On isospin Breaking in τ input for $(g-2)$ from Lattice QCD

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in collaboration with
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for the RBC/UKQCD Collaboration

BROOKHAVEN NATIONAL LABORATORY

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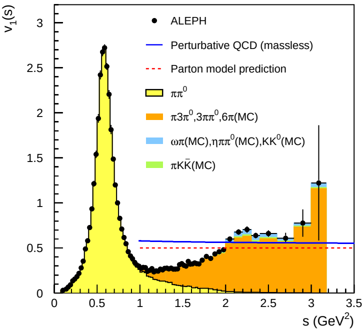
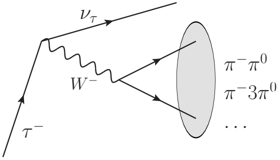
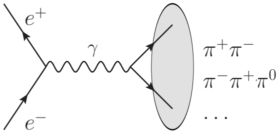
Motivations





$V - A$ current

Final states $I = 1$ charged



EM current

Final states $I = 0, 1$ neutral

τ data can improve $a_\mu[\pi\pi]$

$a_\mu[\text{HVP}] = 693.3(2.5)$ [KNT '18]

$\rightarrow a_\mu[\pi\pi] = 503.9(2.0)$ [KNT '18]

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Isospin Corrections

Restriction to $e^+e^- \to \pi^+\pi^-$ and $\tau^- \to \pi^-\pi^0\,\nu_\tau$

$$v_0(s) = \frac{s}{4\pi\alpha^2}\sigma_{\pi^+\pi^-}(s)$$

$$v_-(s) = \frac{m_\tau^2}{6|V_{ud}|^2}\frac{\mathcal{B}_{\pi\pi^0}}{\mathcal{B}_e}\frac{1}{N_{\pi\pi^0}}\frac{dN_{\pi\pi^0}}{ds}\left(1-\frac{s}{m_\tau^2}\right)^{-1}\left(1+\frac{2s}{m_\tau^2}\right)^{-1}\frac{1}{S_{\text{EW}}}$$

Isospin correction $v_0 = R_{\text{IB}} v_-$ $\quad R_{\text{IB}} = \dfrac{\text{FSR}}{G_{\text{EM}}}\dfrac{\beta_0^3|F_\pi^0|^2}{\beta_-^3|F_\pi^-|^2}$ [Alemani et al. '98]

0. S_{EW} electro-weak radiative correct. [Marciano, Sirlin '88][Braaten, Li '90]

1. Final State Radiation of $\pi^+\pi^-$ system [Schwinger '89][Drees, Hikasa '90]

2. G_{EM} (long distance) radiative corrections in τ decays
- Chiral Resonance Theory [Cirigliano et al. '01, '02]
- Meson Dominance [Flores-Talpa et al. '06, '07]

3. Phase Space ($\beta_{0,-}$) due to $(m_{\pi^\pm} - m_{\pi^0})$

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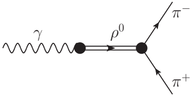
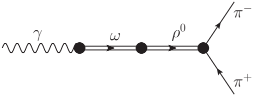
Pion form factors

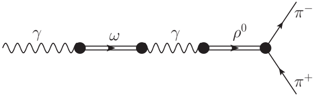
$$F_\pi^0(s) \propto \frac{m_\rho^2}{D_\rho(s)}$$



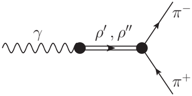
[Gounaris, Sakurai '68]
[Kühn, Santamaria '90]

$$\times\left[1 + \delta_{\rho\omega}\frac{s}{D_\omega(s)}\right]$$

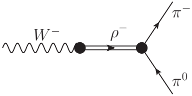




$$+\frac{m_X^2}{D_X(s)} \qquad X = \rho' \,, \rho''$$



$$F_\pi^-(s) \propto \frac{m_{\rho^-}^2}{D_{\rho^-}(s)} + (\rho' \,, \rho'')$$



Sources of IB breaking in phenomenological models

$m_{\rho^0} \neq m_{\rho^\pm}$, $\Gamma_{\rho^0} \neq \Gamma_{\rho^\pm}$, $m_{\pi^0} \neq m_{\pi^\pm}$

$\rho - \omega$ mixing $\delta_{\rho\omega} \simeq O(m_\mathrm{u} - m_\mathrm{d}) + O(e^2)$

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Contribution to a_μ

Time-momentum representation [Bernecker, Meyer, '11]

$$G^\gamma(t) = \frac{1}{3}\sum_k \int d\vec{x}\, \langle j_k^\gamma(x) j_k^\gamma(0)\rangle \quad \rightarrow \quad a_\mu = 4\alpha^2 \sum_t w_t G^\gamma(t)$$

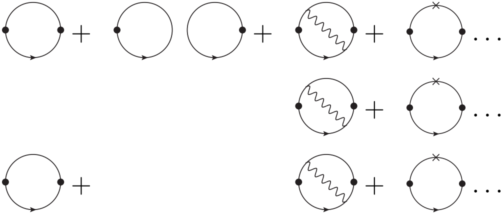
Isospin decomposition of u, d current

$$j_\mu^\gamma = \frac{i}{6}\left(\bar{u}\gamma_\mu u + \bar{d}\gamma_\mu d\right) + \frac{i}{2}\left(\bar{u}\gamma_\mu u - \bar{d}\gamma_\mu d\right) = j_\mu^{(0)} + j_\mu^{(1)}$$

$$G_{00}^\gamma \leftarrow \langle j_k^{(0)}(x) j_k^{(0)}(0)\rangle =$$

$$G_{01}^\gamma \leftarrow \langle j_k^{(0)}(x) j_k^{(1)}(0)\rangle =$$

$$G_{11}^\gamma \leftarrow \langle j_k^{(1)}(x) j_k^{(1)}(0)\rangle =$$



Decompose $a_\mu = a_\mu^{(0,0)} + a_\mu^{(0,1)} + a_\mu^{(1,1)}$

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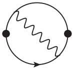
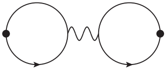
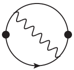
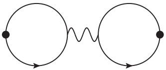
Neutral vs Charged

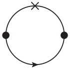
$$\tfrac{i}{2}\left(\bar{u}\gamma_\mu u - \bar{d}\gamma_\mu d\right),\ \begin{bmatrix} I=1 \\ I_3=0 \end{bmatrix} \quad \rightarrow \quad j_\mu^{(1,-)} = \tfrac{i}{\sqrt{2}}\left(\bar{u}\gamma_\mu d\right),\ \begin{bmatrix} I=1 \\ I_3=-1 \end{bmatrix}$$

Isospin 1 charged correlator $G_{11}^{W} = \frac{1}{3}\sum_k \int d\vec{x}\ \langle j_k^{(1,+)}(x) j_k^{(1,-)}(0)\rangle$

$$\delta G^{(1,1)} \equiv G_{11}^{\gamma} - G_{11}^{W}$$

$$= Z_V^4 (4\pi\alpha) \frac{(Q_\mathrm{u} - Q_\mathrm{d})^4}{4} \Big[$$  $+$  $\Big]$

$$G_{01}^{\gamma} = Z_V^4 \frac{(Q_u^2 - Q_d^2)^2}{2} (4\pi\alpha) \Big[$$  $+\ 2\times$  $+$  $+ \dots \Big]$

$$+ Z_V^2 \frac{Q_u^2 - Q_d^2}{2} (m_u - m_d) \Big[2\times$$  $+ \dots \Big]$

$\dots$ = subleading diagrams currently not included

$\Delta a_\mu[\pi\pi,\tau]$

Restriction to 2π $\rightarrow$ neglect pure $I=0$ part $a_\mu^{(0,0)}[\pi^0\gamma, 3\pi\,, \ldots]$

Lattice: $\Delta a_\mu[\pi\pi,\tau] = 4\alpha^2 \sum_t w_t \;\times\; \left[G_{01}^\gamma(t) + G_{11}^\gamma(t) - G_{11}^W(t)\right]$

Pheno: $\Delta a_\mu[\pi\pi,\tau] = \int_{4m_\pi^2}^{m_\tau^2} ds K(s) \;\left[\quad v_0(s) \quad - v_-(s)\right]$

Conversion to Euclidean time for direct comparison

$$\Delta a_\mu[\pi\pi,\tau] = 4\alpha^2 \sum_t w_t \times \left\{ \tfrac{1}{12\pi^2} \int d\omega\; \omega e^{-\omega t} \left[R_{\rm IB}(\omega^2) - 1\right] v_-(\omega^2) \right\}$$

FSR, $G_{\rm EM}$ $\rightarrow$ not computed from lattice but doable [C.Sachrajda Wed]

required for direct comparison v_- vs G_{11}^W

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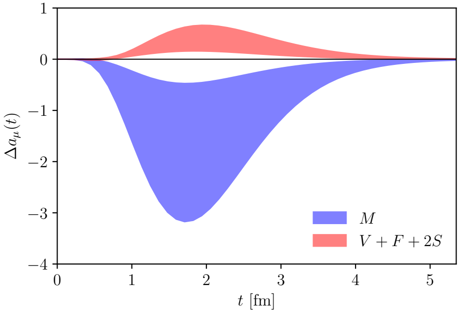
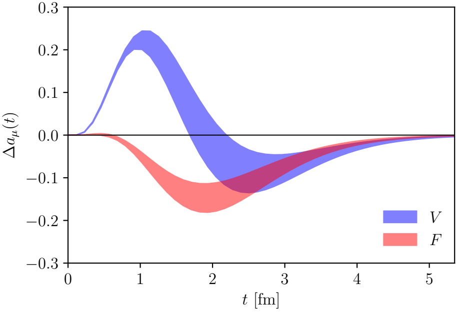
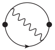
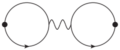
Lattice: Preliminary results

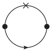
Δa_μ from G_{01}^{γ} (QED and SIB):



Pure $I = 1$ only $O(\alpha)$ terms:



$V =$  $F =$  $S =$ 

$M =$  $\rightarrow$ dominates noise

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Conclusions

For detailed comparison lattice vs pheno:

study systematic errors $\rightarrow$ ongoing finite volume study

improvement of errs $\rightarrow$ high stat. data set from HLbL

Outlook:

1. full lattice calculation of $\Delta a_\mu[\pi\pi, \tau]$ on the way

2. lattice QCD calculation $\rightarrow$ various comparisons

comparison v_- with experiment requires FSR, S_{EW} and G_{EM}
$\rightarrow$ test of long distance QED corrections
$\rightarrow$ direct computation

study G_{01}^{γ} alone $\rightarrow$ $\rho - \omega$ mixing

study $\delta G^{(1,1)}$ alone $\rightarrow$ ρ^0 vs ρ^- properties

Thanks for your attention

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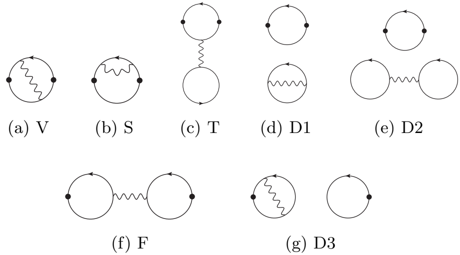
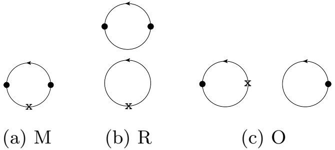
Full QED and SIB



[Blum et. al. '18]
[C. Lehner talk]



Presently only leading diagrams are computed V , F , S , M [Blum et al. '18]

$\rightarrow$ improving precision between 2 and 4 times

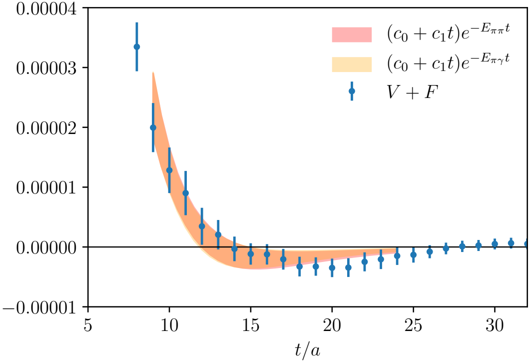
$SU(3)$ and $1/N_c$ diagrams presently not computed

Peeking at the data - I

Lattice fully inclusive $\rightarrow$ comparison with v_- problematic

manipulate correlator to implement energy cut

fit lowest energy state $(c_0 + c_1 t)e^{-Et}$



lattice correlator more precise at short distances

fit with fixed energy $E_{\pi\pi^{I=1}}$, $E_{\pi\gamma}$

temporary solution: not required with better precision

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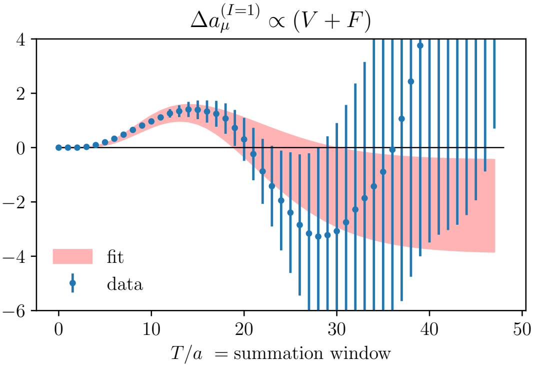
Peeking at the data - II

$\Delta a_\mu = 4\alpha^2 \sum_t w_t \ \delta G(t) \to$ weights suppress short distance



lattice correlator more precise at short distances

fit $(c_0 + c_1 t)e^{-Et}$

$E \to \pi\pi$ or $\pi\gamma$

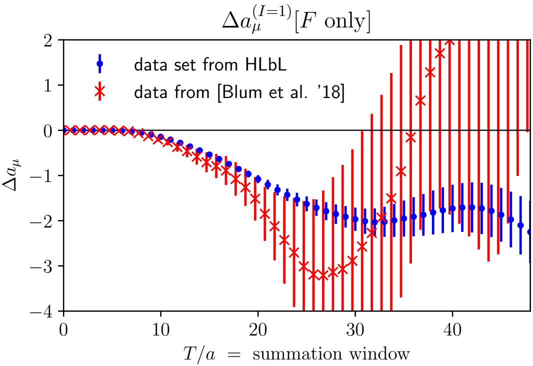
reduction of stat. noise

temporary solution: not required with better precision

Lattice improvements

Stat. improvements from data of HLbL project [Phys.Rev.Lett. 118 (2017)]

contribution of diagram F to pure $I=1$ part of Δa_μ



$O(1000)$ point-src per conf.
$5 \cdot 10^5$ combinations
80 configurations

$\times 4$ reduction in error

finite volume errs relevant
$\rightarrow$ dedicated study

data from [Blum et al. '18]: $O(500)$ point-src per conf.
76 configurations

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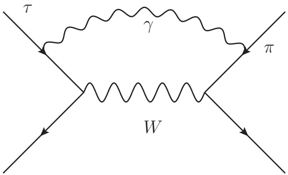
Radiative corrections

Some QED corrections computed in Chiral PT [Cirigliano et al. '01]

e.g. photon exchange between τ and hadrons



relevant to compare lattice data vs v_-

is current precision enough?

alternative calculation from lattice possible [Giusti et al. '17]

[C. Sachrajda Wed] [J. Richings Wed]