High Precision Statistical Landau Gauge Lattice Gluon Propagator Computation

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Abstract

We report on results for the Landau gauge gluon propagator computed from large statistical ensembles and look at the compatibility of the results with the Gribov-Zwanziger tree level prediction for its refined and very refined versions. Our results show that the data is well described by the tree level estimate only up to momenta $p \lesssim 1$ GeV, while clearly favoring the so-called Refined Gribov-Zwanziger scenario. We also provide a global fit of the lattice data which interpolates between the above scenario at low momenta and the usual continuum one-loop renormalization improved perturbation theory after introducing an infrared log-regularizing term.

Lattice Landau gauge gluon propagator

$$\langle A^a_\mu(\hat{p})\, A^b_\nu(\hat{p}')\rangle = L^4\, \delta^{ab}\, \delta(\hat{p}+\hat{p}')\, P_{\mu\nu}(p)\, D(p^2)$$

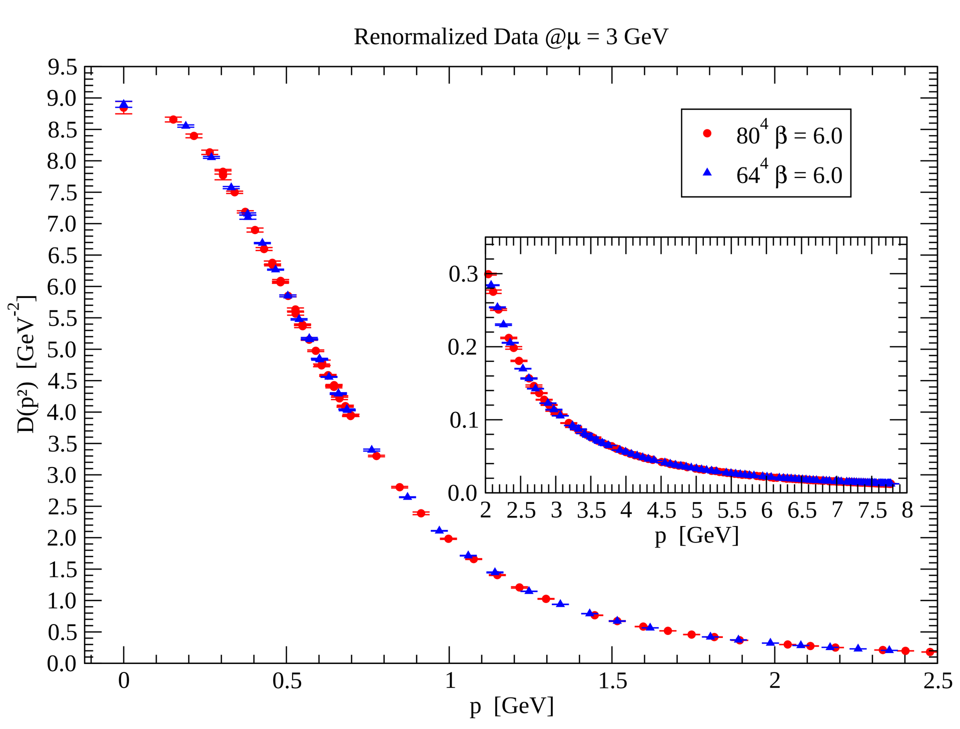
- lattice momentum
$$\hat{p}_\mu = \frac{2\pi}{aL} n_\mu, \qquad n_\mu = 0, 1, \ldots, L-1$$
- continuum momentum
$$p_\mu = \frac{2}{a} \sin\left(\frac{\pi}{L} n_\mu\right)$$
- orthogonal projector
$$P_{\mu\nu}(p) = \delta_{\mu\nu} - \frac{p_\mu p_\nu}{p^2}$$

Lattice simulations

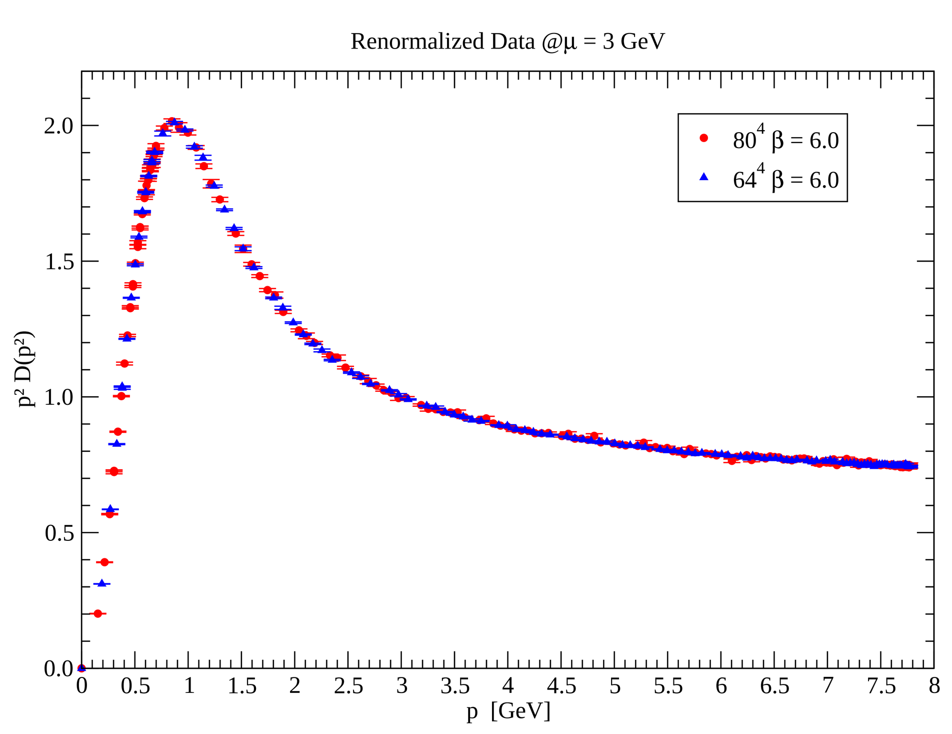
- two large physical volume lattice simulations

  Duarte, Oliveira, Silva, PRD94(2016)014502
  - Wilson gauge action, $\beta = 6.0$
    $1/a = 1.943$ GeV, $a = 0.1016(25)$ fm,
  - 64^4 and 80^4 lattices
  - physical volumes: $(6.57\text{ fm})^4$, $(8.21\text{ fm})^4$
  - number of configurations: 2000, 550
  - rotated to the Landau gauge
  - $p_{min} = 191$ MeV , 153 MeV ; $p_{max} = 7.7$ GeV
- renormalization: MOM scheme, scale $\mu = 3$ GeV
- conical and cylindrical cuts for $p > 0.7$ GeV
- all lattice data for $p < 0.7$ GeV

Gluon Propagator



Gluon Dressing Function



Refined Gribov-Zwanziger (RGZ) framework

- implements the functional restriction to the first Gribov horizon
- defines a local renormalizable quantum field theory

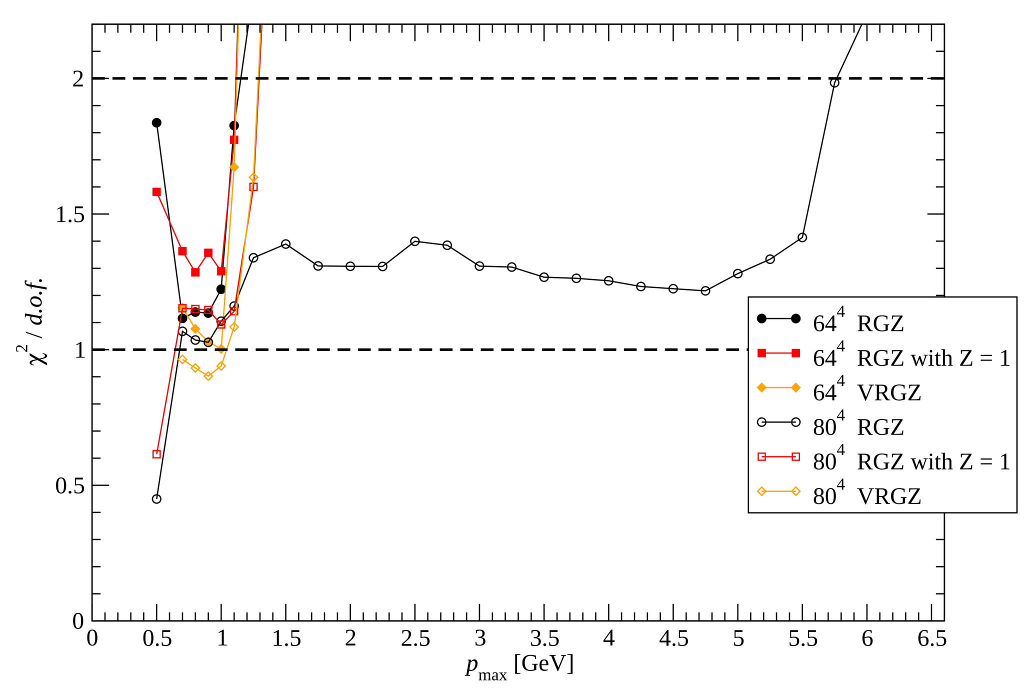
  Dudal, Sorella, Vandersickel, Verschelde, PRD77(2008)071501
  Dudal, Gracey, Sorella, Vandersickel, Verschelde, PRD78(2008)065047
  Capri, Fiorentini, Pereira, Sorella, PRD96(2017)054022
  Gracey, PRD82(2010)085032
  Dudal, Sorella, Vandersickel, Verschelde, PRD84(2011)065039
- RGZ takes into account some $d = 2$ condensates
- we study the compatibility of tree level predictions for the gluon propagator with lattice data
- (Very) Refined Gribov-Zwanziger *ansätze*

$$D_{RGZ}(p^2) = Z \frac{p^2 + M_1^2}{p^4 + M_2^2 p^2 + M_3^4}$$

$$D_{VRGZ}(p^2) = \frac{p^4 + M_2^2 p^2 + M_1^4}{p^6 + M_5^2 p^4 + M_4^4 p^2 + M_3^6}$$

- M_i are fitting parameters with dimensions of mass
- for $D_{VRGZ}(p^2)$ we do not include an overall normalization factor Z
- fits to the lattice data from $p = 0$ up to p_{max}

Quality of the fits



- 80^4 lattice: $p_{max} \sim 6$ GeV (smaller statistics)
- 64^4 lattice: $p_{max} \sim 1$ GeV (larger statistics)

Fitting $D_{RGZ}(p^2)$

$\nu = \chi^2/\text{d.o.f.}$

- $L = 64$

p_{max}	ν	Z	M_1^2	M_2^2	M_3^4
1.10	1.83	0.959(62)	2.67(32)	0.511(26)	0.285(15)
1.19	1.77	1	2.478(27)	0.496(9)	0.275(3)

- $L = 80$

p_{max}	ν	Z	M_1^2	M_2^2	M_3^4
1.10	1.16	0.957(66)	2.73(34)	0.527(29)	0.290(16)
3.00	1.31	0.730(4)	4.16(7)	0.592(10)	0.335(3)
1.25	1.60	1	2.454(35)	0.489(11)	0.273(3)

- lattice data compatible with RGZ up to $p \sim 1$ GeV
- agreement between ensembles for similar p_{max}

Fitting $D_{VRGZ}(p^2)$

- no significant difference between the RGZ and the VRGZ scenarios
- M_1^4, M_3^6 compatible with zero: VRGZ reduces to RGZ

$$D(p^2) = \frac{p^2 + M_2^2}{p^4 + M_5^2 p^2 + M_4^4}$$

$$M_2^2\Big|_{VRGZ} \sim M_1^2\Big|_{RGZ}, \quad M_5^2\Big|_{VRGZ} \sim M_2^2\Big|_{RGZ}, \quad M_4^4\Big|_{VRGZ} \sim M_3^4\Big|_{RGZ}$$

- IR lattice data ($p \lesssim 1$ GeV) well described by tree level Refined Gribov-Zwanziger prediction
- if the VRGZ reduces to the RGZ, the equality between the condensates $\langle \overline{\varphi}\, \overline{\varphi}\rangle = \langle \varphi\, \varphi\rangle$ should hold.

Global Fits: from Infrared to Ultraviolet

- for large p^2 one expects to recover the usual perturbative behaviour

$$D(p^2) \propto \frac{1}{p^2}\left[\ln\left(\frac{p^2}{\Lambda^2_{QCD}}\right)\right]^{\gamma_{gl}}$$

- $\gamma_{gl} = -\frac{13}{22} \longrightarrow$ 1-loop gluon anomalous dimension
- Interpolation between
  - RGZ for low p
  - 1-loop RG-improved expression for high p

$$D(p^2) = Z \frac{p^2 + M_1^2}{p^4 + M_2^2 p^2 + M_3^4}\left[\omega \ln\left(\frac{p^2 + m_g^2(p^2)}{\Lambda^2_{QCD}}\right) + 1\right]^{\gamma_{gl}}$$

- $\omega = 11 N \alpha_s(\mu)/(12\pi)$
- $\Lambda_{QCD} = 0.425$ GeV, $\alpha_s(3\text{ GeV}) = 0.3837$

  Dudal, Felix, Guimarães, Sorella, PRD96(2017)074036
  Aguilar, Binosi, Papavassiliou, JHEP07(2010)002
- regularisation of the leading log — introduction of a “log-regularisation mass” $m_g^2(p^2)$ which should become negligible at high momentum
- several regularizing mass definitions:

$$m_g^2(p^2) = \frac{M_3^4 + \left(M_2^2 - M_1^2\right)p^2}{M_1^2 + p^2} \qquad \text{(RGZ inspired)}$$

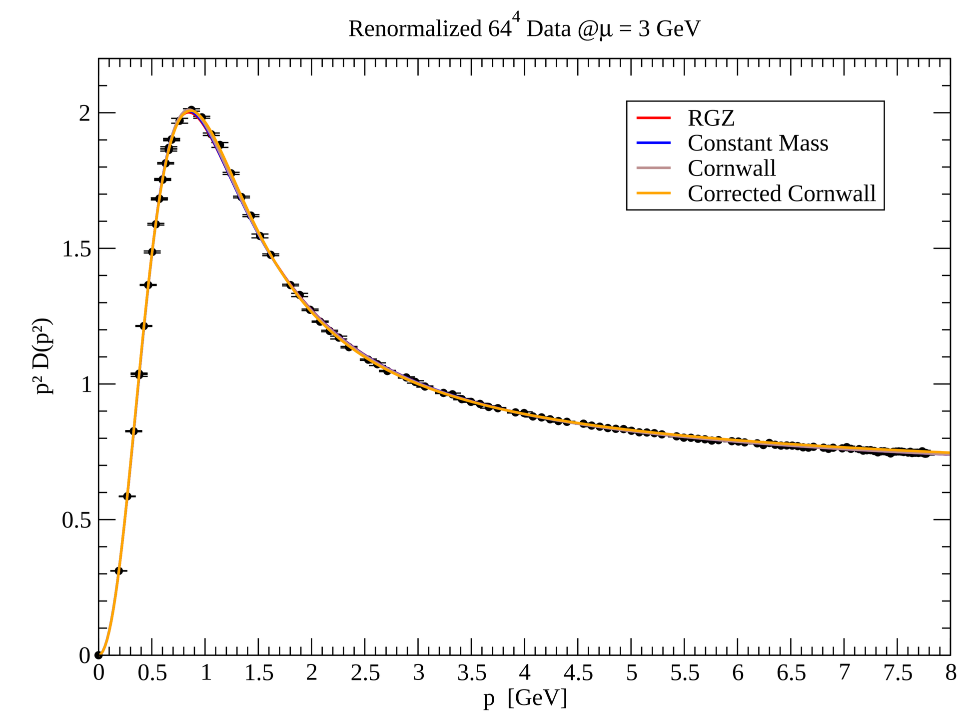
$$m_g^2(p^2) = m_0^2 \qquad \text{(Constant)}$$

$$m_g^2(p^2) = \frac{m_0^4}{p^2 + \lambda^2} \qquad \text{(Cornwall)}$$

$$m_g^2(p^2) = \lambda_0^2 + \frac{m_0^4}{p^2 + \lambda^2} \qquad \text{(Corrected Cornwall)}$$

- best 64^4 fit: corrected Cornwall regularisation mass ($\chi^2/\text{d.o.f.} = 1.11$)

$$\begin{aligned}
Z &= 1.36992 \pm 0.00072 \\
M_1^2 &= 2.333 \pm 0.042 \text{ GeV}^2 \\
M_2^2 &= 0.514 \pm 0.024 \text{ GeV}^2 \\
M_3^4 &= 0.2123 \pm 0.0032 \text{ GeV}^4 \\
m_0^2 &= 1.33 \pm 0.13 \text{ GeV}^4 \\
\lambda^2 &= 0.100 \pm 0.035 \text{ GeV}^2 \\
\lambda_0^2 &= -0.954 \pm 0.070 \text{ GeV}^2
\end{aligned}$$



For more information . . .

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