec{p},T)\bar{O}_N(\vec{p},0)\rangle$$

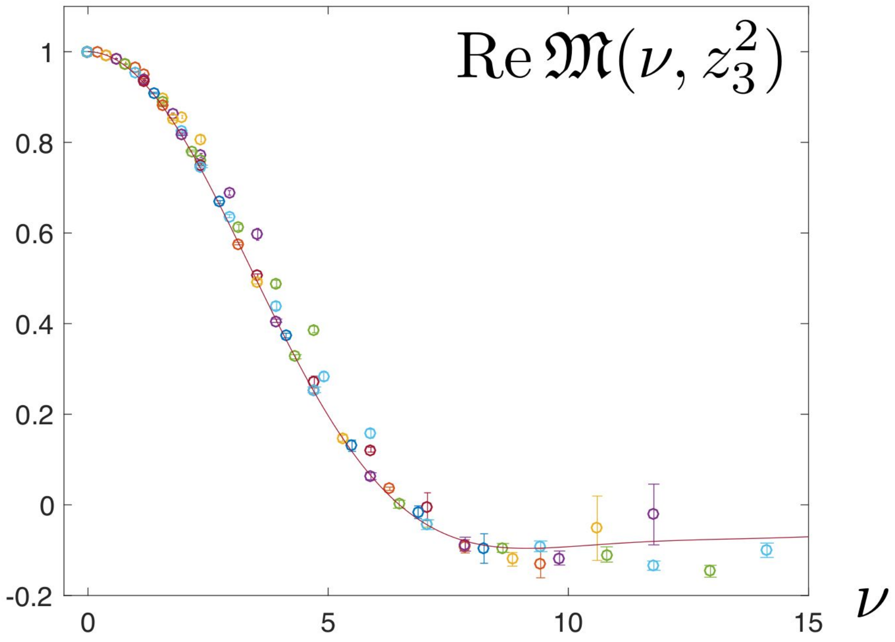
$$C_{op}(O_{op};\vec{p},T) = \sum_t \sum_{\vec{x}} \langle O_N(-\vec{p},T)O_{op}(\vec{x},t)\bar{O}_N(\vec{p},0)\rangle$$

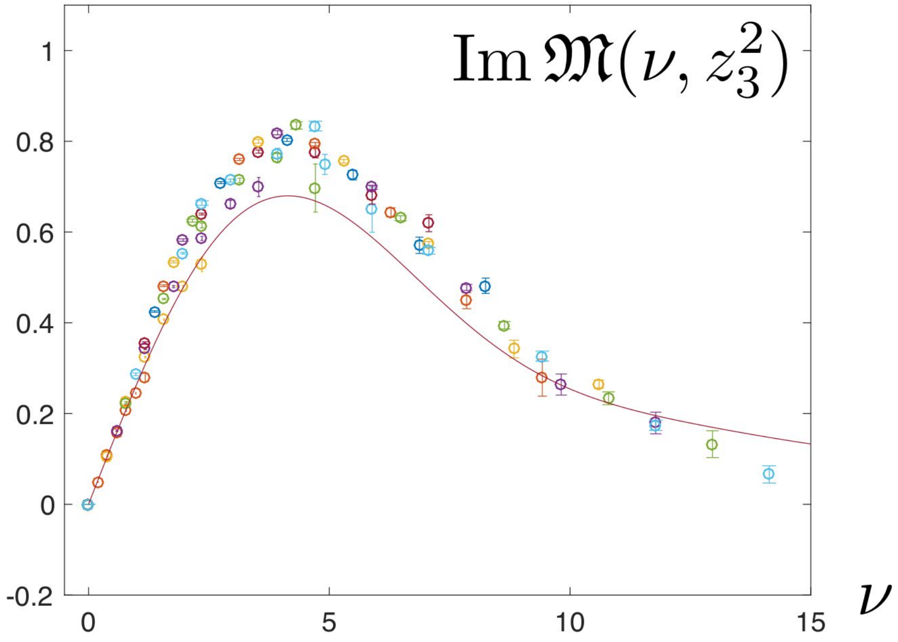
- Summation method extraction C. Bouchard et.al Phys. Rev. D 96, no. 1, 014504 (2017)

$$\frac{\langle N(p)|O_{op}|N(p)\rangle}{2E_{N(p)}} = \lim_{T\to\infty} \frac{1}{\tau}(R(T+\tau) - R(T)) \qquad R(T) = \frac{C_{op}(O_{op};\vec{p},T)}{C_2(\vec{p},T)}$$

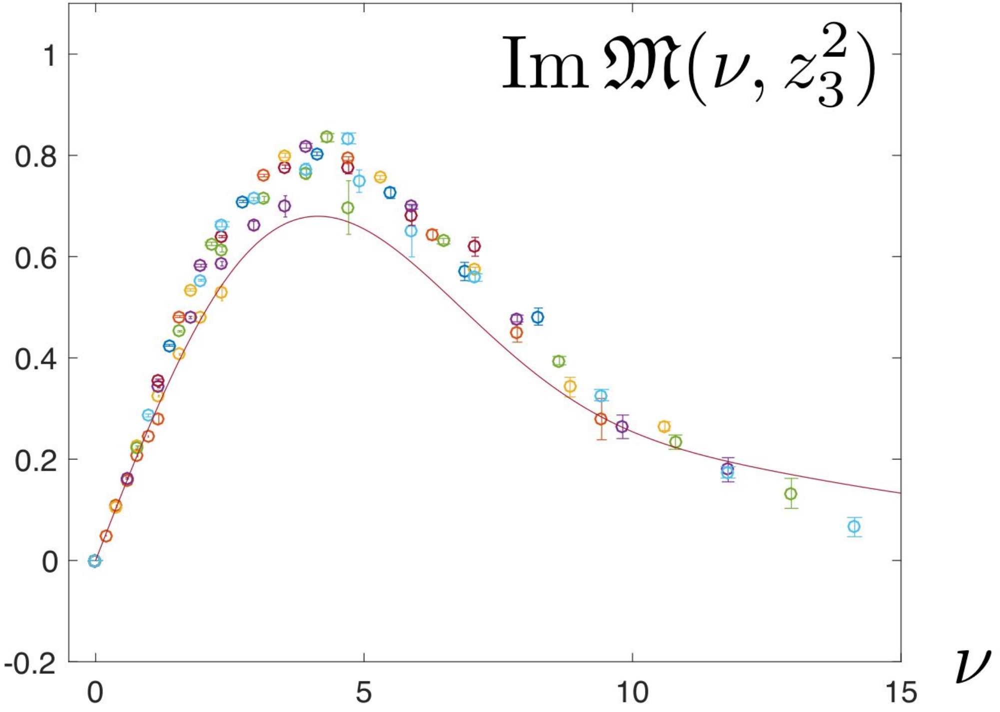
$$O_q^\alpha(z;T) = \sum_{\vec{x}} \bar{\psi}_q(\vec{x}+\vec{z},T)\lambda^3\gamma^\alpha W((\vec{x}+\vec{z},T);(\vec{x},T))\psi_q(\vec{x},T)$$

Quenched Results





Imaginary Component and AntiQuarks



- Imaginary component mixes valence, sea, and antiquark distributions
- Use real component to find valence contribution, the rest is the sea and antiquarks
- Identify anti quark distribution without need of needing to perform inaccurate Fourier transforms and requiring the unreliable low x region
- Qualitatively it gives proper sign for quenched iso-vector quarks

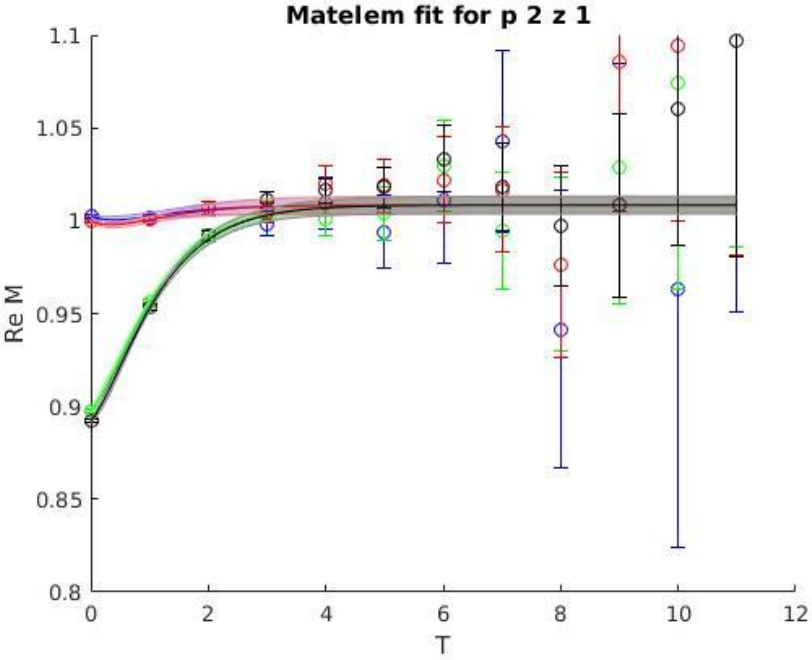
Improved Matrix element Extraction

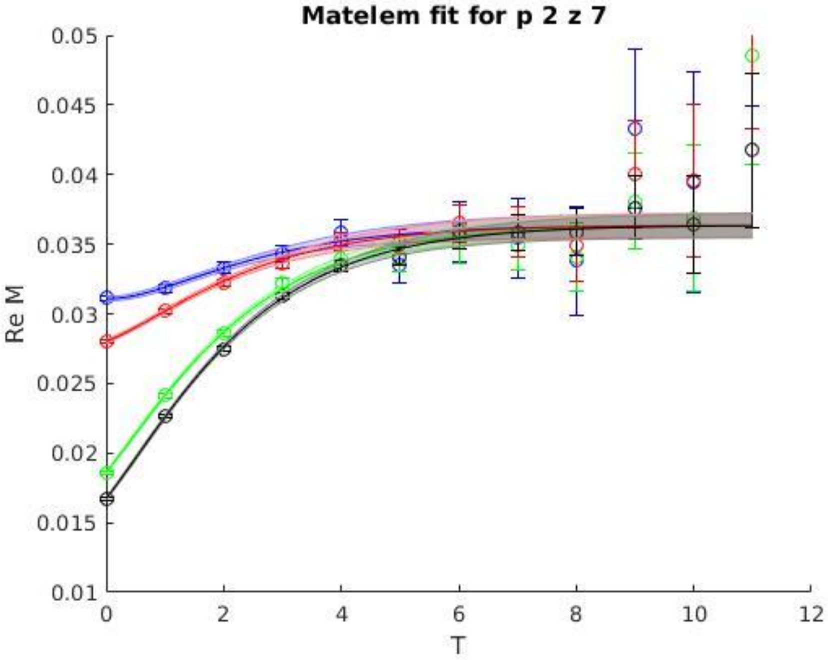
- Use four different correlation functions
    - Regular Gaussian Smearing
        - Smeared-to-Smeared
        - Smeared-to-Point
    - Momentum smearing
        - Smeared-to-Smeared
        - Smeared-to-Point
- Contact terms from T=0 cancel in the ratio
    - Fits can be over the entire range of interpolator field separations
- Form of excited states

$$O(T) = M + C_1 e^{-\Delta E T} + C_2 T e^{-\Delta E T}$$

Some good extractions

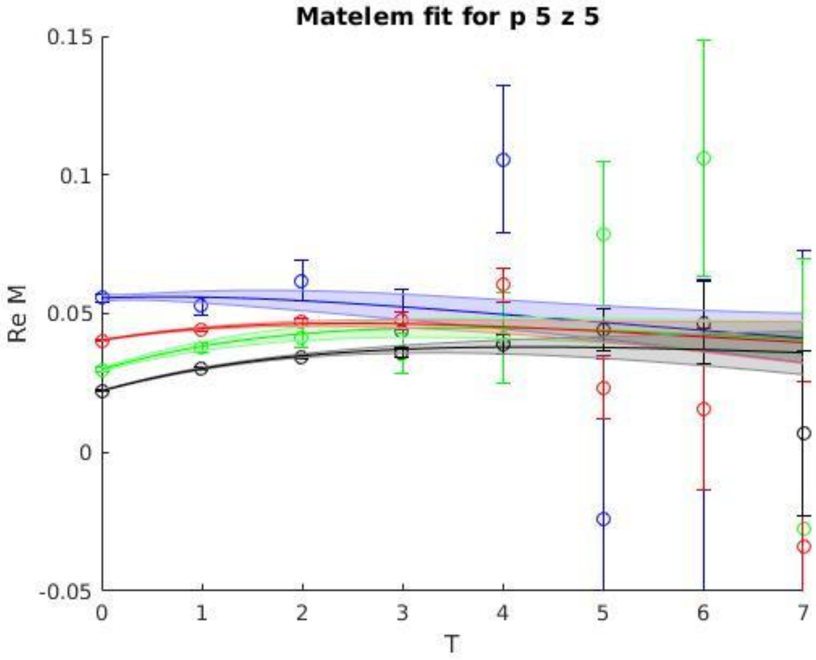
Preliminary

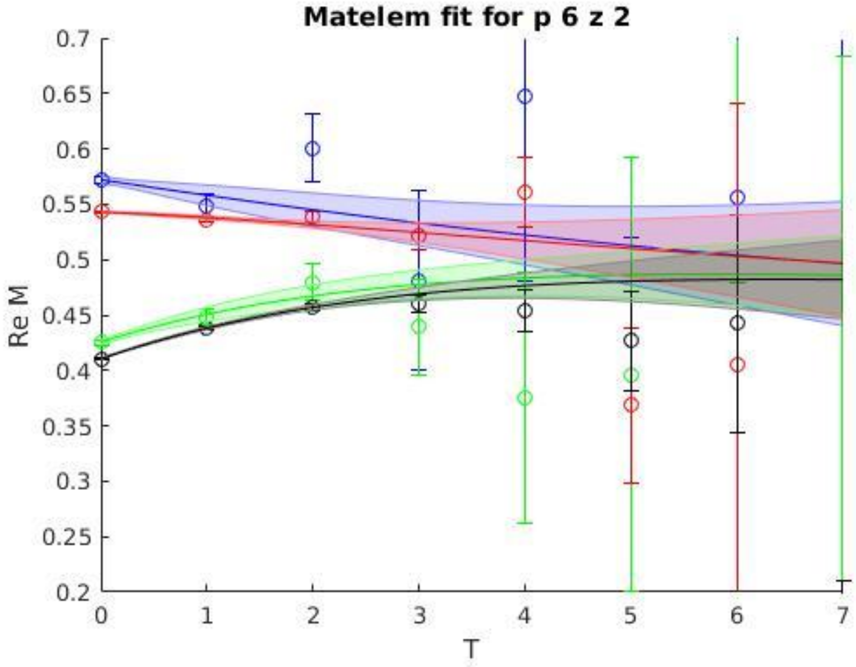




Preliminary

Less good extractions





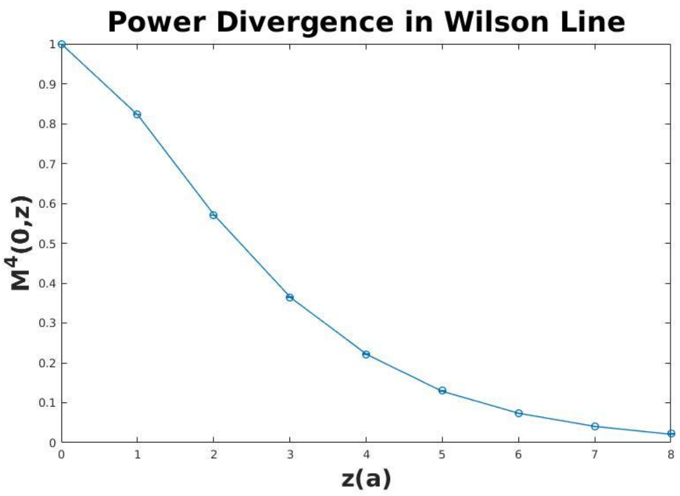
Renormalization and the Reduced distribution

- Vector current $Z_p^{-1} = M^4(0,p)$
  - Forces matrix elements to give unit charge

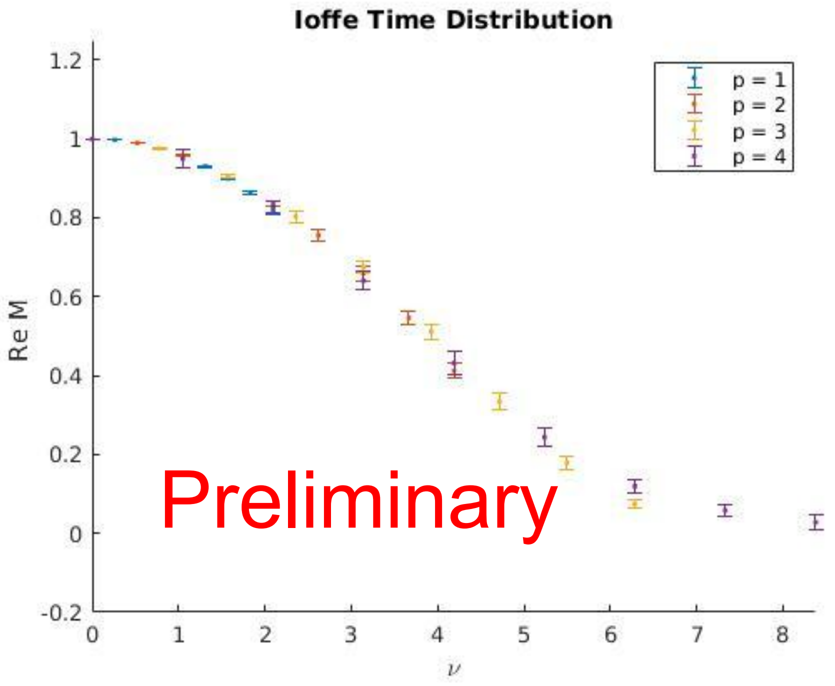
- Reduced distribution $\mathfrak{M}(\nu, z^2) = \dfrac{\mathcal{M}(\nu, z^2)}{\mathcal{M}(0, z^2)}$
  - TMD “Factorization” and suppression of polynomial corrections

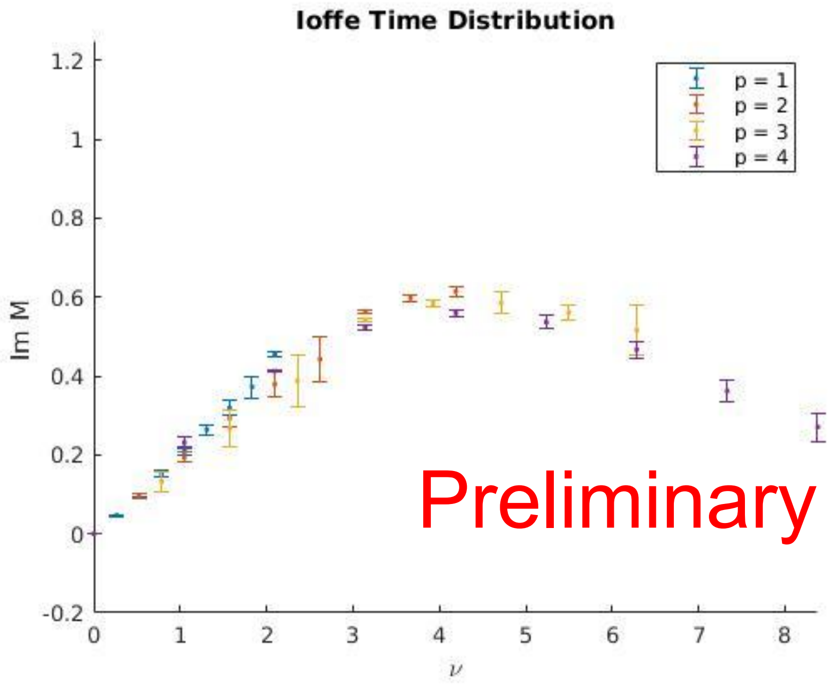
    $F(x,k_T^2) = f(x)g(k_T^2) \Rightarrow M(\nu,z^2) = M(\nu,0)M(0,z^2)$

  - BONUS: Multiplicative UV power divergent corrections from Wilson line cancel away

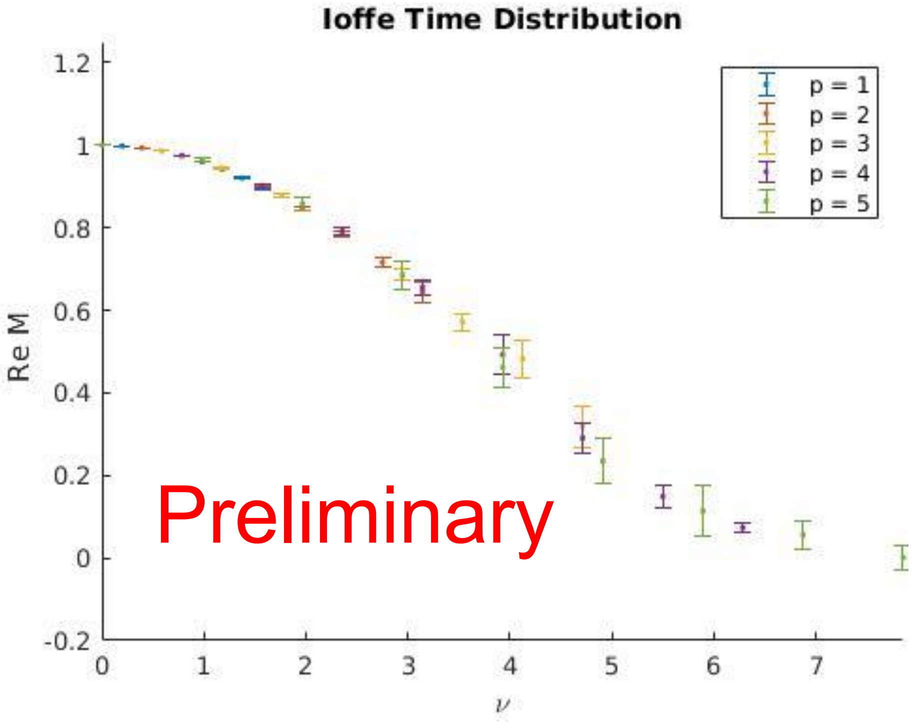


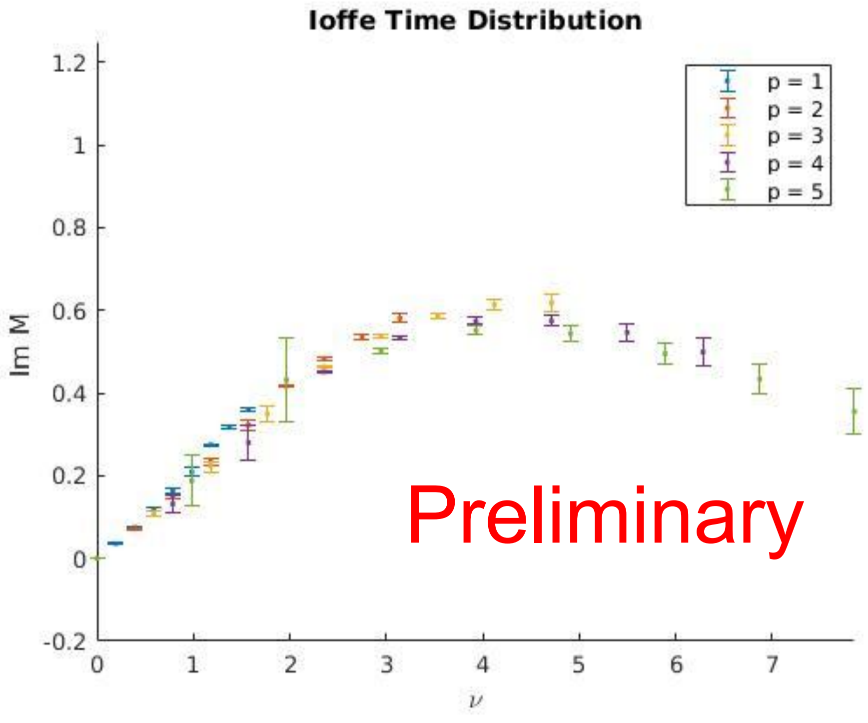
Pseudo-ITDF Results a127m440



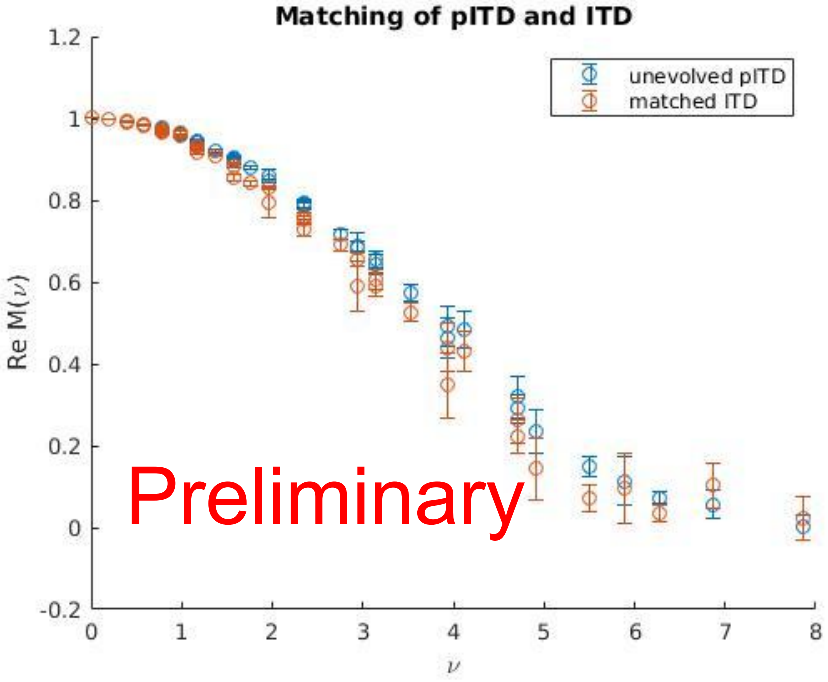


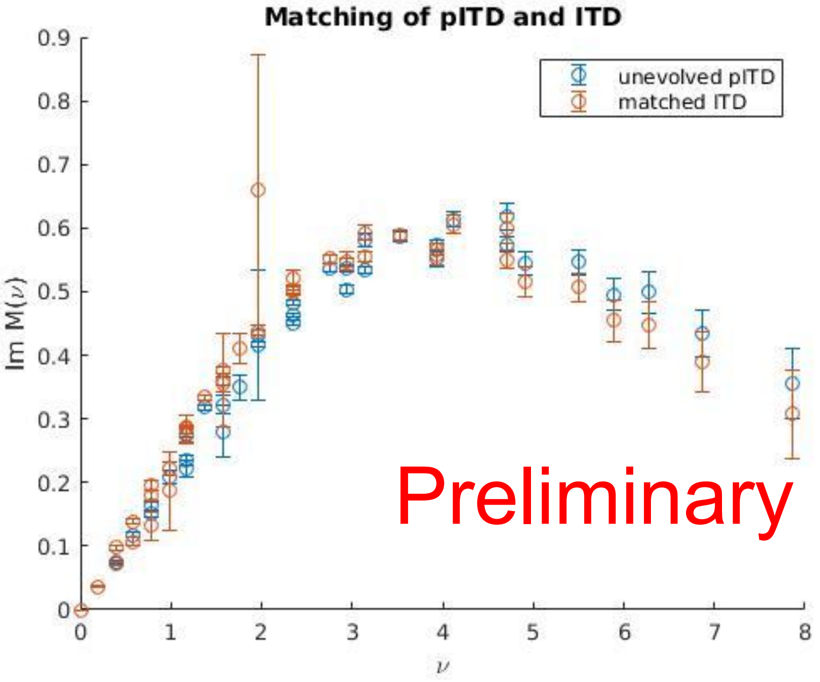
Pseudo-ITDF Results a127m440L





ITDF MS bar matched Results a127m440L



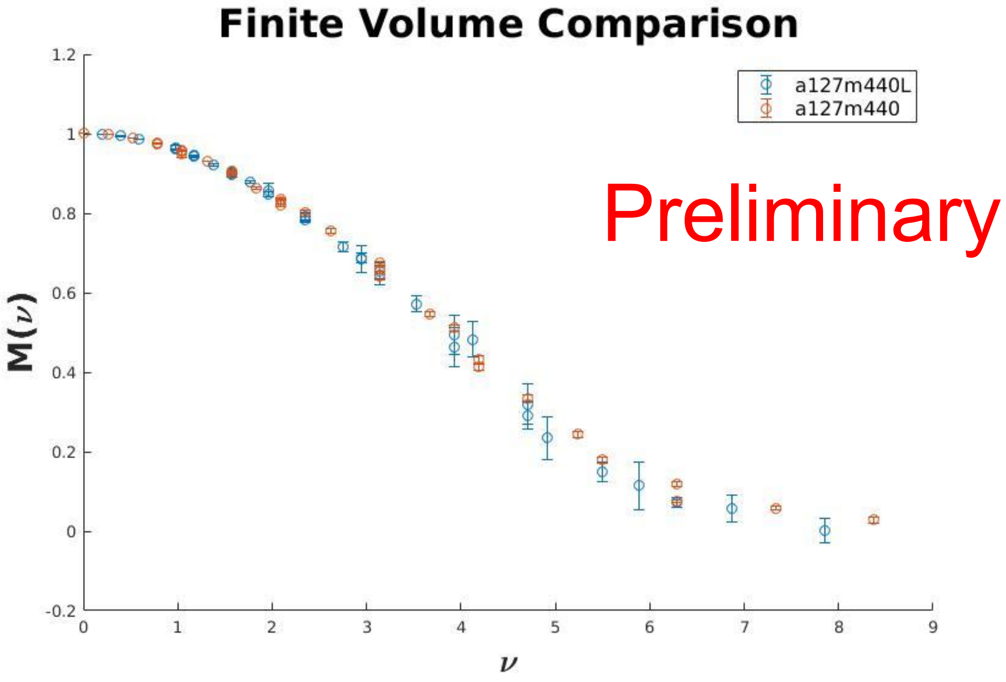


Comparison of Volumes

- Two Current matrix elements can have very large finite volume corrections
  - See talk by Guerrero

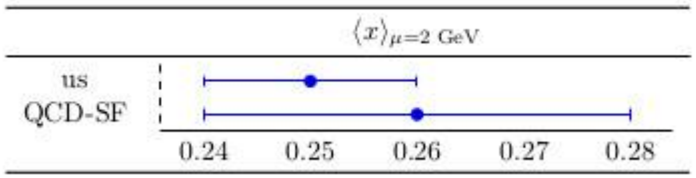
Briceno et al. (2018) 1805.01034
Bali et al. (2018) 1807.03073

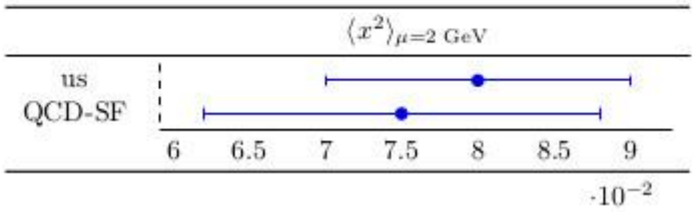
- Finite volume effects of Wilson line operator has been unstudied



Moments of PDFs

- Taylor Expansion coefficients of Ioffe Time Distributions are the moments of PDFs
  - Even moments from Real component of ITDF
  - Odd moments from Imaginary component of ITDF
  - With enough data there is no limit on what moments can be calculated
- Comparison of moments from Quenched data





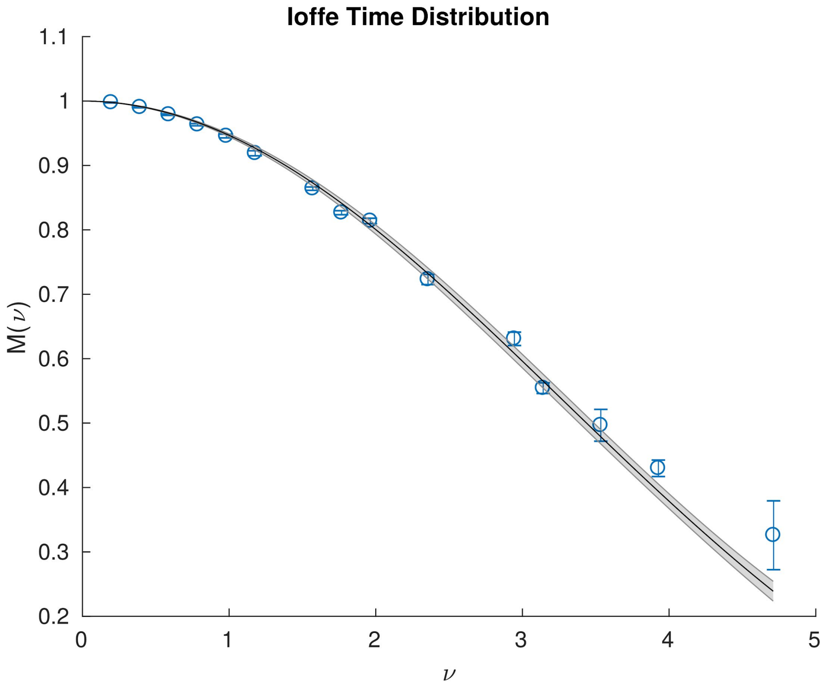
Real component and the Valence Quark distribution

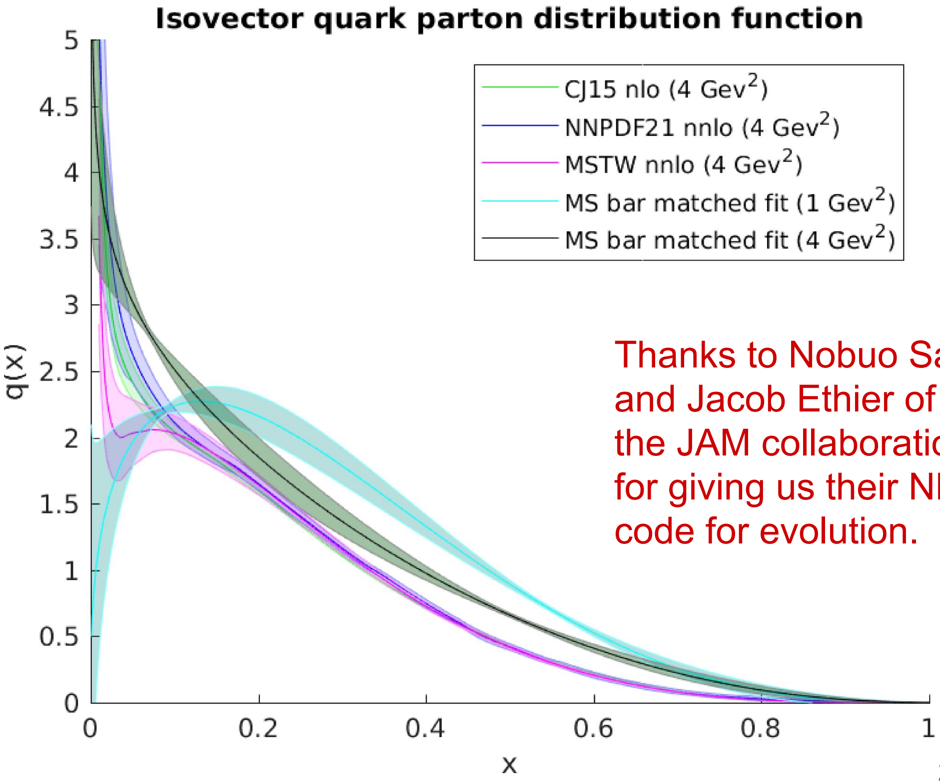
- In first attempt to avoid ill posed inverse Fourier transform
- A general model PDF used by JAM collaboration for fitting

$$f_{abcd}(x) = N_{abcd}\ x^a(1-x)^b(1+c\sqrt{x}+d\,x)$$

- Lowest order behaviors
  - Regge $a = -\frac{1}{2}$
  - Quark counting $b = 3$
  - Small Corrections $c \sim 0 \sim d$

Quenched Pseudo PDF Matched to MS bar Compared to Global fit PDFs





Thanks to Nobuo Sato and Jacob Ethier of the JAM collaboration for giving us their NLO code for evolution.

Summary

- Qualitative agreement with PDFs despite few systematics under control
- Divergent behavior improves, but not recovered, under proper evolution to 4 GeV^2
- To Do List:
  - Systematics left to be thoroughly studied (pion mass, lattice spacing,....)
  - Study PDF reconstruction methods on real lattice data
- Once techniques are understood and controlled then any light cone distribution is within reach of the lattice.

Thank you for your attention!